

Tunneling Current Calculation in Double Quantum Dots*

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Abstract

System of double quantum dots under influence of DC-magnetic field has been discussed and analyzed. The tunneling current is calculated as a function of Dc-magnetic field using the equation of motion for the state occupation probabilities, the results evidence that the resonance energy is dominant factor affected in the current.

Keywords: quantum dots, polarization, tunneling rate, occupation probability.

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1. Introduction

The name “dot or island” is often called artificial atom because of its quantum properties⁽¹⁾, and their electronic characteristics are closely related to the size and shape of the individual crystal⁽²⁾. Two quantum dots can be coupled to form an artificial molecule and the bond of the artificial molecule, depends on the strength of the inter dot coupling which can form ionic like (weak tunnel coupling) or covalent like bonds (strong tunnel coupling).

The quantum dots have very sharp density of state because of their nano-size, they have excellent transport and optical properties, so their transport process gives many interesting phenomena seen in the electronic device like coulomb blockade⁽³⁾, spin blockade⁽⁴⁾, and more applications like spintronic⁽⁵⁾, and quantum information⁽⁶⁾.

Spin degree of freedom of electrons in quantum dots has a great potential as a carrier of classical information⁽⁵⁾ and quantum information, spin quantum bits⁽⁶⁾.

The study of current in quantum dots has attracted much attention in the recent decades⁽⁷⁻¹²⁾.

Following the work of Emary⁽¹³⁾, we study the current between the double dots system connected to a non magnetic leads as shown in the figure(1).

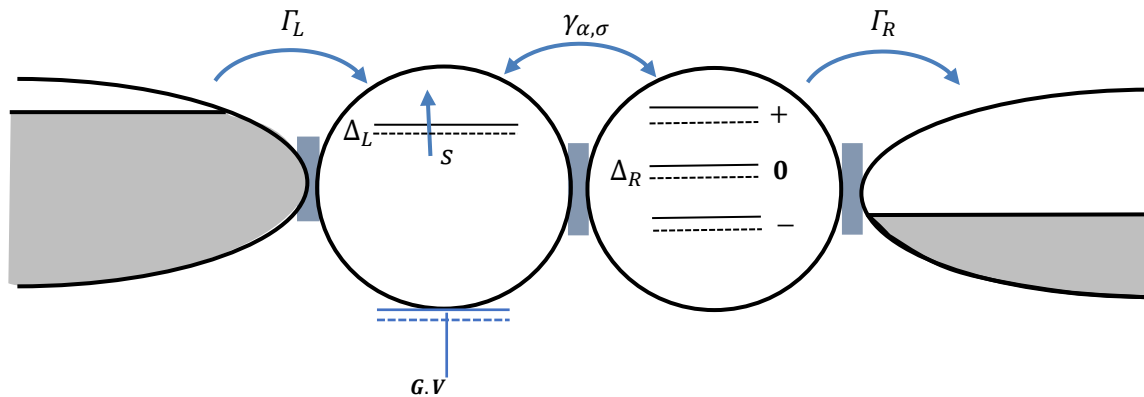


Figure 1: Schematic diagram of a double quantum dots connected to the two leads under the influence of DC magnetic field in the Z-direction.

In Fig(1), consider the left dot has an s-type orbital and the right dot has three orbitals, also consider a DC magnetic field applied on the dots in the Z-direction, and the dots are from different material, so they have different g-factors.

2. System Hamiltonian

The system can be described by the Hamiltonian:

$$\hat{H} = \hat{H}_L + \hat{H}_R + \hat{H}_{LR} + \hat{H}_T + \hat{H}_{leads} \quad (1)$$

Where $\hat{H}_i (i = L, R)$ described the left (right) uncoupled quantum dots.

$$\hat{H}_L = \sum_{\sigma} \epsilon_{s\sigma} |s\sigma\rangle\langle s\sigma|$$

With

$$\epsilon_{s\sigma} = \epsilon_s + \Delta_{L,\sigma}$$

Where ϵ_s is the energy of the level s in the left quantum dot.

$\Delta_i (i = L, R)$ is the Zeeman splitting in the states of the two dots and $\Delta_L \neq \Delta_R$ because of the different g-factor.

$$\Delta_{i,\sigma} = g_i \mu_B B \sigma$$

g_i is the land g-factor, μ_B is the Bohr magneton, $\mu_B = e\hbar/2m$

$$\mu_B = 9.274 \times 10^{-21} \text{ erg per Gauss per partical}$$

B is the applied DC magnetic field, and σ is the spin operator $\sigma = +1$ for spin up and $\sigma = -1$ for spin down. We use for the value of the g-factor $g_L = -0.33$ and $g_R = -0.89$ ⁽¹⁴⁾.

