Sensorless Speed Estimation of Permanent Magnet Synchronous Motor Using Extended Kalman Filter

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Abstract— *This paper presents a new approach in the modeling and the design of sensorless control method for the Permanent Magnet Synchronous Motor (PMSM) based on the use of the Extended Kalman Filter (EKF). The fourth order EKF is proposed to estimates the states: motor speed ω, rotor angular position, θ and the currents from the measured currents and voltages. The linearization of the nonlinear PMSM model is done in the α_β coordinate system to perform the operation of the EKF.The three phases Space Vector Pulse Width Modulation (SVPWM) inverter usually implemented as a part of the Field Oriented Control (FOC) strategy which has become a standard in the control of PMSM.*

Index Terms— *PMSM, EKF,SVPWM, and fuzzy like PI controller.*

I. INTRODUCTION

The PMSM now adays have a great use in the high performance applications because these motors offer many advantages such as high efficiany , high power density , reliablity, low weight and high torque density,some of these applications are Elevators, Electric road vhichles , Pumps, and Power Factor Control.

This FOC method is a more advanced method which gives a fast transient response and a good characteristics performance.

Since we are not using the position sensors in the vector control strategy some methods such as EKF have been developed to estimate the rotor position and the motor speed. The tune of the covariance matrices of the EKF parameters can be adjusted by the trial-and-error procedures. The state equations are demonstrated by normalized state vectors based on the PMSM model.

The EKF is based on the linearizing of the state and the output equations at each sampling instant around the most recent state estimate. For this reason, this type of estimator requires computation of the Jacobian matrices.

The Vector control system structure of PMSM is shown in *Fig.*1 includes Fuzzy like PI controller (FLC), SVPWM inverter, EKF, and PMSM model.

The motor adopts the FOC method with closed-loop speed control scheme which compares the reference speed $\boldsymbol{\omega_r}^*$ with the estimated speed $\widehat{\boldsymbol{\omega_r}}$, the resulting speed error (e) is fed into FLC that is designed by using Mamdani type . It has two inputs e and its rate $\dot{e}$ and one output. The inputs are defined as follows:

$$\boldsymbol{e(k)} = \boldsymbol{\omega_r}^* - \widehat{\boldsymbol{\omega}}_r \qquad\qquad (1)$$
$$\dot{\boldsymbol{e}}(\boldsymbol{k}) = \boldsymbol{e(k)} - \boldsymbol{e(k-1)} \qquad\qquad (2)$$

The organization of this paper is as follows. Section II presents the Simulink model of the SVPWM inverter. Section III presents the structure of PMSM with mathematical model. In Section IV, we introduce the state estimation with EKF. In Section V, we show the fuzzy like PI controller design. Section VI introduces the simulation results of PMSM control system. Finally, we present meaningful conclusion.
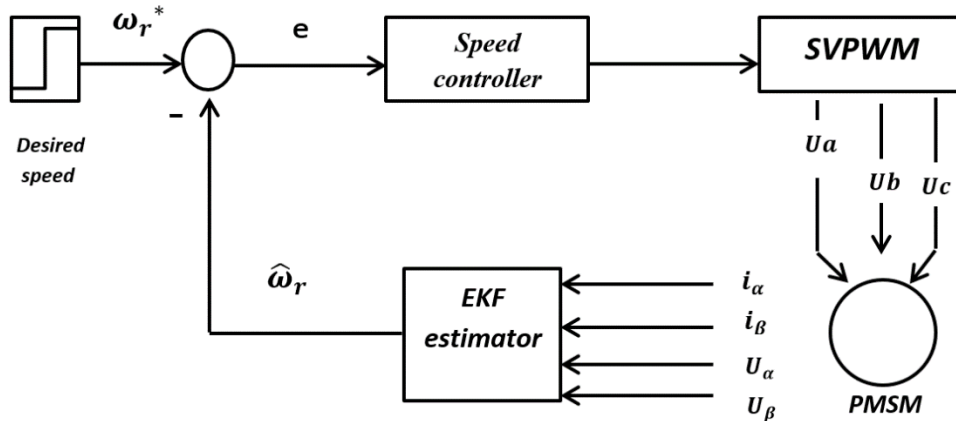

FIG. 1. BLOCK DIAGRAM OF FOC SCHEME BASED ON PMSM DRIVE.

II. SVPWM INVERTER:

The Pulse Width Modulation(PWM) strategies are used to generate sinsusoids of variable voltage and frequency.

The SVPWM is a better technique of the PWM implemention, it has been used as a three-phase voltage source PWM inverter feeding the PMSM . It is often used in the FOC strategy, it has 8 state vectors. The area between these vectors is devided into 6 sectors[1]. *Fig.* 2 shows these vectors and sectors[2].

The SVPWM technique can be implemented in three steps:
1. Determining the reference vecter (V_{ref}) and its angle (α) from the direct and quadratic vecters (V_α , V_β).
2. Determining the switching time duration T_0, T_1, and T_2 .
3. Determining the switching time S_1 to S_6.

