Robust Test Statistics for O ne-Way Munlivariate Analysis of Variance (MANOVA) Model: Simulation Study

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Abstract

One-way multivariate analysis of variance (MANOVA) deals with testing the null hypothesis of equal mean vectors of two or more multivariate normal populations. The classical Wilks', Roy's, Pillai's, and Lawley-Hotelling statistics are used and well-known for testing the hypotheses in one-way MANOVA which are extremely sensitive to the influence of outliers. In this study the robust test statistics based on reweighted minimum covariance determinant (RMCD) estimator with Hampel weighted function have been proposed. Also, the approximate distributions for the robust test statistics have been constructed which are related directly with weights. The distributions of the proposed statistics differ from the classical one. Monte Carlo simulations are used to evaluate the performance of the test statistics under various distributions in terms of the simulated significance levels, its power functions and robustness. The powers of the robust and classical statistics are compared using size-power curves. The results show that, the robust test statistics are close to the classical test statistics in case of normal distribution for the data set. In the case of contaminated distribution, the P-value plots and size-power curves clearly show the advantage of the proposed robust test statistics over the classical statistics.

Keywords: MANOVA, MANOVA Test Statistics, Outliers, Robustness, Minimum Covariance Determinant Estimator.

1 Introduction

The classical test statistics are commonly used for testing the hypotheses in the one-way MANOVA for testing the hypothesis of equal mean vectors of several multivariate normal groups. Under the classical assumptions that all groups arise from multivariate normal distributions, many test statistics are extremely sensitive to the influence of outliers (Beakman and Cook [1]).

Then the robust statistics have been proposed to to reduce the influence of outliers. In the one-group, the Hotelling's statistic is the standard tool for inference about the center of a multivariate normal distribution. Willems et al. [2] proposed the robust Hotelling's statistic based on the minimum covariance determinant (MCD) estimator. Candan and Aktas [3] implemented the robust Hotelling's statistic based on the minimum volume ellipsoid (MVE) estimator to test the hypothesis about the location parameter of one group.

Todorov and Filzmoser [4] introduced the robust Wilks' statistic for the one-way MANOVA based on the RMCD estimator. Van Aelst and Willems [5] proposed a robust Wilks' statistic for testing the hypotheses in the one-way MANOVA based on S and the MM-estimators. Abdul- Sayed [6] proposed the robust Hotelling's statistic based on RMCD to test the hypothesis about location parameter of one and two groups. Abbas [7] proposed the robust Wilks' statistic for the one-way MANOVA based on RMVE and RMCD estimators with Hampel weighted function. Spangl [8] proposed the robust test statistics for the two-way MANOVA based on RMCD estimator. The effect of outliers on the classical test statistics will be illustrated in the simulation study in Section 5.

Therefore, we proposed other robust test statistics which differs from the classical test statistics. For this purpose, the MCD estimator that proposed by Rousseeuw in [9] which is a highly robust estimator of location and scatter is used. And summarized in Section 3. In order to increase the efficiency while retaining high robustness, one can apply RMCD estimator. In Section 4, we construct approximate distributions based on RMCD that differ from the classical distributions. Section 5 describes the Monte Carlo simulation design of the simulation study and its results.

To formalize the hypothesis in the one-way MANOVA, let us assume that $\boldsymbol{y}_{i1}, \boldsymbol{y}_{i2}, \dots, \boldsymbol{y}_{in_i}$ be sample observations of size n_i are obtained from $p-$variate normal populations where $i = 1, 2, \dots, k$ and the number of groups $k \geq 2$, with mean vector $\boldsymbol{\mu}_i$ and equal covariance matrix $\sum$. The model for each observation vector is

$$\mathbf{y}_{ij} = \boldsymbol{\mu}_i + \boldsymbol{\varepsilon}_{ij}, \quad i = 1, 2, \dots, k \ , \qquad j = 1, 2, \dots, n_i,$$

where $\boldsymbol{\varepsilon}_{ij}$ is an error random vector has $\mathrm{N}_p(\mathbf{0}, \sum)$.

Then the hypothesis, that is the null and alternative hypotheses can be expressed as:

$H_0 : \boldsymbol{\mu}_1 = \boldsymbol{\mu}_2 = \dots = \boldsymbol{\mu}_k,$

$H_1 : \boldsymbol{\mu}_i \neq \boldsymbol{\mu}_j$ for at least one $i \neq j$.

2 Test Statistics

In the multivariate case, there are "between" and "within" matrices H and E respectively, defined as [10]:

$$\boldsymbol{H} = \sum_{i=1}^{k} n_i \, (\bar{\mathbf{y}}_{i\cdot} - \bar{\mathbf{y}}_{\cdot\cdot}) \, (\bar{\mathbf{y}}_{i\cdot} - \bar{\mathbf{y}}_{\cdot\cdot})', \tag{1}$$

$$\boldsymbol{E} = \sum_{i=1}^{k} \sum_{j=1}^{n_i} (\mathbf{y}_{ij} - \bar{\mathbf{y}}_{i\cdot}) \, (\mathbf{y}_{ij} - \bar{\mathbf{y}}_{i\cdot})', \tag{2}$$

where

$$\bar{\mathbf{y}}_{i\cdot} = \frac{1}{n_i} \sum_{j=1}^{n_i} \mathbf{y}_{ij}, \ \ \bar{\mathbf{y}}_{\cdot\cdot} = \frac{1}{n} \sum_{i=1}^{k} \sum_{j=1}^{n_i} \mathbf{y}_{ij}, \ n = \sum_{i=1}^{k} n_i \, .$$

For testing the null hypothesis H_0, there are many statistics to be used:

2.1 Wilks' statistic Λ [10, 12] :

$$\Lambda = \frac{|E|}{|E + H|} \, . \tag{3}$$

Wilks' Λ in (3) can be expressed in terms of the eigenvalues $\lambda_1, \lambda_2, \dots, \lambda_s$ of $\mathrm{E}^{-1}\mathrm{H}$, as follows:

$$\Lambda = \prod_{i=1}^{s} \frac{1}{1 + \lambda_i}, \tag{4}$$

where $s = min\{p, v_H\}$. The null hypothesis H_0 is rejected if $\Lambda \leq \Lambda_{\alpha,p,v_{H},v_E}$ where $\Lambda_{\alpha,p,v_H,v_E}$ is the tabulated value with level of significance α and the degrees of freedom are $p, v_E = n - k$, and $v_H = k - 1$.

An approximate F distribution for Wilks' statistic is given by the following [10, 13]:

$$\Lambda_1 = \frac{\left(1 - \Lambda^{1/t}\right)}{\Lambda^{1/t}} \cdot \frac{v_2}{v_1} \approx F_{v_1,v_2}, \tag{5}$$

where

$$v_1 = pv_H, \quad v_2 = ct - \frac{1}{2}(pv_H - 2),$$

$$c = v_E + v_H - \tfrac{1}{2}(p+v_H + 1), and \quad t = \sqrt{\frac{p^2\, v_H{}^2 - 4}{p^2 + v_H{}^2 - 5}},$$

when $p\, v_H = 2$, t is set equal to 1 [10].

2.2 Pillai's statistic V [10, 12] :

$$\mathrm{V} = tr[(E + H)^{-1}H] = \sum_{i=1}^{s} \frac{\lambda_i}{1 + \lambda_i} \cdot \tag{6}$$

The null hypothesis is rejected if $\mathrm{V} \geq \mathrm{V}_{\alpha,s,g,l}$ where $\mathrm{V}_{\alpha,s,g,l}$ is the tabulated value with level of significance α and the degrees of freedom are

$$s, \ g = \tfrac{1}{2}(|v_H - p| - 1) \ and \ l = \tfrac{1}{2}(v_E - p - 1) \cdot$$

An approximate F distribution for Pillai's statistic is given by [10]:

$$\mathrm{V}_1 = \frac{V}{(s - V)} \cdot \frac{(2l + s + 1)}{(2g + s + 1)} \approx F_{s(2g+s+1),s(2l+s+1)}. \tag{7}$$

2.3 Lawley-Hotelling statistic U [10, 12] :

$$U = tr(E^{-1}H) = \sum_{i=1}^{s} \lambda_i \cdot \tag{8}$$

The null hypothesis is rejected if $U \geq U_{\alpha,s,g,l}$ where $U_{\alpha,s,g,l}$ is the tabulated value with level of significance α and the degrees of freedom are s , $g, and\ l$.

An approximate F distribution for Lawley-Hotelling statistic is given by [10]:

$$\mathrm{U}_1 = \frac{2(s\, l + 1)\, U}{s^2(2g + s + 1)} \approx F_{s(2g+s+1),2(sl+1)}. \tag{9}$$

2.4 Roy's largest root statistic θ [14, 10, 12] :

$$\theta = \frac{\lambda_1}{1 + \lambda_1}, \tag{10}$$

where λ_1 is the largest value of eigenvalues of the matrix $E^{-1}H$, the null hypothesis is rejected if $\theta \geq \theta_{\alpha,s,g,l}$ where, $\theta_{\alpha,s,g,l}$ is the tabulated value with level of significance, α and the degrees of freedom are $s, g\ and\ l$.

An approximate F distribution for Roy's statistic is given by [14]:

$$\theta_1 = \frac{(v_E - d + v_H)}{d}\, \lambda_1 \approx F_{d,v_E-d+v_H}, \tag{11}$$

where $d = max\{p, v_H\}$.

3 Robust Estimators

Initially, the theory of estimation was started by using classical ways with maximum likelihood estimator (MLE) and least squares estimator (LSE) which were considered the most important methods. These classical methods are efficient and achieve the required when the assumptions in the statistical model are not likely to breakthrough, but they are biased and inefficient when small deviations are present in this model. Therefore, the classical estimators must be replaced by the robust estimator of parameters to get reliable analysis of data. Many authors have dealt with the problem of the non-robustness of the classical estimates, and they have provided several robust estimators in univariate and multivariate statistics. MCD estimator for multivariate location and scatter parameters which was introduced by Rousseeuw is a highly robust and efficient estimator. So, it is preferable in this study for this reason. For instance, MCD estimator has been computed by the approximate subsamples algorithm which takes a large number of random subsets from the data set. The MCD location estimator T is the mean vector of the subset which has the smallest determinant and the MCD scatter estimate C is a multiple of its covariance matrix. The multiplication factor is selected so that C is consistent at the multivariate normal model and unbiased at small samples - see Willems et al. [2]. For computing the MCD and the related estimators, the FAST-MCD algorithm (FMCD) of Rousseeuw and Van Driessen [15] will be used as implemente d in the Matlab package.

Robust test statistics have been constructed depending on RMCD estimators instead of the classical estimators, in order to obtain a robust procedure for inference about the mean vectors in the one- way MANOVA model.