The second part of the Hamiltonian \hat{H}_R can be described as;

$$\begin{aligned} \hat{H}_R = & (E_B + \Delta_{R,\sigma})|1\sigma\rangle\langle 1\sigma| + \Delta_{R,\sigma}|2\sigma\rangle\langle 2\sigma| - (E_B - \Delta_{R,\sigma})|3\sigma\rangle\langle 3\sigma| \\ & + L_c(|1\sigma\rangle\langle 2\sigma| + |2\sigma\rangle\langle 1\sigma| + |2\sigma\rangle\langle 3\sigma| + |3\sigma\rangle\langle 2\sigma|) \end{aligned} \quad (2)$$

With $1\sigma, 2\sigma$, and 3σ are the levels of the right quantum dot, $E_B = c_B(B - B_0)$ is the magnetic energy corresponding to the applied magnetic field, c_B is a constant describes the magnetic field dependence of the Fock–Darwin (FD)⁽¹⁵⁾ orbitals and L_c is the coupling between the nearest levels of the right quantum dot.

The Hamiltonian \hat{H}_{LR} describe the interaction between the left and right quantum dots and is given by:

$$\hat{H}_{LR} = \sum_{\alpha,\sigma} (\gamma_{\alpha\sigma} |s\sigma\rangle \langle \Psi_{\alpha\sigma}| + H.c.)$$

Where $\gamma_{\alpha\sigma}$ is the hopping rate of an electron between states $|s\sigma\rangle$ and $|\Psi_{\alpha\sigma}\rangle$.

And \hat{H}_T in eq(1) represents the interaction between the double quantum dots (DQD) and the leads.

$$\hat{H}_T = \sum_{l \in \{L,R\} k\sigma} (\gamma_l |lk\sigma\rangle \langle l\sigma| + H.c.)$$

The Hamiltonian of the leads is described by:

$$\hat{H}_{leads} = \sum_{lk\sigma} \epsilon_{lk} |lk\sigma\rangle \langle lk\sigma|$$

ϵ_{lk} described two standard fermionic reservoirs, and $l = L/R$ lead.

3. Energies and wave functions of right quantum dot

In order to calculate the energy and the wave functions for the second dot; we use the Hamiltonian \hat{H}_R in the Schrödinger equation

$$\hat{H}\Psi = \epsilon\Psi \quad (3)$$

We get

$$\epsilon_{0\sigma} = \Delta_{R,\sigma} \quad (4)$$

$$\epsilon_{\pm\sigma} = \pm \sqrt{E_B^2 + 2L_c^2} + \Delta_{R,\sigma} \quad (5)$$

And the wave functions are corresponding to these eigen values are,

$$\begin{aligned}
 |\Psi_0\rangle &= \frac{1}{\sqrt{E_B^2 + 2L_c^2}} (-L_c|1\rangle + E_B|2\rangle + L_c|3\rangle) \\
 |\Psi_{\pm}\rangle &= \frac{L_c^2}{E_B^2 + 2L_c^2 \pm E_B\sqrt{E_B^2 + 2L_c^2}} |1\rangle \pm \frac{L_c}{\sqrt{E_B^2 + 2L_c^2}} |2\rangle \\
 &\quad + \frac{L_c^2}{E_B^2 + 2L_c^2 \mp E_B\sqrt{E_B^2 + 2L_c^2}} |3\rangle
 \end{aligned} \tag{6}$$

The overlap between the state $|s\sigma\rangle$ of the left QD and the states $|\Psi_{\alpha\sigma}\rangle$ of the right QD can be calculated as:

$$\langle s|\Psi_0\rangle = \frac{-L_c}{\sqrt{E_B^2 + 2L_c^2}} \langle s|1\rangle + \frac{E_B}{\sqrt{E_B^2 + 2L_c^2}} \langle s|2\rangle + \frac{L_c}{\sqrt{E_B^2 + 2L_c^2}} \langle s|3\rangle$$

Where $\langle s|i\rangle = s_i$, $i = 1,2,3$, $|i\rangle$ are the Fock-Darwin wave functions.