Fig. 3 shows the simulation model of the SVPWM technique were the overall model , the model for the switching time and sectors are presented .
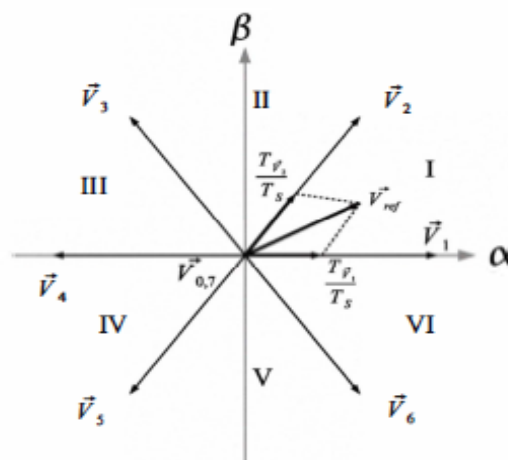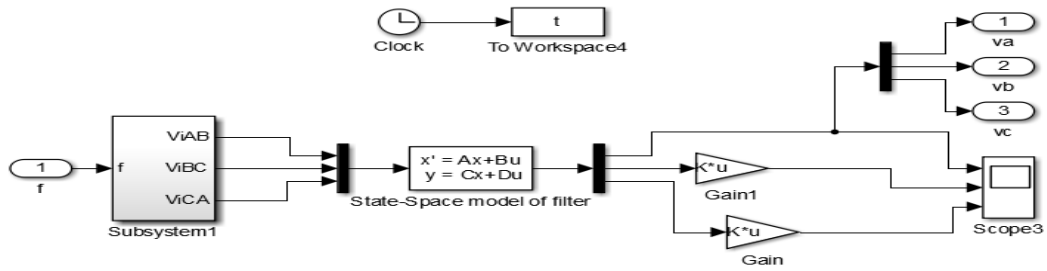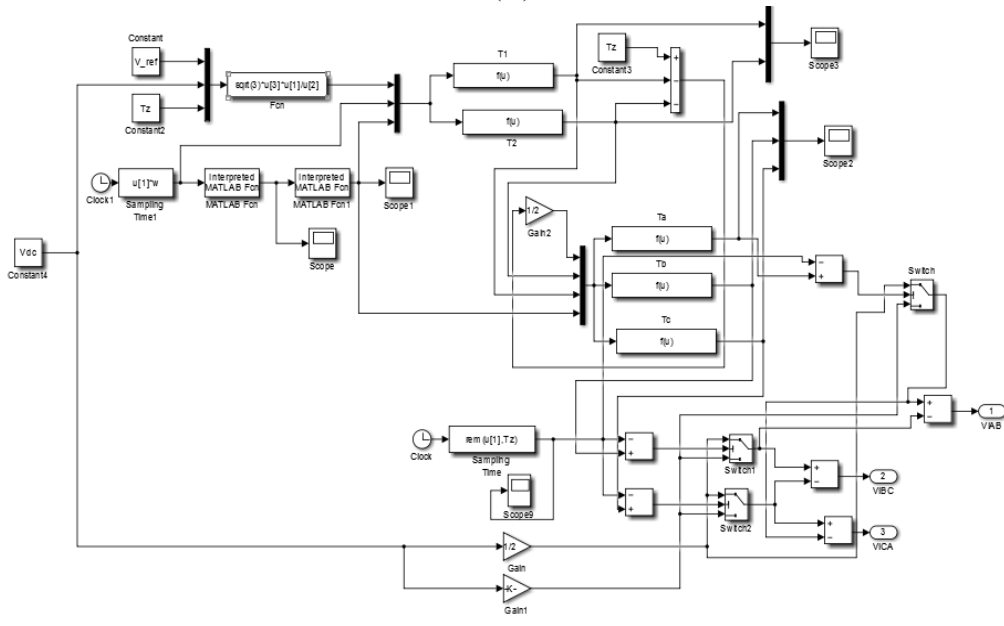

FIG. 2. THE SVPWM BASE VECTORS AND SECTORS IN αβ FRAME.

(A)

(B)

FIG. 3. A) THE SVPWM SIMULATION MODEL.
B) THE SIMULATION INSIDE (SUBSYSTEM1) BLOCK.

III.   MATHEMATICAL MODEL OF PMSM

The voltage equations of the fourth order Three-phase PMSM in the rotating reference frame (d-q reference frame) are derived from the equivalent circuit below[3]:
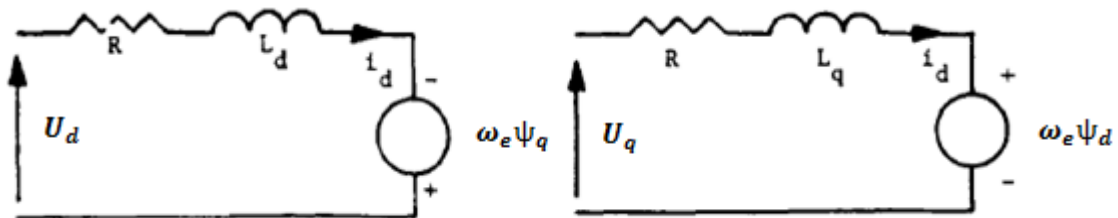

FIG. 4. PMSM EQUIVALENT CIRCUIT.

$$U_d = R_s i_d + \frac{d\psi_d}{dt} - \omega_e \psi_q \qquad\qquad (3)$$

$$U_q = R_s i_q + \frac{d\psi_q}{dt} + \omega_e \psi_d \qquad\qquad (4)$$

Where

$$\psi_d = L_d\, i_d + \psi_f \tag{5}$$

and

$$\psi_q = L_q\, i_q \tag{6}$$

Rearrange these two voltage equations (3) and (4) , the axis currents , electromagnatic torque, angular speed are given by[4]:

$$T_e = \frac{3}{2}\frac{N_p}{2}\left[\psi_f\, i_q + (L_d - L_q)i_d i_q\right] \tag{7}$$

