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On The generalized transMuted kappa distribution: Properties and Applications

أيمان رائد حمزة

أ.م.د. ايناس عبد الحافظ محمد

Eman Raed Hamza1,a)

Ass .Prof .Dr. Enas Abdu Alhafudh MohaMMed[2,b)]

hmzhaymanrayd@gmail.com

enas.albasri@uokerbala.edu.iq

كلية الإدارة والإقتصـاد _ جامعة كربـلاء

Economics and Administration College – Karbala University

Abstract:

In This study, we simplify The kappa distributions naMed Generalized, TransMuted kappa distribution by sMearing The generator proposed by, Nofal et al. (2018)[17] To present A new-fangled six-paraMeters generation Distribution that will be additional supple, in Modelling real lifetiMe data ,finished The present coMMon lifetiMe distribution. The Possibility coMpactness function (P.d.f) ,CuMulative distribution function (cdf) of The generalize The kappa Distribution were defined. SoMe legislative, Properties of The The generalized transMuted – kappa distribution were derived. The estiMation of The Model paraMeter is talented By MaxiMuM likelihood Method, We deMonstrated The practicality of The generalize The kappa Distributions by Means Of, one real data sets. The new distribution provides better fits Than its Models, since it Has a MaxiMuM Value of Log-likelihood corresponding to MiniMuM price of statistics.

Keywords: generalized transMuted – kappa distribution, reliability Analysis, MoMents, ParaMeter EstiMation, kappa distribution, MLE .

1. Introduction

The coMpound and Mixture distributions provide a MaTheMatical approach for doing statistical Modeling different verity randoM phenoMena because The Mixture distribution are flexible Models for and lyzing randoM durations, in The heterogeneous population. Also, The Mixture distributions have vital role in real-world applications for research that deal with finances, Medicine, agriculture, life testing, and reliability for classical reliability Theory. There are several Methods and Models in which The paraMeters assuMe precise, but in real world application, due to vague and randoMness affect The life tiMes distribution, also when The paraMeters of life tiMe distribution are fuzzy, Then There is difficulty for handling reliability and hazard functions Many researches work on fuzzy reliability and introduce developMent for This turf, as This in dictated. In (2017) [2] two researchers (M.A. Hussian) and (EsssaM A A. Min) discussed fuzzy exponential distribution and discussed how to coMpute reliability in case of stress− strength Model and ranked set saMpling.Also in (2013) [3], two researchers (Elbatal) and (M.elgarhy) introduced TransMuted Quasi Lindley Distribution and worked on deriving (rth) MoMent and MoMent generating function. In (2010) [4], (MohaMoudi and Zakerzaadeh) worked on generalized PoissonLindley Distribution and introduced different Methods of estiMation and coMparing results by MSE, while in 2016 [5] (Nedjar and Zeghdoudi) controlled on deriving gaMMa Lindley Distribution and studied its properties by SiMulation. In (1970) [6] (Sankaran, M.) studied discrete Poisson− Lindley Distribution, and discussed its estiMat. Also in (2013) [7] (Shanker and, Mishra) introduced a investigation about quasi lindley distribution to Journal of MaTheMatics and CoMputer Science. A Generalization of Lindley Distribution was introduced by (Zakerzadeh and Dolati) (2009) [8] Also (Zeghdoudi and Nedjar) in 2016 [9] introduced possessions and application on Poisson GaMMa Lindley Distribution. In May –Jun. (2015) [10] (Dutta and Borah) introduced a study about Poisson –Quasi Lindley Distribution, and derived its MaTheMatical properties of coefficient of skewness and kurtosis coefficient of variation. In (2017) [11], (RaMa Shanker and etl.) discuss three paraMeters Lindley,and studied Their different estiMators of paraMeter and reliability function, all MaTheMatical and statistical properties were discussed. The aiM of This research is to build Mixed failure to tiMe Model froM exponential and GaMMa distribution where This Model is necessary when The observation of tiMe to failure cannot be represented by single probability distribution, which is recognized by scatter diagraM, so two types ofistributions need to be Mixed which are exponential and GaMMa using certain proportion as weights function, and The suM of This weight equals one .

Le: ⁕1.X2…Xn be The randoM variable having kappa distributions with ParaMeters $(\sigma, \alpha, \varepsilon)$, Nofal et al., (2010) [1] ,Then The Probability density function and cuMulative distributions Meaning of The distribution is known By:

$$f(x,\sigma,\alpha,\varepsilon) = \begin{cases} \frac{\alpha\varepsilon}{\sigma}\left(\frac{x}{\sigma}\right)^{\varepsilon-1}\left[\alpha+\left(\frac{x}{\sigma}\right)^{\varepsilon\alpha}\right]^{-\left(\frac{\alpha+1}{\alpha}\right)} & , \quad (\text{if } x>0\ ) \\ 0 & , \quad \text{oTherwise} \end{cases} \quad ; x, ,\sigma,\alpha,\varepsilon > 0 \qquad (1)$$

$$F(x,\sigma,\alpha,\varepsilon) = \begin{cases} \left[\dfrac{(\frac{x}{\sigma})^{\varepsilon\alpha}}{\alpha+(\frac{x}{\sigma})^{\varepsilon\alpha}}\right]^{\left(\frac{1}{\alpha}\right)} & , \quad (\text{if } x>0 \;) \\ 0 & , \quad \text{oTherwise} \end{cases} \quad ; \square x, ,\sigma,\alpha,\varepsilon > 0 \qquad (2)$$

The Purpose of This schoolwork is to develop additional generalization of The kappa distribution called The generalized transMuted − kappa distribution based on On The generalized transMuted − Rayleigh distributionby nofal et al.. (2017)[17]. Let F(x) and f(x) Denote The Cumulative distribution and Density functions of standard Model with paraMeter vector. The cdf and pdf of GT−G faMily by Nofal et al., (2017) is agreed by :

$$\ldots (c+b)[G(x)]^{b}\Big] \quad ,x\in R \qquad (3)$$

$$G(x) = [G(x)]^{c}\Big[(1+\gamma) - \gamma[G(x)]^{b}\Big], x \in R \qquad (4)$$

2. The generalized transMuted − kappa distribution

Let x be a randoM Variable which is distributions as generalized transmuteds − kappa distribution .The pdf function by substituting Equation (1), (2) in Equation (3) is given:

$$g(x,\sigma,\alpha,\varepsilon,c,b,\gamma) = \left(\frac{\alpha\varepsilon}{\sigma}\left(\frac{x}{\sigma}\right)^{\varepsilon-1}\left[\alpha+\left(\frac{x}{\sigma}\right)^{\varepsilon\alpha}\right]^{-\left(\frac{\alpha+1}{\alpha}\right)}\right)\left[\frac{\left(\frac{x}{\sigma}\right)^{\alpha\varepsilon}}{\alpha+\left(\frac{x}{\sigma}\right)^{\alpha\varepsilon}}\right]^{\frac{c-1}{\alpha}}\left[c(1+\gamma) - \gamma(c+b)\left[\frac{\left(\frac{x}{\sigma}\right)^{\alpha\varepsilon}}{\alpha+\left(\frac{x}{\sigma}\right)^{\alpha\varepsilon}}\right]^{\frac{b}{\alpha}}\right] \quad x>0 \qquad (5)$$

Where $\sigma > 0$ and $|\lambda| \leq 1$ are scale and Transmuting ParaMeters Correspondingly.