This eigenstate-overlap clearly vanishes for $E_B = T(s_1 - s_3)/s_2$. One can assumed here that $\langle s|i\rangle$ are the same for the all wave functions of the Fock-Darwin levels and for this reason we can set the square of the matrix elements as follow,

$$\begin{aligned}
 \langle s|\Psi_0\rangle &= \frac{E_B}{\sqrt{E_B^2 + 2L_c^2}} s_2 \\
 \langle s|\Psi_{\pm}\rangle &= s_1 \pm \frac{L_c}{\sqrt{E_B^2 + 2L_c^2}} s_2
 \end{aligned}$$

And the squares of the relevant matrix elements are:

$$|\langle s|\Psi_0\rangle|^2 = \frac{E_B^2}{E_B^2 + 2L_c^2} \tag{7}$$

$$|\langle s|\Psi_{\pm}\rangle|^2 = 1 + \frac{L_c^2}{E_B^2 + 2L_c^2} \pm \frac{2L_c}{\sqrt{E_B^2 + 2L_c^2}} \tag{8}$$

4. Tunneling rate

The hopping rate of an electron between states $|s\sigma\rangle$ and $|\Psi_{\alpha\sigma}\rangle$ can be calculated as

$$\gamma_{\alpha\sigma} = \gamma |\langle s\sigma | \Psi_{\alpha\sigma} \rangle|^2 \varrho(\epsilon_{s\sigma}, \epsilon_{\alpha\sigma}; \Gamma_{\sigma}) \quad (9)$$

$\varrho(\epsilon_{s\sigma}, \epsilon_{\alpha\sigma}; \Gamma_{\sigma})$ describes the broadening of the hybridization function of the dots levels at the density of states of leads,

$$\varrho(\epsilon_{s\sigma}, \epsilon_{\alpha\sigma}; \Gamma_{\sigma}) = \frac{\Gamma^2}{(\epsilon_{s\sigma} - \epsilon_{\alpha\sigma})^2 + \Gamma^2}, \text{ and}$$

$$\Gamma_{\sigma} \equiv \Gamma_{L,\sigma} + \Gamma_{R,\sigma}$$

$\Gamma_{L/R,\sigma} = \Gamma$ is the tunnelling rate of an electron with spin σ between the dot and the left/right leads.

Figure 2 shows the hopping rates, $\gamma_{\alpha\sigma}$ as a function of the applied magnetic field B , we represents here the case of a large ϵ_s , which can be achieved by adjusting the gate voltage on the left QD. In this case, the large ϵ_s leads to s-orbit resonating close with the “+” channel, this transition shows that the large peaks for γ_+ in comparison with the other peaks for γ_0 and γ_- comes from the resonance condition.

In this system, large ϵ_s is consider to leads to the s-orbit resonating close with + channel in the right quantum dot, large ϵ_s obtained by adjusting the gate voltage in the left quantum dot. There are two parameters affected the transport in this system, one is the spin-dependent energy level $\epsilon_{+\sigma}$ and the

