5-decomposition Matrix for the Projective Characters of the Symmetric Group S_{18}

Jenan Abd Alreda Resen
Department of Mathematics, College of Sciences,
University of Basrah, Basrah, Iraq
Jenanabdalreda8@gmail.com

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Abstract

In this paper,we determine the irreducible modular spin characters of S_{18}, p=5 which is equivalent determine the decomposition matrix of S_{18},p=5, this work is complete except two columns is still undetermined.

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Notation:

p.s.	The principle spin character.
p.i.s.	The principle indecomposable spin character.
m.s.	The modular spin character.
i.m.s.	The irreducible modular spin character.
d_i	The p.i.s. of S_n.
D_i	The p.i.s. of S_{n-1}.
${D_{n,p}}^{(i)}$	The decomposition matrix of S_n for the prime p of the block B_i.

1. Introduction

The theory of projective (spin) characters of the symmetric groups S_n data back to classical of Schur [1]. He showed the symmetric group S_n has representation group $\overline{S_n}$ of order $2n!$, when $n \geq 4$ [2]. The irreducible characters for $\overline{S_n}$ of $S_n$called ordinary (modular) corresponding of characteristic of field is zero (p>0) [3]. The ordinary characters for $\overline{S_n}$ which are corresponding to the ordinary characters for S_n called the ordinary charactersfor $\overline{S_n}$, the remaining ordinary characters of $\overline{S_n}$ are called projective (spin) characters of S_n [2].
Yassen [4] has given the $5-$ irreducible modular spin characters for S_n, $5 \leq n \leq 13$, Taban $[4-6]$, has given the $5-$ irreducible modular spin characters for S_n, where $n = 14{,}15$, respectively with one ambiguity when $n = 15$, Abdullah $[7]$ has given $5-$irreducible modular spin characters for, where $n = 16$ with one more ambiguity and Resan [8] found 5-irreducible modular spin characters forS_n, where $n = 17$. In this paper, we have determined the $5-$irreducible modular spin characters of S_{18}.

2. Preliminaries

1. Any spin character of S_n can be written as a linear combination, with non-negative integer coefficients of the irreducible spin characters [9].
2. The degree of the spin characters $\langle\alpha\rangle = \langle\alpha_1, \dots, \alpha_m\rangle$ is:

$$deg\langle\alpha\rangle = 2^{\left[\frac{n-m}{2}\right]}\frac{n!}{\prod_{i=1}^{m}(\alpha_i!)}\prod_{1\leq i<j\leq m}\left(\alpha_i - \alpha_j\right)/\left(\alpha_i + \alpha_j\right) \; [10].$$

3. The values of characters $\langle\alpha\rangle$ and $\langle\alpha\rangle'$ differ only on the class corresponding to α on which they have values $\pm i^{\frac{n-m+1}{2}}\sqrt[2]{(\alpha_1 \ldots \alpha_m/2)}$ [2].
4. Let H be a subgroup of S_n. Then:
i) If θ is a spin character of H, then $\theta \uparrow S_n$ is a spin character of S_n.
ii) If θ is a spin character of S_n, then $\theta \downarrow H$ is a spin character of H [11].
5. Let p be odd. If n is odd and $p \nmid n$ or $p \nmid n-1$, then $\langle n-1,1\rangle$ and $\langle n-1,1\rangle'$ are distinct irreducible modular spin characters of degree $2^{[(n-3)/2]} \times (n-2)$ which are denoted by $\varphi\langle n-1,1\rangle$ and $\varphi\langle n-1,1\rangle'$ [4].
6. Let p be an odd prime and let α, β be a bar partitions of n which are not p −bar cores. Then $\langle\alpha\rangle\,(and\ \langle\alpha\rangle'\ if\ \ \alpha\ \ is\ odd)$ and $\langle\beta\rangle\,(and\ \langle\beta\rangle'\ if\ \ \beta\ \ is\ odd)$ are in the same p −block if and only if $\widetilde{\langle\alpha\rangle} = \widetilde{\langle\beta\rangle}$. If α be a bar partition of n and $\langle\alpha\rangle = \widetilde{\langle\alpha\rangle}$, then $\langle\alpha\rangle\,(and\ \langle\alpha\rangle'\ if\ \ \alpha\ \ is\ \ odd)$ forms a p −block of defect 0 [3].
7. Let p be an odd prime and $\alpha = (\alpha_1, \ldots, \alpha_m)$ be a bar partition of n not a p −bar core. Let B be the block containing $\langle\alpha\rangle$. Then