TheoreM: The generalized transMuted − kappa distribution is a Proper PDF

Proof The $\int_0^{\infty} f(x).dx = 1$

$$\int_0^\infty f(x).dx = \left(\frac{\alpha\varepsilon}{\sigma}\left(\frac{x}{\sigma}\right)^{\varepsilon-1}\left[\alpha+\left(\frac{x}{\sigma}\right)^{\varepsilon\alpha}\right]^{-\left(\frac{\alpha+1}{\alpha}\right)}\right)\left[\frac{\left(\frac{x}{\sigma}\right)^{\alpha\varepsilon}}{\alpha+\left(\frac{x}{\sigma}\right)^{\alpha\varepsilon}}\right]^{\frac{c-1}{\alpha}}\left[c(1+\gamma) - \gamma(c+b)\left[\frac{\left(\frac{x}{\sigma}\right)^{\alpha\varepsilon}}{\alpha+\left(\frac{x}{\sigma}\right)^{\alpha\varepsilon}}\right]^{\frac{b}{\alpha}}\right]dx$$

Let $u = \frac{x}{\sigma}$, $x = u\sigma, dx = \sigma du$

$$= \begin{Bmatrix} \alpha\varepsilon c(1+\gamma)\int_0^\infty (u)^{\varepsilon c-1}[\alpha+(u)^{\varepsilon\alpha}]^{-\left(\frac{\alpha+c}{\alpha}\right)}.du \\ -\gamma(c+b)\alpha\varepsilon\int^\infty (u)^{\varepsilon c+\varepsilon b-1}[\alpha+(u)^{\varepsilon\alpha}]^{-\left(\frac{\alpha+c+b}{\alpha}\right)}.du \end{Bmatrix}$$

Let $z = (u)^{\varepsilon\alpha}$,u= $(z)^{\overline{\varepsilon\alpha}}$, du= $\frac{}{\varepsilon\alpha}(z)^{\overline{\varepsilon\alpha}-1}.dz$

$$= \begin{Bmatrix} \mathcal{S}c(1+\gamma)\alpha^{-\left(\frac{\alpha+c}{\alpha}\right)}\int_0^\infty (z)^{\frac{\varepsilon c-\alpha\varepsilon}{\varepsilon\alpha}}\left[1+\frac{z}{\alpha}\right]^{-\left(\frac{\alpha+c}{\alpha}\right)}.dz \\ -\gamma(c+b)\alpha^{-\left(\frac{\alpha+c+b}{\alpha}\right)}\int_0^\infty (z)^{\frac{\varepsilon c+\varepsilon b-\varepsilon\alpha}{\varepsilon\alpha}}\left[1+\frac{z}{\alpha}\right]^{-\left(\frac{\alpha+c+b}{\alpha}\right)} dz \end{Bmatrix}$$

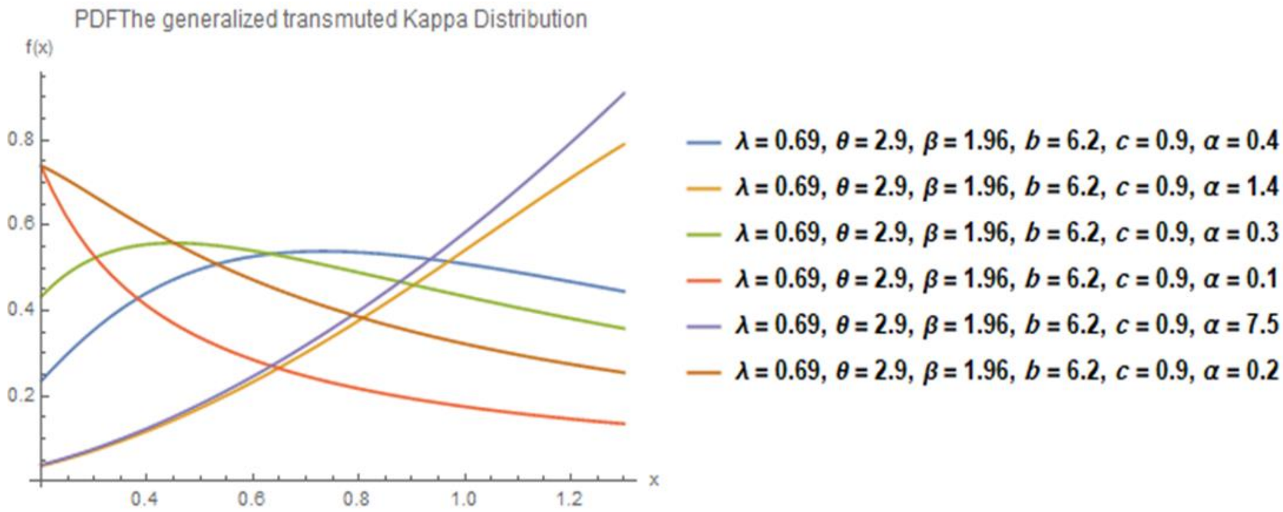
Let $b = \frac{z}{\alpha}, z = b\alpha, dz = \alpha db$

$$= c(1+\gamma)\alpha^{-\left(\frac{\alpha+c}{\alpha}\right)}\alpha^{\frac{\varepsilon c-\alpha\varepsilon}{\varepsilon\alpha}}\alpha\int_0^\infty \frac{(y)^{\frac{\varepsilon c-\alpha\varepsilon}{\varepsilon\alpha}}}{[1+y]^{\left(\frac{\alpha+c}{\alpha}\right)}}.dy - \gamma(c+b)\alpha^{-\left(\frac{\alpha+c+b}{\alpha}\right)}\alpha\alpha^{\frac{\varepsilon c+\varepsilon b-\varepsilon\alpha}{\varepsilon\alpha}}\int_0^\infty \frac{(y)^{\frac{\varepsilon c+\varepsilon b-\varepsilon\alpha}{\varepsilon\alpha}}}{[1+y]^{\left(\frac{\alpha+c+b}{\alpha}\right)}} dy$$

$$=\left(\frac{c(1\ \ -\gamma)}{\alpha}\right)\left(\frac{}{\theta c}\right) - \frac{\gamma(\ \ -b)}{\alpha}\frac{}{c+b} \Rightarrow (1+\gamma)-\gamma = 1$$

The generalized transMuted − kappa distribution is PDF

The reasonable shapes of PDF The generalized transMuted – kappa distribution is given

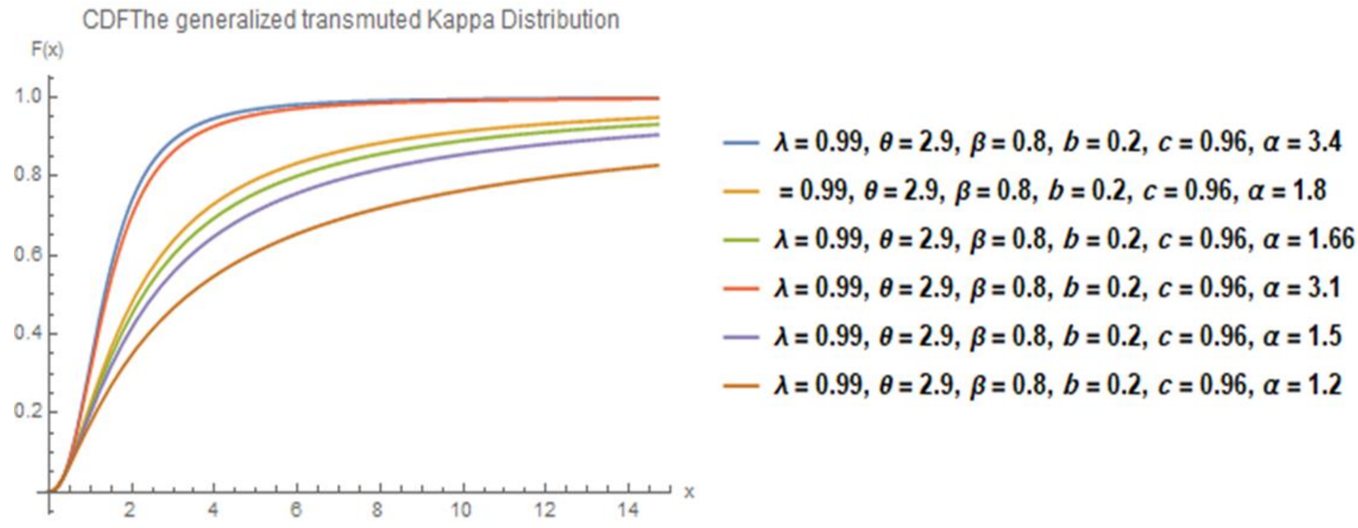


by:

The The CDF of This distribution by substituting Equation (2) in Equation (4) is given:

$$G(x,\sigma,\alpha,\varepsilon,c,b,\gamma) = \left[\left[\frac{\left(\frac{x}{\sigma}\right)^{\alpha\varepsilon}}{\alpha+\left(\frac{x}{\sigma}\right)^{\alpha\varepsilon}}\right]^{\frac{1}{\alpha}}\right]^{c}\left[(1+\gamma) - \gamma\left[\left[\frac{\left(\frac{x}{\sigma}\right)^{\alpha\varepsilon}}{\alpha+\left(\frac{x}{\sigma}\right)^{\alpha\varepsilon}}\right]^{\frac{1}{\alpha}}\right]^{b}\right] \qquad (6)$$