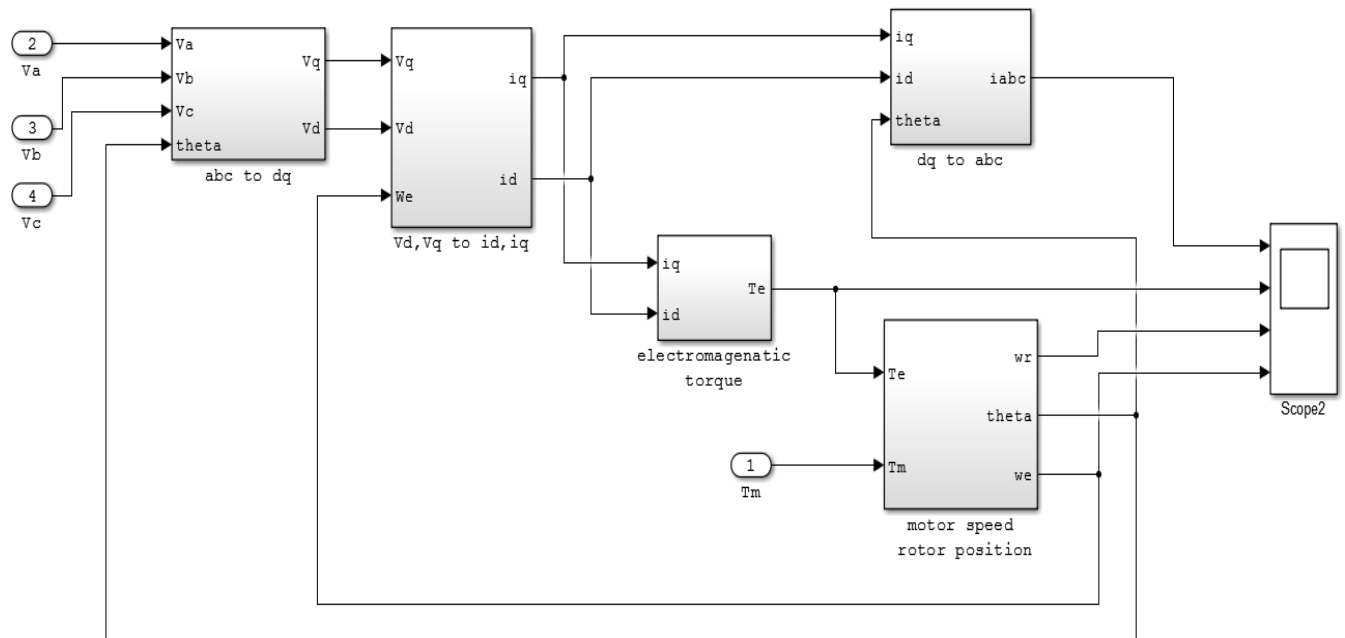$$\frac{di_d}{dt} = -\frac{R_s}{L_d}i_d + \omega_e\frac{(L_q i_q)}{L_d} + \frac{1}{L_d}U_d \tag{8}$$

$$\frac{di_q}{dt} = -\frac{R_s}{L_q}i_q - \omega_e\frac{(L_d i_d + \psi_f)}{L_q} + \frac{1}{L_q}U_q \tag{9}$$

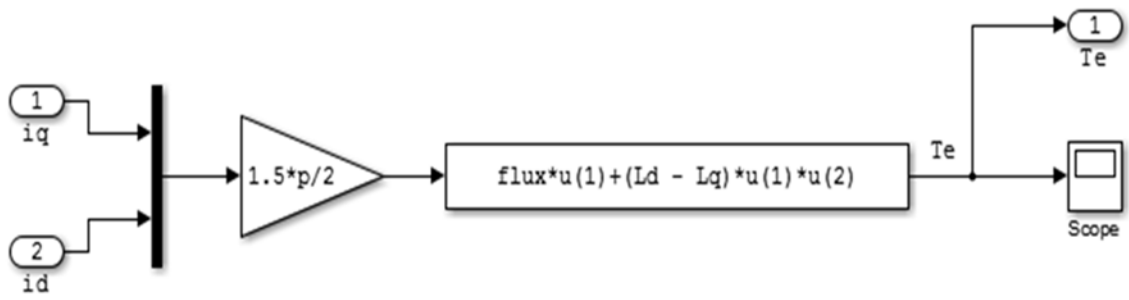$$\frac{d\omega_r}{dt} = \frac{T_e - T_m - F\omega_r}{J} \tag{10}$$