It starts with obtaining the individual group location estimates $\boldsymbol{m}_i,\ i = 1,2,\dots,k$ as the reweighted MCD location estimates of each group. These group means are swept from the original observations to obtain the centered observations

$$\boldsymbol{Z} = \{\boldsymbol{z}_{ij}\}, \quad \boldsymbol{z}_{ij} = \boldsymbol{y}_{ij} - \boldsymbol{m}_i \cdot \tag{12}$$

The common covariance matrix $\hat{\sum}_Z$ and the location estimate $\hat{\boldsymbol{\mu}}_Z$ are estimated from the centered observations $\boldsymbol{Z}$ based on MCD estimator. The location estimate $\hat{\boldsymbol{\mu}}_Z$ is used to adjust the group means $\hat{\boldsymbol{\mu}}_i$ and thus the final group means are

$$\hat{\boldsymbol{\mu}}_i = \boldsymbol{m}_i + \hat{\boldsymbol{\mu}}_Z, \quad i = 1,2,\dots,k. \tag{13}$$

In order to increase the efficiency while retaining high robustness, reweighted estimators for the RMCD can be applied by using the following steps:

1. By using the obtained robust estimates $\hat{\boldsymbol{\mu}}_i$ and $\hat{\sum}_Z$ for MCD estimators, the initial robust distances can be computed as:

$$MD(\boldsymbol{y}_{ij}) = \sqrt{(\boldsymbol{y}_{ij} - \hat{\boldsymbol{\mu}}_i)^t \hat{\sum}_Z^{-1} (\boldsymbol{y}_{ij} - \hat{\boldsymbol{\mu}}_i)}, i = 1, 2, \dots, k, j = 1, 2, \dots, n_i \cdot \tag{14}$$

2. With these initial robust distances, a weight for each observation $\mathbf{y}_{ij}$, $i = 1,2,\dots,k\ \ and\ \ j = 1,2,\dots,n_i$ based on Hampel weighted function (1974) [16] can be defined as:

$$w_{ij} = \begin{cases} 1 & MD(\boldsymbol{y}_{ij}) \le d_0, \\ d/MD(\boldsymbol{y}_{ij}) & otherwise, \end{cases} \tag{15}$$

where

$$d = d_0\, exp\left[-0.5\, ((MD(\mathbf{y}_{ij}) - d_0)/b_2)^2\right],$$

$$d_0 = \sqrt{p} + \frac{b_1}{\sqrt{2}}, \quad b_1 = 2\ \ and\ \ b_2 = 1.25 \cdot$$

3. The weighted group means $\hat{\boldsymbol{\mu}}_{w_i}, i = 1, 2, \ldots, k$ and weighted covariance matrix $\hat{\Sigma}_w$ are to be computed as the following [7]:

$$\hat{\boldsymbol{\mu}}_{w_i} = \frac{1}{w_i}\sum_{j=1}^{n_i} w_{ij}\boldsymbol{y}_{ij}, \text{ where } w_i = \sum_{j=1}^{n_i} w_{ij}, \; i = 1, 2, \ldots, k \; \; and$$

$$\hat{\Sigma}_w = \frac{1}{\sum_{i=1}^{k} r_i - 1} \sum_{i=1}^{k}\sum_{j=1}^{n_i} w_{ij}^2 \left(\mathbf{y}_{ij} - \hat{\boldsymbol{\mu}}_{w_i}\right)\left(\mathbf{y}_{ij} - \hat{\boldsymbol{\mu}}_{w_i}\right)^t, \text{where } r_i = \sum_{j=1}^{n_i} w_{ij}^2 \cdot$$

4. Above steps are to be repeated until $\hat{\boldsymbol{\mu}}_{w_i} = \hat{\boldsymbol{\mu}}_i$ and $\hat{\Sigma}_w = \hat{\Sigma}_Z$.

By using the final weights wij $(i = 1,2, \ldots, k, j = 1,2, \ldots, n_i)$ in (15), the final reweighted estimates can be computed, namely the group means $\hat{\boldsymbol{\mu}}_{w_i}$ for $i = 1,2, \ldots, k$, the total sum of means $\hat{\boldsymbol{\mu}}_w$, the within matrix E_w and the between matrix H_w which are necessary for constructing the robust test statistics [4].

$$H_w = \sum_{i=1}^{k} w_i \left(\hat{\boldsymbol{\mu}}_{w_i} - \hat{\boldsymbol{\mu}}_w\right)\left(\hat{\boldsymbol{\mu}}_{w_i} - \hat{\boldsymbol{\mu}}_w\right)^t, \quad (16)$$

$$E_w = \sum_{i=1}^{k}\sum_{j=1}^{n_i} w_{ij} \left(\boldsymbol{y}_{ij} - \hat{\boldsymbol{\mu}}_{w_i}\right)\left(\boldsymbol{y}_{ij} - \hat{\boldsymbol{\mu}}_{w_i}\right)^t, \quad (17)$$

where $w_i = \sum_{j=1}^{n_i} w_{ij}$, $\hat{\boldsymbol{\mu}}_{w_i} = \frac{1}{w_i}\sum_{j=1}^{n_i} w_{ij}\, \boldsymbol{y}_{ij}$, $\hat{\boldsymbol{\mu}}_w = \frac{1}{w}\sum_{i}\sum_{j} w_{ij}\, \boldsymbol{y}_{ij}$ and $w = \sum_{i=1}^{k} w_i$ $\cdot$

4 The Robust Test Statistics

The effect of outliers on the quality of the hypothesis test, which is based on the classical statistics, makes us propose to use robust estimators instead of the classical ones for computing the test statistics for one-way MANOVA. Therefore, a robust version of the test statistics can be obtained by substituting the estimates of the matrices E_w and H_w into formulas (3), (6), (8), and (10):

4.1 Robust Wilks' statistic RΛ

$$\text{R}\Lambda = \frac{|E_w|}{|E_w + H_w|} \approx \Lambda_{\alpha,p,v_{H_w},v_{E_w}}, \quad (18)$$

where $v_{E_w} = w - \sum_{i=1}^{k} \frac{r_i}{w_i}$, $v_{H_w} = \sum_{i=1}^{k} \frac{r_i}{w_i} - \frac{\sum_{i=1}^{k} r_i}{w}$.

The approximate distribution of robust Wilks' statistic RΛ which is similar to F approximation in formula (5), is given by the following:

$$\text{R}\Lambda_1 = \frac{\left(1 - \text{R}\Lambda^{1/t}\right)}{\text{R}\Lambda^{1/t}} \cdot \frac{v_{w_2}}{v_{w_1}} \approx F_{v_{w_1},v_{w_2}}, \quad (19)$$

where

$$v_{w_1} = p\, v_{H_w}\,,\ \ v_{w_2} = c_w t_w - \frac{1}{2}\left(p\, v_{H_w} - 2\right), t_w = \sqrt{\frac{p^2\, {v_{H_w}}^2 - 4}{p^2 + \, {v_{H_w}}^2 - 5}}\,,$$

$$and\ \ c_w = v_{E_w} + v_{H_w} - \frac{1}{2}\left(p + v_{H_w} + 1\right).$$

4.2 Robust Pillai's statistic RV

$$\mathrm{RV} = tr[(E_w + H_w)^{-1} H_w] \approx \mathrm{V}_{\alpha, s_w, g_w, l_w}\,, \quad (20)$$

where

$$s_w = min\{p, v_{H_w}\}, g_w = \frac{1}{2}\left(\left|v_{H_w} - p\right| - 1\right)\ and\ \ l_w = \frac{1}{2}\left(v_{E_w} - p - 1\right).$$

The approximate distribution of robust Pillai's statistic RV which is similar to F approximation in formula (7) is given by the following:

$$\mathrm{RV}_1 = \frac{\mathrm{RV}}{(s - \mathrm{RV})} \cdot \frac{2l_w + s_w + 1}{2g_w + s_w + 1} \approx\ F_{s_w(2g_w + s_w + 1), s_w(2l_w + s_w + 1)}. \quad (21)$$

4.3 Robust Lawley-Hotelling statistic RU

$$\mathrm{RU} = \ tr(E_w^{-1} H_w) \approx U_{\alpha, s_w, g_w, l_w}. \quad (22).$$

The approximate distribution of robust Lawley-Hotelling statistic RU which is similar to F approximation in formula (9) is given by the following:

$$\mathrm{RU}_1 = \frac{2(s_w l_w + 1)}{s_w^2(2g_w + s_w + 1)} \cdot \mathrm{RU} \approx F_{s_w(2g_w + s_w + 1), 2(s_w l_w + 1)}. \quad (23)$$

4.4 Robust Roy's statistic Rθ

$$\mathrm{R\theta} = \frac{\lambda_1}{1 + \lambda_1} \approx \theta_{\alpha, s_w, g_w, l_w}\,, \quad (24)$$

where λ_1 is the largest eigenvalues of the matrix $E_w^{-1} H_w$.

The approximation distribution of robust Roy's statistic Rθ which is similar to F approximation in formula (11) is given by the following:

$$\mathrm{R\theta}_1 = \frac{(v_{E_w} - d_w + v_{H_w})}{d_w} \cdot \lambda_1 \approx F_{d_w, v_{E_w} - d_w + v_{H_w}}, \quad (25)$$

where $d_w = max\{p, v_{H_w}\}$.

Now, the accuracy of these approximations will be verified. For the various dimension values p, number of groups k, and sample sizes for each group n_i, $i = 1, 2, \ldots, k, m = 3000$ samples from the normal standard distribution will be generated and for each of them $\mathrm{R\Lambda}, \mathrm{RV}, \mathrm{RU}$ and $\mathrm{R\theta}$ are calculated. Using the QQ-plots [4], the empirical distribution of these statistics will be compared to the approximate distributions given in formulas (19), (21), (23) and (25) respectively. Some of them are shown in figures (1) and (2) for two and three groups. The usual cutoff values of tests, the $99\%, 95\%$ and 90% quantiles are shown in these plots by vertical lines. It is very clear through the plots that the approximations are very accurate for sample sizes, various dimensions, and equal and unequal group's sizes, except the Roy's approximation which is bad when $s > 1$.