i) If $n-m-m_0$ is even, then all irreducible modular spin characters in B are double.
ii) If $n-m-m_0$ is odd, then all irreducible modular spin characters in B are associate.
$(here\ \ m_0\ \ the\ number\ of\ parts\ of\ \alpha\ \ divisible\ by\, p)$ [11].

8. If C is a principal character of G for an odd prime p and all the entries in C are divisible by a non-negative integer q, then $(1\backslash q)C$ is a principal character of G [11].
9. If C is a principal character of G for a prime p, then deg$C \equiv 0\ \ mod\ p^A$, where o$(G) = p^A m\,, (p,m) = 1$ [6].
10. The number of inequivalent irreducible modular characters of G is always less than or equal to the number of inequivalent irreducible ordinary characters of G [12].
11. If the decomposition matrix $D_{n-1,p} = \left(d_{ij}\right)$ for S_{n-1} is know, then we can induced columns $\left(\psi_j \uparrow^{(r,\bar{r})} S_n\right)$ for S_n [12], these columns are a linear combination with non-negative coefficients from the columns of $D_{n,p}$ [12]. The inducing columns (we take only linear independent) form a matrix $R_{n,p}$ which is called approximation matrix to the decomposition matrix $D_{n,p}$ and the coefficients of $R_{n,p}$ is equal or large than the coefficients $D_{n,p}$ [12]. This known is technique for finding the approximation matrix to the decomposition matrix of the symmetric group S_n.

3. The Spin Blocks of $\overline{S_{18}}$

The group S_{18} has 69 of irreducible spin characters and $\overline{S_{18}}$ has 39 of $(5,\alpha)$ −regular classes, then the decomposition matrix of the spin characters of S_{18}, $p=5$ has 69 rows and 39columns [12].

By using (preliminaries 6), there are seven blocks of S_{18} for $p=5$, these blocks are B_1 ,B_2, B_3, B_4,B_5 ,B_6, B_7. Three of them of defect 0 which are B_5 ,B_6, B_7. The blocks of defect 0 B_5 ,B_6, B_7 includes the spin character $\langle 11,6,1\rangle, \langle 11,6,1\rangle', \langle 8,6,3,1\rangle^*$ respectively, these characters are i.m.s preliminaries (6). The spin characters contained in the block $B_4$are:$\langle 12,4,2\rangle, \langle 12,4,2\rangle'$,$\langle 9,7,2\rangle, \langle 9,7,2\rangle'$,$\langle 7,5,4,2\rangle^*$. The block B_3 includes the spin characters $\langle 14,4\rangle^*, \langle 9,5,4\rangle, \langle 9,5,4\rangle', \langle 9,4,3,2\rangle^*$. The spin characters contained in the block

B_2 are: $\langle 17,1\rangle^*, \langle 16,2\rangle^*, \langle 15,2,1\rangle, \langle 15,2,1\rangle', \langle 12,6\rangle^*, \langle 12,5,1\rangle, \langle 12,5,1\rangle', \langle 12,3,2,1\rangle^*$, $\langle 11,7\rangle^*, \langle 11,5,2\rangle, \langle 11,5,2\rangle', \langle 11,4,2,1\rangle^*, \langle 10,7,1\rangle, \langle 10,7,1\rangle', \langle 10,6,2\rangle, \langle 10,6,2\rangle', \langle 10,5,2,1\rangle^*$, $\langle 9,6,2,1\rangle^*, \langle 8,7,2,1\rangle^*, \langle 7,6,5\rangle, \langle 7,6,5\rangle', \langle 7,6,4,1\rangle^*, \langle 7,6,3,2\rangle^*, \langle 7,5,3,2,1\rangle, \langle 7,5,3,2,1\rangle'$, $\langle 6,5,4,2,1\rangle, \langle 6,5,4,2,1\rangle'$. Finally, the principal block B_1 (the block which contains the character $\langle n\rangle$) includes the other spin characters.