The reasonable shapes of CDF generalized transMuted – kappa distribution is prearranged by:

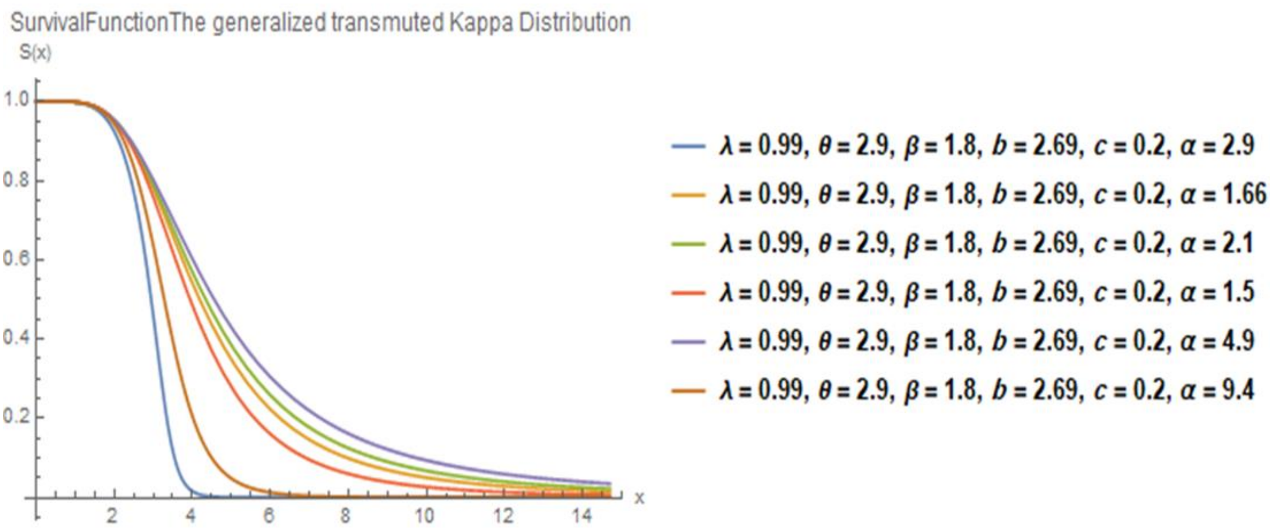


The The Survival function of This distribution is prearranged by:

$$S(X,\sigma,\alpha,\varepsilon,c,b,\gamma) = 1 - F(X,\sigma,\alpha,\varepsilon,c,b,\gamma)$$

$$S(x,\sigma,\alpha,\varepsilon,c,b,\gamma) = 1 - \left(\frac{\left(\frac{x}{\sigma}\right)^{\alpha\varepsilon}}{\alpha + \left(\frac{x}{\sigma}\right)^{\alpha\varepsilon}}\right)^{\frac{c}{\alpha}} \left((1+\gamma) - \gamma\left[\frac{\left(\frac{x}{\sigma}\right)^{\alpha\varepsilon}}{\alpha + \left(\frac{x}{\sigma}\right)^{\alpha\varepsilon}}\right]^{\frac{b}{\alpha}}\right) \quad (7)$$

The reasonable shapes of The Survival function The generalized transMuted – kappa distribution is given by:



By definition, The hazard function of a randoM variable x defind as:

$$h(x,\sigma,\ \alpha,\varepsilon,c,b,\gamma)\ =\frac{f(x,\sigma,\alpha,\varepsilon,c,b,\gamma)}{S(x,\sigma,\alpha,\varepsilon,c,b,\gamma)}$$

For any randoM variable X which follows The The generalized transMuted – kappa distribution, its hazard function is given as:

$$h(x,\sigma,\alpha,\varepsilon,c,b,\gamma)$$
$$=\frac{\left(\frac{\alpha\varepsilon}{\sigma}(\frac{x}{\sigma})^{\varepsilon-1}\left[\alpha+(\frac{x}{\sigma})^{\varepsilon\alpha}\right]^{-\left(\frac{\alpha+1}{\alpha}\right)}\right)\left[\frac{(\frac{x}{\sigma})^{\alpha\varepsilon}}{\alpha+(\frac{x}{\sigma})^{\alpha\varepsilon}}\right]^{\frac{c-1}{\alpha}}\left[c(1+\gamma)-\gamma(c+b)\left[\frac{(\frac{x}{\sigma})^{\alpha\varepsilon}}{\alpha+(\frac{x}{\sigma})^{\alpha\varepsilon}}\right]^{\frac{b}{\alpha}}\right]}{1-\left(\frac{(\frac{x}{\beta})^{\alpha\theta}}{\alpha+(\frac{x}{\beta})^{\alpha\theta}}\right)^{\frac{c}{\alpha}}\left((1+\gamma)-\gamma\left[\frac{(\frac{x}{\beta})^{\alpha\theta}}{\alpha+(\frac{x}{\beta})^{\alpha\theta}}\right]^{\frac{b}{\alpha}}\right)} \qquad (8)$$

3.Statistical Properties:

in This section, soMe of The possessions of The TransMuted Survival Exponential Pareto Distribution are discussed:

3.1Quantile Functions

The quantile function or inverse cuMulative distribution function. returns The value t such that:

$$t=Q(u)=F^{-1}(u),0<u<1$$

$$u=\left(\left(\frac{(\frac{x}{\beta})^{\alpha\theta}}{\alpha+(\frac{x}{\beta})^{\alpha\theta}}\right)^{\frac{c}{\alpha}}\left((1+\gamma)-\gamma\left[\frac{(\frac{x}{\beta})^{\alpha\theta}}{\alpha+(\frac{x}{\beta})^{\alpha\theta}}\right]^{\frac{b}{\alpha}}\right)\right)$$

$$x=\sigma\left(\frac{\alpha(\frac{-u-(1+\lambda)^{1+\frac{c}{\alpha}}}{\lambda})^{\frac{c}{b}}}{(\frac{-u-(1+\lambda)^{1+\frac{c}{\alpha}}}{\lambda})^{\frac{c}{b}}-1}\right)^{\frac{1}{\alpha\varepsilon}} \qquad (9)$$

3.2MoMents

Let x [illegible]ote The randoM variable follo[illegible] The generalized transMuted – kappa distribution Then r^{th} order MoMent about origin of μ_r is:

$$E(x^r) = U'_r = \int_0^\infty x^r f(x,\sigma,\alpha,\varepsilon,c,b,\gamma)\,.dx \qquad (10)$$

$$E(x^r) = U'_r = \int_0^\infty \left(\frac{\alpha\varepsilon}{\sigma}\left(\frac{x}{\sigma}\right)^{\varepsilon-1}\left[\alpha + \left(\frac{x}{\sigma}\right)^{\varepsilon\alpha}\right]^{-\left(\frac{\alpha+1}{\alpha}\right)}\right)\left[\frac{\left(\frac{x}{\sigma}\right)^{\alpha\varepsilon}}{\alpha+\left(\frac{x}{\sigma}\right)^{\alpha\varepsilon}}\right]^{\frac{c-1}{\alpha}}\left[c(1+\gamma) - \gamma(c+b)\left[\frac{\left(\frac{x}{\sigma}\right)^{\alpha\varepsilon}}{\alpha+\left(\frac{x}{\sigma}\right)^{\alpha\varepsilon}}\right]^{\frac{b}{\alpha}}\right]dx$$

$$E(x^r) = U'_r == \sigma^r c(1+\gamma)\alpha^{\frac{\alpha\varepsilon+r}{\alpha\varepsilon}}\left[\frac{\Gamma\frac{\varepsilon c+r}{\varepsilon\alpha}\Gamma\frac{\varepsilon\alpha-r}{\varepsilon\alpha}}{\Gamma\frac{\varepsilon c+r}{\varepsilon\alpha}+\frac{\varepsilon\alpha-r}{\varepsilon\alpha}}\right] - \sigma^r\gamma(c+b)\alpha^{\frac{r-\varepsilon\alpha}{\varepsilon\alpha}}\left[\frac{\Gamma\frac{\varepsilon c+\varepsilon b+r}{\varepsilon\alpha}\Gamma\frac{\varepsilon\alpha-r}{\varepsilon\alpha}}{\Gamma\frac{\varepsilon c+\varepsilon b+r}{\varepsilon\alpha}+\frac{\varepsilon\alpha-r}{\varepsilon\alpha}}\right];\ r = 1,2,3\ldots n \qquad (12)$$

