P-Modules and Related Concepts

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Abstract
In this work, we introduce the concept of P-Module as a generalization of the concept Q-Module. Many characterizations and properties of P-Modules are obtained. We investigate conditions for P-Modules to be Q-Modules. Modules which are related to P-Modules are studied. Some classes of modules which are P-Modules are given. Furthermore, characterizations of P-Modules in some classes of modules are obtained.

Introduction
Throughout this paper, R will denote an associative ring with identity, and all R-modules are unitary (left) R-modules. An R-module M is called a Q-Module, if every submodule of M is a quasi-injective [12]. An R-module M is called a quasi-injective, if for each submodule N of M and each R-homomorphism from N into M can be extended to an R-homomorphism from M into M [9]. An R-module M is called a pseudo-injective, if for each submodule N of M and each R-monomorphism from N-into M can be extended to an R-homomorphism from M into M. For an R-module M, E(M) stand for the injective envelope of M. A submodule of an R-module M is called a fully invariant if $f(N) \subseteq N, for\ each\ f \in End(M)$ [18]. An R-module M is called uniform, if every submodule of M is essential in M, where we said that a submodule N of M is essential in M if $N \cap K \neq (0)$ for each submodule K of M. which is equivalent to say that $0 \neq m \in M$, there exists $0 \neq r \in R$ such that $0 \neq mr \in N$. [6].

§1 Basic properties of P-Modules
In this section, we introduce the definition of P-Module and give examples characterizations and some basic properties of this concept.

Definition 1.1
An R-module M is called a P-Module, if every submodule of M is a pseudo-injective.

Examples and Remarks 1.2
1. Every submodule of P-Module is a P-Module.
2. A direct summand of P-Module is a P-Module.
3. Z_n as a Z-module is a P-Module for every n
4. Every simple R-module is a P-Module.
5. Z_{p^∞} as a Z-module is a P-Module.
6. Z as a Z-module is not a P-Module, and Q as a Z-module is a quasi-injective, but not a P-module
7. The inverse image of a P-Module is not necessary P-Module. For example the Z-module Z_2 is a P-Module and if we let $f: Z \to Z_2$ defined by $f(x) = \begin{cases} o, & if\ x\ is\ even \\ 1, & if\ x\ is\ odd \end{cases}$
It is clear that f is Z-homomorphism and $f^{-1}(Z_2) = Z$ is not a P-Module.
8. The direct sum of two P-Modules is not necessary a P-Module. For example the Z-modules Z_2 and Z_4 are P-Modules, but $Z_2 \oplus Z_4$ is not a P-Module, (since $Z_2 \oplus Z_4$ itself is not a pseudo-injective Z-module.)
9. If M is a P-Module, then $M \oplus M$ is not necessary P-Module. For example, since Z_4 as a Z-module is a P-Module, but $Z_4 \oplus Z_4$ is not a P-Module.

Before we give the main result of this section we introduce the following lemma.

Lemma 1.3
Any fully invariant submodule of a pseudo-injective module is a pseudo-injective.

Proof
Let K be a fully invariant submodule of pseudo- injective module M, let L be any submodule of K, and $f: L \to K$ be any R-monomorphism. Since M is a pseudo injective, then there exists an R-homomorphism $g: M \to M$ such that g extend f But K is a fully invariant submodule of M, then $g(K) \subseteq K$. Let $g|K = h: K \to K$. Then for all $x \in K, h(x) = g(x) = f(x)$. That is h is extends f. Hence K is a pseudo injective. ■

Theorem 1.4
Let M be an R-module. Then the following statements are equivalent.
1. M is a P-Module.
2. M is a pseudo-injective and every essential submodule of M is a fully invariant under monomorphisms of $End_R(M)$.
3. Every essential submodule of M is a pseudo-injective.