4. The decomposition Matrix for the Block B_4 of Defect One

All i.m.s. of the decomposition matrix for the block B_4 are associate (preliminaries 7) and $\langle\beta\rangle \neq \langle\beta\rangle'$ on $(5,\alpha)$-regular classes (preliminaries 3).

Lemma (4.1): The Decomposition Matrix for this Block is $D^{(4)}_{18,5}$

Proof: When we inducing d_6,d_7 (see table 7) to S_{18} and take the inducing in the block B_4 we have k_6 , k_7 respectively. Now since $\langle 12,4,2\rangle \neq \langle 12,4,2\rangle'$, then k_6 must be split to c_{33}, c_{34} [15] .Also since $\langle 9,7,2\rangle \neq \langle 9,7,2\rangle'$, then k_7 must be split to c_{35}, c_{36} [12] .Now since $c_{33}, c_{34}, c_{35}, c_{36}$ are linearly independent and $c_i - c_j$ is not p.s. for S_{18} for all $1 \leq i,j \leq 4$, then the decomposition matrix for the block B_4 is given in table (1).

5. The Decomposition Matrix for the Block B_3 of Defect One

In this block, we can calculate the decomposition matrix by the Braure tree.

Lemma (5.1): The Braure tree for the block B_3 is:

$$\langle 14,4\rangle^* - \langle 9,5,4\rangle = \langle 9,5,4\rangle' - \langle 9,4,3,2\rangle^*$$

Proof: Since $deg\langle 14,4\rangle^* \equiv deg\langle 9,4,3,2\rangle^* \equiv -50$, $\deg\{\langle 9,5,4\rangle + \langle 9,5,4\rangle'\} \equiv 50$ and by $(r,\bar{r})$-inducing of p.i.s d_{23}, d_{25} of S_{17} (see table 6) to S_{18} we have $c_{31} = \langle 14,4\rangle^* + \langle 9,5,4\rangle + \langle 9,5,4\rangle'$, $c_{32} = \langle 9,5,4\rangle + \langle 9,5,4\rangle' + \langle 9,4,3,2\rangle^*$, then we have the Braure tree for this block, and the decomposition matrix for this block in table (2).

6. The Decomposition Matrix for the Block B_2 of Defect Three

In this block, we have $\langle\beta\rangle = \langle\beta\rangle'$ on $(5,\alpha) - regular\ classes$ (preliminaries 3) and all i.m.s. of the decomposition matrix for this block are double (preliminaries 7).

Lemma (6.1): The decomposition matrix for the block B_2 is $D^{(2)}_{18,5}$.

Proof: When we inducing d_{11}, d_2, $d_3 d_{13}$, d_5, d_7,d_6,d_9,d_{10},d_{19} (seetable 5) to S_{18} and take the inducing in the block B_2 we have c_{11}, c_2, c_3, c_{13}, c_5 , c_7, c_6 ,c_9 ,k_{10} ,c_{19} respectively.

Case(1): c_{13} is not subtracted from c_{11}.

Suppose c_{13} is subtracted from c_{11}, then

$(c_{11} - c_{13}) \downarrow_{(r,\bar{r})} S_{17} = 2d_{11} + 2d_{12} - 2d_{13} - 2d_{14}$ is not p.s for S_{17} (see table 5).Then c_{13} is not subtracted from c_{11}.

Case(2): c_{19} is not subtracted from c_9.

Suppose c_{19} is subtracted from c_9, then

$(c_9 - c_{19}) \downarrow_{(r,\bar{r})} S_{17} = 2d_{17} + 2d_{18} - 2d_{19} - 2d_{20}$ is not p.s forS_{17} (see table 5). Then c_{19} is not subtracted from c_9.

Case(3): $k_{10} - c_{19}$ is not subtracted fromc_7.

