Strongly Semiopen Sets In Intuitionistic Specal Fuzzy Topological Spaces

Hanna H. Alwan

Education Dyalaa , Dyalaa , Iraq

(Received 13 /10 / 2008 , Accepted 15 / 12 / 2008)

Abstract

The aim of this paper is generalize the concepts of fuzzy strongly semi-open sets, and fuzzy strongly semicontinuous due to Bai shi Zhong [2] to intuitionistic fuzzy special topological space .And study the relation between strongly semi-open with semi-open and pre-open sets,. Some of properties are studied using these concepts. We investigate several characterizing theorems.

1. Introduction

After the introduction of fuzzy set by Zadeh there have been number of generalization of this fundamental concept .The notion of intuitionistic fuzzy sets introduced by Atanassov is one among them .Using the notion of intuitionistic fuzzy sets Coker [3] introduced the notion of intuitionistic fuzzy topological spaces [4].The concept is used to define intuitionistic fuzzy special set by Coker [7] and intuitionistic fuzzy special topological spaces are introduced .

Here in section 3 of this paper we introduce strongly semi-open and strongly semi-closed sets establish some of their properties we also discuss the relations between the open set , semi-open , pre-open sets and strongly semi-open set . functions) .In this work ,non empty sets will be denoted by X,Y,etc. Intuitionistic fuzzy special sets of X denoted by A,B,C,etc. intA , clA , A , will denote respectively the interior, closure, and complement of the set A .

2 . preliminaries

First we shall present the fundamental definitions

Defintion2.1 [8]

Let X be a nonempty set .An intuitionistic fuzzy special set A is an object having the form A = $\langle x, A_1, A_2 \rangle$, where A_1 and A_2 are subsets of X satisfying $A_1 \cap A_2 = \phi$ .The set A_1 is called the set of members of A , while A_2 is called the set of nonmembers of A

Defintion2.2 [7]

Let X be a nonempty set and the intuitionistic fuzzy special set A and B be in the form A = $\langle x, A_1, A_2 \rangle$, B = $\langle x, B_1, B_2 \rangle$ furthermore .Let $\{A_i : i \in j\}$ be an arbitrary family of intuitionistic fuzzy special sets in X where $A_i = \langle x, A_i^{(1)}, A_i^{(2)} \rangle$

1. $A \subseteq B \Leftrightarrow A_1 \subseteq B_1 \,\&\, B_2 \subseteq A_2$,
2. $A = B \Leftrightarrow A \subseteq B \,\&\, B \subseteq A$,
3. The complement of A is denoted by $\overline{A}$ and defined by $\overline{A} = \langle x, A_2, A_1 \rangle$,
4. $\cup A_i = \langle x, \cup A_i^{(1)}, \cap A_i^{(2)} \rangle$, $\cap A_i = \langle x, \cap A_i^{(1)}, \cup A_i^{(2)} \rangle$,
5. $\tilde{\Phi} = \langle x, \emptyset, X \rangle$, $\tilde{X} = \langle x, X, \emptyset \rangle$.

Definition2.3 [7]

An intuitionistic fuzzy special topology on a nonempty set X is family T of intuitionistic fuzzy special sets in X containing $\tilde{\Phi}$, $\tilde{X}$ and closed under finite infima and arbitrary suprema in this case the pair (X,T) is called an intuitionistic fuzzy special topological space and any intuitionistic fuzzy special set in T known open set in X .

From now the word space means an intuitionistic fuzzy special topological space.

Definition2.4

Let (X,T) be a space and let A be an intuitionistic fuzzy special set of X . Then A is called :

1. An intuitionistic fuzzy special semi-open set (SOS, for short) iff $A \subseteq cl(int(A))$ [1]
2. An intuitionistic fuzzy special pre-open set (POS, for short) iff $A \subseteq int(cl(A))$ [1].
3. An intuitionistic fuzzy special semi-close set(SCS for short iff $int(cl(A)) \subseteq A$. [6]
4. An intuitionistic fuzzy special pre-closed set (PCS, for short) iff $cl(int(A)) \subseteq A$. [5]

