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Analysis and Design of Beams Reinforced with Steel Fiber and Composite Plates

Mereen H. Fahmi Rasheed [a], Ayad Z. Saber Agha [b]*

[a] Civil Engineering Department, Erbil Technical Engineering College, Erbil Polytechnic University, Erbil, Iraq. mereen.akrawi@epu.edu.iq

[b] Civil Engineering Department, Erbil Technical Engineering College, Erbil Polytechnic University Erbil, Iraq. ayad.saber@epu.edu.iq

*Corresponding author.

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Beams, epoxy bonded composite plate, steel fiber

ABSTRACT

The addition of epoxy bending plate to the tension and compression faces or web is an effective technique for flexural or shear strengthening of reinforced concrete beams, also using of steel fibers with concrete improves the structural behavior and increases the moment capacity and shear resistance of the beams. General equations are derived for predicting the bending moment capacity, maximum and balancing reinforcement ratio the equivalent depth of the compression zone for singly and doubly reinforced rectangular sections and T-beams. The results indicate that bending moment capacity increased in beams with steel fibers and composite plates, while the maximum balancing reinforcement ratios are decreased.

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1. Introduction

Reinforced concrete beams with (FRP) bars are used instead of steel reinforcement in many cases to overcome the problems of corrosion and electromagnetic interference because FRP are made from fibers embedded in a polymeric resin. In addition, FRP materials have a high tensile strength in the direction of the fibers, which can be used as a structural reinforcement. The Japan Society of Civil Engineers (JSCE) [1] modified the Japanese steel-reinforced concrete code for FRP reinforced concrete. The Canadian Standard Association (CAN/CSA - 5806 - 02) [2], are used for the design and construction of buildings with FRP, in addition to including information about characterization tests for FRP internal reinforcement as well. American Concrete Institute (ACI) presented a guideline for structural concrete reinforced with FRP bars. The philosophy of the design is based on the concept that concrete and FRP materials are brittle; therefore, the failure mechanisms are based on the concrete crushing or FRP rupture.

2. Review of Literature

Toutanji and Saafi [4]; studied the bending behavior of GFRP reinforced concrete beams, they proposed equations for the effective moment of inertia and deflection considering the effect of the ratio of reinforcement and Young modulus of elasticity (E) of GPRP. Ferreira et al. [5]; presented a finite element analysis for concrete beams reinforced with FRP rebars, using the first-order shear deformation theory in the analysis of concrete. Salih and Abdel-Sayed [6]; presented an analytical formula that predicts the crack width in beams reinforced with FRP-bars, considering the bond and mechanical properties of FRP bars. Abdul Rahman and Swamy [7]; showed that reinforced concrete beams with GFRP bars give lower ultimate load, lower stiffness and larger deflection at the same load level compared with conventional reinforced concrete beams with steel bars. Ashour [8]; Studied flexural and shear capacities of concrete beams reinforced with GFRP bars, he presented a simplified method for estimating the flexural and shear capacities of the beams. Rafi et al. [9]; presented a study on the RC beams with carbon (CFRP) tension bars and conventional steel bars. The authors studied the flexural behavior, failure modes, load carrying capacity and cracking pattern. Al Sayed and Al-Hozaimy [10]; presented an experimental study on reinforced concrete beams with steel fibers and reinforced with FRP bars. These authors studied the effect of steel fibers on the ductility of the beams, the ductility of FRP beams under 50% less than that of the corresponding beams with steel fibers. Also Li and Wang [11]; improved the ductility and serviceability of FRP reinforced structural using steel fibers with concrete. Issa et al. [12]; considered the effect of steel fibers on the flexural behavior and ductility of concrete beams with GFRP bars, they found that using steel fibers with beams reinforced with GFRP bars achieved reasonable flexural strength and showed close results to the calculated using (ACI 440) [3]. Khalil and Abdul Raziq [13]; studied the mechanical properties of carbon fiber concrete, their results showed that the properties of concrete being significantly improved.

Ezeldin et al. [14]; analyzed the effect of steel fibers in shear and moment of beams using a computer program, the effect of (beam width, depth, fiber content, bonding reinforcement ratio, stirrups content ratio and stirrup spacing) are considered. An experimental study is presented by Swamy et al. [15]; on the flexural behavior and deformation characteristics of beams considering the effect of steel fiber. They concluded that ultimate strength and post cracking stiffness are increased at all stages of loading up to failure, which results in narrower crack width and less deformation.

The parameter of the stress block and ultimate concrete strain are given by Aziz et al. [16] for normal-weight concrete with steel fiber. These parameters are shown below:

$$\epsilon_{uf} = 0.003 + 0.008\ (Q_f \cdot \frac{L_f}{D_f}) \quad (1)$$

$$\gamma_f = 0.85 + 0.02\ (Q_f \cdot \frac{L_f}{D_f}) \quad (2)$$

$$\beta_f = 0.85 + 0.03\ (Q_f \cdot \frac{L_f}{D_f}) \quad \text{for} \quad f_c' < 27.58\ Mpa \quad (3)$$

$$\beta_f = 0.85 + 0.03\left(Q_f \cdot \frac{L_f}{D_f}\right) - 0.05(1 - 0.25 Q_f \cdot \frac{L_f}{D_f})(\frac{f_c' - 27.58}{6.89}) \text{ for } 55.2 > f_c' > 27.58\ Mpa \quad (4)$$

$$\beta_f = 0.65 \text{ for } f_c' > 55.2\ Mpa \quad (5)$$

Where:
Q_f = Steel fiber volumetric percent (%).
L_f = Steel fiber length (mm).