Suppose $k_{10} - c_{19}$ is subtracted from c_7, then

$(c_7 - (k_{10} - c_{19})) \downarrow_{(r,\bar{r})} S_{17}$=$d_{13} + d_{14} + d_{17} + d_{18} - d_{19} - d_{20}$ it is not p.s for S_{17} (see table 5).

S , $k_{10} - c_{19}$ is not subtracted from c_7.

Case (4): k_{19} is subtracted from c_{10}.

Suppose k_{19} is not subtracted from c_{10}, then$\langle 9,6,2,1\rangle^* - \langle 10,6,2\rangle + \langle 12,6\rangle^*$ is m.s for S_{18}, but the restriction is not m.s for S_{17} (see table 5), then k_{19} is subtracted from c_{10}. Now since c_{11} , c_2 , c_3 , c_{13}, c_5, c_7 , c_6 , c_9,$k_{10} - c_{19}$, and c_{19} are linearly independent and $c_i - c_j$ is not p.s. for S_{18} for all $1 \le i,j \le 10$, then the decomposition matrix for the block B_2 is as given in table (3).

7. The Decomposition Matrix for the Principle Block B_1 of Defect Three

All i.m.s. of the decomposition matrix for the principle block B_1 are associate (preliminaries 7) and $\langle\beta\rangle \neq \langle\beta\rangle'$ on $(5,\alpha)$ −regular classes (preliminaries 3).

Theorem (7.1)

The decomposition matrix for the spin characters of S_{18} is

$D_{18,5} = D_{18,5}{}^{(1)} \oplus D_{18,5}{}^{(2)} \oplus D_{18,5}{}^{(3)} \oplus D_{18,5}{}^{(4)} \oplus D_{18,5}{}^{(5)} \oplus D_{18,5}{}^{(6)} \oplus D_{18,5}{}^{(7)}$

Proof: Until now we find the decomposition matrices for all blocks of S_{18} except the principle block, so to complete our proof we must find the decomposition matrix for the principle block.

By using $(r,\bar{r})$ −inducing of p.i.s. $d_1, d_{21}, d_{22}, d_3, d_{23}, d_{24}, d_5, d_6, d_{27}, d_{28}, d_{29}, d_{30}, d_9, d_{10}$ of S_{17} (see table 6,7) to S_{18} and take the inducing in the block B_1 we have $k_1, k_{21}, k_{22}, k_3, k_{23}, k_{24}, k_5, k_6, k_{27}, k_{28}, k_{29}, k_{30}, k_9, k_{10}$ respectively.

Case(1): By (preliminaries 5), then k_1 is split to l_1, l_2 .

Case(2): k_{10} is not subtracted from l_1, since

1- when $c_1 = k_1$ we have $(\langle 10,4,3,1\rangle^* + \langle 15,3\rangle^* + \langle 13,3,2\rangle - \langle 11,4,3\rangle)$ is m.s for S_{18}, but $(\langle 10,4,3,1\rangle^* + \langle 15,3\rangle^* + \langle 13,3,2\rangle - \langle 11,4,3\rangle) \downarrow_{(3,3)} S_{17} = \langle 10,4,2,1\rangle + \langle 10,4,2,1\rangle' + \langle 15,2\rangle + \langle 15,2\rangle' + \langle 12,3,2\rangle^* - \langle 11,4,2\rangle^*$ is not m.s for S_{17} (see table7), similarly for $c_1 = k_1 - d_4$

2-when $c_1 = k_1 - 2d_4$ we have $(\langle 10,4,3,1\rangle^* + 2\langle 18\rangle + 2\langle 18\rangle' + \langle 13,3,2\rangle - \langle 11,4,3\rangle)$ is m.s for S_{18}, but $(\langle 10,4,3,1\rangle^* + 2\langle 18\rangle + 2\langle 18\rangle' + \langle 13,3,2\rangle - \langle 11,4,3\rangle) \downarrow_{(3,3)} S_{17} = \langle 10,4,2,1\rangle + \langle 10,4,2,1\rangle' + 4\langle 17\rangle^* + \langle 12,3,2\rangle^* - \langle 11,4,2\rangle^*$ is not m.s for S_{17} (see table 7)

Case(3): Since $\langle 13,3,2\rangle \neq \langle 13,3,2\rangle'$ on $(5,\alpha)$-regular classes, then k_3, k_5 are split [12] as $k_3 = l_3 + l_4$ and $k_5 = l_5 + l_6$.