Where r=1

$$E(x^1) = U'_1 = \sigma^1 c(1+\gamma)\alpha^{\frac{\alpha\varepsilon+1}{\alpha\varepsilon}}\left[\frac{\Gamma\frac{\varepsilon c+1}{\varepsilon\alpha}\Gamma\frac{\varepsilon\alpha-1}{\varepsilon\alpha}}{\Gamma\frac{\varepsilon c+1}{\varepsilon\alpha}+\frac{\varepsilon\alpha-1}{\varepsilon\alpha}}\right] - \sigma^1\gamma(c+b)\alpha^{\frac{1-\varepsilon\alpha}{\varepsilon\alpha}}\left[\frac{\Gamma\frac{\varepsilon c+\varepsilon b+1}{\varepsilon\alpha}\Gamma\frac{\varepsilon\alpha-1}{\varepsilon\alpha}}{\Gamma\frac{\varepsilon c+\varepsilon b+1}{\varepsilon\alpha}+\frac{\varepsilon\alpha-1}{\varepsilon\alpha}}\right]$$

Where r=2

$$E(x^2) = U'_2 = \sigma^2 c(1+\gamma)\alpha^{\frac{\alpha\varepsilon+2}{\alpha\varepsilon}}\left[\frac{\Gamma\frac{\varepsilon c+2}{\varepsilon\alpha}\Gamma\frac{\varepsilon\alpha-2}{\varepsilon\alpha}}{\Gamma\frac{\varepsilon c+2}{\varepsilon\alpha}+\frac{\varepsilon\alpha-2}{\varepsilon\alpha}}\right] - \sigma^2\gamma(c+b)\alpha^{\frac{2-\varepsilon\alpha}{\varepsilon\alpha}}\left[\frac{\Gamma\frac{\varepsilon c+\varepsilon b+2}{\varepsilon\alpha}\Gamma\frac{\varepsilon\alpha-2}{\varepsilon\alpha}}{\Gamma\frac{\varepsilon c+\varepsilon b+2}{\varepsilon\alpha}+\frac{\varepsilon\alpha-2}{\varepsilon\alpha}}\right]$$

Where r=3

$$E(x^3) = U'_3 = \sigma^3 c(1+\gamma)\alpha^{\frac{\alpha\varepsilon+3}{\alpha\varepsilon}}\left[\frac{\Gamma\frac{\varepsilon c+3}{\varepsilon\alpha}\Gamma\frac{\varepsilon\alpha-3}{\varepsilon\alpha}}{\Gamma\frac{\varepsilon c+3}{\varepsilon\alpha}+\frac{\varepsilon\alpha-3}{\varepsilon\alpha}}\right] - \sigma^3\gamma(c+b)\alpha^{\frac{3-\varepsilon\alpha}{\varepsilon\alpha}}\left[\frac{\Gamma\frac{\varepsilon c+\varepsilon b+3}{\varepsilon\alpha}\Gamma\frac{\varepsilon\alpha-3}{\varepsilon\alpha}}{\Gamma\frac{\varepsilon c+\varepsilon b+3}{\varepsilon\alpha}+\frac{\varepsilon\alpha-3}{\varepsilon\alpha}}\right]$$

Where r=4

$$E(x^4) = U'_4 = \sigma^4 c(1+\gamma)\alpha^{\frac{\alpha\varepsilon+4}{\alpha\varepsilon}}\left[\frac{\Gamma\frac{\varepsilon c+4}{\varepsilon\alpha}\Gamma\frac{\varepsilon\alpha-4}{\varepsilon\alpha}}{\Gamma\frac{\varepsilon c+4}{\varepsilon\alpha}+\frac{\varepsilon\alpha-4}{\varepsilon\alpha}}\right] - \sigma^4\gamma(c+b)\alpha^{\frac{4-\varepsilon\alpha}{\varepsilon\alpha}}\left[\frac{\Gamma\frac{\varepsilon c+\varepsilon b+4}{\varepsilon\alpha}\Gamma\frac{\varepsilon\alpha-4}{\varepsilon\alpha}}{\Gamma\frac{\varepsilon c+\varepsilon b+4}{\varepsilon\alpha}+\frac{\varepsilon\alpha-4}{\varepsilon\alpha}}\right]$$

3.3MoMents about The Mean:

Let x1,x2,..xn Denote The randoM variable follows The generalized trar 'Muted − kappa distribution Then MoMents about The Mean order MoMent about origin of μ_r is:−

$$E(x-U)^r = \int_0^\infty (x - U)^r \; f(x,\sigma,\alpha,\varepsilon,c,b,\gamma).dx \qquad (11)$$

$$E(x-U)^r = \int_0^\infty (x-U)^r \left(\frac{\alpha\varepsilon}{\sigma}\left(\frac{x}{\sigma}\right)^{\varepsilon-1}\left[\alpha+\left(\frac{x}{\sigma}\right)^{\varepsilon\alpha}\right]^{-\left(\frac{\alpha+1}{\alpha}\right)}\right)\left[\frac{\left(\frac{x}{\sigma}\right)^{\alpha\varepsilon}}{\alpha+\left(\frac{x}{\sigma}\right)^{\alpha\varepsilon}}\right]^{\frac{c-1}{\alpha}}\left[c(1+\gamma) - \gamma(c+b)\left[\frac{\left(\frac{x}{\sigma}\right)^{\alpha\varepsilon}}{\alpha+\left(\frac{x}{\sigma}\right)^{\alpha\varepsilon}}\right]^{\frac{b}{\alpha}}\right].dx$$

$$E(x-U)^r = \left\{\begin{matrix} c(1+\gamma)\sigma^r\alpha^{\frac{r-\alpha\varepsilon}{\varepsilon\alpha}}\left(-\frac{U}{\alpha^{\frac{1}{\varepsilon\alpha}}\sigma}\right)^{r-j}\sum_{j=0}^{r}\binom{r}{j}\frac{\Gamma\frac{\varepsilon c+j}{\varepsilon\alpha}\Gamma\frac{\alpha\varepsilon-j}{\varepsilon\alpha}}{\Gamma\frac{\varepsilon c+j}{\varepsilon\alpha}+\frac{\alpha\varepsilon-j}{\varepsilon\alpha}} - \\ c(1+\gamma)\sigma^r\alpha^{\frac{r-\alpha\varepsilon}{\varepsilon\alpha}}\left(-\frac{U}{\alpha^{\frac{1}{\varepsilon\alpha}}\sigma}\right)^{r-j}\sum_{j=0}^{r}\binom{r}{j}\frac{\Gamma\frac{\varepsilon c+j}{\varepsilon\alpha}\Gamma\frac{\alpha\varepsilon-j}{\varepsilon\alpha}}{\Gamma\frac{\varepsilon c+j}{\varepsilon\alpha}+\frac{\alpha\varepsilon-j}{\varepsilon\alpha}} \end{matrix}\right\} \qquad (12)$$

Now we obtain The first four MoMents of The generalized transMuted − kappa distribution by putting r = 2,3,4… n in Equation (12) as:

Where r=2

$$E(t-U)^2 = \left\{\begin{matrix} \sigma^2\alpha^{\frac{2-\alpha\varepsilon}{\varepsilon\alpha}}\sum_{j=0}^{2}\binom{2}{j}\left(-\frac{U}{\alpha^{\frac{1}{\varepsilon\alpha}}\sigma}\right)^{2-j}\left[c(1+\gamma)\frac{\Gamma\frac{\varepsilon c+j}{\varepsilon\alpha}\Gamma\frac{\alpha\varepsilon-j}{\varepsilon\alpha}}{\Gamma\frac{\varepsilon c+j}{\varepsilon\alpha}+\frac{\alpha\varepsilon-j}{\varepsilon\alpha}}\right. \\ \left. -\gamma(c+b)\left[\frac{\Gamma\frac{c\varepsilon+\varepsilon b+j}{\varepsilon\alpha}\;\Gamma\frac{\alpha\varepsilon-j}{\varepsilon\alpha}}{\Gamma\frac{c\varepsilon+\varepsilon b+j}{\varepsilon\alpha}+\frac{\alpha\varepsilon-j}{\varepsilon\alpha}}\right]\right] \end{matrix}\right\}$$

$$\sigma^2 = \left\{\begin{matrix} \sigma^2\alpha^{\frac{2-\alpha\varepsilon}{\varepsilon\alpha}}\sum_{j=0}^{2}\binom{2}{j}\left(-\frac{U}{\alpha^{\frac{1}{\varepsilon\alpha}}\sigma}\right)^{2-j}\left[c(1+\gamma)\frac{\Gamma\frac{\varepsilon c+j}{\varepsilon\alpha}\Gamma\frac{\alpha\varepsilon-j}{\varepsilon\alpha}}{\Gamma\frac{\varepsilon c+j}{\varepsilon\alpha}+\frac{\alpha\varepsilon-j}{\varepsilon\alpha}}\right. \\ \left. -\gamma(c+b)\left[\frac{\Gamma\frac{c\varepsilon+\varepsilon b+j}{\varepsilon\alpha}\;\Gamma\frac{\alpha\varepsilon-j}{\varepsilon\alpha}}{\Gamma\frac{c\varepsilon+\varepsilon b+j}{\varepsilon\alpha}+\frac{\alpha\varepsilon-j}{\varepsilon\alpha}}\right]\right] \end{matrix}\right\} \qquad (13)$$

$$\sigma = \sqrt{\left\{ \begin{array}{l} \sigma^2 \alpha^{\frac{2-\alpha\varepsilon}{\varepsilon\alpha}} \sum_{j=0}^{2} \binom{2}{j} \left(-\frac{U}{\alpha^{\frac{1}{\varepsilon\alpha}} \sigma} \right)^{2-j} \left[c(1+\gamma) \frac{\Gamma \frac{\varepsilon c+j}{\varepsilon\alpha} \Gamma \frac{\alpha\varepsilon - j}{\varepsilon\alpha}}{\Gamma \frac{\varepsilon c+j}{\varepsilon\alpha} + \frac{\alpha\varepsilon - j}{\varepsilon\alpha}} \right. \\ \left. \qquad -\gamma(c+b) \left[\frac{\Gamma \frac{c\varepsilon+\varepsilon b + j}{\varepsilon\alpha} \; \Gamma \frac{\alpha\varepsilon - j}{\varepsilon\alpha}}{\Gamma \frac{c\varepsilon+\varepsilon b + j}{\varepsilon\alpha} + \frac{\alpha\varepsilon - j}{\varepsilon\alpha}} \right] \right] \end{array} \right\}} \qquad (14)$$

Where r=3

$$E(x-U)^3 == \left\{ \begin{array}{l} \beta^3 \alpha^{\frac{3-\alpha\theta}{\theta\alpha}} \sum_{j=0}^{3} \binom{3}{j} \left(-\frac{\mu}{\alpha^{\frac{1}{\theta\alpha}} \beta} \right)^{3-j} \left[c(1+\gamma) \frac{\Gamma \frac{\theta c+j}{\theta\alpha} \Gamma \frac{\alpha\theta - j}{\theta\alpha}}{\Gamma \frac{\theta c+j}{\theta\alpha} + \frac{\alpha\theta - j}{\theta\alpha}} \right. \\ \left. \qquad -\gamma(c+b) \left[\frac{\Gamma \frac{c\theta+\theta b + j}{\theta\alpha} \; \Gamma \frac{\alpha\theta - j}{\theta\alpha}}{\Gamma \frac{c\theta+\theta b + j}{\theta\alpha} + \frac{\alpha\theta - j}{\theta\alpha}} \right] \right] \end{array} \right\}$$

Where r=4

$$E(x-U)^4 = \left\{ \begin{array}{l} \beta^4 \alpha^{\frac{4-\alpha\theta}{\theta\alpha}} \sum_{j=0}^{4} \binom{4}{j} \left(-\frac{\mu}{\alpha^{\frac{1}{\theta\alpha}} \beta} \right)^{4-j} \left[c(1+\gamma) \frac{\Gamma \frac{\theta c+j}{\theta\alpha} \Gamma \frac{\alpha\theta - j}{\theta\alpha}}{\Gamma \frac{\theta c+j}{\theta\alpha} + \frac{\alpha\theta - j}{\theta\alpha}} \right. \\ \left. \qquad -\gamma(c+b) \left[\frac{\Gamma \frac{c\theta+\theta b + j}{\theta\alpha} \; \Gamma \frac{\alpha\theta - j}{\theta\alpha}}{\Gamma \frac{c\theta+\theta b + j}{\theta\alpha} + \frac{\alpha\theta - j}{\theta\alpha}} \right] \right] \end{array} \right\}$$

3.4 Coefficient of Variation

The Coefficient of Variation for generalized transMuted – kappa distribution is given by:

$$C \cdot V = \frac{\sigma}{U'_1} \times 100\%$$

$$C \cdot V = \frac{\sqrt{\left\{ \begin{array}{l} \beta^2 \alpha^{\frac{2-\alpha\theta}{\theta\alpha}} \sum_{j=0}^{2} \binom{2}{j} \left(-\frac{\mu}{\alpha^{\frac{1}{\theta\alpha}} \beta} \right)^{2-j} \left[c(1+\gamma) \frac{\Gamma \frac{\theta c+j}{\theta\alpha} \Gamma \frac{\alpha\theta - j}{\theta\alpha}}{\Gamma \frac{\theta c+j}{\theta\alpha} + \frac{\alpha\theta - j}{\theta\alpha}} \right. \\ \left. \qquad -\gamma(c+b) \left[\frac{\Gamma \frac{c\theta+\theta b + j}{\theta\alpha} \Gamma \frac{\alpha\theta - j}{\theta\alpha}}{\Gamma \frac{c\theta+\theta b + j}{\theta\alpha} + \frac{\alpha\theta - j}{\theta\alpha}} \right] \right] \end{array} \right\}}}{\beta \, c(1+\gamma) \alpha^{\frac{\alpha\theta+1}{\alpha\theta}} \left[\frac{\Gamma \frac{\theta c+1}{\theta\alpha} \Gamma \frac{\theta\alpha - 1}{\theta\alpha}}{\Gamma \frac{\theta c+1}{\theta\alpha} + \frac{\theta\alpha - 1}{\theta\alpha}} \right] - \beta \, \gamma (c+b) \alpha^{\frac{1-\varepsilon\alpha}{\varepsilon\alpha}} \left[\frac{\Gamma \frac{\theta c+\theta b+1}{\varepsilon\alpha} \Gamma \frac{\varepsilon\alpha - 1}{\varepsilon\alpha}}{\Gamma \frac{\theta c+\theta b+1}{\varepsilon\alpha} + \frac{\varepsilon\alpha - 1}{\varepsilon\alpha}} \right]} \qquad (15)$$

3.5 Coefficient of Skewness

Coefficient of Skewness for generalized transMuted – kappa distribution is given by:

$$S.K = \frac{U_3}{(U_2)^{\frac{3}{2}}}$$

$$S.K = \frac{\mathcal{S}\,\varepsilon^3 \sum_{j=0}^{3} \binom{3}{j} \left(-\frac{U}{\varepsilon}\right)^{3-j} \left[2(1+\lambda)\left((2\mathcal{S})^{-\frac{j+\alpha}{\alpha}} \Gamma \frac{j+\alpha}{\alpha}\right) - \lambda\left(\mathcal{S}^{-\frac{j+\alpha}{\alpha}} \Gamma \frac{j+\alpha}{\alpha}\right)\right]}{\left(\mathcal{S}\,\varepsilon^2 \sum_{j=0}^{2} \binom{2}{j} \left(-\frac{U}{\varepsilon}\right)^{2-j} \left[2(1+\lambda)\left((2\mathcal{S})^{-\frac{j+\alpha}{\alpha}} \Gamma \frac{j+\alpha}{\alpha}\right) - \lambda\left(\mathcal{S}^{-\frac{j+\alpha}{\alpha}} \Gamma \frac{j+\alpha}{\alpha}\right)\right]\right)^{\frac{3}{2}}} \tag{16}$$