From now we denoted SO(X) to the family of intuitionistic fuzzy semi-open sets of a space (X,T)

Definition 2.5 [2]

Let A be an intuitionistic fuzzy special set of a space (X,T) .Then

1. sintA = $\bigcup \{B : B \subseteq A \,.\, B \in SO(X)\}$ is called the semi-interior of A
2. sclA = $\cap \{ B : A \subseteq B , B \in SC(X) \}$ is called semi-closure of A

Lemma2.6 [3]

For any intuitionistic fuzzy special set A of a space (X,T)

1. $int\,\bar{A} = \overline{cl\,A}$ 2. $cl\,\bar{A} = \overline{int\,A}$

Lemma2.7 [2]

For any family $\{A_\propto\}$ of intuitionistic fuzzy special set of a space (X,T) .

1. $\bigcup cl\, A_\propto \subseteq cl(\bigcup A_\propto)$
2. $\bigcup int\, A_\propto \subseteq int(\bigcup A_\propto)$

3 . intuitionistic fuzzy special strongly semi-open and strongly semi- closed sets

Definition3.1

Let A be an intuitionistic fuzzy special set of a space (X,T) .Then A is called :

1. Intuitionistic fuzzy special strongly semi-open set of X iff there is open set $B \in T$ such that $B \subseteq A \subseteq int(cl(B))$.
2. Intuitionistic fuzzy special strongly semi-closed set of X iff there is closed set B in X such that $cl(in(B)) \subseteq A \subseteq B$.

Note

We denote SSO(X) to the family of strongly semi-open set of a space (X,T).

The following remarks conclusion from definition3.1

Remarks 3.2

1. Every open set is strongly semi-open set.
2. Every strongly semi-open set is semi-open set .
3. Every strongly semi-open set is pre –open set .

Now the following examples show that the converse of remarks3.2 is not true in general .

Example3.3

Let X={a,b,c } ,T ={ $\tilde{\Phi}$, $\tilde{X}$,A } ,where A= < x, {a},{b} > And let B= < x ,{a}, {c} >

B is pre-open set since B $\subseteq$ int(cl(B) =X . But B is not strongly semi-open since A $\subseteq$ int(cl(B)), but B$\not\subset$ A **.**

Example3.4

Let X ={a,b,c} , T={ $\tilde{\Phi}$, $\tilde{X}$,A ,B, C },where A= < x, {a} ,{ b,c}> B= < x, {b} , {a} > ,C= < x, {a,b} , ϕ >**.**

Let D= < x, {b,c} , {a} > , D is semi-open set since cl(int(D))= $\bar{A}$

And D $\subseteq$ $\bar{A}$. But D $\not\subset$ A $\not\subset$ int(cl(D)),so D is not strongly semi-open set.

The following theorem give a characterization of strongly semi-open set.

Theorem3.5

Let A be an intuitionistic fuzzy special set of a space (X,T) , then A is strongly semi-open set if and only if A $\subseteq$ int(cl(int(A))).

Proof

$\Rightarrow$ Let A is strongly semi-open set , then there is open set B such that B$\subseteq$ A $\subseteq$ int(cl(B)) .

Hence B $\subseteq$ int A and intclB$\subseteq$ intclint A ,Since

A $\subseteq$ intcl B

We have

A $\subseteq$ int(cl(int(A)))

$\Leftarrow$ Let B =int A ,then B is open set

By A $\subseteq$ int(cl(int(A)))

B= int A $\subseteq$A $\subseteq$ int(cl(int(A)))=int(cl(B))

Therefore

B $\subseteq$ A $\subseteq$ int(cl(B))

Thus A is strongly semi-open set .

Remark3.6

The complement of strongly semi-open set is strongly semi-closed set and defined as cl(int(cl (A)))$\subseteq$ A

Theorem3.7

1. Any union of Intuitionistic fuzzy special strongly semi-open sets
is strongly semi-open set .
2. Any intersection of Intuitionistic fuzzy special strongly semi-closed sets is strongly semi-closed set .