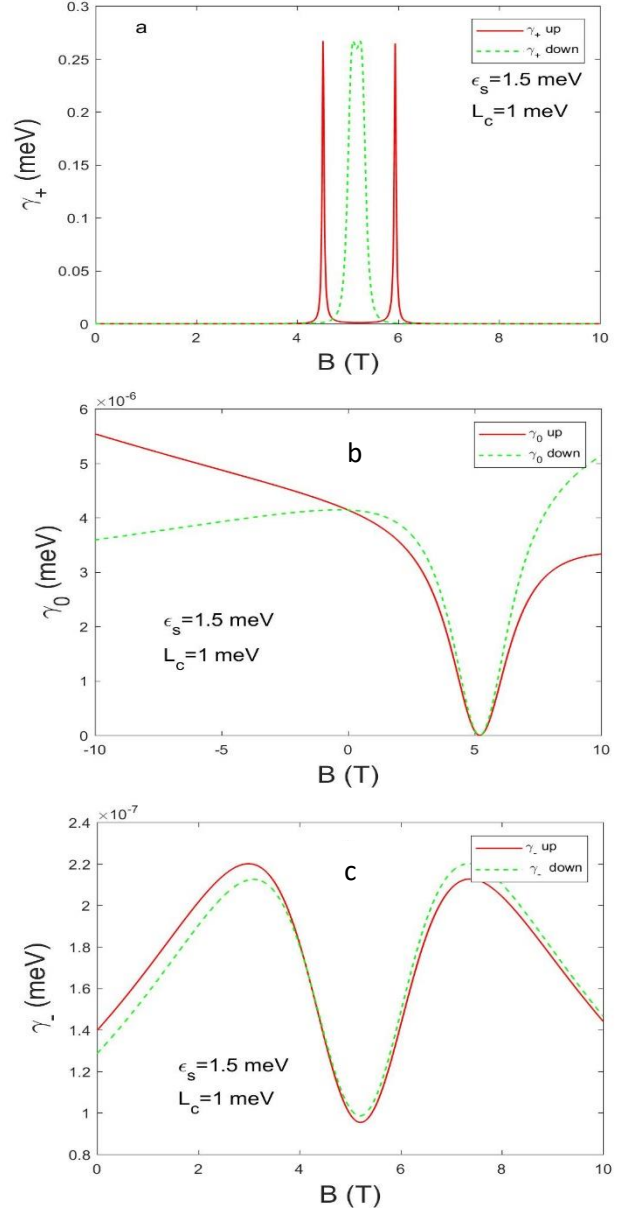


Figure 2: The spin-dependent hopping rates $\gamma_{\alpha\sigma}$ versus magnetic field B_{dc} . (a) $\gamma_{+\sigma}$, (b) $\gamma_{0\sigma}$ and (c) $\gamma_{-\sigma}$. Parameters used are $L_c = 1 \text{ meV}$, $\Gamma = 10 \mu\text{eV}$, $\gamma = 100 \mu\text{eV}$, $\epsilon_s = 1.5 \text{ meV}$, and $B_0 = 5.2 \text{ T}$.

other is the Dc-magnetic field that applied on the system.

From Fig(2), one can see that tunneling rate depend on the spin σ also the peaks which appears in this figure is due to resonance tunneling when the s-orbit in the left quantum dot is aligned to the $\epsilon_{+\sigma}$ level in the right quantum dot.

In order to see the effects of the different parameters on the hopping rate, we take different values to the ϵ_s as shown in Fig. (3a) and (3b).

Fig.(3a) shows more brooding in the spin down rate when ϵ_s is less than 1.5 meV and when ϵ_s greater than 1.5 meV the spin down rate shows two peaks. The effect of the parameters L_c , which is responsible the coupling strength between the levels due to anisotropy of the right quantum dot, is shown in figure (4a) and (4b).

The change effect of the leads in the system can be achieved by changing the parameters Γ which is defined the hybridization of the level of the right quantum dot with the density of state of the right lead, that is to say the rate between right quantum dot and the right lead as shown in the figure (5a), and (5b).

5. The basis states

The basis states in double quantum dots containing two electrons are distributed under the consideration of one electron is confined in the left dot and the other electron participating if transport process, so these bases are:

For one electron

$$|1\rangle = |\uparrow, 0\rangle, |2\rangle = |\downarrow, 0\rangle$$

And for two electrons

$$|3\rangle = |\uparrow\downarrow, 0\rangle, |4\rangle = |\uparrow, \downarrow_+\rangle, |5\rangle = |\uparrow, \downarrow_0\rangle, |6\rangle = |\uparrow, \downarrow_-\rangle, |7\rangle = |\uparrow, \uparrow_+\rangle, |8\rangle = |\uparrow, \uparrow_0\rangle, |9\rangle = |\uparrow, \uparrow_-\rangle$$

$$|10\rangle = |\downarrow, \uparrow_+\rangle, |11\rangle = |\downarrow, \uparrow_0\rangle, |12\rangle = |\downarrow, \uparrow_-\rangle, |13\rangle = |\downarrow, \downarrow_+\rangle, |14\rangle = |\downarrow, \downarrow_0\rangle, |15\rangle = |\downarrow, \downarrow_-\rangle$$