D_f = Steel fiber diameter (mm).
Hannant [17] and Swamy et al. [18] presented the following equation to determine the tensile strength of fibrous concrete.

$$\sigma_{fu} = 0.82\,\tau\,F \quad (6)$$

Where:
τ = Bond strength of the matrix.
F = Fiber factor which is given by (Narayanan and Palanjian) [19, 20].

$$= Q_f \cdot d_f \frac{L_f}{D_f} \quad (7)$$

Where d_f = Bond coefficient which is a function of fiber type.
Shear strength of concrete corbels without shear reinforcement is analyzed by Kumar [21]. A semi-empirical equation is proposed based on the experimental data to predict the shear strength and ductility of the specimens showing the effect of fibers. A new method and procedure are being used by Baluguru et al. [22] to determine the deflection and bending behavior of fibrous concrete. The authors reported that the ductility is significantly improved by using steel fibers in both normal and high strength concrete. Malek and Saadatmanesh [23]; presented an experimental and analytical study on concrete beams strengthened with epoxy banded fiber composite plate, they considered different modes of failure in the development of equations of strengthening of concrete beams. Theoretical results are unified and compared with experimental results.

3. Theory and Analysis

I. Single R.C. beams

Depth of Equivalent Compression Zone is obtained from the horizontal equilibrium of stress diagram shown in Figure 1.

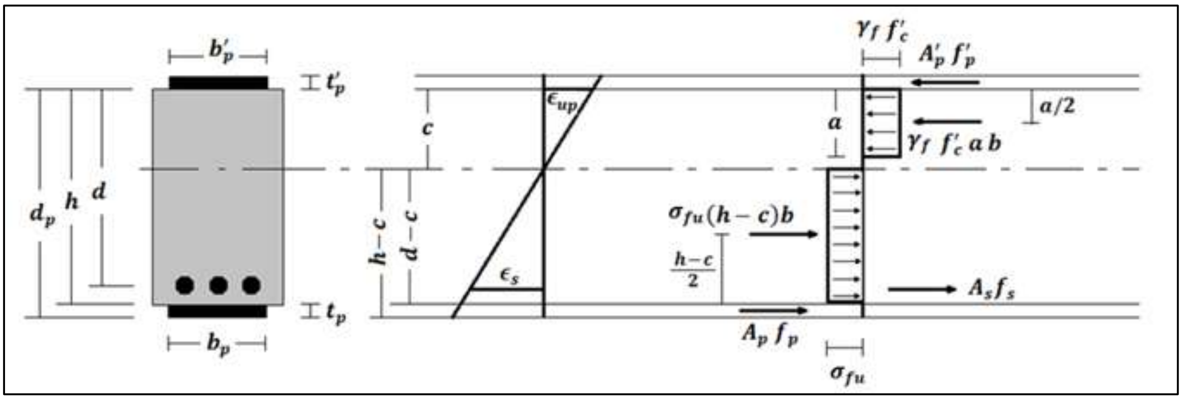

Figure 1: Singly reinforced concrete rectangular beams with fibers and composite plate

$$a_{fp} = \frac{(A_s f_y + A_p f_p - A'_p f'_p + \sigma_{fu} b h)}{\left(\gamma_f f'_c + \frac{\sigma_{fu}}{\beta_f}\right) b} \quad (8)$$

If $\sigma_{fu} = 0$ and $A_p = A'_p = 0$; the same original equation of (a) is obtained

$$a = \frac{A_s f_y}{(\gamma_f f'_c b)} \quad (9)$$

Where:
σ_{fu} = Ultimate strength of concrete in tension. (Mpa)
f'_c = Cylindrical concrete compressive strength. (Mpa)
f_y = Steel bar yield strength. (Mpa)
a = Equivalent rectangular stress distribution depth in compression. (mm)
a_{fp} = Equivalent rectangular compression stress distribution depth for beams with steel fiber and composite plates. (mm)
b = Beam width. (mm)
h = Beam height. (mm)
d = Effective depth of the beams. (mm)
A_s = Area of tensile steel reinforcement (mm^2)
$A_p = b_p t_p$ = Area of the epoxy bonded plate in tension Zone. (mm^2)
$A'_p = b'_p t'_p =$ Area of the epoxy bonded plate in Compression Zone. (mm^2)
f_p = Stress in epoxy-bonded plate in tension Zone. (Mpa)
f'_p = Stress in epoxy-bonded plate in the comp. Zone. (Mpa)
t_p = Thickness of epoxy-bonded plate in tension Zone. (mm)
t'_p = Thickness of epoxy-bonded plate in the comp. Zone. (mm)
b_p = Width of epoxy-bonded plate in tension Zone. (mm)
b'_p = Width of epoxy-bonded plate in the comp. Zone. (mm)
The stress in epoxy-bonded plate f_p & f'_p are determined from the strain distribution as the following:

$$f_p = E_p \epsilon_{uf} \left(\frac{\frac{dp}{d} - K}{K}\right) \quad (10)$$

$$f'_p = E_p \epsilon_{uf} \left(\frac{K + \frac{tp'}{2d}}{K}\right) \quad (11)$$

$$\mathrm{K} = \frac{\epsilon_{uf}}{\epsilon_{uf} + \epsilon_s} \quad (12)$$

Where:
E_p = Modulus of Elasticity of the epoxy bonded plate. (Mpa).
ϵ_{uf} = Ultimate Strain of the Fibrous Concrete. (mm/mm).
ϵ_s = Strain of the reinforcement steel. (mm/mm).
Another form can be used for equation (8):

$$a_{fp} = a \, . \lambda \quad (13)$$

Where:

$$\lambda = \frac{\left[1 + \frac{\rho p f p - \rho'_p f'_p + \sigma_{fu} \frac{h}{d}}{\rho f y}\right]}{\left[1 + \frac{\sigma_{fu}}{\gamma_f \beta_f f'_c}\right]} \quad (14)$$

Where:
$\rho_p = \frac{A_p}{bd}$, $\rho'_p = \frac{A'_p}{bd}$ & $\rho = \frac{A_s}{bd}$ are reinforcement indices.
The depth of Compression Zone is increased by a factor (λ).
The effect of fiber factor (F) and the ratio $\lambda = \frac{a_{fp}}{a}$ are shown in Figure 2 and Table 1, for different values of (ρ_p).

As shown the depth of compression Zone (a_{fp}) is increased with increasing fiber factor (F) & ratio of the reinforcement epoxy-bonded plate (ρ_p).
The nominal bending moment is written as:

$$M_{nfp} = A_s f_s \left(d - \frac{a_{fp}}{2}\right) + \sigma_{fu} b \left(h - \frac{a_{fp}}{\beta_f}\right)\left(\frac{h}{2} - \frac{a_{fp}}{2} + \frac{a_{fp}}{2\beta_f}\right) + A_p f_p \left(h + \frac{tp}{2} - \frac{a_{fp}}{2}\right) + A'_p f'_p \left(\frac{a_{fp}}{2} + \frac{t'_p}{2}\right) \quad (15)$$

Or in another form:

$$M_{nfp} = M_n + M_{fiber} + M_p + M'_p \quad (16)$$

$$M_n = A_s f_s \left(d - \frac{a_{fp}}{2}\right) \quad (17)$$

$$M_{fiber} = \frac{\sigma_{fu}\, a_{fp}^2\, b}{2\beta_f^2}\left(\frac{\beta_f\, h}{a_{fp}} - 1\right)\left(\frac{\beta_f\, h}{a_{fp}} - \beta_f + 1\right) \quad (18)$$

$$M_p = A_p f_p \left(h + \frac{t_p}{2} - \frac{a_{fp}}{2}\right) \quad (19)$$

$$M'_p = A'_p\, f'_p \left(\frac{a_{fp}}{2} + \frac{t'_p}{2}\right) \quad (20)$$

Where:
M_n = Internal moment capacity of the beam. ($N.mm$)
M_{nfp} = Internal moment capacity of the fibrous R.C. beam with composite plates. ($N.mm$)
M_{fiber} = Nominal bending moment contribution of steel fibers. ($N.mm$)
M_p = Nominal bending moment contribution of epoxy-bonded plate in tension Zone. ($N.mm$)
M'_p = Nominal bending moment contribution of epoxy-bonded plate in the comp. Zone. ($N.mm$)
For tension failure condition, the steel stress $f_s = f_y$.
Table 2 and Figure 3 show the contribution of steel fiber and epoxy-bonded plate in the total nominal bending moment of the beam. The ratio (M_{nfp}/M_n) is determined for different values of Fiber factor (F) and plate reinforcement ratios (ρ_p), as shown, the value of (M_{nfp}) is increased with increasing of the epoxy plate ratio (ρ_p) & fiber content which is represented by the fiber factor (F).
To achieve ductile failure, the reinforcement amount shall not exceed the permitted limit (ρ_{max} or ρ_b), [24], i.e. ($\rho < \rho_{max}$ or ρ_b), where ρ_{max} is the maximum allowed reinforcement index and ρ_p is the balance condition reinforcement ratio.

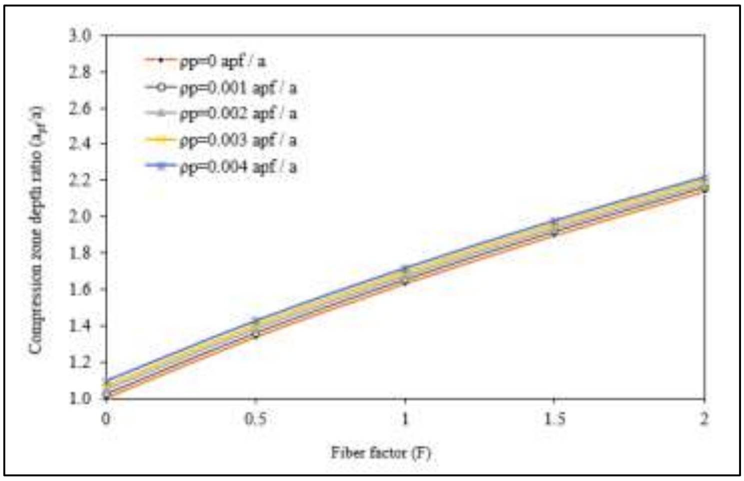

Figure 2: Effect of epoxy composite plate and steel fiber on effective compression zone depth ratio