Proof
$(1) \Rightarrow (2)$ Let N be an essential submodule of M , then N is a pseudo-injective . Let $f: M \to M$ be an R-monomorphism and $K = \{x \in N: f(x) \in N\}$, that is $K = f^{-1}(N)$. Since N is a pseudo-injective, then there exists $g: N \to N$ which extends f. Since M is a pseudo-injective, then there exists an R-homomorphism $h: M \to M$ which extend g. We claim that $(h - f)(N) = (0)$. Suppose that $(h - f)(N) \neq (0)$, then $(h - f)(N) \cap N \neq (0)$, for N is an essential submodule of M, which implies that $(h - f)(n) = l$ for some n , l in N. Thus $(h - f)(n) = l$ implies that $(g - f)(n) = l$, then $f(n) = g(n) - l \in N$. This shows that $n \in K$. so $(h - f)(n) = (0)$ which is contradicts the assumption, hence $(h - f)(N) = (0)$ implies that h(N)=f(N). But f(N)=h(N)=g(N)⊆N. then f(N)⊆N.

$(2) \Rightarrow (3)$ Let N be an essential submodule of M. Then by hypothesis N is a fully invariant under monomorphism of $End_R(M)$. Hence by Lemma 1.3 N is a pseudo-injective.
$(3) \Rightarrow (1)$ Let N be a submodule of M, then $N \oplus C$ is a pseudo-injective ,where C is a relative complement of N in M, which implies that N is a pseudo-injective [8]. Hence M is a P-Module.■

Now, we look at the injective hull of P-Module. It turns out that under certain condition it's also P-Module.

Proposition 1.5

Let M be a P-Module such that every submodule of E(M) is isomorphic to subquotient of M. Then M is a P-Module if and only if E(M) is a P-Module.

Proof $\Rightarrow$ Let N be a submodule of E(M). Then N is isomorphic to a subquotient of M. Hence by [10] N is a submodule of M. therefore N is a pseudo-injective.
$\Leftarrow$ trivial. ■

§2 Relationships between P-Modules and pseudo-injective modules

It's clear that every P-Module is a pseudo-injective, but the converse is not true (see Example and Remarks 1.2 (6).). In the following propositions, we give conditions under which pseudo-injective modules become P-Modules.
Recall that an R-module M is duo module if every submodule of M is a fully invariant [18].

Proposition 2.1

Let M be duo module. Then M is a P-Module if and only if M is a pseudo-injective.

Proof:

Let N be a submodule of M, then N is a fully invariant submodule of M. Hence by lemma 1.3 N is a pseudo-injective. Therefore M is a P-Module.■

Recall that an R-module M satisfies Baer's Criterion, if every submodule of M satisfies Baer's criterion, where we say that a submodule N of M satisfies Baer's Criterion, if for each R-homomorphism $f: N \to M$ there exists r in R such that $f(n) = rn, \forall n \in N$ [1]..■

Proposition 2.2

Let M be an R-module which satisfies Bears criterion. Then M is a P-Module if and only if M is a pseudo-injective.

Proof

Let N be a submodule of M, then N satisfies Baer's criterion. Hence N is a fully invariant submodule of M (since for each $f \in End(M)$, and fore each $n \in N, f(n) = rn \in N \text{ for some } r \in R$). Hence by lemma 1.3 N is a pseudo-injective. Therefore M is a P-Module.
Recall that a submodule N of an R-module M is annihilator, if $N = ann_M(I)$ for some ideal I of R [14]..■

Proposition2.3

Let M be an R-module in which all its submodules are annihilator. Then M is a P-Module if and only if M is a pseudo-injective.

Proof

Let N be a submodule of M, then N is an annihilator submodule. That is $N = ann_M(I)$ for some ideal I of R. We claim that N is a fully invariant submodule of M. Let $f \in End(M)$, then $0 = f(IN) = If(N). Hence\ f(N) \subseteq ann_M(I) = N$.
Thus N is a fully invariant submodule of M. Therefore by Lemma 1.3 N is a pseudo-injective. Hence M is a P-Module. .■

Proposition2.4

Let M be an R-module such that every cyclic submodule of M is fully invariant. Then M is a P-Module if and only M is a pseudo-injective.

Proof

Let N be a submodule of M. Since every cyclic submodule of M is a fully invariant in M, then for each $f \in End(M)$ and for each x in N, $f((x)) \subseteq (x) \subseteq N. Thus\ f(x) \in N$. Hence N is a fully invariant submodule of M. Thus by Lemma 1.3 N is a pseudo injective. Hence M is a P-Module.■

Recall that a submodule N of an R-module M is closed, if N has no proper essential extension. [6]

Proposition2.5

Let M be an R-module, such that every submodule of M is closed. Then M is a P-Module if and only if M is a pseudo injective.