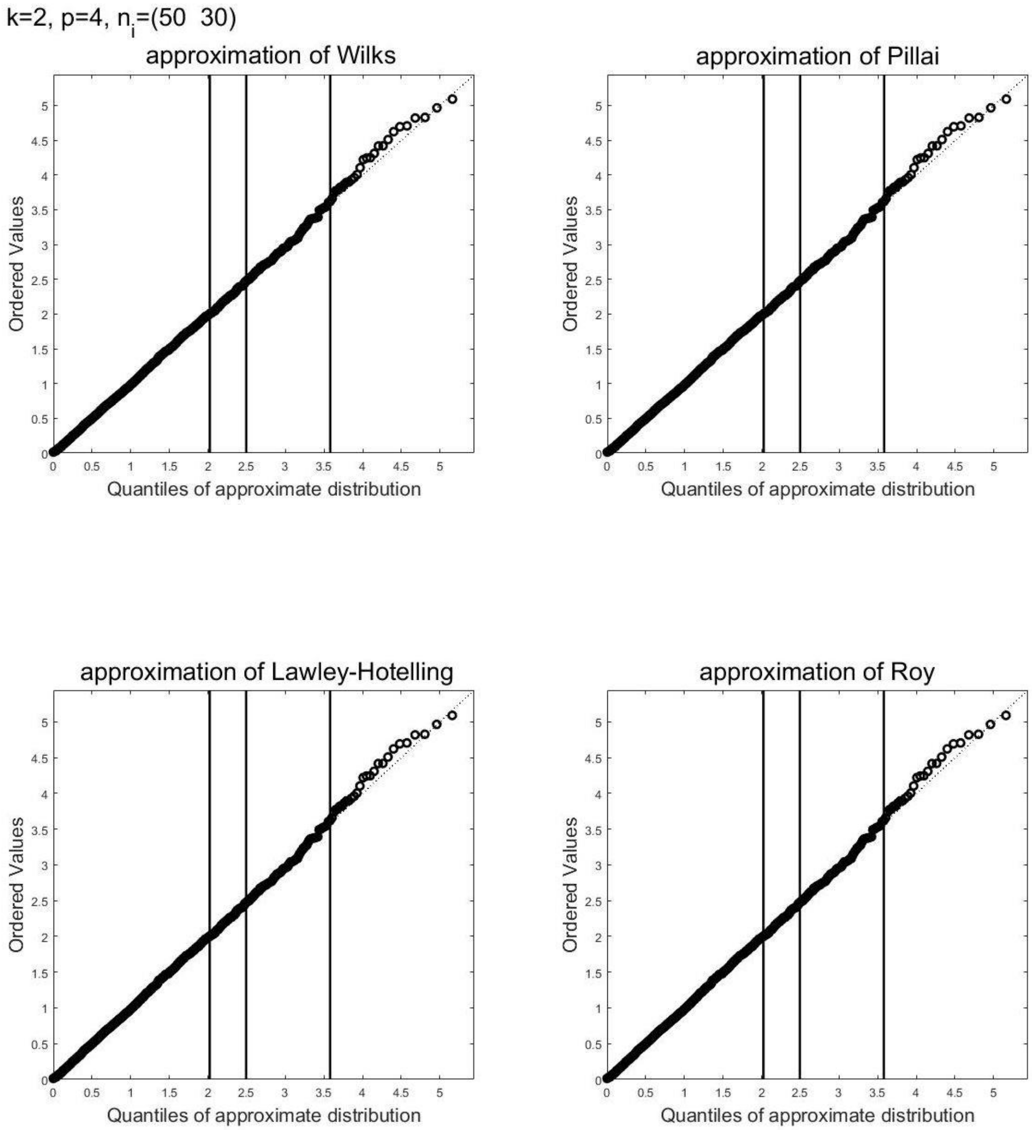

Figure (1): QQ-plots for the robust test statistics in the case of two groups and the four dimensions and $n = \sum_{i=1}^{k} n_i$.

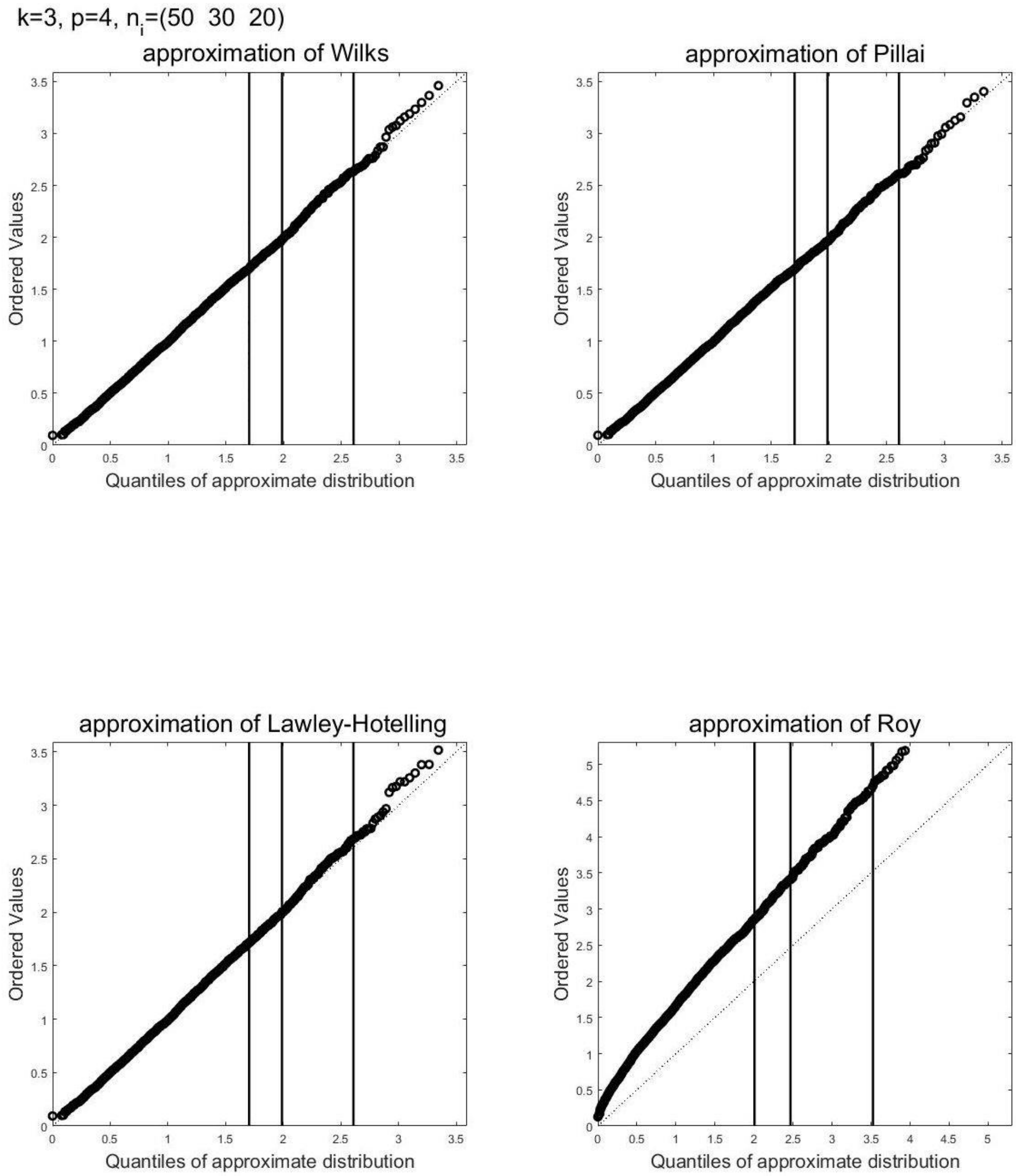

Figure (2): QQ-plots for the robust test statistics in the case of three groups and the four dimensions and $n = \sum_{i=1}^{k} n_i$.

5 Monte Carlo Simulations

Monte Carlo simulation can deal with a large number of random variables, various distribution types and nonlinear models; furthermore it is a powerful statistical analysis tool. This is done by generating a large number of simulated samples and conducting the experiments in the computer by using numerical techniques. Monte Carlo study is conducted to evaluate the performance of the proposed statistics. The evaluation of the performance of any test statistics involves two measures: significance level (the type I error rate) and the power of the test. To investigate the simulated significance level (type I error rate) and the power of the test for classical statistics and proposed robust statistics under various distributions, normal and contaminated distributions, the number of groups $k = \{2, 3, 4\}$, equal and unequal sample sizes $n_i,\ i = 1,2, \dots, k$ and several dimensions $p = \{2,4,8\}$ will be taken into consideration. The sample sizes for two, three and four groups are selected as shown in Table (1).

Table (1): Selected group sizes for the simulation study.

Two groups (n_1, n_2)	Three groups (n_1, n_2, n_3)	four groups (n_1, n_2, n_3, n_4)
(10, 10)	(10, 10, 10)	(20, 20, 20,20)
(20, 20)	(20, 20, 20)	(30, 30, 30,30)
(30, 30)	(30, 30, 20)	
(50, 50)		

5.1 Significance level

To investigate the type I error rates $\hat{\alpha}$ of the test statistics, observations are to be generated from the multivariate normal distribution $\boldsymbol{y}_{ij} \sim N_P(0, \mathbf{I}_{\mathrm{p}})$ under the null hypothesis $H_0 :\ \boldsymbol{\mu}_1 = \boldsymbol{\mu}_2 = \ \dots = \boldsymbol{\mu}_k = 0$. Thus, the classical statistics and the robust version based on RMCD estimates are calculated for test statistics. The approximations of classical statistics are compared with the approximations of the proposed robust statistics, and a comparison of classic and strong test statistics among them. This is repeated $m = 3000$ and the percentages of values $\hat{\alpha} = L(T)/m$ (where $L(T)$ is the number of times of the test statistic rejects the hypothesis H_0 when H_0 is true) of the test statistics which are above the appropriate critical value of the corresponding approximate distribution are taken as an estimate of the true significance level. The true significance level $\alpha = 0.01, 0.05$ and 0.10 with the number of replications $\mathrm{m} = 3000$, and from the standard error formula of Saltier and Fawcett $\alpha \pm 2\sqrt{\alpha(1-\alpha)/\mathrm{m}}$ [17] yields the standard deviation interval around the nominal levels as follows: (0.006, 0.014), (0.042, 0.058), (0.089, 0.111) respectively.

Multivariate analysis of variance generates a P-value that is used to determine whether or not the null hypothesis can be rejected. In addition to using the approximations for the test statistics based on F or chi-square distributions, it will be easier to calculate P-value [18].

Also we will use the P-value plots proposed by Davidson and McKinnon, which gives more complete picture of how the test statistics follow the approximate distribution under the null hypothesis in the simulated samples [4].

5.2 Power of Test

In order to assess the power of the test $\hat{\pi}$ of our statistics we will generate data $\boldsymbol{y}_{ij,}\ i = 1,2,\dots,k,\ j = 1,2,\dots,n_i$ under an alternative hypothesis (H_1: not all $\boldsymbol{\mu}_i, i = 1,\dots,k$ are equal). The same combinations of number of groups k, dimensions p, sample sizes $n_i, i = 1,2,\dots,k$ used previously in the experiments for studying the significance level will be used. All groups $\pi_{i,}\ i = 1,\dots,k$, have the same covariance matrix $\mathbf{I}_p$ and each of them has a different mean $\boldsymbol{\mu}_i$ $\left(\text{i.e. } \pi_i \sim N_P(\boldsymbol{\mu}_i, \mathbf{I}_p), i = 1,\dots,k\right)$, with $\boldsymbol{\mu}_1 = (0, 0,\dots, 0)^t$, $\boldsymbol{\mu}_2 = (0.5, 0,\dots, 0)^t, \boldsymbol{\mu}_3 = (0, 0.5,\dots, 0)^t,\dots, \boldsymbol{\mu}_k = (0, 0,\dots, 0.5, 0)^t$.

Again, the classical and the robust test statistics are computed. This is repeated $m = 3000$ times and then the power of test is calculated $\hat{\pi} = K(T)/m$ (where $K(T)$ is the number of times of the test statistic rejects the hypothesis H_0 when H_0 is false) of the test statistics when the statistic exceeds its appropriate critical value it will be the estimate of the power for the specific configuration. The power of the statistics can be visually compared by simulating size-power curves under constant alternatives, as proposed by Davidson and McKinnon [19].