Table 1: Effect of epoxy composite plate and steel fiber on Effective compression zone depth ratio

Q_f %	F	ρ_p=0	ρ_p=0.001	ρ_p=0.002	ρ_p=0.003	ρ_p=0.004

		a_{pf} / a	a_{pf} / a	a_{pf} / a	a_{pf} / a	a_{pf} / a
0	0	1.0000	1.0233	1.0465	1.0698	1.0930
0.5	0.5	1.3364	1.3582	1.3800	1.4019	1.4237
1	1	1.6320	1.6529	1.6735	1.6943	1.7150
1.5	1.5	1.8979	1.9176	1.9374	1.9573	1.9771
2	2	2.1400	2.1594	2.1784	2.1974	2.2164

Table 2: Effect of epoxy composite plate and steel fiber on Ultimate strength moment

Q_f %	F	ρ_p=0 Mn_{pf} / Mn	ρ_p=0.001 Mn_{pf} / Mn	ρ_p=0.002 Mn_{pf} / Mn	ρ_p=0.003 Mn_{pf} / Mn	ρ_p=0.004 Mn_{pf} / Mn	ρ_p=0.005 Mn_{pf} / Mn	ρ_p=0 Mn_{pf} / Mn_f	ρ_p=0.001 Mn_{pf} / Mn_f	ρ_p=0.002 Mn_{pf} / Mn_f	ρ_p=0.003 Mn_{pf} / Mn_f	ρ_p=0.004 Mn_{pf} / Mn_f	ρ_p=0.005 Mn_{pf} / Mn_f
0	0	1.0000	1.0259	1.0517	1.0776	1.1035	1.1293	1.0000	1.0259	1.0517	1.0776	1.1035	1.1293
0.5	0.5	1.2296	1.2258	1.2820	1.3084	1.3346	1.3609	1.0000	1.0214	1.0427	1.0640	1.0854	1.1068
1	1	1.4503	1.4770	1.5037	1.5303	1.5570	1.5837	1.0000	1.0184	1.0368	1.0552	1.0736	1.0920
1.5	1.5	1.6767	1.7038	1.7308	1.7579	1.7850	1.8120	1.0000	1.0161	1.0323	1.0484	1.0646	1.0807
2	2	1.9037	1.9312	1.9587	1.9862	2.0136	2.0411	1.0000	1.0144	1.0289	1.0433	1.0577	1.0721

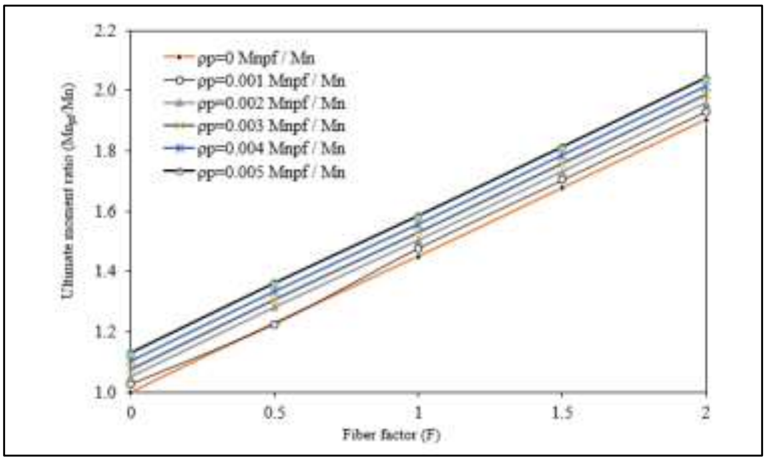

Figure 3: Effect of epoxy composite plate and steel fiber on Ultimate moment ratio

To find balance condition reinforcement ratio, Strain Compatibility Condition is applied as the following:

$$c_b = \frac{\epsilon_{uf}}{\epsilon_{uf}+\epsilon_y}\, d \qquad (21)$$

$$a_b = \frac{\beta_f\, \epsilon_u}{\epsilon_u+\epsilon_y}\, d \qquad (22)$$

The balancing reinforcement index can be determined by equating equation 8 & 23 and taking ($\rho = \frac{A_s}{bd}$)

$$\rho_{bfp} = \left(\frac{\gamma_f\ \beta_f\ f_c'+\sigma_{fu}}{f_y}\right)\left(\frac{\epsilon_{uf}}{\epsilon_{uf}+\epsilon_y}\right) - \frac{\sigma_{fu}\, h}{f_y\, d} + \rho_p' \left\{\frac{E_p \epsilon_{uf}}{f_y}\left(\frac{K+\frac{t_p'}{2d}}{K}\right)\right\} - \rho_p \left\{\frac{E_p \epsilon_{uf}}{f_y}\left(\frac{\frac{d_p}{a}-K}{K}\right)\right\} \qquad (23)$$