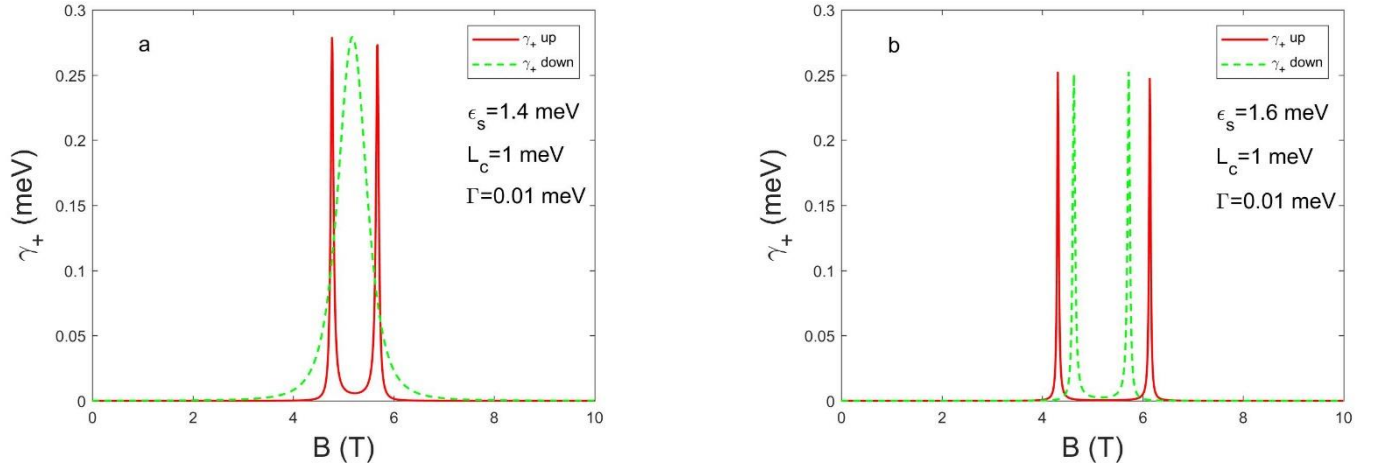


Figure 3: The spin-dependent hopping rates $\gamma_{\alpha\sigma}$ versus magnetic field B_{dc} for deferent values of ϵ_s .

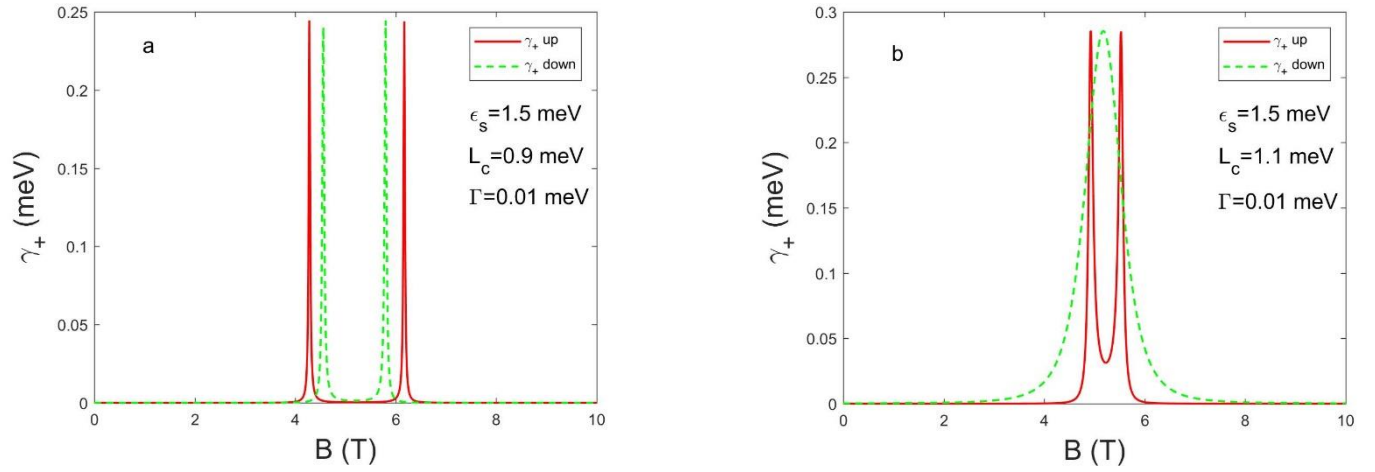


Figure 4: The spin-dependent hopping rates $\gamma_{\alpha\sigma}$ versus magnetic field B_{dc} for deferent values of L_c .