Proof

Let N be submodule of M, then N is a closed submodule of M. Since M is a pseudo injective, then by [4, Cor.1.3] N is a direct summand of M, and by [8, Lemma 1] N is a pseudo injective. Hence M is a P-Module.■

Since a direct summand of any module is closed [6] we get the following.

Corollary 2.6

Let M be an R-module, such that every submodule of M is a direct summand. Then M is a P-Module if and only if M is a pseudo injective.
Recall that a submodule N of an R-module is quasi-stable if for every submodule K of M with K⊆N and every R-homomorphism $g: K \to M$ such that $Img \subseteq N$, then $h(N) \subseteq N$ for each R-homomorphism $h: N \to M$ such that $g = h \circ i_K$.**[1].**

Since a quasi-stable submodule inherit a pseudo injectivity[1], we get the following.

Proposition2.7

Let M be an R-module such that all submodules of M are quasi-stable. Then M is a P-Module if and only if M is a pseudo injective.

§3: Relationships between P-Modules and Q-Modules

In this section we study the relation between P-Modules and Q-Modules.
Since every quasi-injective module is a pseudo injective, but the converse is not true [9], then every Q-Module is a

P-Module but the converse is not true. Thus under certain conditions P-Module become Q-Modules.

Proposition3.1

Let M be an R-module over a principle ideal domain. Then M is a Q-Module if and only if M is a P-Module.

Proof

Let N be a submodule of M. Since M is an R-module over a principle ideal domain, then N is a submodule over a principle ideal domain. But M is a P-Module, and then N is a pseudo injective. Thus by [15, Th.3.3] N is a quasi-injective. Hence M is a Q-Module. ■

Recall that an R-module M is torsion free if $T(M) = \{m \in M : mr = 0, for\ some\ r \in R\} = (0)$.

It is given in [15, Cor. 3.9] that any torsion free module which is a pseudo injective is a quasi-injective, we get the following proposition.

Proposition 3.2

Let M be torsion free R-module. Then M is a Q-Module if and only if M is a P-Module.

Proposition 3.3

Let M be a torsion module over quasi-Dedekind ring. Then M is a Q-Module if and only if M is a P-Module.

Proof

Let N be a submodule of M, then N is a pseudo injective module over quasi-Dedekind ring. Since M is torsion module, then N is a torsion submodule. Thus by [16, Th. 2] N is a quasi-injective. Hence M is a Q-Module. ■

A ring R is called a generalized uniserial ring, if every primitive idempotent element $e \in R, eR\ (Re)$ have unique composition series as right (left) R-module.

The following proposition shows that over a generalized uniserial ring, P-Modules and Q-Modules are equivalent.

Proposition 3.4

Let M be an R-module over a generalized uniserial ring R. Then M is a Q-Module if and only if a P-Module.

Proof

Let N be submodule of M, then N is a pseudo injective submodule over a generalized uniserial ring R. Hence by [8, Th.4] N is a quasi-injective. Therefore M is a Q-Module. ■

Proposition3.5

Let M be a uniform non-singular module. Then M is a Q-Module if and only if M is a P-Module.

Proof

Let N be a submodule of M. Since M is a uniform, then N is a uniform, also , since M is a non-singular, then by [6] N is a non-singular. Let L be a submodule of N and $f: L \to N$ be an R-homomorphism, then since N is non-singular, uniform, so $Kerf = (0)$ or $Kerf = L$. If $Kerf = L$, then f can be trivially extended to a homomorphism from N into N. If $Kerf = (0)$, then f is monomorphism and from pseudo-injectivity of N, f can be extended to an R-homomorphism from N into N. hence N is a quasi-injective and then M is a Q-Module. ■

It is well-known a pseudo injective torsion module over a multiplication ring or hereditary ring is a quasi-injective [16, Cor.1].

We end this section by the following result.

Proposition 3.6

Let M be a torsion module over a multiplication ring or hereditary ring R. Then M is a Q-Module if and only if M is a P-Module.

§4 Modules imply P-Modules

In this section we establish modules which imply p-Modules. Recall that an R-module M is a semi-simple, if every submodule of M is a direct summand [6].

The following proposition shows that semi-simple modules imply P-Modules

Proposition 4.1

If M is a semi-simple R-module, then M is a P-Module.