5.3 Robustness comparisons

Now, the robustness for the one-way MANOVA hypothesis test will be investigated based on the test statistics. For this purpose, data sets will be generated under the null hypothesis $H_0 : \boldsymbol{\mu}_1 = \boldsymbol{\mu}_2 = \dots = \boldsymbol{\mu}_k$ and they will contaminated by adding outliers. More precisely the data will be generated from the following contamination model:

$$\boldsymbol{y}_{ij} \sim (1-\epsilon)\ \mathrm{N_P}(0, \mathbf{I}_\mathrm{p}) + \epsilon\ \mathrm{N_P}(\widehat{\boldsymbol{\mu}}_{i,}\ 0.25^2\mathbf{I}_\mathrm{p}),\ \ i = 1,2,\dots,k,$$

where $\widehat{\boldsymbol{\mu}}_i = (xQ_p,\dots, xQ_p)^t$, $Q_p = \sqrt{\chi^2{}_{p,0.001}/p}$, $\epsilon = 0.1$ and $x = 5$.

The same combinations of dimensions p, number of groups k, and sample sizes $n_i,\ i = 1,\dots,k$ will be used.

6 Discussion of the Results

6.1 In Case of normal distribution for the data set

The P-value plots in figures (3) to (5) show that all the proposed approximate statistics are close to the approximate classical statistics, i.e. the approximations are capable to keep the significance levels for all cases. And these figures also show all the classical approximate statistics are identical when $s = 1$, and the same that for the proposed approximate statistics, and they are very close to each other when $s > 1$, except approximate statistics of Roy's where the figures show the failure of Roy's statistic in all cases when s is greater than one.

Size-power curves plots in figures (6) to (8) show that all the approximate statistics give almost the same power and are very close to the classical.

6.2 Case of contaminated distribution for the data set

The P-value plots in figures (9) to (11) show us that all the classical approximate statistics are very bad while the robust statistics are accurate and very close of the 45° line in all cases.

Size-power curves in figures (12) to (14) show us that the power of all approximate statistics is increasing as the length of the sample size increases. The power of the approximate robust statistics are the best compared to the classical approximation in all cases.

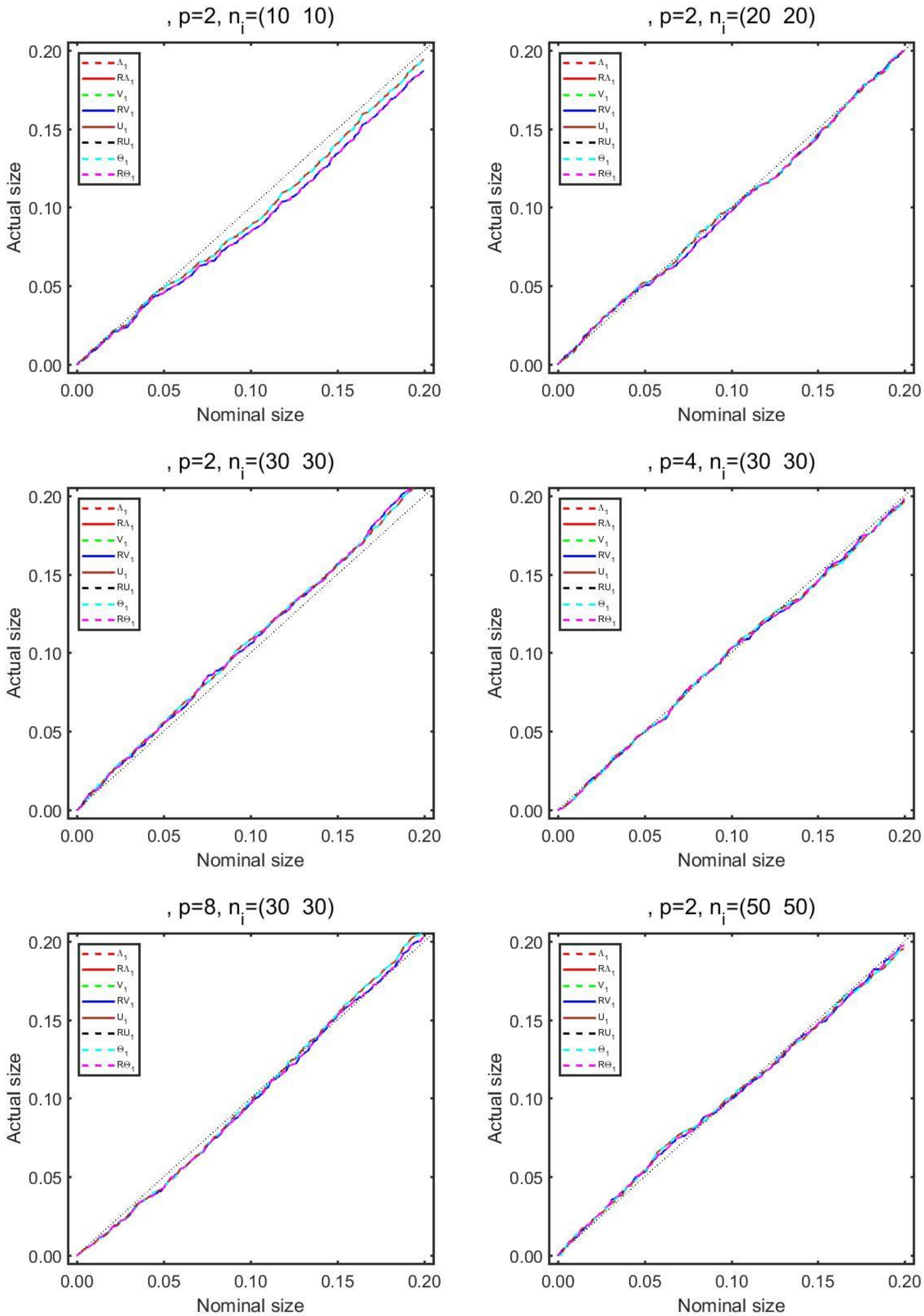

Figure 3: P-value plots for the approximate classical statistics $\Lambda_1, \mathrm{V}_1, \mathrm{U}_1$ and θ_1, and approximate robust statistics $\mathrm{R}\Lambda_1, \mathrm{RV}_1, \mathrm{RU}_1, and\ \mathrm{R}\theta_1$ for two groups and several values for p and $n = \sum_{i=1}^{k} n_i$. The 45° line is given too, represented by a black dotted line. The results of classical statistics are identical, also the same case for the proposed statistics and therefore, some of statistics are not shown.

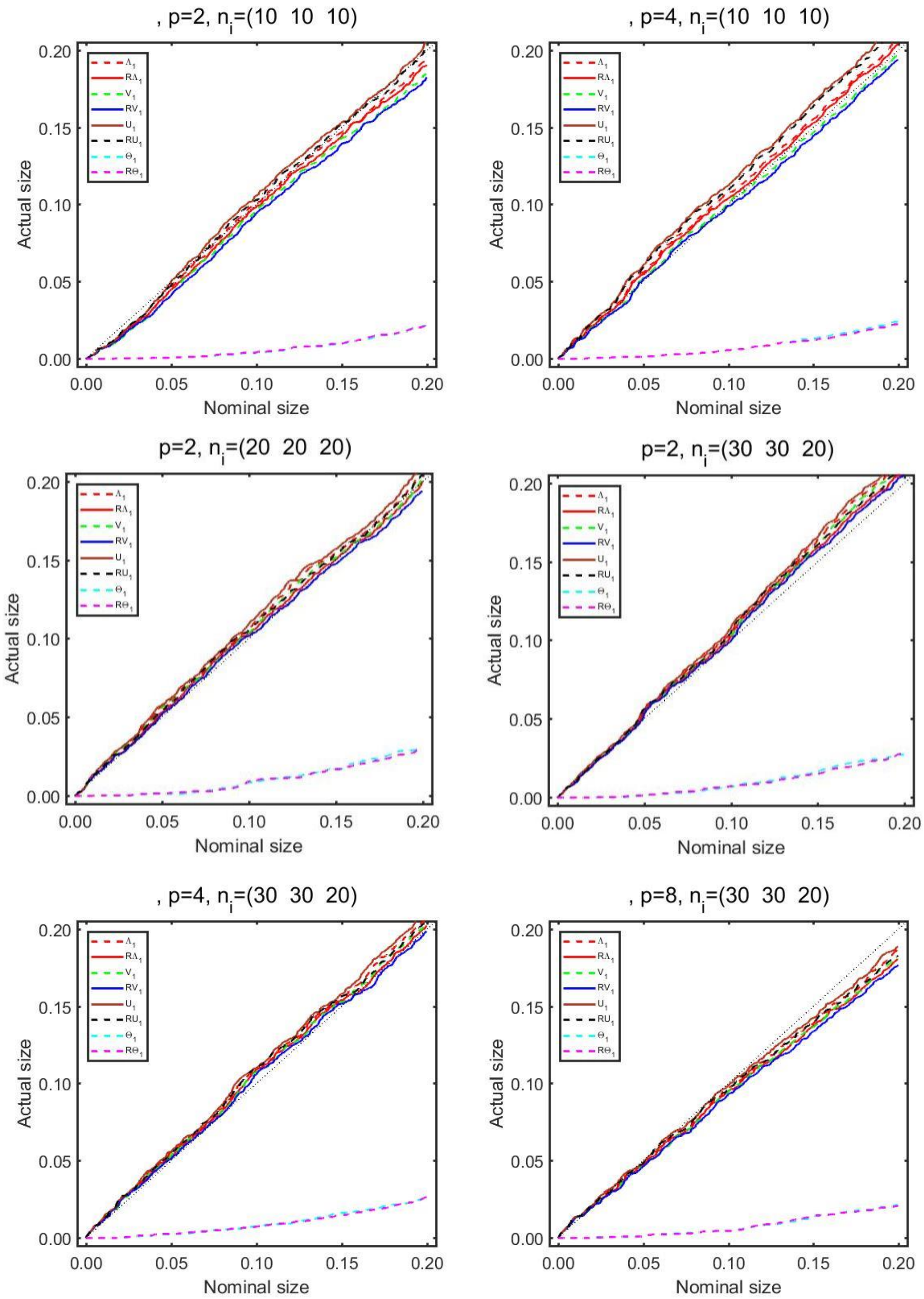

Figure 4: P-value plots for the approximate classical statistics Λ_1, V_1, U_1 and θ_1, and approximate robust statistics $R\Lambda_1, RV_1, RU_1, and\ R\theta_1$ for three groups and several values for p and $n = \sum_{i=1}^{k} n_i$. The 45° line is given too, represented by a black dotted line.