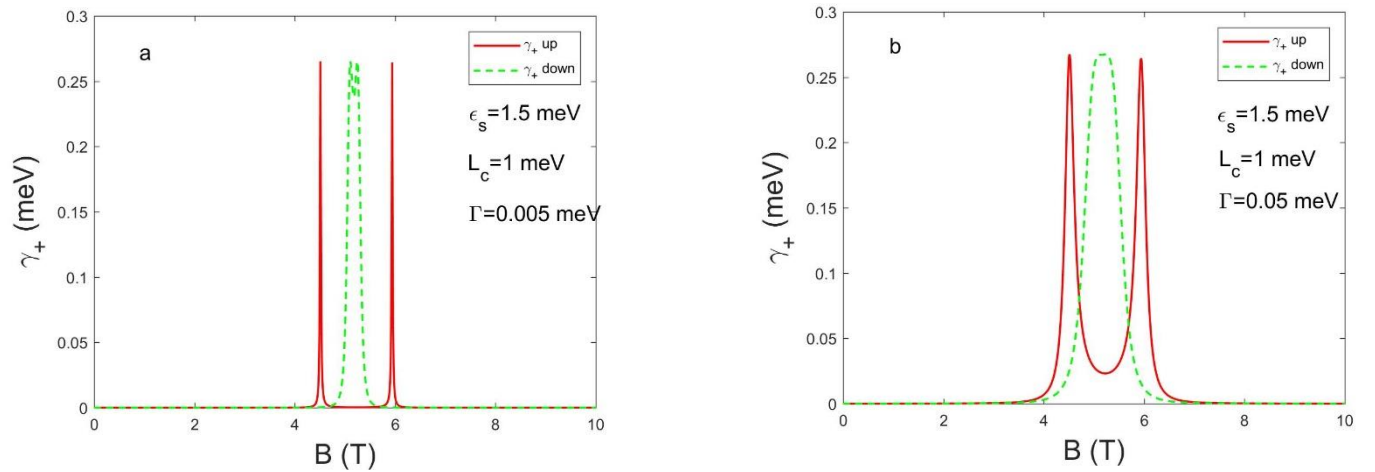


Figure 5: The spin-dependent hopping rates $\gamma_{\alpha\sigma}$ versus magnetic field B_{dc} for deferent values of Γ .

Equation of motion⁽¹⁶⁾ [16] can be used to described the dynamics of the double dots system

$$\dot{\rho}(t)_s = \sum_{m \neq s} W_{sm} \rho_m - \sum_{k \neq s} W_{ks} \rho_s$$

ρ is the state occupation probability

W_{ij} is the transition rate from state j to state i and it's defined by different methods according to processes in the system such as

$$|\downarrow, 0\rangle \rightarrow |\uparrow\downarrow, 0\rangle \Rightarrow W_{3,2} = \Gamma_{L,\uparrow},$$

$$|\downarrow, \downarrow_0\rangle \rightarrow |\uparrow, \downarrow_0\rangle \Rightarrow W_{5,14} = \left[\frac{1}{T_{sf}} \right]_L,$$

$$|\uparrow\downarrow, 0\rangle \rightarrow |\uparrow, \downarrow_-\rangle \Rightarrow W_{6,3} = \gamma - \downarrow$$

Where T_{sf} is the time for electron to reverse (flip) it's spin. The other transition rates can be calculated in the same way. With the inclusion of the spin-flip interactions, the rate equations for the occupation probabilities of $|2\rangle, |3\rangle, |4\rangle$ and $|5\rangle$ are:

$$\begin{aligned} \dot{\rho}_2 = & W_{2,1}\rho_1 + W_{2,10}\rho_{10} + W_{2,11}\rho_{11} + W_{1,12}\rho_{12} + W_{1,13}\rho_{13} + W_{1,14}\rho_{14} + W_{1,15}\rho_{15} - W_{1,2}\rho_2 \\ & - W_{3,2}\rho_2 \end{aligned}$$

$$\begin{aligned} \dot{\rho}_3 = & W_{3,1}\rho_1 + W_{3,2}\rho_2 + W_{3,4}\rho_4 + W_{3,5}\rho_5 + W_{3,6}\rho_6 + W_{3,10}\rho_{10} + W_{3,11}\rho_{11} + W_{3,12}\rho_{12} - W_{4,3}\rho_3 \\ & - W_{5,3}\rho_3 - W_{6,3}\rho_1 - W_{10,3}\rho_3 - W_{11,3}\rho_3 - W_{12,3}\rho_3 \end{aligned}$$

$$\dot{\rho}_4 = W_{4,3}\rho_3 + W_{4,7}\rho_7 + W_{4,13}\rho_{13} - W_{1,4}\rho_4 - W_{3,4}\rho_4 - W_{7,4}\rho_4 - W_{13,4}\rho_4$$

$$\dot{\rho}_5 = W_{5,3}\rho_3 + W_{5,8}\rho_8 + W_{5,14}\rho_{14} - W_{1,5}\rho_5 - W_{3,5}\rho_5 - W_{8,5}\rho_5 - W_{14,5}\rho_5$$

For the steady state one can calculate every ρ_m , ($m = 1, \dots, 15$) in the system.