Proof

Let N be a submodule of M, then N is a semi-simple, also let L be a submodule of N and $f: L \to N$ be an R-monomorphism. Since N is a semi-simple, then L is a direct summand of N. that is $N = L \oplus K$ for some submodule K of N. Now, we can extend f to an R-homomorphism $g: N \to N$ by setting

$$g(n) = \begin{cases} f(n), & if\ n \in L \\ 0, & if\ n \in K \end{cases}$$

This gives that N is a pseudo-injective. Hence M is a P-Module. ■

The converse of Prop. 4.1 is not true in general. In fact the Z-module Z_8 is a P-Module, but not semi-simple.

The following proposition gives a condition under which P-Modules are Q-Modules.

Proposition 4.2

If M is a P-Module such that every submodule of M is a closed, then M is a semi-simple.

Proof

Let N be a submodule of M. Then by hypothesis N is closed. Since M is a P-Module, then M is a pseudo-injective. Therefore by [4, Cor. 13] N is a direct summand of M. Hence M is a semi-simple. ■

From proposition 2.5, proposition 4.1 and proposition 4.2, we get the following result.

Proposition 4.3

Let M be an R-module such that every submodule of M is a closed. Then the following statements are equivalent.

1. M is a semi-simple module.
2. M is a P-Module.
3. M is a pseudo-injective module.

Recall that an R-module M is anti-hopfain if every proper submodule of M is a non-hopf kernel. Where, a submodule N of M is called a non-hopf kernel if there exists an isomorphism between M/N and M [7].

It is well-known that anti-hopfain module, is a quasi-injective (pseudo-injective) [2]. Also every submodule of anti-hopfain module is anti-hopfain [2] we get the following results.

Proposition 4.4

If M is an anti-hopfain R-module, then M is a P-Module.

Corollary 4.5

If M is an anti-hopfain R-module, then M/N is a P-Module for any submodule N of M.

The following proposition shows that the homomorphic image of anti-hopfain module is a p-Module.

Proposition 4.6

If M is an anti-hopfain R-module, then f(M) is a P-Module for each R-homomorphism $f: M \to M'$ Where M' is any R-module.

Proof

Suppose that M is an anti-hopfain module and $f: M \to M'$ be an R-homomorphism. Thus $M/kerf \cong f(M)$. Since M is an anti-hopfain, then by Corollary 4.5 $M/kerf$ is P-Module. Hence f(M) is P-Module. ■

§5 P-Modules and Multiplication modules

An R-module M is called multiplication module, if every submodule of M is of the form IM for some ideal I of R [3].

In this section we study the relation of multiplication modules with P-Modules.

We preface our section by the following theorem which gives the relationship between P-Modules over R and P-Modules over $End_R(M)$.

Theorem 5.1

If M is a multiplication module, then M is a P-Module over R if and only if M is a P-Module over S where$S = End_R(M)$.

Proof

$(\Rightarrow)$ Let N be S-submodule of M. Since M is a multiplication, then N is an R-submodule of M, then N is a pseudo-injective submodule of M. Hence M is a P-Module over S.

$(\Leftarrow)$ Let N be R-submodule of M . Since M is a multiplication, then by [13, Prop. 1.1] N is an S-submodule of M. Then N is a pseudo-injective submodule of M. Hence M is a P-Module over R. ■

In the following theorem we give a characterization of P-Module in class of multiplication modules.

A submodule N of an R-module M is called a quasi-invertible if $Hom\left(\frac{M}{N}, M\right) = (0)$ [11].

Theorem 5.2

Let M be a multiplication module with $ann_R(M)$ is a prime ideal of R. Then M is a P-Module if and only if every quasi-invertible submodule of M is a pseudo-injective.

Proof

$(\Rightarrow)$ Trivial..