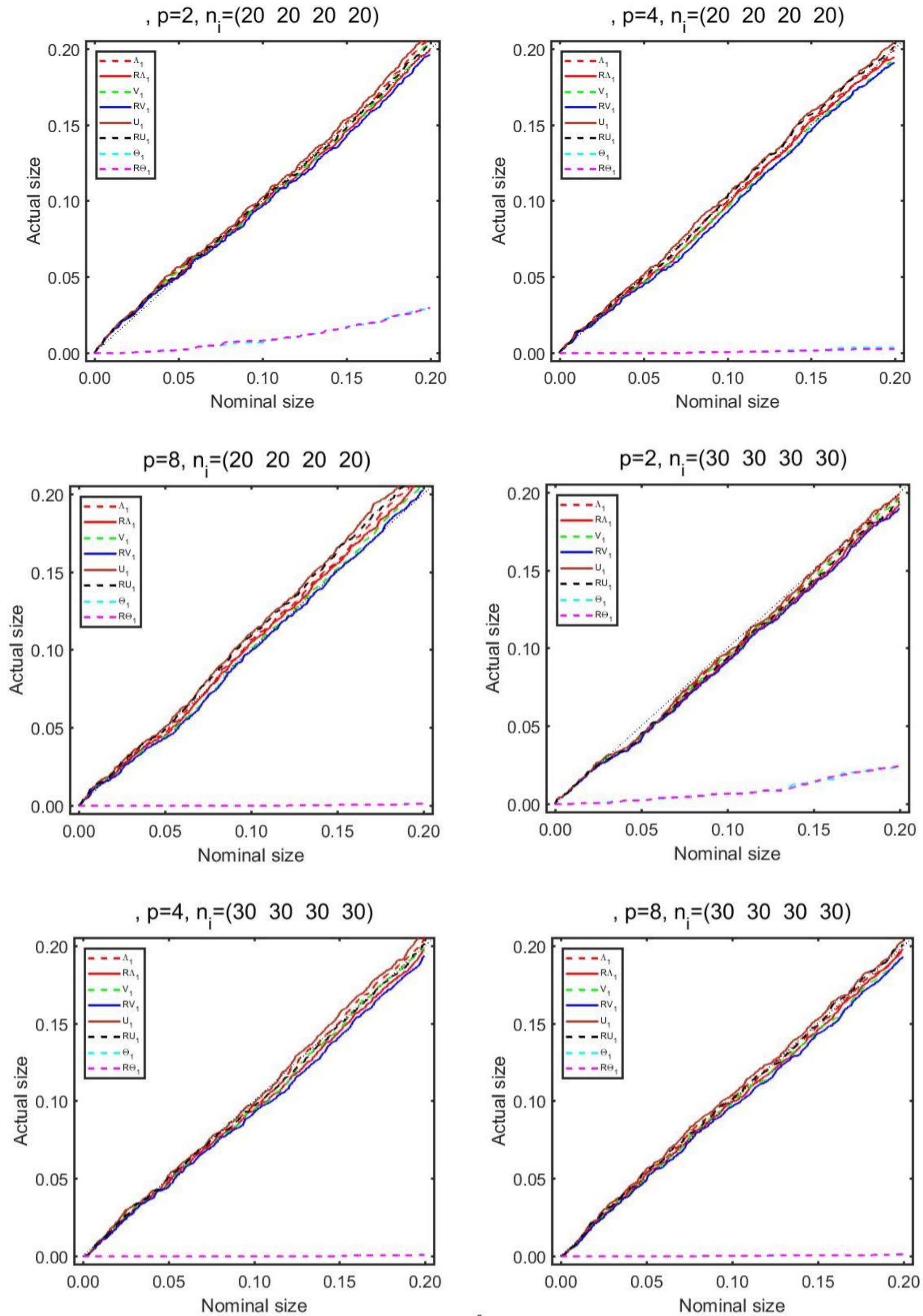

Figure 5: P-value plots for the approximate classical statistics $\Lambda_1, \mathrm{V}_1, \mathrm{U}_1$ and θ_1, and approximate robust statistics $\mathrm{R}\Lambda_1, \mathrm{RV}_1, \mathrm{RU}_1, and\ \mathrm{R}\theta_1$ for four groups and several values for p and $n = \sum_{i=1}^{k} n_i$. The 45° line is given too, represented by a black dotted line.

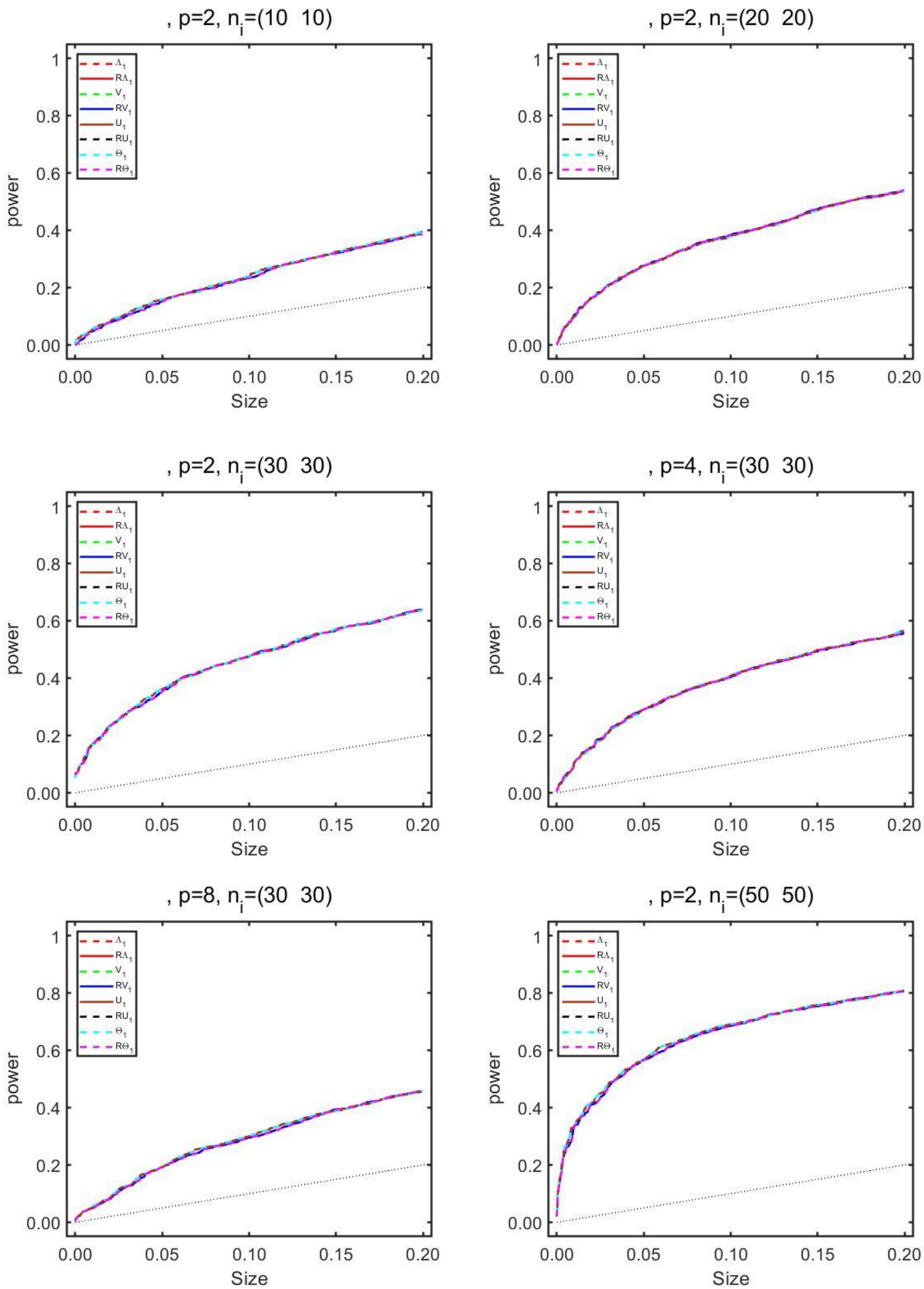

Figure 6: Size-power curves for the approximate classical statistics $\Lambda_1, \mathrm{V}_1, \mathrm{U}_1$ and θ_1, and approximate robust statistics $\mathrm{R}\Lambda_1, \mathrm{RV}_1, \mathrm{RU}_1, and\ \mathrm{R}\theta_1$ for two groups and several values for p and $n = \sum_{i=1}^{k} n_i$. The 45° line is given too, represented by a black dotted line. The results of test statistics are identical and therefore, some of statistics are not shown.

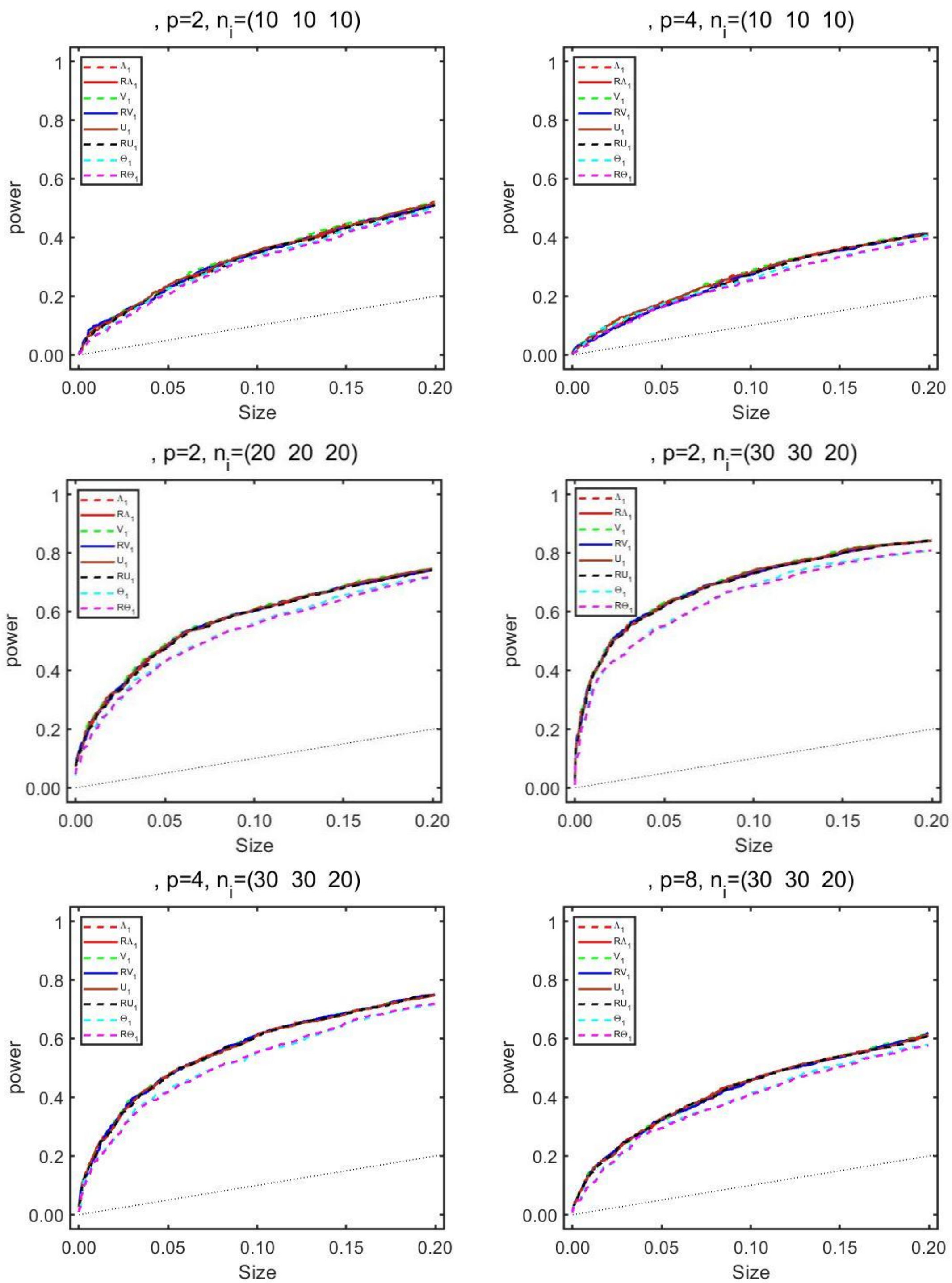

Figure 7: Size-power curves for the approximate classical statistics $\Lambda_1, \mathrm{V}_1, \mathrm{U}_1$ and θ_1, and approximate robust statistics $\mathrm{R}\Lambda_1, \mathrm{RV}_1, \mathrm{RU}_1,$ *and* $\mathrm{R}\theta_1$ for three groups and several values for p and $n = \sum_{i=1}^{k} n_i$. The 45° line is given too, represented by a black dotted line. The results of some test statistics are identical and therefore, some of statistics are not shown.