$$\omega_e = \frac{N_p}{2}\omega_r \tag{11}$$

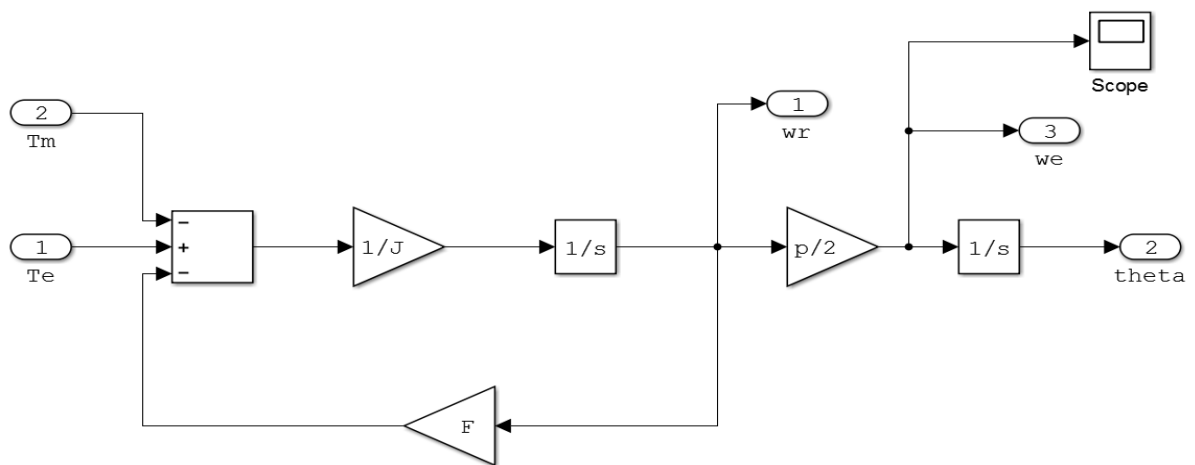From equations (7) - (11) the PMSM Simulink model is developed, and it is shown in *Fig*. 5.
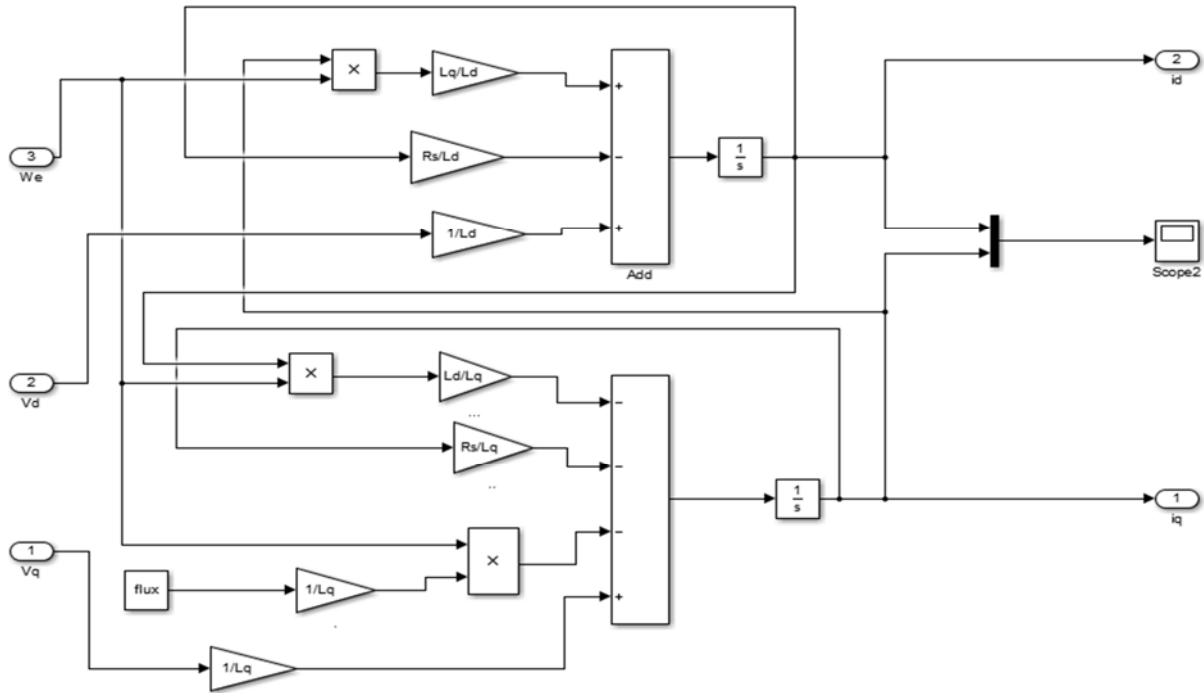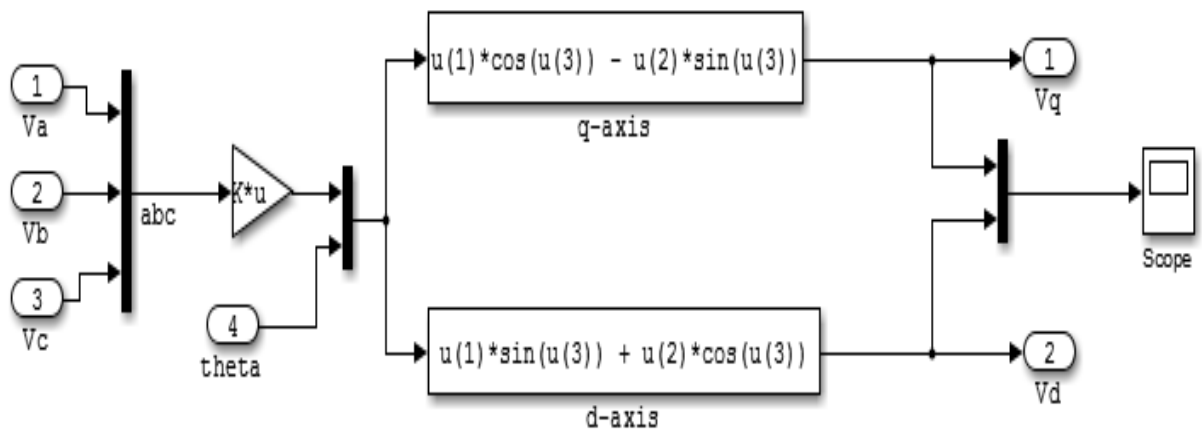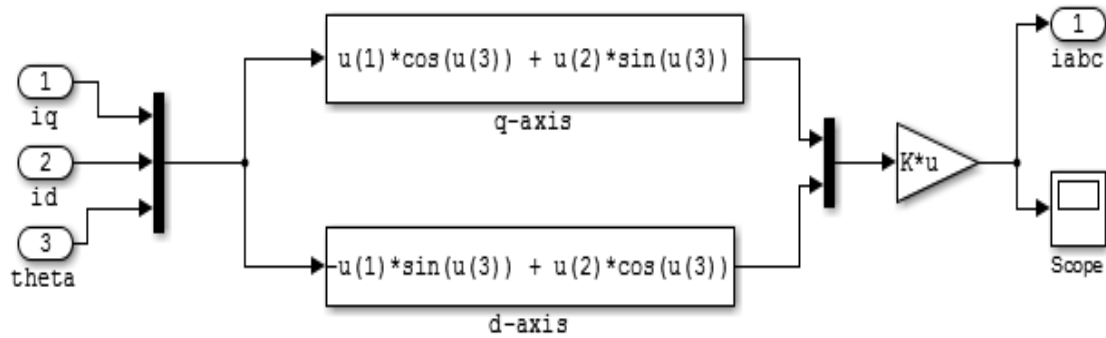
(a)