$(\Leftarrow)$ Let N be a submodule of M. Then $N \oplus K$ is an essential submodule of M, where K is an intersection relative complement of N in M. We claim that $N \oplus K$ is a quasi-invertible submodule of M. Let $f \in Hom(M/N \oplus K, M), f \neq 0$. Thus, there exists an element $m + (N \oplus K) \in M/N \oplus K$ such that $f(m + (N \oplus K)) = y \neq 0, y \in M$. Since $N \oplus K$ is an essential submodule of M, then there exists a non zero element r in R such that $rm \neq (0) \in N \oplus K. Hence\ 0 = rf(m + N \oplus K) = ry$ and hence $r \in ann_R(y)$ .Since M is multiplication module then by[5, Prop.1] $Ry = IM$ for some ideal I of R. Thus $0 = rIM$ and hence $rI \subseteq ann_R(M)$. Since $ann_R(M)$ is a prime ideal of R, then either $I \subseteq ann_R(M)$ or $r \in ann_R(M)$. If $I \subseteq ann_R(M), then\ IM = Ry = (0)$ and hence $y = 0$, and this is a contradiction. If $r \in ann_R(M)$, then $rm = 0$, for all m in M, this is a contradiction a gain. Thus $f \in Hom(M/N \oplus K, M)$ must be zero. Hence $Hom(M/N \oplus K, M) = (0)$, which implies that $N \oplus K$ is a quasi-invertible submodule of M. Then by hypothesis $N \oplus K$ is a pseudo-injective submodule of M. Hence by [8, lemma1] N is a pseudo-injective submodule of M. Therefore M is a P-Module. ■

As an immediate consequence of Th.5.2 we have the following result.

An R-module M is prime if, $ann_R(M) = ann_R(N)$ for every R-submodule N of M.

Corollary 5.3

Let M be a prime multiplication module. Then M is a P-Module if and only if every a quasi-invertible submodule of M is a pseudo-injective.

Proposition 5.4

If M is a pseudo-injective multiplication module, then M is a P-Module.

Proof

Let N be a submodule of M. Since M is a multiplication module then $N = IM$ for some ideal I of R. Let $f \in End_R(M)$, then $f(N) = f(IM) = If(M) \subseteq IM = N$. Hence N is a fully invariant submodule of M. Since M is a pseudo-injective, therefore by lemma 1.3 N is a pseudo-injective. Thus M is a P-Module. ■

The following corollary is an immediate consequence of Prop. 5.4.

Corollary 5.5

If M is a cyclic pseudo-injective R-module, then M is a P-module.

§6 Characterizations of P-Modules in some types of modules.

Definition 6.1

An R-module M is called a pseudo-duo module, if every submodule of M is a fully invariant under monomorphisms of $End_R(M)$.

Proposition 6.2

Let M be a uniform module, then M is a P-Module if and only if M is a pseudo-injective and pseudo due module.

Proof

$(\Rightarrow)$ Since M is a P-Module, then M is a pseudo-injective. Let N be a submodule of M. Since M is a uniform module, then N is essential submodule of M. Hence by Theorem 1.4 N is a fully invariant under monomorphisms of $End_R(M)$. Therefore, M is a pseudo-duo module.

$(\Leftarrow)$ Let N be a submodule of M. Since M is a uniform module, then N is an essential submodule of M. And since M is pseudo-duo module, then N is fully invariant under a monomorphism $End_R(M)$. Now, every essential submodule is fully invariant under monomorphism of $End_R(M)$. Hence by Theorem 1.4 M is a P-Module. ■

Recall that an R-module M is a monoform, if every non-zero homomorphism $f \in Hom(N, M)$(where N is any submodule is a monomorphism [17].

It is well-known that a monoform module is a uniform we get the following immediate consequence of prop. 6.2.

Corollary 6.3

Let M be a monoform module. Then M is a P-Module if and only if M is a pseudo-injective and pseudo-duo.

Recall that an R-module M is a rational extension of an R-submodule N of M, provided that $Hom_R\left(\frac{K}{N}, M\right) = (0), whenever\ N \subseteq K \subseteq M$. [6]

Proposition 6.4

Let M be a rational extension of every submodule of M. Then M is a P-Module if and only if M is a pseudo-injective and pseudo-duo module.

Proof

$(\Rightarrow)$ Since M is a P-Module, then M is a pseudo-injective module. Let N be a submodule of M. Since M is a rational extension of N, then clearly is an essential submodule of M, then by Theorem 1.4 N is a fully invariant under monomorphisms of $End_R(M)$. Hence M is a pseudo-duo module.