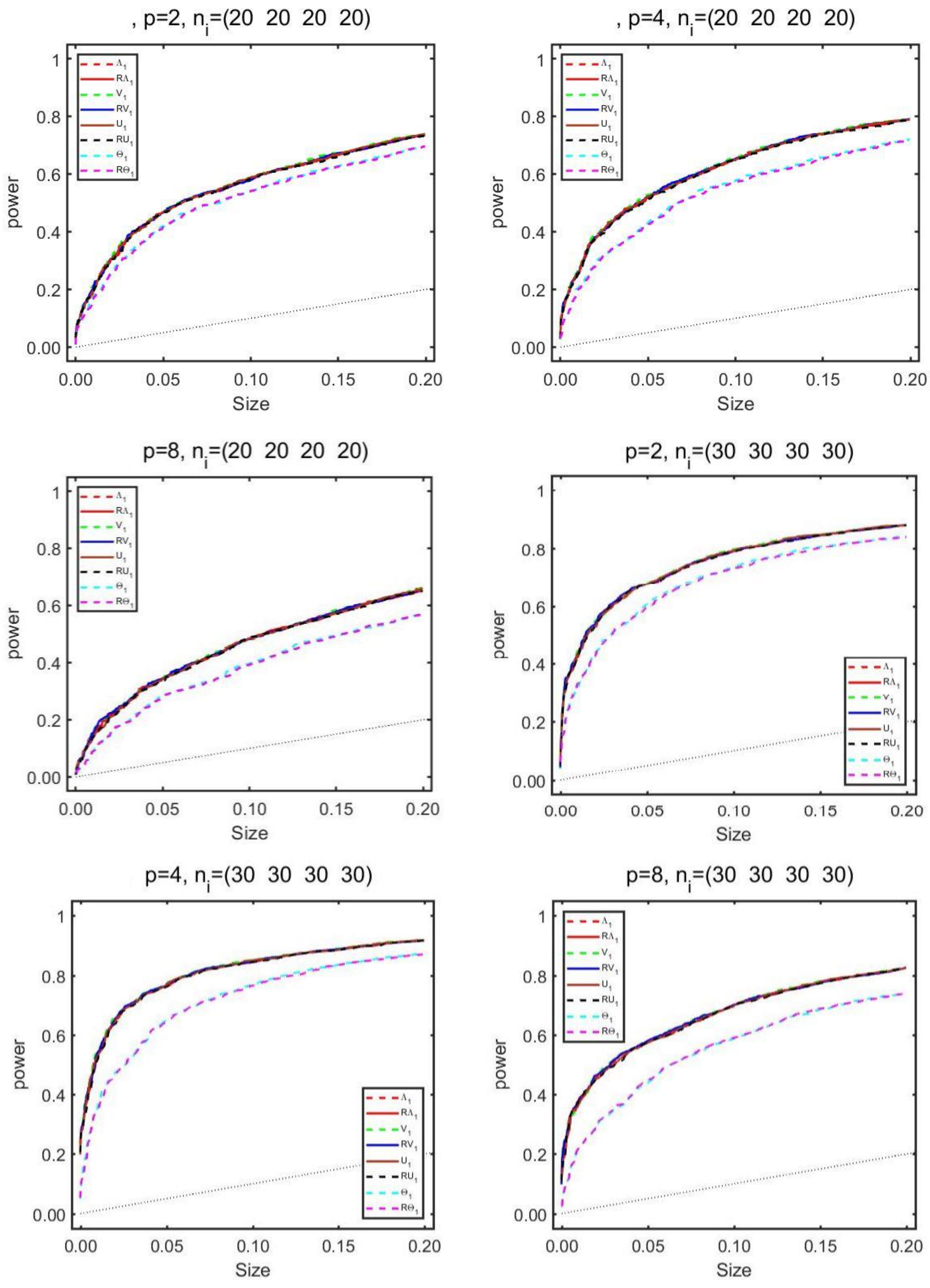

Figure 8: Size-power curves for the approximate classical statistics $\Lambda_1, \mathrm{V}_1, \mathrm{U}_1$ and θ_1, and approximate robust statistics $\mathrm{R}\Lambda_1, \mathrm{RV}_1, \mathrm{RU}_1, and\ \mathrm{R}\theta_1$ for four groups and several values for p and $n = \sum_{i=1}^{k} n_i$. The 45° line is given too, represented by a black dotted line. The results of some test statistics are identical and therefore, some of statistics are not shown.

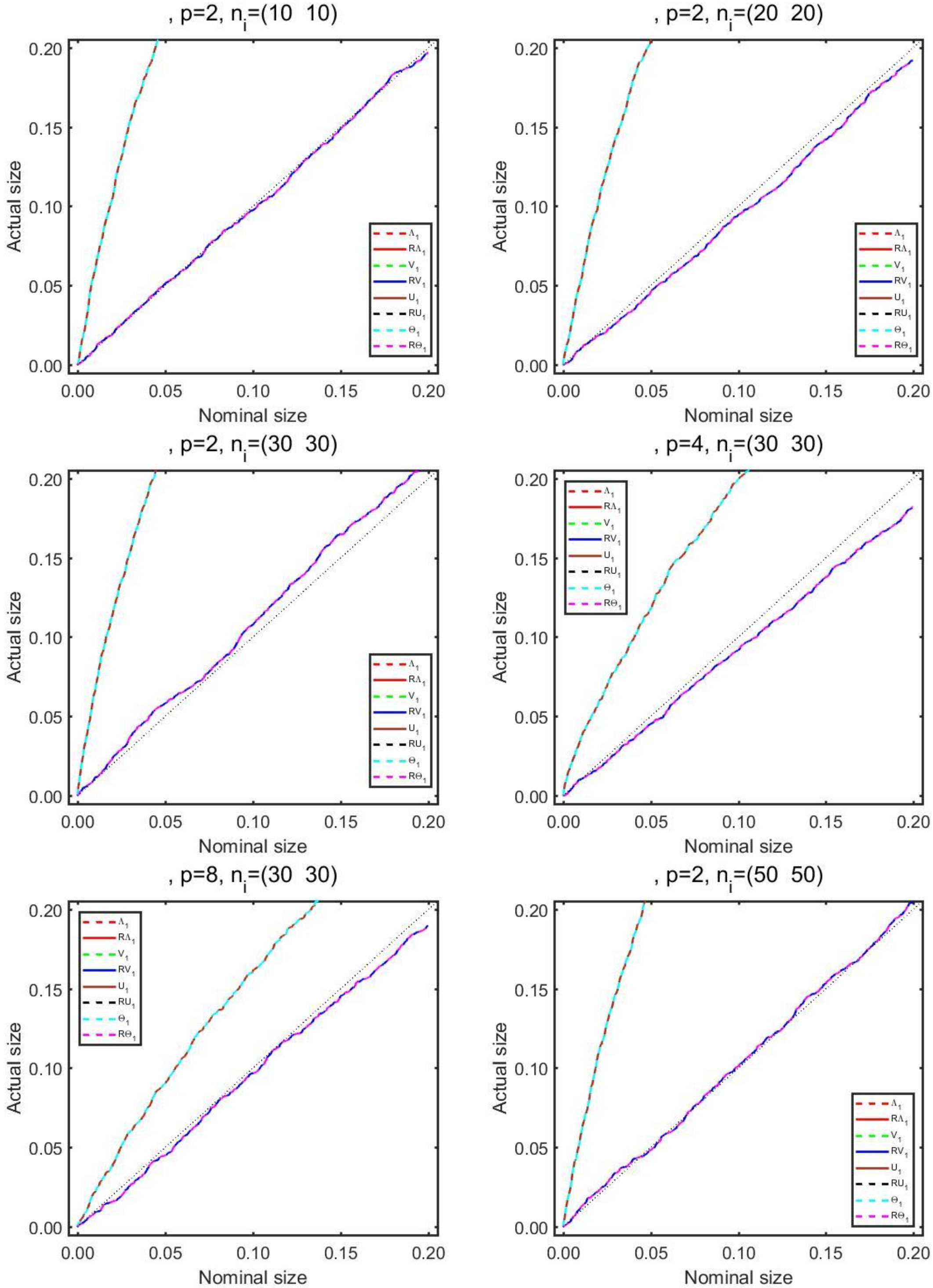

Figure 9: P-value plots for the approximate classical statistics $\Lambda_1, \mathrm{V}_1, \mathrm{U}_1$ and θ_1, and approximate robust statistics $\mathrm{R}\Lambda_1, \mathrm{RV}_1, \mathrm{RU}_1, and\ \mathrm{R}\theta_1$ for two groups and several values for p and $n = \sum_{i=1}^{k} n_i$. The 45° line is given too, represented by a black dotted line. The results of classical statistics are identical, also the same case for the proposed statistics and therefore, some of statistics are not shown.