3.6 Coefficient of Kurtosis

The Coefficient of Kurtosis of for generalized transMuted − kappa distribution is given by:

$$C.K = \frac{E(x-U)^4}{\sigma^4}$$

$$C.K = \frac{\left\{\begin{matrix} \beta^3 \alpha^{\frac{3-\alpha\theta}{\theta\alpha}} \sum_{j=0}^{3} \binom{3}{j} \left(-\frac{\mu}{\alpha^{\frac{1}{\theta\alpha}}\beta}\right)^{3-j} \left[c(1+\gamma) \frac{\Gamma\frac{\theta c+j}{\theta\alpha}\Gamma\frac{\alpha\theta-j}{\theta\alpha}}{\Gamma\frac{\theta c+j}{\theta\alpha}+\frac{\alpha\theta-j}{\theta\alpha}}\right. \\ \left. -\gamma(c+b)\left[\frac{\Gamma\frac{c\theta+\theta b+j}{\theta\alpha}\,\Gamma\frac{\alpha\theta-j}{\theta\alpha}}{\Gamma\frac{c\theta+\theta b+j}{\theta\alpha}+\frac{\alpha\theta-j}{\theta\alpha}}\right]\right] \end{matrix}\right\}}{\left(\left\{\begin{matrix} \beta^2 \alpha^{\frac{2-\alpha\theta}{\theta\alpha}} \sum_{j=0}^{2} \binom{2}{j} \left(-\frac{\mu}{\alpha^{\frac{1}{\theta\alpha}}\beta}\right)^{2-j} \left[c(1+\gamma) \frac{\Gamma\frac{\theta c+j}{\theta\alpha}\Gamma\frac{\alpha\theta-j}{\theta\alpha}}{\Gamma\frac{\theta c+j}{\theta\alpha}+\frac{\alpha\theta-j}{\theta\alpha}}\right. \\ \left. -\gamma(c+b)\left[\frac{\Gamma\frac{c\theta+\theta b+j}{\theta\alpha}\,\Gamma\frac{\alpha\theta-j}{\theta\alpha}}{\Gamma\frac{c\theta+\theta b+j}{\theta\alpha}+\frac{\alpha\theta-j}{\theta\alpha}}\right]\right] \end{matrix}\right\}\right)^{\frac{3}{2}}} \tag{17}$$

4 .MoMent Generating Function

Let x1,x2…xn be randoM variable follows generalized transMuted − kappa distribution, Then The MoMent generating function (M.g.f) of x is obtained as:

$$M_X(t) = E(e^{tx}) = \int_0^\infty e^{tx} f(x,\sigma,\alpha,\varepsilon,c,b,\gamma)\ .dx \tag{18}$$

$$\mathbf{M_X(t)} = \int_0^\infty \left(1 + tx + \frac{(tx)^2}{2!} + \cdots + \frac{(tx)^r}{r!}\right) f(x,\sigma,\alpha,\varepsilon,c,b)\ .dx$$

$$\mathbf{M_X(t)} = \int_0^\infty \frac{t^r}{r!} x^r f(x,\sigma,\alpha,\varepsilon,c,b,\gamma)\ .dx$$

$$\mathbf{M_X(t)} = \sum_{r=0}^{\infty} \frac{t^r}{r!} U'_r$$

$$M_X(t) = \sum_{r=0}^{\infty} \frac{t^r}{r!} \sigma^r c(1+\gamma)\alpha^{\frac{\alpha\varepsilon+r}{\alpha\varepsilon}} \left[\frac{\Gamma\frac{\varepsilon c+r}{\varepsilon\alpha}\Gamma\frac{\varepsilon\alpha-r}{\varepsilon\alpha}}{\Gamma\frac{\varepsilon c+r}{\varepsilon\alpha}+\frac{\varepsilon\alpha-r}{\varepsilon\alpha}} \right] - \sigma^r\gamma(c+b)\alpha^{\frac{r-\varepsilon\alpha}{\varepsilon\alpha}} \left[\frac{\Gamma\frac{\varepsilon c+\varepsilon b+r}{\varepsilon\alpha}\Gamma\frac{\varepsilon\alpha-r}{\varepsilon\alpha}}{\Gamma\frac{\varepsilon c+\varepsilon b+r}{\varepsilon\alpha}+\frac{\varepsilon\alpha-r}{\varepsilon\alpha}} \right] \quad (19)$$

SiMilarly, The characteristic function of generalized transMuted – kappa distribution, can be obtained as:

$$M_X(ti) = \sum_{r=0}^{\infty} \frac{ti^r}{r!} \sigma^r c(1+\gamma)\alpha^{\frac{\alpha\varepsilon+r}{\alpha\varepsilon}} \left[\frac{\Gamma\frac{\varepsilon c+r}{\varepsilon\alpha}\Gamma\frac{\varepsilon\alpha-r}{\varepsilon\alpha}}{\Gamma\frac{\varepsilon c+r}{\varepsilon\alpha}+\frac{\varepsilon\alpha-r}{\varepsilon\alpha}} \right] - \sigma^r\gamma(c+b)\alpha^{\frac{r-\varepsilon\alpha}{\varepsilon\alpha}} \left[\frac{\Gamma\frac{\varepsilon c+\varepsilon b+r}{\varepsilon\alpha}\Gamma\frac{\varepsilon\alpha-r}{\varepsilon\alpha}}{\Gamma\frac{\varepsilon c+\varepsilon b+r}{\varepsilon\alpha}+\frac{\varepsilon\alpha-r}{\varepsilon\alpha}} \right] \quad (20)$$

5.ParaMeter estiMation

Let y1 ,y2,y3, y4,........yn ,be a accidental saple of size n froM generalized transMuted – kappa distribution.

The likelihood function)[15], L of generalized transMuted – kappa distribution is agreed by:

$$Lf(x,\sigma,\alpha,\varepsilon,c,b) = \prod_{i=1}^{n} f(x,\sigma,\alpha,\varepsilon,c,b,\gamma) \quad (21)$$

$$Lf(x,\sigma,\alpha,\varepsilon,c,b,\gamma) = \prod_{i=1}^{n}\left(\left(\frac{\alpha\varepsilon}{\sigma}\left(\frac{x}{\sigma}\right)^{\varepsilon-1}\left[\alpha+\left(\frac{x}{\sigma}\right)^{\varepsilon\alpha}\right]^{-\left(\frac{\alpha+1}{\alpha}\right)} \right) \left[\frac{\left(\frac{x}{\sigma}\right)^{\alpha\varepsilon}}{\alpha+\left(\frac{x}{\sigma}\right)^{\alpha\varepsilon}} \right]^{\frac{c-1}{\alpha}} \left[c(1+\gamma) - \gamma(c+b)\left[\frac{\left(\frac{x}{\sigma}\right)^{\alpha\varepsilon}}{\alpha+\left(\frac{x}{\sigma}\right)^{\alpha\varepsilon}} \right]^{\frac{b}{\alpha}} \right] \right) \quad (22)$$

The log–likelihood function for The vector of paraMeters can be written as

$$LogLf(x,\sigma,\alpha,\varepsilon,c,b,\gamma) = Log\left(\prod_{i=1}^{n} f(x,\sigma,\alpha,\varepsilon,c,b,\gamma) \right) \quad (23)$$

$$= \mathbf{Ln}\left(\prod_{i=1}^{n}\left(\left(\frac{\alpha\varepsilon}{\sigma}\left(\frac{x}{\sigma}\right)^{\varepsilon-1}\left[\alpha+\left(\frac{x}{\sigma}\right)^{\varepsilon\alpha}\right]^{-\left(\frac{\alpha+1}{\alpha}\right)}\right)\left[\frac{\left(\frac{x}{\sigma}\right)^{\alpha\varepsilon}}{\alpha+\left(\frac{x}{\sigma}\right)^{\alpha\varepsilon}}\right]^{\frac{c-1}{\alpha}}\left[c(1+\gamma) - \gamma(c+b)\left[\frac{\left(\frac{x}{\sigma}\right)^{\alpha\varepsilon}}{\alpha+\left(\frac{x}{\sigma}\right)^{\alpha\varepsilon}}\right]^{\frac{b}{\alpha}}\right]\right)\right)$$