6. Current calculation

We are mainly interested in the calculation of the current from the right dot to the right lead in the system, for different electron spin

$$I_{\uparrow} = e\Gamma_{R\uparrow}(\rho_7 + \rho_8 + \rho_9 + \rho_{10} + \rho_{11} + \rho_{12})$$

$$I_{\downarrow} = e\Gamma_{R\downarrow}(\rho_4 + \rho_5 + \rho_6 + \rho_{13} + \rho_{14} + \rho_{15})$$

The calculation under small spin-flip rates leads to an approximate formula for the current

$$I_{\uparrow} = \Gamma \left(\frac{\gamma_{+\uparrow}\rho_3}{\Gamma + \gamma_{+\uparrow}} \right) \quad (10)$$

$$I_{\downarrow} = \Gamma \left(\frac{\gamma_{+\downarrow}\rho_3}{\Gamma + \gamma_{+\downarrow}} \right) \quad (11)$$

And for the average total current is given by:

$$\langle I \rangle = \frac{\Gamma^2(\gamma_{+\uparrow} + \gamma_{+\downarrow}) + 2\Gamma\gamma_{+\uparrow}\gamma_{+\downarrow}}{\Gamma^2 + \Gamma(\gamma_{+\uparrow} + \gamma_{+\downarrow}) + \gamma_{+\uparrow}\gamma_{+\downarrow}} \quad (12)$$

The relation between the current and the static magnetic field for spin up is shown in figure (6a), the current shows two peaks because its depends on the a magnetic field with a complicated relation and the maximum of these peaks are related to the resonance energy of the level $\epsilon_{s\uparrow}$ and the level $\epsilon_{+\uparrow}$ in the right quantum dot.

Figure (6b) show the spin down current which has a peak around B_0 , the average total current is listed in figure (6c) for the same parameters values of ϵ_s, L_c and γ . From figure (6) no spin blockade are observed.

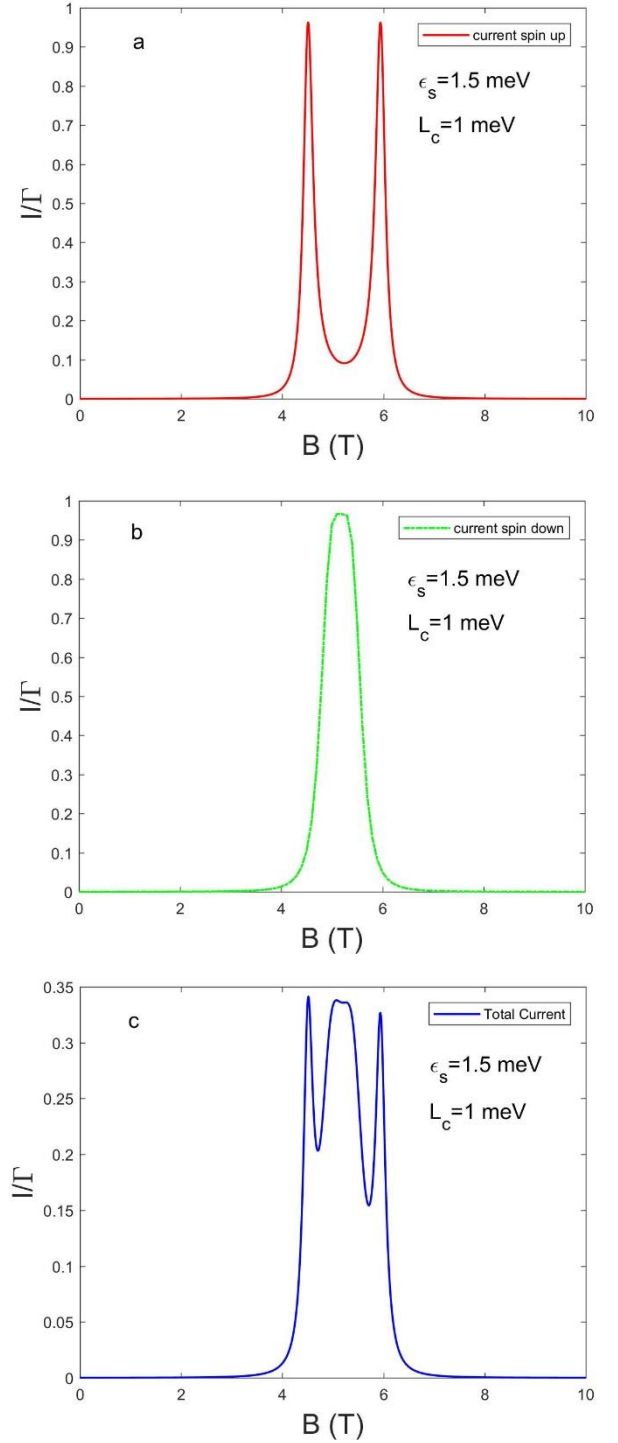


Figure 6: Current I versus magnetic field B .
(a) $I_{+\uparrow}$, (b) $I_{+\downarrow}$ and (c) $I_{+\uparrow} + I_{+\downarrow}$