Where: $d_p = h + \frac{tp}{2}$, $\rho_p' = \frac{A_p'}{bd}$, $\rho_p = \frac{A_p}{bd}$ & $K = \frac{\epsilon_{uf}}{\epsilon_{uf}+\epsilon_y}$

Or in another form:

$$\rho_{bfp} = \rho_b - \rho_{fiber} + \rho_{plate}' - \rho_{plate} \qquad (24)$$

And

$$\rho_{maxfp} = \rho_{max} - \rho_{fiber} + \rho_{plate}' - \rho_{plate} \qquad (25)$$

Where:

$$\rho_b = \gamma_f\ \beta_f\ \frac{f_c'}{f_y}\ \left(\frac{\epsilon_{uf}}{\epsilon_{uf}+\epsilon_y}\right) \qquad (26)$$

$$\rho_{fiber} = \frac{\sigma_{fu}}{f_y}\left(\frac{h}{d} - \frac{\epsilon_{uf}}{\epsilon_{uf}+\epsilon_y}\right) \qquad (27)$$

$$\rho'_{plate} = \rho'_p \left\{ \frac{E_p \epsilon_{uf}}{f_y} \left(\frac{K + \frac{t'_p}{2d}}{K} \right) \right\} \quad (28)$$

$$\rho_{plate} = \rho_p \left\{ \frac{E_p \epsilon_{uf}}{f_y} \left(\frac{\frac{d_p}{d} - K}{K} \right) \right\} \quad (29)$$

Where:

ϵ_{uf} = Maximum strain of compression zone.

ϵ_y = Steel bar yielding strain.

a_b = Depth of equivalent comp. zone in balance condition. (mm)

c_b = Depth of comp. zone in balance condition. (mm)

ρ = Tensile reinforcement index.

ρ_b = Balancing condition reinforcement index.

ρ_{max} = Max. allowed reinforcement index.

ρ_{bfp} = Balancing condition reinforcement index of fibrous R.C. beam with epoxy-bonded plate.

ρ_{maxfp} = Max. allowed reinforcement index of fibrous R.C. beam with epoxy-bonded plate.

ρ_{fiber} = Steel fiber reinforcement index.

Table 3 and Figure 4; show the effect of steel fiber content represented by Fiber factor (F) and epoxy plate ratio (ρ_p) on the balance reinforcement ratio (ρ_{bfp}/ρ_b), as shown the value of balance reinforcement ratio of a beam with steel fiber and bonded plates reduced with increasing fiber factor (F) & (ρ_p), this means that the ductility of the beam is increased with the addition of steel fiber and using epoxy-bonded plates.

Table 3: Effect of epoxy composite plate and steel fiber on Balance reinforcement ratio

Q_f%	F	ρ_p=0	ρ_p=0.001	ρ_p=0.002	ρ_p=0.003	ρ_p=0.004	ρ_p=0.005	ρ_p=0	ρ_p=0.001	ρ_p=0.002	ρ_p=0.003	ρ_p=0.004	ρ_p=0.005
		$\rho b_{pf}/\rho b$	$\rho b_{pf}/\rho b$	$\rho b_{pf}/\rho b$	$\rho b_{pf}/\rho b$	$\rho b_{pf}/\rho b$	$\rho b_{pf}/\rho b$	$\rho b_{pf}/\rho b_f$	$\rho b_{pf}/\rho b_f$	$\rho b_{pf}/\rho b_f$	$\rho b_{pf}/\rho b_f$	$\rho b_{pf}/\rho b_f$	$\rho b_{pf}/\rho b_f$
0	0	1.0000	0.9908	0.9836	0.9754	0.9672	0.9590	1.0000	0.99178	0.98359	0.97538	0.96718	0.95895
0.5	0.5	0.9363	0.9286	0.9209	0.9132	0.9055	0.8979	1.0000	0.99178	0.98356	0.97534	0.96712	0.95890
1	1	0.8868	0.8771	0.8722	0.8649	0.8576	0.8503	1.0000	0.99177	0.98353	0.97530	0.96707	0.95890
1.5	1.5	0.8495	0.8426	0.8356	0.8287	0.8217	0.8148	1.0000	0.99182	0.98362	0.97544	0.96726	0.95907
2	2	0.8112	0.8045	0.7978	0.7912	0.7845	0.7779	1.0000	0.99179	0.98357	0.97536	0.96714	0.95893

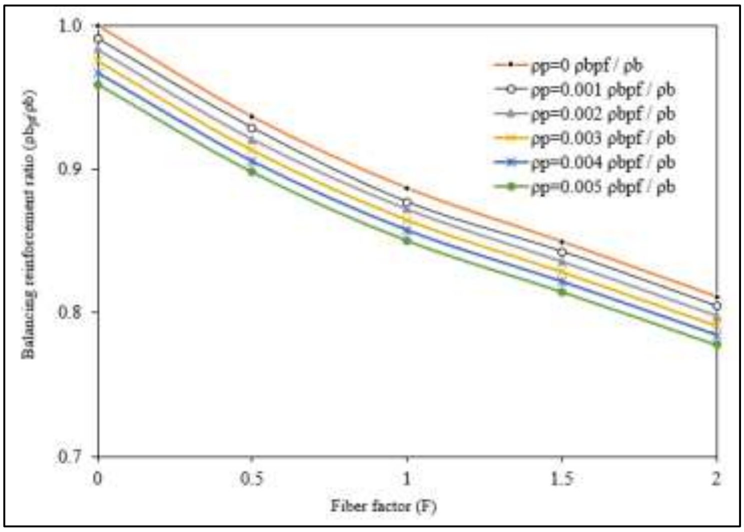

Figure 4: Effect of epoxy composite plate and steel fiber on balanced reinforcement