(b)

(c)

(d)

(e)

(f)

FIG 5. THE SIMULATION MODEL OF THE PERMANENT MAGNET SYNCHRONOUS MOTOR (PMSM)
(a) THE OPEN LOOP MODEL.
(b)) INSIDE THE (ELECTROMAGNETIC TORQUE) BLOCK.
(c) INSIDE THE (MOTOR SPEED AND ROTOR POSITION) BLOCK.
(d)INSIDE THE(V_d, V_q TO i_d , i_q) BLOCK.
(e) INSIDE THE (abc TO dq) BLOCK.
(f) INSIDE THE (dq TO abc) BLOCK.

IV.   STATE ESTIMATION WITH EKF:

The model of the PMSM in the stationary reference frame (α_β) is derived from the voltage equations below **[5]**and**[6]**:

$$U_\alpha = R_s i_\alpha + L_s \frac{di_\alpha}{dt} - \omega_r \psi_f N_p \, sin(\theta_e) \qquad (12)$$

$$U_\beta = R_s i_\beta + L_s \frac{di_\beta}{dt} + \omega_r \psi_f N_p \, cos(\theta_e) \qquad (13)$$

From these two voltage equations (12) and (13)   the stator current , the mechanical speed and the electical position are:

$$\frac{di_\alpha}{dt} = -\frac{R_s}{L_s} i_\alpha + \omega_r \frac{\psi_f N_p}{L_s} sin(\theta_e) + \frac{1}{L_s} U_\alpha \qquad (14)$$

$$\frac{di_\beta}{dt} = -\frac{R_s}{L_s} i_\beta - \omega_r \frac{\psi_f N_p}{L_s} cos(\theta_e) + \frac{1}{L_s} U_\beta \qquad (15)$$

$$\frac{d\omega_r}{dt} = 0 \qquad (16)$$

$$\frac{d\theta_e}{dt} = \omega_r \qquad (17)$$

From the equations ('4)-(17), we have the state equation and output equation of PMSM in equations (18) and (19):

$$\frac{dx}{dt} = Ax + Bu \qquad (18)$$

$$y = Hx \qquad (19)$$

Defining

$$x = \begin{bmatrix} i_\alpha \\ i_\beta \\ \omega_r \\ \theta_e \end{bmatrix}, \quad U = \begin{bmatrix} U_\alpha \\ U_\beta \end{bmatrix}, \quad Y = \begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix},$$

$$A = \begin{bmatrix} -\frac{R_s}{L_s} & 0 & \frac{\psi_f N_p}{L_s}sin(\theta_e) & 0 \\ 0 & -\frac{R_s}{L_s} & \frac{\psi_f N_p}{L_s}cos(\theta_e) & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & N_p & 0 \end{bmatrix}, \quad B = \begin{bmatrix} \frac{1}{L_s} & 0 \\ 0 & \frac{1}{L_s} \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad H = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}.$$

The discrete equations of (18) and (19) can be written as [5]:

$$\tilde{x}_k = f(\hat{x}_{k-1}, u_{k-1}) \qquad (20)$$
$$y_k = h(x_k) \qquad (21)$$

Where

$$f(\tilde{x}_{k-1}, u_{k-1}) = (I + A.T)\hat{x}_{k-1} + B.T.u_{k-1}$$

$$y_k = H.\hat{x}_{k-1}$$

The system matrix can be defined as:

$$\Phi_k = \frac{\partial f}{\partial \hat{x}_{k-1}}$$

$$\Phi_k = \begin{bmatrix} 1 - T\frac{R_s}{L_s} & 0 & T\frac{\psi_f N_p}{L_s}sin(\theta_e) & T\frac{\psi_f N_p}{L_s}\omega_r cos(\theta_e) \\ 0 & 1 - T\frac{R_s}{L_s} & -T\frac{\psi_f N_p}{L_s}cos(\theta_e) & T\frac{\psi_f N_p}{L_s}\omega_r sin(\theta_e) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & T & 0 \end{bmatrix}$$