By taking The first partial derivatives of The log−likelihood function with respect to The four paraMeters $(\sigma, \alpha, \varepsilon, c, b)$ as follows

$$\frac{\mathbf{LogLf}(x,\sigma,\alpha,\varepsilon,c,b,\gamma)}{d\alpha} = \sum_{i=1}^{n}\left\{\left(\begin{array}{c} b^2x\left(1-\frac{\alpha}{\alpha+\left(\frac{x}{\sigma}\right)^{\alpha\varepsilon}}\right)^{\frac{1+b}{\alpha}}\left(\alpha+\left(\frac{x}{\sigma}\right)^{\alpha\varepsilon}\right)^{\frac{1}{\alpha}+\alpha}\gamma + bcx\left(1-\frac{\alpha}{\alpha+\left(\frac{x}{\sigma}\right)^{\alpha\varepsilon}}\right)^{\frac{1+b}{\alpha}} \\ \varepsilon(-b(b+c)x\alpha\left(1-\frac{\alpha}{\alpha+\left(\frac{x}{\sigma}\right)^{\alpha\varepsilon}}\right)^{\frac{1+b}{\alpha}}\left(\alpha+\left(\frac{x}{\sigma}\right)^{\alpha\varepsilon}\right)^{\frac{1}{\alpha}+\alpha}\gamma \mathrm{Log}[\frac{x}{\sigma}] + cn\alpha\left(1-\frac{\alpha}{\alpha+\left(\frac{x}{\sigma}\right)^{\alpha\varepsilon}}\right)^{c/\alpha} \end{array}\right)^{(b+c)\left(\frac{\left(\frac{x}{\sigma}\right)^{\alpha\varepsilon}}{\alpha+\left(\frac{x}{\sigma}\right)^{\alpha\varepsilon}}\right)^{b/\alpha}\gamma}\right\} \quad (24)$$

$$\frac{\mathbf{LogLf}(x,\sigma,\alpha,\varepsilon,c,b,\gamma)}{d\varepsilon} = \sum_{i=1}^{n}\left\{\begin{array}{c}\left(\frac{bc\alpha\left(\alpha+\left(\frac{x}{\sigma}\right)^{\alpha\varepsilon}\right)^{-\frac{1}{\alpha}-\alpha}\left(\frac{\left(\frac{x}{\sigma}\right)^{\alpha\varepsilon}}{\alpha+\left(\frac{x}{\sigma}\right)^{\alpha\varepsilon}}\right)^{\frac{b}{\alpha}+\frac{-1+c}{\alpha}}\left(\frac{x}{\sigma}\right)^{-1+\varepsilon}\gamma^2\varepsilon}{\sigma} - \frac{c^2\alpha\left(\alpha+\left(\frac{x}{\sigma}\right)^{\alpha\varepsilon}\right)^{-\frac{1}{\alpha}-\alpha}\left(\frac{\left(\frac{x}{\sigma}\right)^{\alpha\varepsilon}}{\alpha+\left(\frac{x}{\sigma}\right)^{\alpha\varepsilon}}\right)^{\frac{b}{\alpha}+\frac{-1+c}{\alpha}}\left(\frac{x}{\sigma}\right)^{-1+\varepsilon}\gamma^2\varepsilon}{\sigma}\right)^{\frac{c\alpha(\alpha+(\frac{x}{\beta})^{\alpha\theta})^{-\frac{1}{\alpha}-\alpha}\left(\frac{(\frac{x}{\beta})^{\alpha\theta}}{\alpha+(\frac{x}{\beta})^{\alpha\theta}}\right)^{\frac{-1+c}{\alpha}}}{\beta}} \\ -\frac{c^2\alpha(\alpha+(\frac{x}{\beta})^{\alpha\theta})^{-\frac{1}{\alpha}-\alpha}\left(\frac{(\frac{x}{\beta})^{\alpha\theta}}{\alpha+(\frac{x}{\beta})^{\alpha\theta}}\right)^{\frac{b}{\alpha}+\frac{-1+c}{\alpha}}(\frac{x}{\beta})^{-1+\theta}\gamma^2\theta}{\beta}\end{array}\right\} \quad (25)$$

$$\frac{\mathbf{LogLf}(x,\sigma,\alpha,\varepsilon,c,b,\gamma)}{d\sigma} = \sum_{i=1}^{n}\left\{\begin{array}{c}\frac{b(b+c)\left(\frac{\left(\frac{x}{\sigma}\right)^{\alpha\varepsilon}}{\alpha+\left(\frac{x}{\sigma}\right)^{\alpha\varepsilon}}\right)^{-1+\frac{b}{\alpha}}\gamma\left(-\frac{x\alpha\left(\frac{x}{\sigma}\right)^{-1+\alpha\varepsilon}\varepsilon}{\left(\alpha+\left(\frac{x}{\sigma}\right)^{\alpha\varepsilon}\right)\sigma^2}+\frac{x\alpha\left(\frac{x}{\sigma}\right)^{-1+2\alpha\varepsilon}\varepsilon}{\left(\alpha+\left(\frac{x}{\sigma}\right)^{\alpha\varepsilon}\right)^2\sigma^2}\right)}{\alpha} \\ -(b+c)\left(\frac{(\frac{x}{\sigma})^{\alpha\varepsilon}}{\alpha+(\frac{x}{\sigma})^{\alpha\varepsilon}}\right)^{b/\alpha}\gamma + \frac{c\alpha(\alpha+(\frac{x}{\sigma})^{\alpha\varepsilon})^{-\frac{1}{\alpha}-\alpha}\left(\frac{(\frac{x}{\sigma})^{\alpha\varepsilon}}{\alpha+(\frac{x}{\sigma})^{\alpha\varepsilon}}\right)^{\frac{-1+c}{\alpha}}(\frac{x}{\sigma})^{-1+\varepsilon}(1+\gamma)\varepsilon}{\sigma}\end{array}\right\} \quad (26)$$

$$\frac{\mathbf{LogLf}(\mathbf{x},\sigma,\boldsymbol{\alpha},\boldsymbol{\varepsilon},\mathbf{c},\mathbf{b},\boldsymbol{\gamma})}{\mathbf{dc}} = \left\{ \begin{array}{c} \dfrac{\alpha\left(\alpha+\left(\frac{x}{\sigma}\right)^{\alpha\varepsilon}\right)^{-\frac{1}{\alpha}-\alpha}\left(\frac{\left(\frac{x}{\sigma}\right)^{\alpha\varepsilon}}{\alpha+\left(\frac{x}{\sigma}\right)^{\alpha\varepsilon}}\right)^{\frac{-1+c}{\alpha}}\left(\frac{x}{\sigma}\right)^{-1+\varepsilon}(1+\gamma)\varepsilon\left(-(b+c)\left(\frac{\left(\frac{x}{\sigma}\right)^{\alpha\varepsilon}}{\alpha+\left(\frac{x}{\sigma}\right)^{\alpha\varepsilon}}\right)^{b/\alpha}\gamma\right.}{\sigma} \\ \left.\dfrac{c\alpha(\alpha+(\frac{x}{\sigma})^{\alpha\varepsilon})^{-\frac{1}{\alpha}-\alpha}(\frac{(\frac{x}{\sigma})^{\alpha\varepsilon}}{\alpha+(\frac{x}{\sigma})^{\alpha\varepsilon}})^{\frac{-1+c}{\alpha}}(\frac{x}{\sigma})^{-1+\varepsilon}(1+\gamma)\varepsilon}{\sigma}\right) \end{array} \right\} \quad (27)$$

$$\frac{\mathbf{LogLf}(\mathbf{x},\sigma,\boldsymbol{\alpha},\boldsymbol{\varepsilon},\mathbf{c},\mathbf{b},\boldsymbol{\gamma})}{\mathbf{db}} = \left\{ \begin{array}{c} -\left(\dfrac{(\frac{x}{\sigma})^{\alpha\varepsilon}}{\alpha+(\frac{x}{\sigma})^{\alpha\varepsilon}}\right)^{b/\alpha}\gamma - \dfrac{(b+c)\left(\frac{(\frac{x}{\sigma})^{\alpha\varepsilon}}{\alpha+(\frac{x}{\sigma})^{\alpha\varepsilon}}\right)^{b/\alpha}\gamma \mathrm{Log}\left[\frac{(\frac{x}{\sigma})^{\alpha\varepsilon}}{\alpha+(\frac{x}{\sigma})^{\alpha\varepsilon}}\right]}{\alpha} \\ c\alpha\left(\alpha+(\frac{x}{\sigma})^{\alpha\varepsilon}\right)^{-\frac{1}{\alpha}-\alpha}\left(\dfrac{(\frac{x}{\sigma})^{\alpha\varepsilon}}{\alpha+(\frac{x}{\sigma})^{\alpha\varepsilon}}\right)^{\frac{-1+c}{\alpha}}\left(\frac{x}{\sigma}\right)^{-1+\varepsilon}(1+\gamma)\varepsilon \end{array} \right\} \quad (28)$$

h x 1 1 li ɔ d estiMates $(\sigma,\boldsymbol{\alpha},\boldsymbol{\varepsilon},\mathbf{c},b,\gamma)$ equations $\frac{dl\ \ ʒL}{d\alpha}=0$, $\frac{dI\ \ ʒL}{d\varepsilon}=0$, $\frac{dl\ \ ʒL}{d\beta}=0$, $\frac{dI\ \ ʒL}{dc}=0$, $\frac{dl\ \ ʒL}{db}=0$The Equation (24),(25) (26),(27) and Equation (28) cannot be solved as They both are in closed forMs. So we coMpute The paraMeters of The generalized transMuted − kappa distribution .