7. References

- (1) Yadav, M. and A. Chaudhary (2014). Quantum Dots: An Introduction. National Conference "IAEISDISE.
- (2) Akopian, N., Patriarche, G., Liu, L., Harmand, J.-C., & Zwiller, V. (2010). Crystal phase quantum dots. *Nano letters*, 10(4), 1198-1201.
- (3) Romero, H. E. and M. Drndic (2005). "Coulomb blockade and hopping conduction in PbSe quantum dots." *Physical review letters* 95(15): 156801.
- (4) Shaji, N., Simmons, C., Thalakulam, M., Klein, L. J., Qin, H., Luo, H., . . . Joynt, R. (2008). Spin blockade and lifetime-enhanced transport in a few-electron Si/SiGe double quantum dot. *Nature Physics*, 4(7), 540-544.
- (5) Wolf, S., Awschalom, D., Buhrman, R., Daughton, J., Von Molnar, S., Roukes, M., . . . Treger, D. (2001). Spintronics: a spin-based electronics vision for the future. *Science*, 294(5546), 1488-1495.
- (6) Loss, D. and D. P. DiVincenzo (1998). "Quantum computation with quantum dots." *Physical Review A* 57(1): 120..
- (7) Cho, S. Y., McKenzie, R. H., Kang, K., & Kim, C. K. (2003). Magnetic polarization currents in double quantum dot devices. *Journal of Physics: Condensed Matter*, 15(7), 1147.
- (8) Świrkowicz, R., Wilczyński, M., & Barnaś, J. (2006). Spin-polarized transport through a single-level quantum dot in the Kondo regime. *Journal of Physics: Condensed Matter*, 18(7), 2291.
- (9) Trocha, P. and J. Barnaś (2007). "Quantum interference and Coulomb correlation effects in spin-polarized transport through two coupled quantum dots." *Physical Review B* 76(16): 165432.
- (10) Busl, M. and G. Platero (2010). "Spin-polarized currents in double and triple quantum dots driven by ac magnetic fields." *Physical Review B* 82(20): 205304.
- (11) AL-Mukh, J. M. S. I. Easa and L. F. Al-Badry (2014) "Theoretical Treatment for Electron Transport throughout Quantum Dots Bridge" *Basra journal of science*, Vol. 32, 1A 71-85.

- (12) Bacon, M., Bradley, S. J., & Nann, T. (2014). Graphene quantum dots. *Particle & Particle Systems Characterization*, 31(4), 415-428.
- (13) Emary, C., Pörtl, C., & Brandes, T. (2009). Three-level mixing and dark states in transport through quantum dots. *Physical Review B*, 80(23), 235321.
- (14) Huang, S., Tokura, Y., Akimoto, H., Kono, K., Lin, J., Tarucha, S., & Ono, K. (2010). Spin bottleneck in resonant tunneling through double quantum dots with different Zeeman splittings. *Physical review letters*, 104(13), 136801.
- (15) Kouwenhoven, L. P., Austing, D., & Tarucha, S. (2001). Few-electron quantum dots. *Reports on Progress in Physics*, 64(6), 701.
- (16) Li, A.-X., Duan, S.-Q., & Zhang, W. (2012). Spin and orbital blockade in quantum transport through double quantum dots. *EPL (Europhysics Letters)*, 98(4), 47009.

حساب تيار النفق في نقطتين كميتين

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البصرة – العراق

تم مناقشة وتحليل نظام مكون من نقطتين كميتين تحت تأثير فيض مجال مغناطيسي ثابت. حيث استخدمت معادلة الحركة لاشغال الحالات الالكترونية لحساب تيار النفق كدالة للمجال المغناطيسي الثابت. أظهرت النتائج ان طاقة الرنين هي العامل السائد المؤثر في التيار.

الكلمات المفتاحية: النقاط الكمية، الاستقطاب، معدل الانتقال، احتمالية الاشغال.