The flow chart as shown in *Fig*. 6 Represents the computing cycle of the Extended Kalman Filter which consist of two steps (Prediction and Correction) with the equations of these two steps where the Extended Kalman   Filter model is built based on these equations with the appropriate Values of Q and R to obtain the desired stable and accurate estimated results.
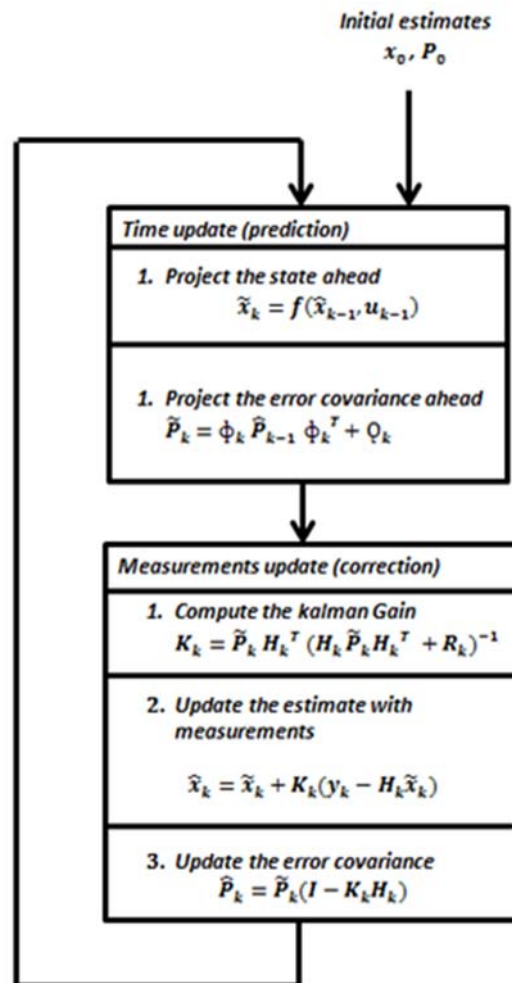
Initial estimates
$$x_0, P_0$$

Time update (prediction)

1. Project the state ahead
$$\tilde{x}_k = f(\hat{x}_{k-1}, u_{k-1})$$

1. Project the error covariance ahead
$$\tilde{P}_k = \phi_k \hat{P}_{k-1} \phi_k{}^T + Q_k$$

Measurements update (correction)

1. Compute the kalman Gain
$$K_k = \tilde{P}_k H_k{}^T (H_k \tilde{P}_k H_k{}^T + R_k)^{-1}$$

2. Update the estimate with measurements
$$\hat{x}_k = \tilde{x}_k + K_k(y_k - H_k\tilde{x}_k)$$

3. Update the error covariance
$$\hat{P}_k = \tilde{P}_k(I - K_k H_k)$$

FIG **6**. THE FLOW CHART OF AN EKF ESTIMATOR.

V.   FUZZY LIKE PI CONTROLLER DESIGN:

The structure of the FLC[7] is shown in *Fig*. 7 and the base rules required for computing the output are listed in TABLE 1. The output of the controller represents the frequency which indicates the speed of the PMSM. A trial and error technique is used to tune the controller gains (Kp,Kd) to obtain the desired performance.
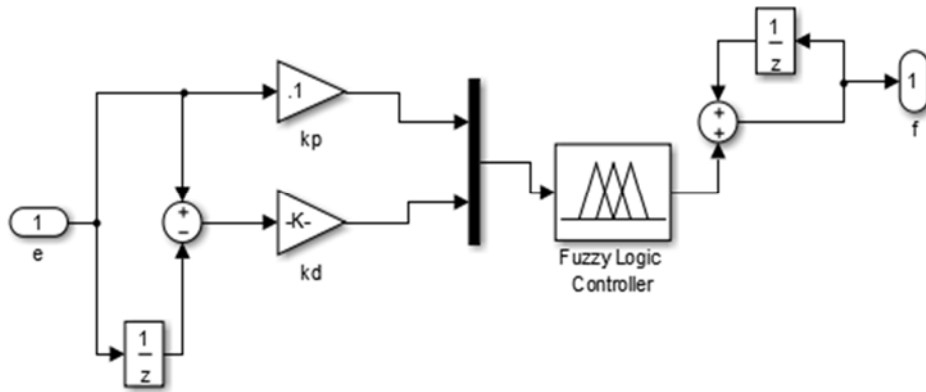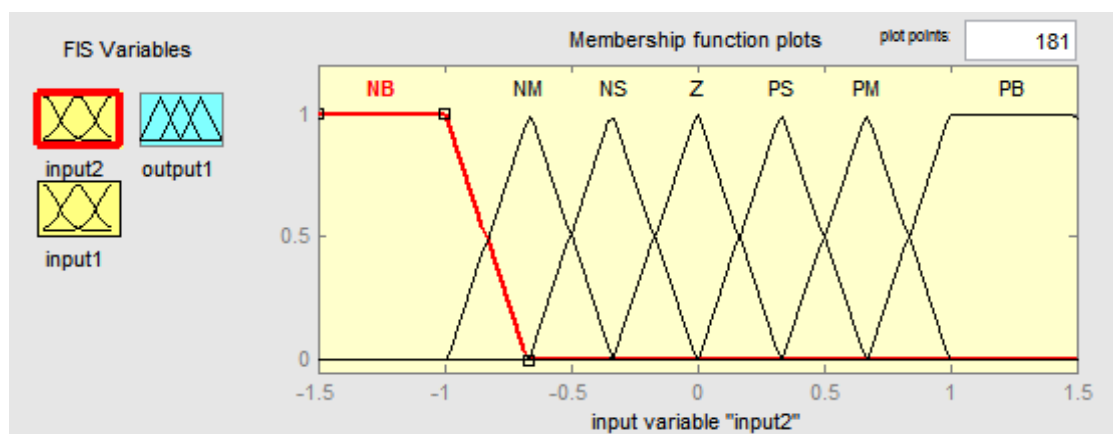
FIG. 7. THE STRUCTURE OF THE FUZZY LOGIC CONTROLLER.