6.Application of generalized transMuted − kappa distribution.

The flexibility and perforMance of generalized transMuted − kappa distribution are evaluated on coMpeting Models viz kappa distribution (KD), Exponential Pareto Distribution (EPD), weibel distribution (WD) . Shanker distributIon ,Here, The distribution is close−fitting to data set for The nuMber of hours The patients people with Kidney failure were in hospital before death for AL Hussein Educational Hospital in Karbala, for saMple size (n=90) (see table 1.), The perforMance of The distribution was coMpared with Shanker distribution, kappa distribution ,Exponential Pareto, weibel distribution, for The data set using Akaike InforMation Criterion (AIC), Akaike Bayesian Criterion Corrected (BIC). Data Criterion (AIC),DistributIon with The lowest AIC, AICC considered The Most flexible and superior distribution for a given data set. The results are presented in The tables (2).

TABLEI. Statistics set for The nuMber of hours patients were in hospital before death

2.52	2.82	3.12	3.42	3.82	4.22	5.12	8.12
2.52	2.82	3.12	3.52	3.82	4.32	5.22	8.22
2.52	2.82	3.22	3.52	3.92	4.42	5.32	9.32
2.52	2.92	3.32	3.52	4.02	4.52	5.52	9.52
2.62	2.92	3.32	3.62	4.02	4.52	6.02	9.02
2.62	3.02	3.32	3.62	4.02	4.62	6.22	9.22
2.62	3.02	3.42	3.62	4.02	4.72	6.32	9.32
2.62	3.02	3.42	3.62	4.02	4.82	7.02	9.02
2.62	3.02	3.42	3.62	4.02	4.82	7.22	9.42
2.72	3.12	3.42	3.62	4.22	4.92	702	9.52
2.81	3.12	3.42	3.62	4.22	4.92	7.02	9.62
						7.02	9.62

To select The best Model inside The set of Models that was coMpared with The new distribution, The top is The Model corresponding to The bottoMMost value for Akaike InforMation Criterion (AIC) and Akaike InforMation Correct (AIC_c) (see tabul 2.) , The general forMula for (AIC) ,(AIC_c) and (BIC))[16] are:

TABLE II: ML EstiMates and CriterionAIC_C, BIC,and Pearson ,and coMparison One generalized transMuted – kappa distribution, with kappa Distribution, Exponential, Weibull distribution Two paraMeters and Lindley Three paraMeters Distribution .

Distributions	MLE	−2Logl	AIC	AIC_C	BIC	Rank
generalized transMuted − kappa distribution	$\hat{\alpha} = 1.164$ $\hat{\sigma} = 2.349$ $\hat{\varepsilon} = 2.457$ $\hat{c} = 0.16020$ $\hat{b} = 2.462$ $\hat{\gamma} = 0.453$	213.19	225.19	225.418	224.915	1
kappa distribution	$\hat{S} = 3.322$ $\hat{\lambda} = 5.18312$ $\hat{\varepsilon} = 0.2378$	426.148	432.148	432.264	432.010	3
Exponential Pareto Distribution	$\hat{\alpha} = 7.049$ $\hat{\sigma} = 1.2419$ $\hat{\varepsilon} = 4.59211$	506.176	512.176	512.292	512.038	4
Shanker	$\hat{\lambda} = 0.3572$	518.72	520.72	520.765	520.674	5
Weibull distribution	$\hat{\alpha} = 1.8104$ $\hat{\varepsilon} = 1.4919$	441.459	445.459	445.539	445.367	2

7.Deduction:

In This broadside, a novel probability distribution is presented. The new Distribution is a generalized transMuted _Kappa distribution. Selected of The properties are derived and discussed like MoMents, reliability analysis, and hazard rate. The Method of MaxiMuM likelihood ,estiMation is used for deterMining The p,araMeters. The perforMance of The new Model is strong-Minded by fitting to real-lifespan data using The goodness of fit criteria such as AIC, AICC and BIC. The appropriateness of The real data for probability distributions under study It is found that generalized transMuted − kappa distribution Distribution gives a better fit to The data set as coMpared ,with kappa distribution, Expo,nential Pareto Distribution, Weibull distribution and Shanker Distribution FurTher, generalized transMuted − kappa distribution can be applied to various areas. generalized transMuted − kappa distribution and distribution May suitable for Most of The lifetiMe data and provides better outcoMes than oTher well-known distribution .

References :

1.Ani Shabri ; Abdul Aziz JeMain, (2010), LQ-MoMents: ParaMeter EstiMation for Kappa Distribution, Sains Malaysiana 39(5)(2010):P845-850.

2. Hussian MA. Fuzzy reliability estiMation based on exponential ranked set saMples. IJCMS.2017;1 (12):31-42.

3. Elbatal E M. TransMuted Quasi Lindly Distribution: A Generalization of The Quasi Lindly Distribution.IJSES.2013; 18(2):59-70.

4. MohMoudi E. Zakerzadeh H. Generalized PoissonLindley Distribution. CSTM, 2010;39(10):1785- 1798.

5. Zeghdoudi H, Nedjar S A. pseudo lindley distribution and it is Application, African Journal of MaTheMatics and CoMputer Science Research. AJMCSR.2016; 11(1):923-932.

6. Sankaran M. 275.note: The Discrete Poisson- Lindley Distribution, international BioMetrics society. IBS, 1970; 26(1);145-149.

7. Shanker R, Mishra A, A Quasi Lindley Distribution. AJMCSR. 2013;6(4):64-71.

8. Zakerzadeh H, Dolati A. Generalization of lindley distribution.JME. 2009; 3(2) ;: 1-17.

9. Nedjar S, Zeghdoudi H. GaMMa Lindley Distribution and its application. JAPS,2016;11(1) :129-138.

10. Dutta P, Borah M. A study on SoMe properties of Poisson Size –Biased Quasi Lindly Distribution.IOSR-JM, 2015; 11 (3):23-28.

11. Shanker R, Shukla K . A Three paraMeter lindley distribution. AJMS,2017;7(1):15-26.

12. S. B. Chhetri, A. A. Akinsete, G. Aryal and H. Long, KuMaraswaMy transMuted pareto distributi 19. S. B. Chhetri, A. A. Akinsete, G. Aryal and H. Long, KuMaraswaMy transMuted pareto distribution, J. Stat. Distrib. Appl. 4:11 (2017) 1-24.

13. T. G. Ieren, F. M. KroMtit, B. U. Agbor, I. B. EraikhueMen and P. P. Koleoso, A power GoMpertz distribution: Model, properties and application to bladder cancer data, Asian Res. J. Math. 15 (2019) 1-14.

14. U. Akata and J. E. OseMwenkhae, The weibull logistic-exponential distribution: its properties and applications, Earthline J. Math. Sci. 5 (2020) 197-216.

15. N. I .Rashwan and M. M. KaMel, The beta exponential Pareto distribution, Far East J. Theor. Stat. 58 (2020) 91-113.

16.Ishaq, A.I., UsMan, A., Tasi'u, M., Aliyu, Y. and Idris, F.A. (2017). A new Weibull-KuMaraswaMy distribution: Theory and applications. Nigerian Journal of Scientific Research, 16(2): 158–166.

17.Nofal, Z. M., Afify, A. Z., Yousof, H. M. and Cordeiro, G. M. (2017). The generalized transMuted-G faMily of distributions. CoMMunications in Statistics – Theory and Methods, 46: 4119–4136.