Case(4): Since $\langle 11,4,3\rangle \neq \langle 11,4,3\rangle'$ on $(5,\alpha)$-regular classes, then k_6 is split [12] as $k_6 = l_7 + l_8$.

Case(5): l_8 is not subtracted from l_{23}.

Suppose l_8 is subtracted from l_{23}, since $\varphi\langle 18\rangle, \varphi\langle 18\rangle'$ is complex and $\langle 13,4,1\rangle, \langle 13,4,1\rangle'$ is real, then we have contradiction with (preliminaries 5).

Then l_8 is not subtracted from l_{23}. Similarity for l_7 is not subtracted from l_{24}.

Case(6): Since $\langle 9,6,3\rangle \neq \langle 9,6,3\rangle'$ on $(5,\alpha)$-regular classes, then k_9, k_{10} are split [12] as $k_9 = l_9 + l_9'$ and $k_{10} = l_{10} + l_{10}'$.

Hence, each $k_{21}, k_{22}, l_3, l_4, k_{23}, k_{24}, l_5, l_6, l_7, l_8, k_{27}$ $k_{28}, k_{29}, k_{30}, l_9, l'_9, l_{10}, l'_{10}$ is p.i.s .

Case(7): when $d_1 - 2d_4$ is p.i.s for S_{17} (see table 7), then:

If $c_1 = l_1 - (l_{10}' + l_{10}'')$, since $\langle 10,4,3,1\rangle^* + \langle 15,3\rangle^* + \langle 13,3,2\rangle - \langle 11,4,3\rangle$ is m.s for S_{18} , but the restriction is not m.s for S_{17}, then $c_1 \neq l_1 - (l_{10}' + l_{10}'')$

Case(8): when $d_1 - d_4$ is p.i.s for S_{17} (see table 7), then:

If $c_1 = l_1 - l'_{10}$, since $\langle 10,4,3,1\rangle^* + \langle 15,3\rangle^* + \langle 13,3,2\rangle - \langle 11,4,3\rangle$ is m.s for S_{18} , but the restriction is not m.s for S_{17}, then $c_1 \neq l_1 - l'_{10}$.

Case(9): when d_1 is p.i.s for S_{17} (see table7), then:
If $c_1 = l_1 - l'_{10}$, since $\langle 10,4,3,1\rangle^* + \langle 15,3\rangle^* + \langle 13,3,2\rangle - \langle 11,4,3\rangle$ is m.s for S_{18}, but the restriction is not m.s for S_{17}, similarly when we take $c_1 = l_1 - (l'_{10} + l''_{10})$, then $c_1 = l_1$.
Hence from case 7,8,9 we have $c_1 = l_1$ or $c_1 = l_1 - l'_{10}$ or $c_1 = l_1 - (l'_{10} + l''_{10})$.
Similarly $c_2 = l_2$ or $c_2 = l_2 - l'_{10}$ or $c_2 = l_2 - (l'_{10} + l''_{10})$. Then the decomposition matrix for the principle block is $D^{(1)}_{18,5}$ is as given in table (4).

Table(1)

The degree of the spin characters	The spin characters	The decomposition matrix for the block B_4			
4243200	$\langle 12,4,2\rangle$	1			
4243200	$\langle 12,4,2\rangle'$		1		
19801600	$\langle 9,7,2\rangle$	1		1	
19801600	$\langle 9,7,2\rangle'$		1		1
85542912	$\langle 7,5,4,1\rangle^*$			1	1
		d_{33}	d_{34}	d_{35}	d_{36}

where $c_{33} = d_{33}$, $c_{34} = d_{34}$, $c_{35} = d_{35}$, $c_{36} = d_{36}$

Table (2)

The degree of the spin characters	The spin characters	The decomposition matrix for the block $B_3 D_{18,5}{}^{(3)}$	
435200	$\langle 14,4\rangle^*$	1	
9574400	$\langle 9,5,4\rangle$	1	1
9574400	$\langle 9,5,4\rangle'$	1	1
913920	$\langle 9,4,3,2\rangle^*$		1
		d_{31}	d_{32}

where $c_{31} = d_{31}$, $c_{32} = d_{32}$.