TABLE .1. The base rules of the FLC.

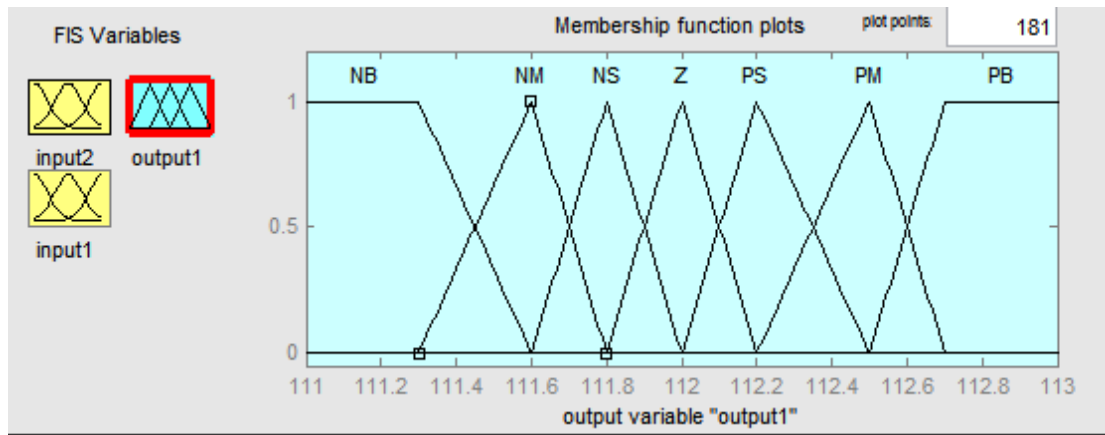| e / ė	NB	NM	NS	Z	PS	PM	PB
NB	PB	PB	PB	PB	PM	PS	Z
NM	PB	PB	PB	PM	PS	Z	NS
NS	PB	PB	PM	PS	Z	NS	NM
Z	PB	PM	PS	Z	NS	NM	NB
PS	PM	PS	Z	NS	NM	NB	NB
PM	PS	Z	NS	NM	NB	NB	NB
PB	Z	NS	NM	NB	NB	NB	NB

a)

b)

FIG. **8**. a) MEMBERSHIP FUNCTIONS FOR INPUT VARIABLES e AND $e^{\bullet}$.

b) MEMBERSHIP FUNCTIONS FOR OUTPUT VARIABLE.

VI.   SIMULATION RESULTS

To control the speed of a three phase Permanent Magnet Synchronous Motor closed loop system using the Field Oriented Control strategy the simulation model is established with Matlab/Simulink program and parameters used in this model are shown in Table.2.[4]

TABLE 2. PARAMETERS OF PMSM

Parameters	Values	Units
Stator Resistance (R_s)	2.875	Ω
Inductance (L_d=$L_q = L_s$)	0.0085	H
Induced Flux (ψ_f)	0.175	Wb
Friction Factor (F)	*0*	none
Inertia (J)	*0.0008*	$Kg * m^2$
Pairs of poles (Np)	4	none

The initialization required for the initial state and state error covariance matrix are x=[0;0;0;0], P=diag([1 1 100 10]);. Varying the value of the covariance matrix P may affect either the system performance or the system steady state condition. In this research, the values of the covariance matrix tuned manually by running several simulations, and this is one of the major drawbacks of the Kalman filtering. Another drawback and critical step in the EKF is to search for the best value of covariance

matrices Q and R with out making the estimates too noisy , the values set up was by trail-error procedure.

Q= diag([20 20 0.001 0.11]);

R = diag([0.5 0.5]) ;

In the EKF algorithm the kalman gain calculation depends on P, Q and R, where Q represents the covariance matrix of system noise and R represents the covariance matrix of the observation noise.The values of $q_{11}, q_{22}, q_{33}, q_{44}$ in Q=diag($[q_{11}; q_{22}; q_{33}; q_{44}]$) affects the performance of the EKF.

   Fig. 9, *Fig*. 10 and *Fig*. 11 show the responses of the measured and estimated state variables (the angular speed, rotor position, and stator currents) via the EKF for step reference of 176 rad/s with a load torque applied at time t = 0.3s.