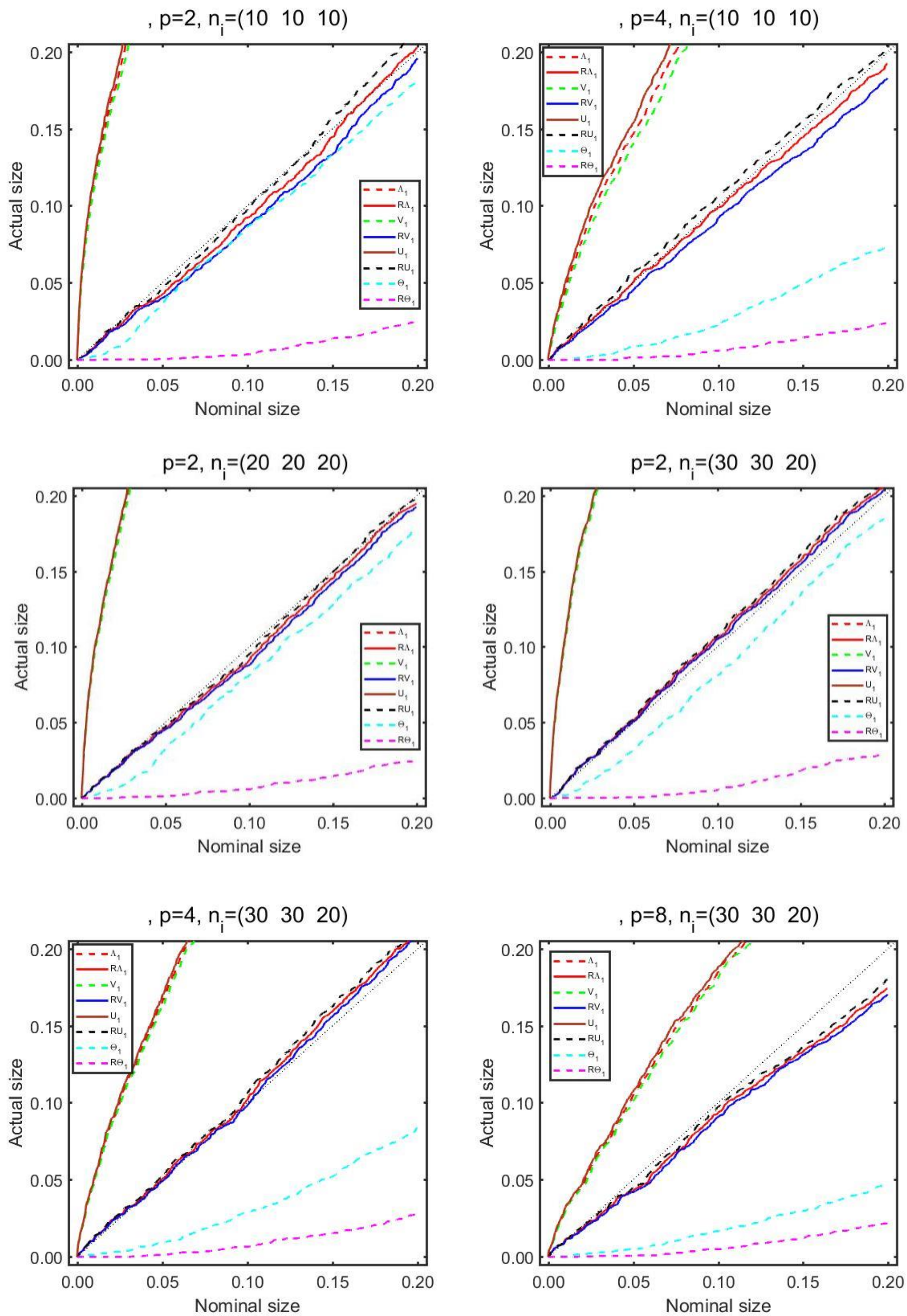

Figure 10: P-value plots for the approximate classical statistics Λ_1, V_1, U_1 and θ_1, and approximate robust statistics $R\Lambda_1, RV_1, RU_1,$ *and* $R\theta_1$ for three groups and several values for p and $n = \sum_{i=1}^{k} n_i$. The 45° line is given too, represented by a black dotted line.

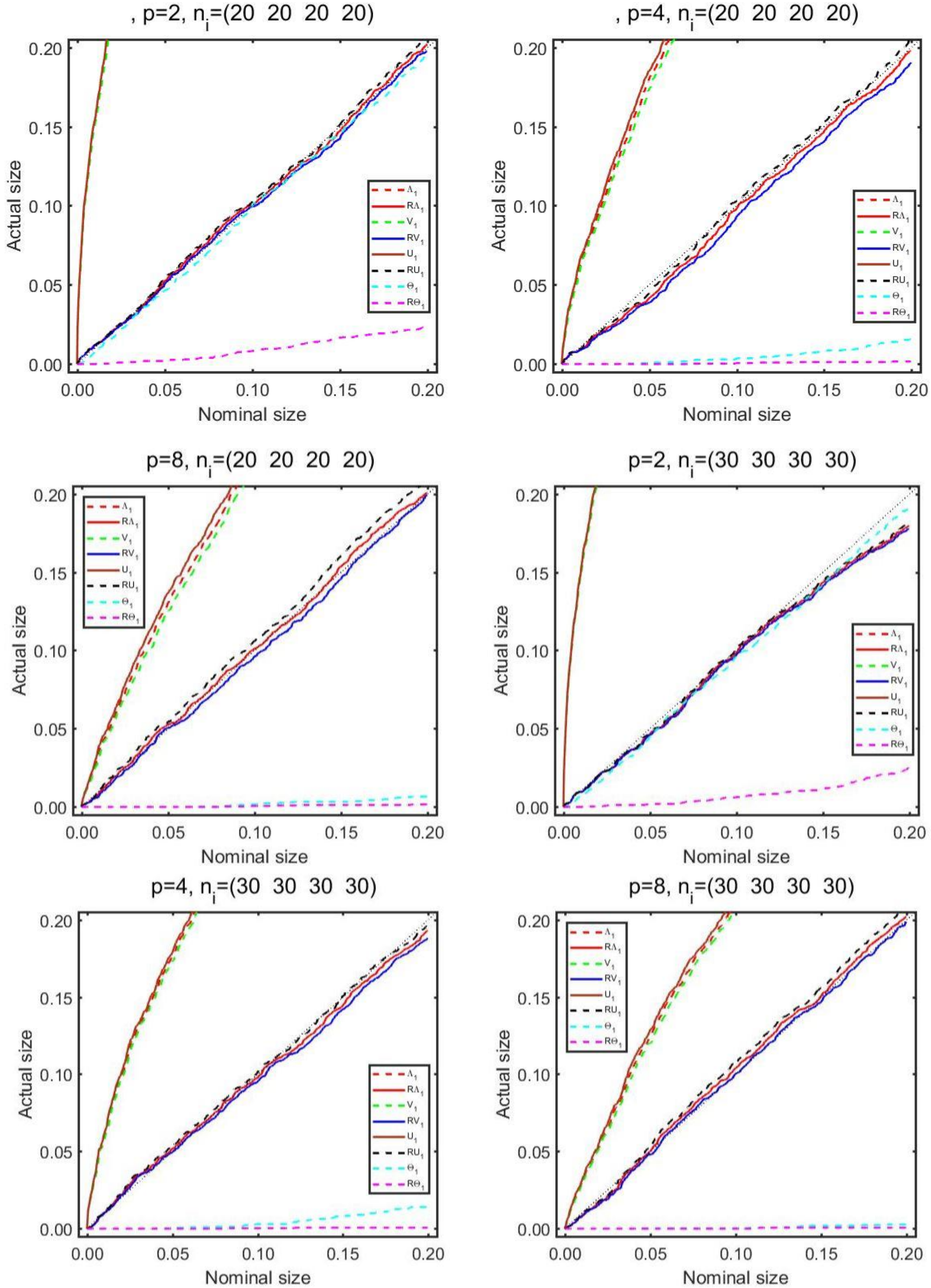

Figure 11: P-value plots for the approximate classical statistics $\Lambda_1, \mathrm{V}_1, \mathrm{U}_1$ and θ_1, and approximate robust statistics $\mathrm{R}\Lambda_1, \mathrm{RV}_1, \mathrm{RU}_1, and$ $\mathrm{R}\theta_1$ for four groups and several values for p and $n = \sum_{i=1}^{k} n_i$. The 45° line is given too, represented by a black dotted line.

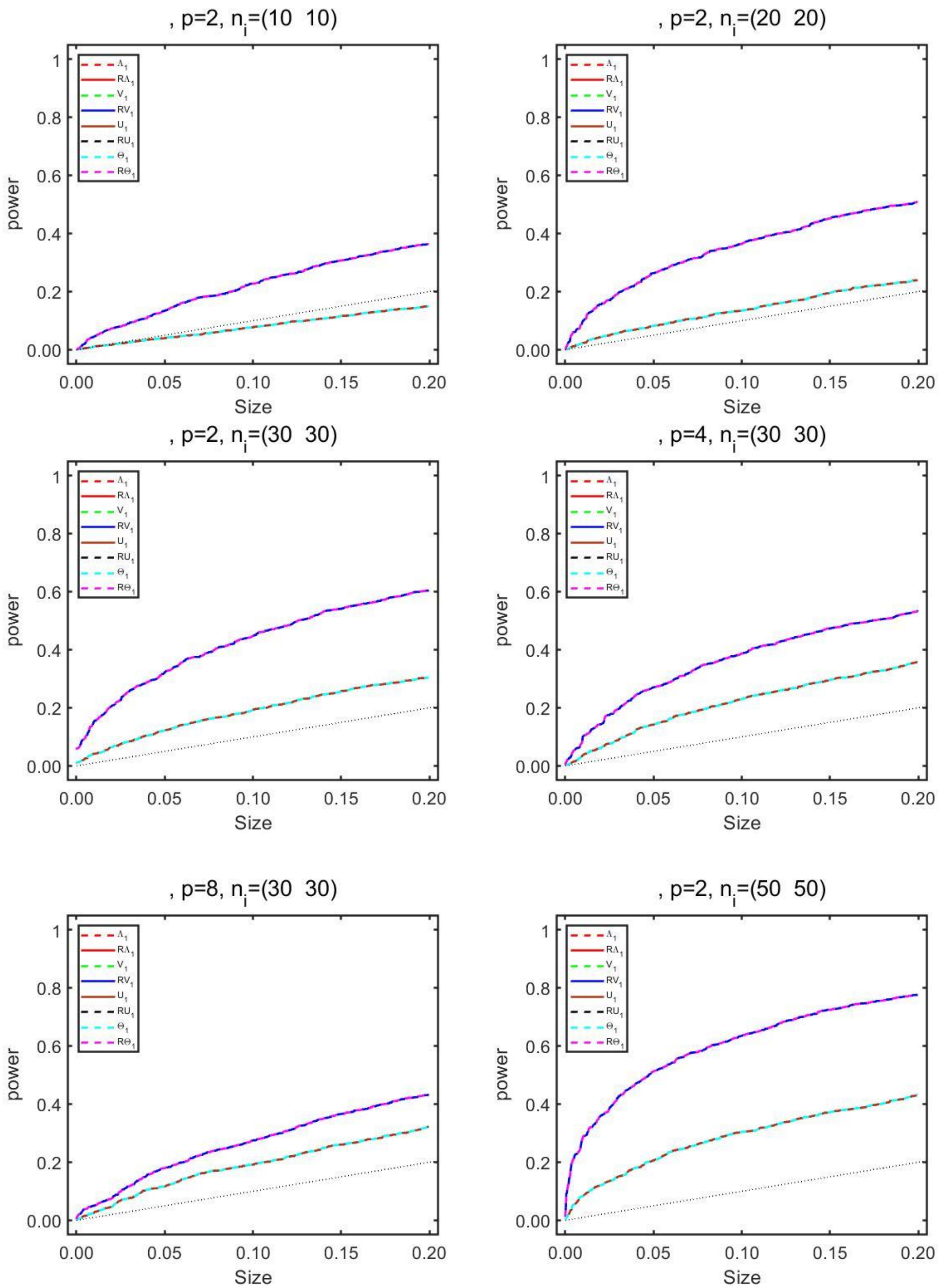

Figure 12: Size-power curves for the approximate classical statistics Λ_1, V_1, U_1 and θ_1, and approximate robust statistics $R\Lambda_1, RV_1, RU_1, and\ R\theta_1$ for two groups and several values for p and $n = \sum_{i=1}^{k} n_i$. The 45° line is given too, represented by a black dotted line. The results of classical statistics are identical, also the same case for the proposed statistics and therefore, some of statistics are not shown.