Table (3)

The degree of the spin characters	The spin Characters	The decomposition matrix for the block $B_2D_{18,5}{}^{(2)}$									
4096	$\langle 17,1\rangle^*$	1									
30464	$\langle 16,2\rangle^*$	1	1								
69888	$\langle 15,2,1\rangle$		1	1							
69888	$\langle 15,2,1\rangle'$		1	1							
1584128	$\langle 12,6\rangle^*$	1	1		1						
3311616	$\langle 12,5,1\rangle$	2	1	1	1	1					
3311616	$\langle 12,,5,1\rangle'$	2	1	1	1	1					
1723392	$\langle 12,3,2,1\rangle^*$			1		1					
1810432	$\langle 11,7\rangle^*$	1			1		1				
9517824	$\langle 11,5,2\rangle$	2	1		1	1	1	1			
9517824	$\langle 11,5,2\rangle'$	2	1		1	1	1	1			
7676928	$\langle 11,4,2,1\rangle^*$					1		1			
4852224	$\langle 10,7,1\rangle$	2			1	1	1		1		
4852224	$\langle 10,7,1\rangle'$	2			1	1	1		1		
13069056	$\langle 10,6,2\rangle$	2	2	1	1	1	2	1	1	1	
13069056	$\langle 10,6,2\rangle'$	2	2	1	1	1	2	1	1	1	
16293888	$\langle 10,5,2,1\rangle^*$		2				2	2		2	
190009.536	$\langle 9,6,2,1\rangle^*$	2	2	1		1	2	1	1	2	1
8712704	$\langle 8,7,2,1\rangle^*$	2		1		1			1		1
2193408	$\langle 7,6,5\rangle$		1	1			1			1	
2193408	$\langle 7,6,5\rangle'$		1	1			1			1	
203046912	$\langle 7,6,4,1\rangle^*$	2	2	1			2		2	2	1
10723328	$\langle 7,6,3,2\rangle^*$	2					2		2	1	1
3734016	$\langle 7,5,3,2,1\rangle$						2		1	1	
3734016	$\langle 7,5,3,2,1\rangle'$						2		1	1	
1357824	$\langle 6,5,4,2,1\rangle$								1		
1357824	$\langle 6,5,4,2,1\rangle'$								1		
		d_{21}	d_{22}	d_{23}	d_{24}	d_{25}	d_{26}	d_{27}	d_{28}	d_{29}	d_{30}

where $c_{11} = d_{21}$, $c_2 = d_{22}$, $c_3 = d_{23}$, $c_{13} = d_{24}$, $c_5 = d_{25}$, $c_7 = d_{26}$, $c_6 = d_{27}$, $c_9 = d_{28}$ and $k_{10} - c_{19} = d_{29}$, $c_{19} = d_{30}$.

Table(4)