II. Doubly reinforced concrete beam

For doubly R.C. beam, depth of equivalent stress block is obtained:

$$a_{fp} = \frac{\beta_f[A_s f_s - A_s' f_s' + A_p f_p - A_p' f_p' + \sigma_{fu} bh]}{[\gamma_f\, \beta_f\, f_c' + \sigma_{fu}]b} \quad (30)$$

In case $f_s = f_s' = f_y$ & taking $\rho = \frac{A_s}{bd}$ & $\rho' = \frac{A_s'}{bd}$

$$a_{fp} = \frac{\beta_f[(\rho-\rho')f_y d + \sigma_{fu} h + (\rho_p\, f_p - \rho_p'\, f_p')d]}{[\gamma_f\, \beta_f\, f_c' + \sigma_{fu}]} \quad (31)$$

or in a simpler form:

$$a_{fp} = a\,.\lambda \quad (32)$$

Where:

$$a = \frac{(\rho-\rho')f_y d}{\gamma_f\, f_c'} \quad (33)$$

and

$$\lambda = \frac{\left[1 + \frac{\rho_p\, f_p - \rho_p'\, f_p' + \sigma_{fu} \frac{h}{d}}{(\rho-\rho')f_y}\right]}{\left[1 + \frac{\sigma_{fu}}{\gamma_f\, \beta_f\, f_c'}\right]} \quad (34)$$

Where:

A_s' = Area of steel reinforcement in compression Zone. (mm^2)
f_s' = Steel bar stress in comp. zone. (Mpa)
f_s = Steel bar stress in tension zone. (Mpa)
ρ' = Compression reinforcement index.

The internal moment capacity of R.C. beam reinforced with steel fiber and composite plates can be written as:

$$M_{nfp} = A_s' f_s'(d-d') + \left(A_s - \frac{A_s' f_s'}{f_s}\right) f_s \left(d - \frac{a_{fp}}{2}\right) + \sigma_{fu} d \left(h - \frac{a_{fp}}{\beta_f}\right)\left(\frac{h}{2} - \frac{a_{fp}}{2} + \frac{a_{fp}}{2\beta_f}\right) + A_p f_p \left(h + \frac{t_p}{2} - \frac{a_{fp}}{2}\right) + A_p' f_p' \left(\frac{a_{fp}}{2} + \frac{t_p'}{2}\right) \quad (35)$$

Or in a simpler form:

$$M_{nfp} = M_n + M_{fiber} + M_p + M_p' \quad (36)$$

Where:

$$M_n = A_s' f_s'(d-d') + \left(A_s - \frac{A_s' f_s'}{f_s}\right) f_s \left(d - \frac{a_{fp}}{2}\right) \quad (37)$$

$$M_{fiber} = \frac{\sigma_{fu}\, a_{fp}^2\, b}{2\beta_f^2}\left(\frac{\beta_f\, h}{a_{fp}} - 1\right)\left(\frac{\beta_f\, h}{a_{fp}} - \beta_f + 1\right) \quad (38)$$

$$M_p' = A_p'\, f_p'\left(\frac{a_{fp}}{2} + \frac{t_p'}{2}\right) \quad (39)$$

$$M_p = A_p f_p \left(h + \frac{t_p}{2} - \frac{a_{fp}}{2}\right) \quad (40)$$

d' = Distance from top comp. face to the center of the bar in the compression zone (mm).
In tension condition, f_s & f_s' equal to f_y, then the equation of balancing and maximum reinforcement index is determined and shown below:

$$\rho_{bfp}' = \rho_b' - \rho_{fiber} + \rho_{plate}' - \rho_{plate} \quad (41)$$

And

$$\rho'_{maxbfp} = \rho'_{max} - \rho_{fiber} + \rho'_{plate} - \rho_{plate} \quad (42)$$

Where:

$$\rho'_{b} = \gamma_f \ \beta_f \ \frac{f'_c}{f_y}\left(\frac{\epsilon_{uf}}{\epsilon_{uf}+\epsilon_y}\right) + \frac{\rho' \ f'_s}{f_y} \quad (43)$$

$$\rho_{fiber} = \frac{\sigma_{fu}}{f_y}\left[\frac{h}{d} - \left(\frac{\epsilon_{uf}}{\epsilon_{uf}+\epsilon_y}\right)\right] \quad (44)$$

$$\rho'_{plate} = \rho'_{p}\left\{\frac{E_p \epsilon_{uf}}{f_y}\left(\frac{K+\frac{t'_p}{2d}}{K}\right)\right\} \quad (45)$$

$$\rho_{plate} = \rho_{p}\left\{\frac{E_p \epsilon_{uf}}{f_y}\left(\frac{\frac{d_p}{d}-K}{K}\right)\right\} \quad (46)$$