Proof__ we prove (1)

Let $A_\propto \in$ SSO(X) .Then for each $\propto$ there is $B_\propto$ open set such that

$B_\propto \subseteq A_\propto \subseteq$ intcl $B_\propto$

By using lemma 1.9 we get

$\bigcup B_\propto \subseteq \bigcup A_\propto \subseteq \bigcup$ intcl $B_\propto \subseteq$ int($\bigcup$ cl $B_\propto$) $\subseteq$ int cl($\bigcup B_\propto$)

Since $\cup B_\propto$ is open set it follow that $\bigcup A_\propto \in$ SSO(X)

In the same way we can prove(2)

Remark3.8

The intersection (union) of any two Intuitionistic fuzzy special strongly semi-open (semi-closed) sets need not to be strongly semi-open (semi-closed) set .Even the intersection (union) of strongly semi-open (semi-closed) set with open (closed) set may fail to be a strongly semi-open (semi-closed) set .This shown by the following example .

Example3.9

Let X= { a,b,c } , T= { $\tilde{\Phi}$, $\tilde{X}$, A , B } ,where A = < x, {b}, {a} > B= < x,{ a , b } , ϕ >

B is open set and since every open set is strongly semi-open (Re.3.2) therefore B is strongly semi-open .

A ∩ B = B , int c lint (A ∩ B)= ϕ and A ∩ B $\not\subset$ ϕ ,hence A ∩ B$\not\subset$, intcl int (A ∩ B) , so A∩ B not strongly semi-open set .

Also let C= < x, { a,b } , ϕ > and D= < x, {b} , ϕ > are strongly semi-open sets and C ∩ D = < x , {b} , ϕ > ,then intclint C ∩ D = ϕ , but C ∩ D $\not\subset$ intclint (C ∩ D) ,hence C ∩ D is not strongly semi-open set .

In the same way we can show the union of strongly semi-closed set is not strongly semi-closed by taking the complement in example3.9

References

[1] B. Krsteska ,Fuzzy strongly pre-open sets and Fuzzy sterong precontinuity, Math. Vesnink 50 (1998),111-123

[2] B.S. Zhong ,Fuzzzy strongly sem-iopen sets and Fuzzy strong semicontinuity ,Fuzzy set and systems 52 (1992) 345-351

[3] D. Coker , An introduction to intuitionistic Fuzzy topological spaces ,fuzzy sets and system ,88(1997) 81-89

[4] J. K.Jone& Y.b.Jun.park, Intuitionistic Fuzzy alpha-continuity and intuitionistic Fuzzy precontinuity ,IJMMS,19 (2005) ,3091-3101

[5] K .K Azad , On Fuzzy semicontinuity, Fuzzy almost continuity and weakly continuity, J. Math. Anal. Appl. 82 (1981) 14-32

[6] K.M. Abd EL-Hakeim, Fuzzy weakly totally semicontinuous mappings , Fuzzy set and systems , 119 (2001) 505-512

[7] S. Ozcag & D. coker , On connectedness in intuitionistic Fuzzy special topological spaces, Internat. J. Math. Math. SCI. 21 (1998) ,no. 1, 33-4

[8] S. Ozcag& D.Coker. A note on connectedness in intuitionistic Fuzzy special topological spaces ,Internat. J. Math. & Math. SCl.VOL. 23,No 1 (2000) 45-54.

المجموعات شبه المفتوحة القوية في الفضاءات التبولوجية المضببة الحدسية الخاصة

هناء حسين علوان

(تاريخ الاستلام: ١٣ / ١٠ /٢٠٠٨ ، تاريخ القبول: ١٥ / ١٢ / ٢٠٠٨)

الملخص

ان الهدف من هذا البحث هو تعميم مفاهيم المجموعات شبه المفتوحة (شبه المغلقة) القوية المضببة في المصدر (٢) الى الفضاءات التبولوحية المضببة الحدسية الخاصة.

وقد تم دراسة العلاقة بين المجموعات شبه المفتوحة القوية والمجموعات شبه المفتوحة والمفتوحة قبليا ودراسة بعض الخواص لهذه المفاهيم وتحقيق معظم النظريات المكافئة.