The degree of the spin characters	The spin characters		The decomposition matrix for the principle block B_1																		
256	$\langle 18\rangle$	1																			
256	$\langle 18\rangle'$		1																		
139264	$\langle 15,3\rangle^*$	2	2	1	1																
439296	$\langle 14,3,1\rangle$			1		1															
439296	$\langle 14,3,1\rangle'$				1		1														
974848	$\langle 13,5\rangle^*$	3	3	1	1			1	1												
1492992	$\langle 13,4,1\rangle$	2	2	1		1		1		1											
1492992	$\langle 13,4,1\rangle'$	2	2		1		1		1		1										
1005312	$\langle 13,3,2\rangle$					1				1											
1005312	$\langle 13,3,2\rangle'$						1				1										
5431296	$\langle 11,4,3\rangle$	2	2					1	1	1		1									
5431296	$\langle 11,4,3\rangle'$	2	2					1	1		1		1								
1244672	$\langle 10,8\rangle^*$	3	3					1	1					1	1						
14074368	$\langle 10,5,3\rangle$	4	4	1	1			2	2			1	1	1	1	1	1				
14074368	$\langle 10,5,3\rangle'$	4	4	1	1			2	2			1	1	1	1	1	1				
12690432	$\langle 10,4,3,1\rangle^*$							1	1			1	1			1	1				
2050048	$\langle 9,8,1\rangle$	2	2					1		1				1				1			
2050048	$\langle 9,8,1\rangle'$	2	2						1		1				1				1		
17425408	$\langle 9,6,3\rangle$	3	3	1	1	1		1	1	1		1		1	1	1	1	1		1	
17425408	$\langle 9,6,3\rangle'$	3	3	1	1		1	1	1		1		1	1	1	1	1		1		1
29872128	$\langle 9,5,3,1\rangle^*$	2	2	1	1			1	1			1	1	1	1	2	2			1	1
8146944	$\langle 8,7,3\rangle$	1	1			1				1								1		1	
8146944	$\langle 8,7,3\rangle'$	1	1				1				1								1		1
11202048	$\langle 8,6,4\rangle$	1	1	1	1	1								1	1	1	1			1	
11202048	$\langle 8,6,4\rangle'$	1	1	1	1		1							1	1	1	1				1
18765824	$\langle 8,5,4,1\rangle^*$	2	2	1	1									1	1	1	1	1	1	1	1
19035648	$\langle 8,5,3,2\rangle^*$	2	2											2	2	1	1	1	1	1	1
2376192	$\langle 8,4,3,2,1\rangle$													1	1	1					
2376192	$\langle 8,4,3,2,1\rangle'$													1	1		1				
1584128	$\langle 6,5,4,3\rangle^*$																	1	1		
		c_1	c_2	d_3	d_4	d_5	d_6	d_7	d_8	d_9	d_{10}	d_{11}	d_{12}	d_{13}	d_{14}	d_{15}	d_{16}	d_{17}	d_{18}	d_{19}	d_{20}

where $c_1 = l_1$ or $c_1 = l_1 - l'_{10}$ or $c_1 = l_1 - (l'_{10} + l''_{10})$. Similarly $c_2 = l_2$ or $c_2 = l_2 - l'_{10}$ or $c_2 = l_2 - (l'_{10} + l''_{10})$, $k_{21} = d_3, k_{22} = d_4, l_3 = d_5, l_4 = d_6, k_{23} = d_7, k_{24} = d_8, l_5 = d_9, l_6 = d_{10}, l_7 = d_{11}, l_8 = d_{12}, k_{27} = d_{13}, k_{28} = d_{14}, k_{29} = d_{15}, k_{30} = d_{16}, l'_9 = d_{17}, l''_9 = d_{18}, l'_{10} = d_{19}, l''_{10} = d_{20}$.

Table(5)

The degree of the spin characters	The spin characters	The decomposition matrix for the block B_2									
1920	$\langle 16,1\rangle$	1									
1920	$\langle 16,1\rangle'$		1								
465920	$\langle 11,6\rangle$	1		1							
465920	$\langle 11,6\rangle'$		1		1						
1980160	$\langle 11,5,1\rangle^*$	2	2	1	1	1	1				
1305600	$\langle 11,3,2,1\rangle$					1					
1305600	$\langle 11,3,2,1\rangle'$						1				
2545920	$\langle 10,6,1\rangle^*$	2	2	1	1	1	1	1	1		
622336	$\langle 8,6,2,1\rangle$	1	1			1		1		1	
622336	$\langle 8,6,2,1\rangle'$	1	1				1		1		1
2872320	$\langle 7,6,3,1\rangle$	1	1					1	1	1	
2872320	$\langle 7,6,3,1\rangle'$	1	1					1	1		1
565760	$\langle 6,5,3,2,1\rangle^*$							1	1		
		d_{11}	d_{12}	d_{13}	d_{14}	d_{15}	d_{16}	d_{17}	d_{18}	d_{19}	d_{20}