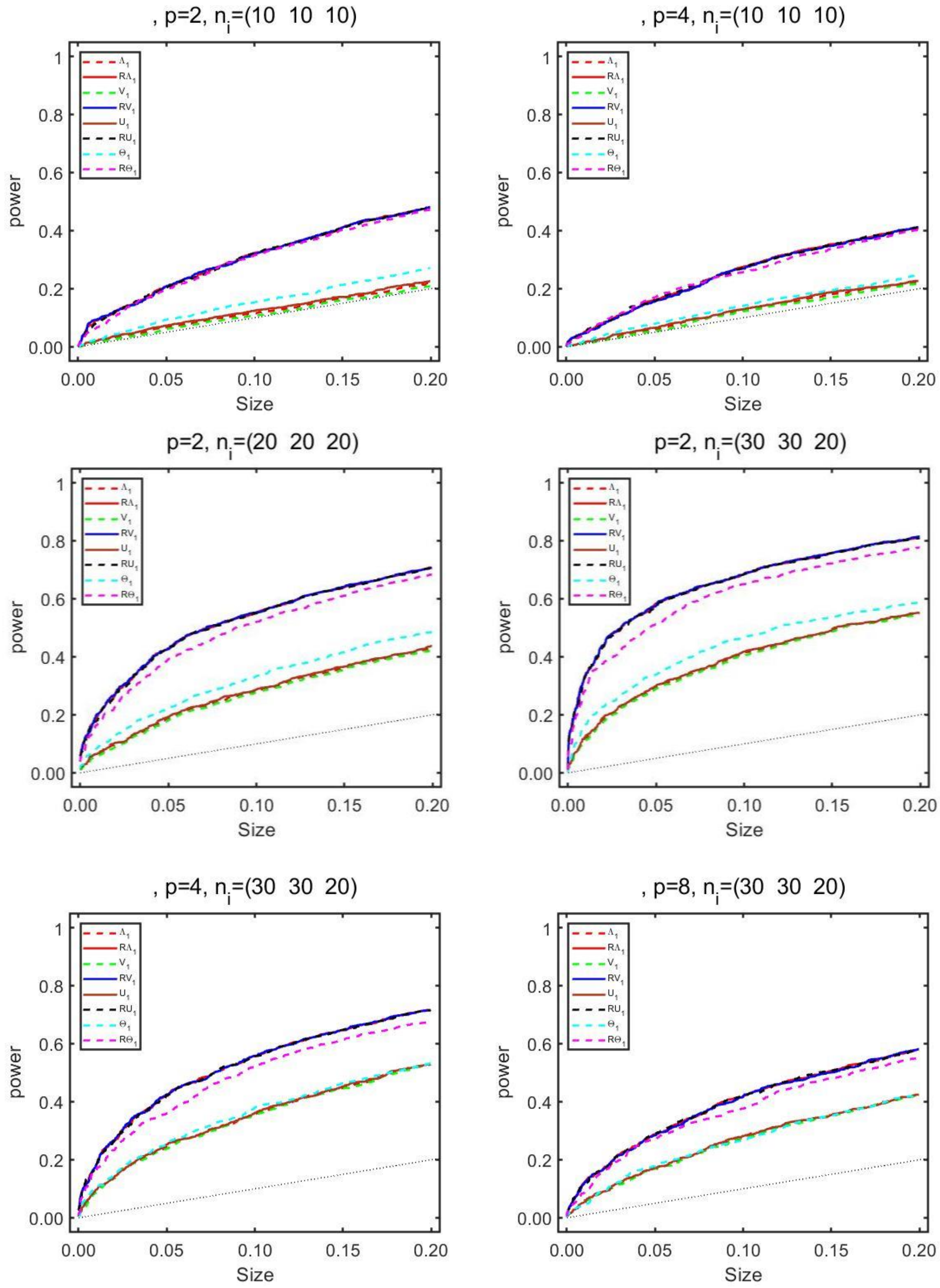

Figure 13: Size-power curves for the approximate classical statistics $\Lambda_1, \mathrm{V}_1, \mathrm{U}_1$ and θ_1, and approximate robust statistics $\mathrm{R}\Lambda_1, \mathrm{RV}_1, \mathrm{RU}_1, and\ \mathrm{R}\theta_1$ for three groups and several values for p and $n = \sum_{i=1}^{k} n_i$. The 45° line is given too, represented by a black dotted line. The results of some test statistics are identical and therefore, some of statistics are not shown.

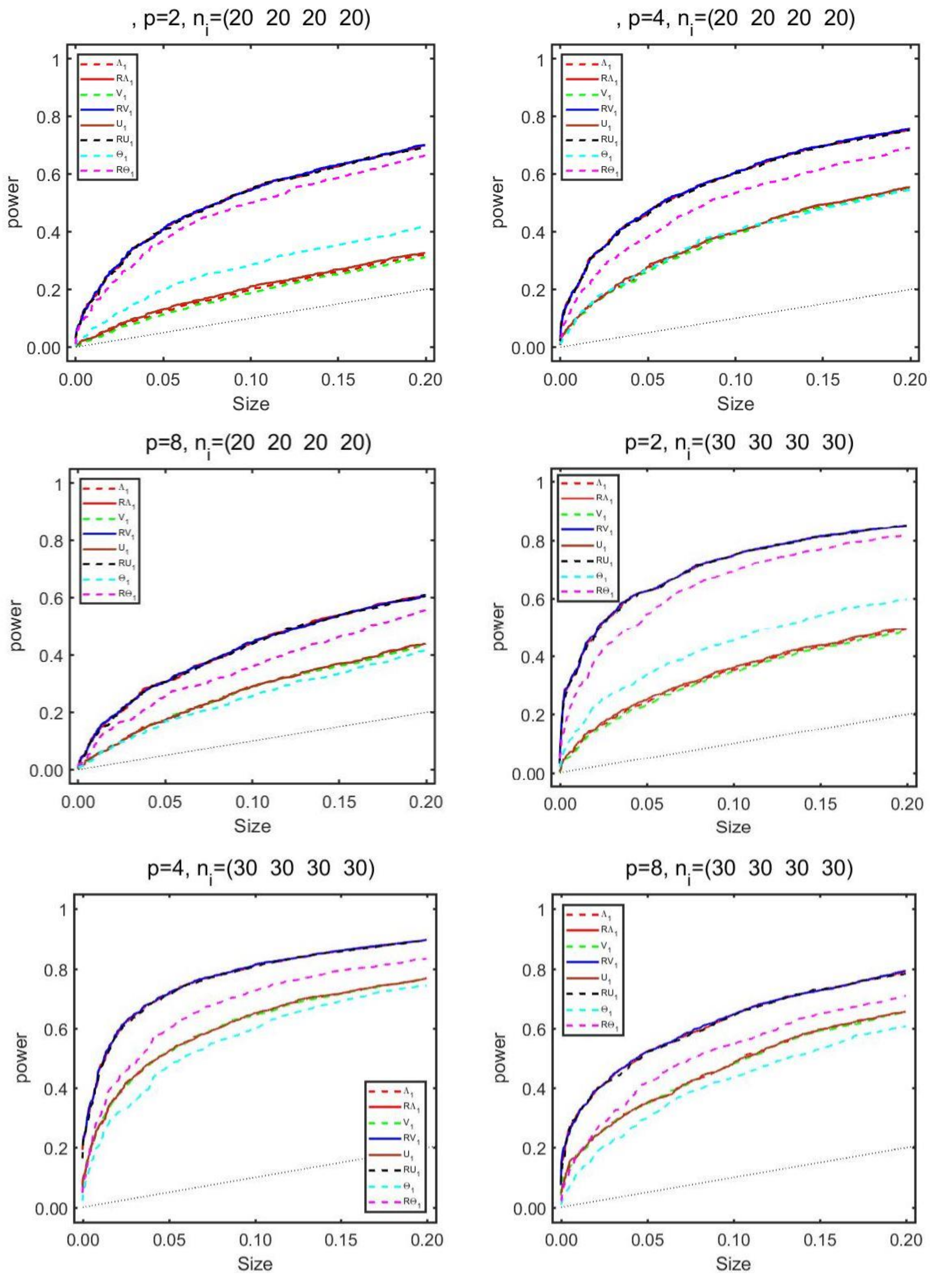

Figure 14: Size-power curves for the approximate classical statistics $\Lambda_1, \mathrm{V}_1, \mathrm{U}_1$ and θ_1, and approximate robust statistics $\mathrm{R}\Lambda_1, \mathrm{RV}_1, \mathrm{RU}_1, and\ \mathrm{R}\theta_1$ for four groups and several values for p and $n = \sum_{i=1}^{k} n_i$. The 45° line is given too, represented by a black dotted line. The results of some test statistics are identical and therefore, some of statistics are not shown.

7 Conclusions

The results show us that the proposed robust approximate statistics are close to the classical statistics in case of normal distribution for the data set. In the case of contaminated distribution the p-value and size-power curves clearly show the advantage of the proposed robust test statistics over the classical.

In conclusion, all classical approximations in case normal distribution and the proposed approximate statistics in case normal and contaminated distributions are accurate statistics, but none of them is always the best.

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إحصاءات الاختبار الحصينة لنموذج تحليل التباين المتعدد المتغيرات ذي العامل الواحد: دراسة المحاكاة

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المستخلص

تحليل التباين المتعدد المتغيرات ذو عامل واحد يتعامل مع اختبار فرضية العدم حول تساوي متوسطات لمجتمعين طبيعيين متعددي المتغيرات او أكثر . الاحصاءات 'Wilks، Roy's، Lawley-Hotelling Pillai's الكلاسيكية هي الأكثر استخداماً لاختبار الفرضية في تحليل التباين المتعدد المتغيرات ذي العامل الواحد ، وتكون حساسة للغاية إزاء تأثير النقاط الشاذة. في هذه الدراسة تم اقتراح إحصاءات حصينة معتمدة على مقدر أصغر محدد تباين مشترك معاد الاوزان (RMCD) مع دالة أوزان Hample. كما تم بناء التوزيعات التقريبية لإحصائيات الاختبار القوية التي ترتبط مباشرة بالأوزان. تختلف توزيعات الإحصاءات المقترحة عن التوزيعات الكلاسيكية. تستخدم محاكاة مونت كارلو لتقييم أداء إحصاءات الاختبار في ظل توزيعات مختلفة من حيث مستويات الأهمية المحاكاة ووظائفها القوية ومتانة قوتها. تتم مقارنة قوة الإحصائيات القوية والكلاسيكية باستخدام منحنيات حجم القوة. تظهر النتائج أن إحصائيات الاختبار القوية قريبة من إحصائيات الاختبار الكلاسيكي في حالة التوزيع الطبيعي لمجموعة البيانات. اما في حالة التوزيع الملوث، تُظهر (P-value) ومنحنيات حجم القوة بوضوح ميزة إحصائيات الاختبار القوية المقترحة على الإحصائيات الكلاسيكية.