Compatibility equations are used to derive the equation (ρ'_{cy}):

$$c_b = \frac{\epsilon_{uf}}{\epsilon_{uf}-\epsilon_y} \ d' \quad (47)$$

And

$$a_b = \beta_f \ c_b \quad (48)$$

Equating equations (49 & 8), the following equation is determined

$$\rho'_{cyfp} = \rho'_{cy} - \rho'_{fiber} + \rho'_{plate} - \rho_{plate} \quad (49)$$

Where:

$$\rho'_{cy} = \gamma_f \ \beta_f \ \frac{f'_c}{f_y}\frac{d'}{d}\left(\frac{\epsilon_{uf}}{\epsilon_{uf}-\epsilon_y}\right) + \frac{\rho' \ f'_s}{f_y} \quad (50)$$

$$\rho'_{fiber} = \frac{\sigma_{fu}}{f_y}\left[\frac{h}{D} - \left(\frac{\epsilon_{uf}}{\epsilon_{uf}-\epsilon_y}\right)\frac{d'}{d}\right] \quad (51)$$

Where:

ρ'_{bfp} = Balancing reinforcement index for fibrous doubly R.C. beam with epoxy-bonded Plates.
ρ'_{b} = Balancing reinforcement index for ordinary doubly R.C. beam.
ρ'_{maxfp} = Max. permitted reinforcement index for fibrous R.C. beam with epoxy-bonded plates.
ρ'_{max} = Max. permitted reinforcement index for ordinary double R.C. beam.
ρ'_{cyfp} = Limit reinforcement index for doubly R.C. Fibrous beam with epoxy-bonded plates.
ρ'_{cy} = Limit reinforcement index for ordinary doubly R.C. beam.
ρ'_{fiber} = Steel fiber reinforcement index

III. Reinforced concrete T-beam

Using the same procedure, which is used in previous singly, & doubly R.C. beams, the depth of equivalent rectangular compression stress is:

$$a_{fp} = \frac{\beta_f[(A_s-A_{sf})f_s+\sigma_{fu}b_w d+A_p f_p-A'_p f'_p]}{(\gamma_f \ \beta_f \ f'_c+\sigma f_u)b_w} \quad (52)$$

In case $f_s = f_y$ and taking $\rho = \frac{A_s}{b_w d}$ & $\rho_f = \frac{A_{sf}}{b_w d}$

$$a_{fp} = \frac{\beta_f[(\rho-\rho_f)f_y d+\sigma_{fu} d+\rho_p\, f_p d-\rho_p'\; f_p' d]}{[\gamma_f\, \beta_f\, f_c'+\sigma f_u]} \tag{53}$$

Or

$$a_{fp} = a\,.\lambda \tag{54}$$

Where:

$$a = \frac{(\rho-\rho_f)f_y d}{\gamma_f\; f_c'} \tag{55}$$

And

$$\lambda = \frac{\left[1+\frac{\sigma_{fu}}{(\rho-\rho_f)f_y d}+\frac{(\rho_p\, f_p-\rho_p'\; f_p')}{(\rho-\rho_f)f_y}\right]}{\left[1+\frac{\sigma_{fu}}{\gamma_f\, \beta_f\, f_c'}\right]} \tag{56}$$

The nominal bending moment for T-beam with steel fiber and epoxy-bonded plates is determined as:

$$M_{nfp} = M_{nw} + M_{fiber} + M_p + M_p' \tag{57}$$

Where:

$$M_{nw} = A_{sf}\; f_s\left(d-\frac{h_f}{2}\right)+\left(A_s - A_{sf}\right)f_s\left(d-\frac{a_{fp}}{2}\right) \tag{58}$$

$$M_{fiber} = \frac{\sigma_{fu}\; a_{fp}^2\; b_w}{2\beta_f^2}\left(\frac{\beta_f\; h}{a_{fp}}-1\right)\left(\frac{\beta_f\; h}{a_{fp}}-\beta_f+1\right) \tag{59}$$

$$M_p = A_p f_p\left(h+\frac{t_p}{2}-\frac{a_{fp}}{2}\right) \tag{60}$$

$$M_p' = A_p'\; f_p'\left(\frac{a_{fp}}{2}+\frac{t_p'}{2}\right) \tag{61}$$

The balancing reinforcement index is determined by using the same procedures that before.

$$\rho_{bwfp} = \rho_{bw} - \rho_{fiber} + \rho_{plate}' - \rho_{plate} \tag{62}$$

Where:

$$\rho_{bw} = \gamma_f\;\; \beta_f\; \frac{f_c'}{f_y}\left(\frac{\epsilon_{uf}}{\epsilon_{uf}+\epsilon_y}\right)+\rho_f \tag{63}$$

$$\rho_{fiber} = \frac{\sigma_{fu}}{f_y}\left(\frac{h}{d}-\frac{\epsilon_{uf}}{\epsilon_{uf}+\epsilon_y}\right) \tag{64}$$

$$\rho_{plate}' = \rho_p'\left\{\frac{E_p\epsilon_{uf}}{f_y}\left(\frac{K+\frac{t_p'}{2d}}{K}\right)\right\} \tag{65}$$

$$\rho_{plate} = \rho_p\left\{\frac{E_p\epsilon_{uf}}{f_y}\left(\frac{\frac{d_p}{d}-K}{K}\right)\right\} \tag{66}$$

$\rho_f = \frac{A_{sf}}{b_w d}$

ρ_{bwfp} = Balance reinforcement index for fibrous R.C. T-beam with epoxy-bonded plate.