Table(6)

The degree of the spin characters	The spin character	The decomposition matrix for the block B_3									
56320	$\langle 14,3\rangle$	1									
56320	$\langle 14,3\rangle'$		1								
161280	$\langle 13,4\rangle$	1		1							
161280	$\langle 13,4\rangle'$		1		1						
2872320	$\langle 10,4,3\rangle^*$			1	1	1	1				
183040	$\langle 9,8\rangle$			1				1			
183040	$\langle 9,8\rangle'$				1				1		
6223360	$\langle 9,5,3\rangle^*$	1	1	1	1	1	1	1	1	1	1
2872320	$\langle 9,4,3,1\rangle$					1				1	
2872320	$\langle 9,4,3,1\rangle'$						1				1
3351040	$\langle 8,5,4\rangle^*$	1	1					1	1	1	1
1697280	$\langle 8,4,3,2\rangle$							1	1	1	
1697280	$\langle 8,4,3,2\rangle'$							1	1		1
		d_{21}	d_{22}	d_{23}	d_{24}	d_{25}	d_{26}	d_{27}	d_{28}	d_{29}	d_{30}

Table(7)

The degree of the spin characters	The spin characters	The decomposition matrix for the principle block $B_1D_{17,5}{}^{(1)}$									
256	$\langle 17\rangle^*$	1									
13312	$\langle 15,2\rangle$	1	1								
13312	$\langle 15,2\rangle'$	1	1								
56576	$\langle 14,2,1\rangle^*$		1	1							
326144	$\langle 12,5\rangle$	3-x	1		1						
326144	$\langle 12,5\rangle'$	3-x	1		1						
1005312	$\langle 12,4,1\rangle^*$	4-x	1	1	1	1					
678912	$\langle 12,3,2\rangle^*$			1		1					
2558976	$\langle 11,4,2\rangle^*$	4-x			1	1	1				
439296	$\langle 10,7\rangle$	3-x			1			1			
439296	$\langle 10,7\rangle'$	3-x			1			1			
4978688	$\langle 10,5,2\rangle^*$	8-x	2		2		2	2	2		
10183680	$\langle 10,4,2,1\rangle$						1		1		
10183680	$\langle 10,4,2,1\rangle'$						1		1		
1867008	$\langle 9,7,1\rangle^*$	4-x			1	1		1		1	
5544448	$\langle 9,6,2\rangle^*$	6-x	2	1	1	1	1	2	2	1	1
3620864	$\langle 9,5,2,1\rangle$	2	1				1	1	2		1
3620864	$\langle 9,5,2,1\rangle'$	2	1				1	1	2		1
4978688	$\langle 8,7,2\rangle^*$	2		1		1				1	1
2193408	$\langle 7,6,4\rangle^*$	2	2	1				2	2		1
14257152	$\langle 7,5,4,1\rangle$	2	1					1	1	1	1
14257152	$\langle 7,5,4,1\rangle'$	2	1					1	1	1	1
2489344	$\langle 7,5,3,2\rangle$	2						2	1	1	1
2489344	$\langle 7,5,3,2\rangle'$	2						2	1	1	1
1357824	$\langle 7,4,3,2,1\rangle^*$							2	1		
792064	$\langle 6,5,4,2\rangle$									1	
792064	$\langle 6,5,4,2\rangle'$									1	
		c_1	d_2	d_3	d_4	d_5	d_6	d_7	d_8	d_9	d_{10}

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مصفوفة التجزئة للمشخصات الاسقاطية لزمرة التناظرية$\boldsymbol{S_{18}}$معيار $\mathbf{5}$

م.م. جنان عبد الرضا رسن
قسم الرياضيات – كلية العلوم
جامعة البصرة

المستخلص
في هذا البحث تم أيجاد مصفوفة التجزئة للمشخصات الاسقاطية لزمرة التناظريةS_{18}معيار 5 ، بأستثناء مشخصين أسقاطيين معياريين لم يتم تحديدهما.