$(\Leftarrow)$ Let N be a submodule. Since M is a rational extension of N, then N is an essential submodule of M. And since M is a pseudo-duo module, then N is a fully invariant under a monomorphisms of $End_R(M)$. Hence by Theorem 1.4 M is a P-Module. ■

Recall that a submodule N of an RF-module M is a quasi-invertible if $Hom_R\left(\frac{M}{N}, M\right) = (0)$[11]. And the submodule N of an R-module M is dense in M if, for every $y \in M\ and\ x \in M, xy^{-1}N \neq (0)$[6].

The following theorem gives many characterization of P-Module in class of a non-singular modules.

Theorem 6.5

Let M be a non-singular R-module. Then the following statements are equivalent.

1. M is a P-Module.
2. Every a quasi-invertible submodule of M is a pseudo-injective.
3. Every dense submodule of M is a pseudo-injective.

Proof

$(1) \Rightarrow (2)$ Trivial.

$(2) \Rightarrow (3)$ Let N be a dense submodule of M. Since M is a non-singular, then by [10] N is an essential submodule of M. We claim that N a quasi-invertible submodule of M. Let $g \in Hom_R\left(\frac{M}{N}, M\right), g \neq 0$, thus there exists $x \in M$ such that $g(x+N) = m \neq 0, where\ m \in M$. Let $r \in R\ and\ r \notin ann(m)$. Hence $rm \neq 0\ and\ rx \notin N$. Since N is an essential submodule of M, then there exists a non-zero element $s \in R\ such\ that\ srx$ is a non-zero element of N. thus $0 = g(srx+N) = srg(x+N) = srm$, this implies that $sr \in ann(m)$. Therefore $ann(m)$ is an essential ideal of R. Since M is non-singular, then $m = 0$ and hence $g = 0$. Therefore $Hom_R(M/N, M) = (0)$ which implies that N is a quasi-invertible submodule of M. Hence by hypothesis N is a pseudo-injective.

$(3) \Rightarrow (1)$ Let N be a submodule of M, then $N \oplus K$ is an essential submodule of M (where K is the relative intersection complement.) Since M is non-singular, then by [10] $N \oplus K$ is dense submodule of M. Thus by hypothesis $N \oplus K$ isa pseudo-injective submodule of M. Hence by [8] N is a pseudo-injective submodule of M. Therefore M is a P-Module. ■

Before we give the last result of this suction, we introduce the following lemma

Lemma 6.6 [15, Th. 4.3]

For any pseudo-injective module M if, $S = End_R(M)$, then $J(S) = \{\alpha \in S: Ker\alpha\ is\ essential\ in\ M\}$

Theorem 6.7

Let M be an R-module such that $J(End_R(M)) = (0)$ then M is a P-Module if and only if M is a pseudo-injective and every quasi-invertible submodule of M is a pseudo-injective.

Proof

$(\Rightarrow)$ Trivial.

$(\Leftarrow)$ Let N be a submodule of M, then $N \oplus K$ is essential submodule of M (where K is the relative intersection complement of N). We claim that $N \oplus K$ is a quasi-invertible submodule of M. Let $g \in Hom_R(M/N \oplus K, M)\ and\ g \neq 0$. Define $f = g \circ \pi$ where $\pi: M \to M/N \oplus K$ a natural homomorphism is. Hence $f \in End_R(M)$ and $f \neq 0\ and\ N \oplus K \subseteq ker f$. Since $N \oplus K$ is an essential submodule of M, then $Kerf$ is essential submodule of M. Since M is a pseudo-injective, then $f \in J(End_R(M))$ and $f = 0$, this implies that $g = 0$, this is a contradiction. Therefore $Hom_R(M/N \oplus K, M) = (0)$, and hence $N \oplus K$ is a quasi-invertible submodule of M. Thus by hypothesis $N \oplus K$ is a pseudo-injective submodule of M. Hence by [8] N is a pseudo-injective. Thus M is a P-Module.

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ألمقاسات من النمط P ومفاهيم اخرى

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الملخص

قدمنا في هذا البحث مفهوم جديد سمي المقاسات من النمط–P كتعميم للمقاسات من النمط–Q. العديد من التشخيصات والصفات لهذا المفهوم وجدت. المقاسات التي لها علاقة مع المقاسات من النمط–P درست. فضلاً عن ذلك تشخيصات اخرى للمقاسات من النمط–P في بعض اصناف المقاسات وجدت.