ρ_{bw} = Balance reinforcement index for ordinary R.C. T-beam.

b_w = Width of the web of T-beam (mm).

IV. Doubly R.C. T-beam

Using the same procedures used in previous types, the following equations are derived for equivalent comp. Zone depth, nominal bending moment and balancing reinforcement index.

$$a_{fp} = a \cdot \lambda \quad (67)$$

$$a = \frac{(\rho - \rho_f - \rho')f_y\, d}{\gamma_f\; f_c'} \quad (68)$$

And $$\lambda = \frac{\left[1 + \frac{\rho_p\, f_p - \rho_p'\; f_p' + \sigma_{fu} \frac{h}{d}}{(\rho - \rho_f - \rho')f_y}\right]}{\left[1 + \frac{\sigma_{fu}}{\gamma_f\, \beta_f\, f_c'}\right]} \quad (69)$$

The nominal bending moment;

$$M_{nfp} = M_n + M_{fiber} + M_p + M_p' \quad (70)$$

Where:

$$M_n = A_s\, f_s\left(d - \frac{h_f}{2}\right) + A_s' f_s'(d - d') + \left(A_s - A_{sf} - \frac{A_s' f_s'}{f_s}\right) f_s\left(d - \frac{a_{fp}}{2}\right) \quad (71)$$

Balancing reinforcement index;

$$\rho_{bwfp}' = \rho_{bw}' - \rho_{fiber} + \rho_{plate}' - \rho_{plate} \quad (72)$$

Where:

$$\rho_{bw}' = \gamma_f\;\beta_f\,\frac{f_c'}{f_y}\left(\frac{\epsilon_{uf}}{\epsilon_{uf} + \epsilon_y}\right) + \rho_f + \frac{\rho' f_s'}{f_y} \quad (73)$$

M_{fiber} , M_p , M_p' , ρ_{fiber} , ρ_{plate}' and ρ_{plate} are the same as derived before.
The derived equations are verified and applied on beam with steel fiber only and without bonded plates founded in ref. [15], the theoretical bending moment is very close to the experimental values as shown in Table 4.

Table 4: Comparison between theoretical and experimental results [15]

Beam	$M_{exp.}\ kN.m$	$M_{theo.}\ kN.m$ [15]	$Ratio\ \frac{M_{theo.}}{M_{exp.}}$
DR 12	23.81	23.82	1.0004
DR 22	35.06	34.187	0.975
DR 32	30.83	29.55	0.959

4. Conclusions

1. The study presents a theoretical study on the flexural strength of different types of singly and doubly reinforced concrete also T-beam; the beams are reinforced also with steel fiber and epoxy-bonded plates at the tension and compression face.

2. General modified equations are derived to determine the depth of equivalent rectangular compression stress (a), internal moment capacity (M_n) balancing and maximum reinforcement index (ρ_b &ρ_{max}), including the effect of steel fiber and epoxy-bonded plates.
3. Figure 2, shows the effect of fiber factor (F) and reinforcement index of epoxy-bonded plate (ρ_p) on the equivalent compression depth (a_{fp}) as shown the ratio of (a_{fp}/a) is increased with increasing the steel fiber content or Fiber factor (F) and bonded plate reinforcement index (ρ_p).
4. Figure 3 shows that the nominal bending moment of beams with steel fiber and epoxy-bonded plate (M_{nfp}) or the ratio (M_{nfp}/M_n) is increased by increasing the steel fiber content of Fiber content and bonded plate reinforcement index.
5. Figure 4 shows that the balanced reinforcement index (ρ_{bfp}) or the ratio (ρ_{bfp}/ρ_b) decreased with increasing steel fiber content or fiber factor and epoxy-bonded plate reinforcement index.
6. The results shown in Table 2 indicate that using steel fiber and composite plates has a significant contribution in the internal moment capacity of the beams.
7. The nominal bending moment is increased by the amount (M_{fiber} & M_{plate}) which represent the contribution of steel fiber and epoxy-bonded plate respectively.
8. The balancing and maximum reinforcement indexes (ρ_{bfp} & ρ_{maxfp}) are decreased by the amount (ρ_{fiber} & ρ_{plate}), this means that the ductility of the beam is increased by using steel fiber and epoxy-bonded plate.
9. For beams with epoxy-bonded plate reinforcement index ($\rho_p = 0.001$) and without steel fiber, (a_{fp}) increased by (2.3%), (ρ_{bfp}), while decreased by (1%) and nominal bending moment (M_{nfp}) increased by 2.6%. However when (1%) steel fiber is used with the same epoxy-bonded plate reinforcement index ($\rho_p = 0.001$), the increasing of (a_{fp}) becomes (65%), (ρ_{bfp}) decreased by an amount of (12%) and bending moment increased by an amount of (48%). Therefore, these results indicate the great effect of epoxy-bonded plate on the flexure behavior of the beam.
10. For beams with steel fiber only and without bonded plates, the theoretical results are compared with the experimental results found in ref. [15]. The theoretical bending moment showed very good agreement with the experimental values as shown in Table 4.

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