 As we can see the estimated states are capable of tracking the actual one and they meet the requirements of our control system. It's very important to know that the sampling rate for the limitations of the EKF is 1ms for measurements. If the sample time decreases, the EKF estimate will be reversed and the EKF becomes unstable.
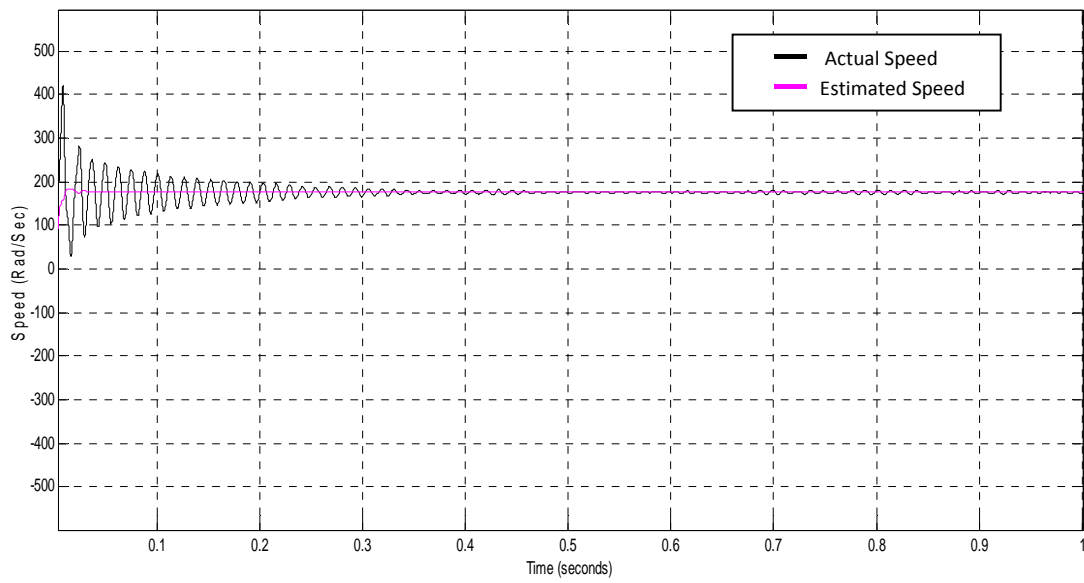
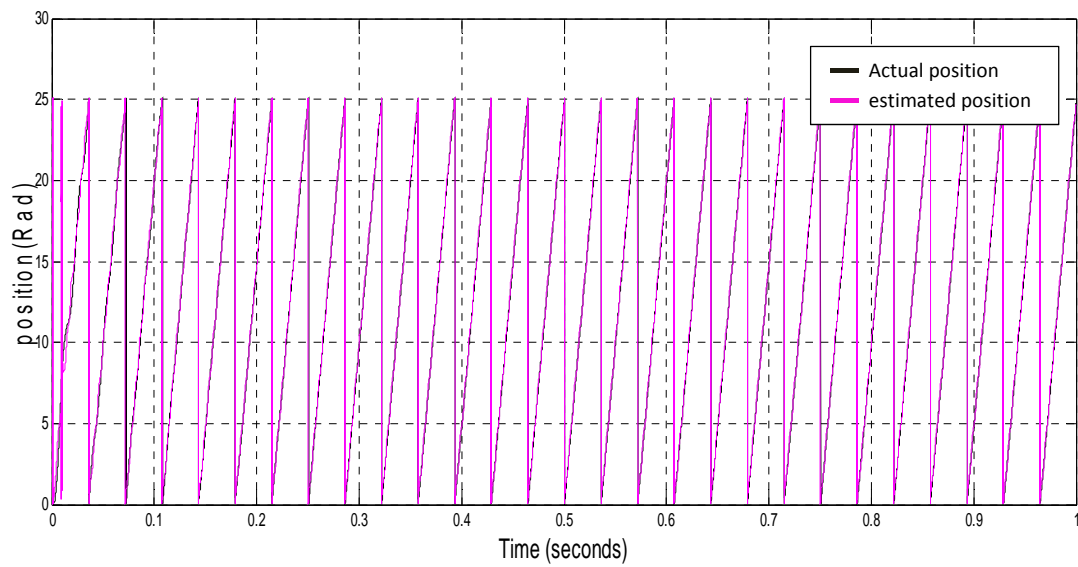FIG. 9.   THE SIMULATION RESULTS OF THE SPEED.
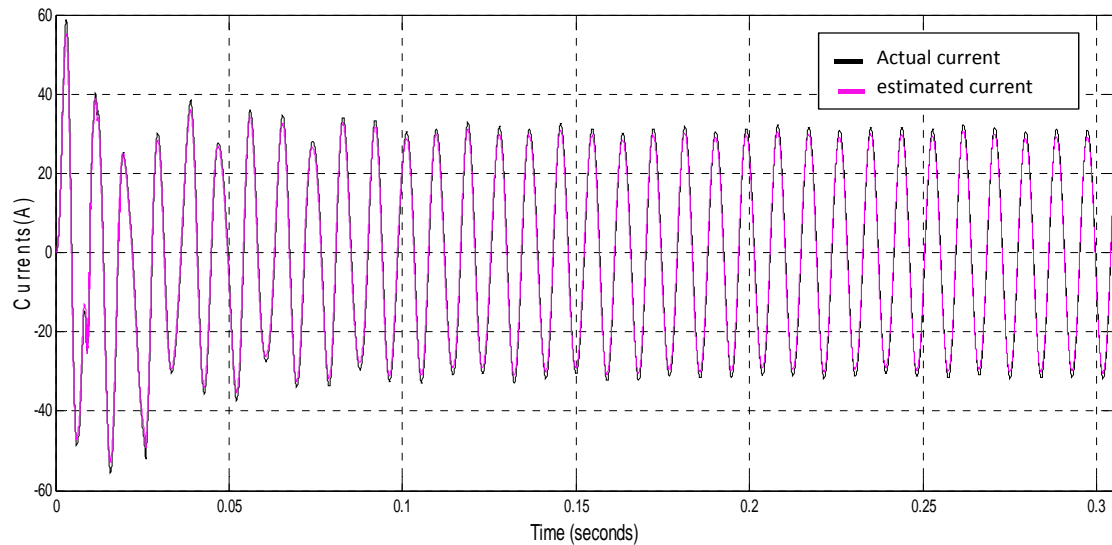

FIG. 10. THE SIMULATION RESULT OF ROTOR POSITION.

FIG. 11. THE SIMULATION RESULTS OF THE CURRENTS (αβ CURRENTS).

The estimation of the Kalman gain tells how much we need to change the estimate with the given measurement.
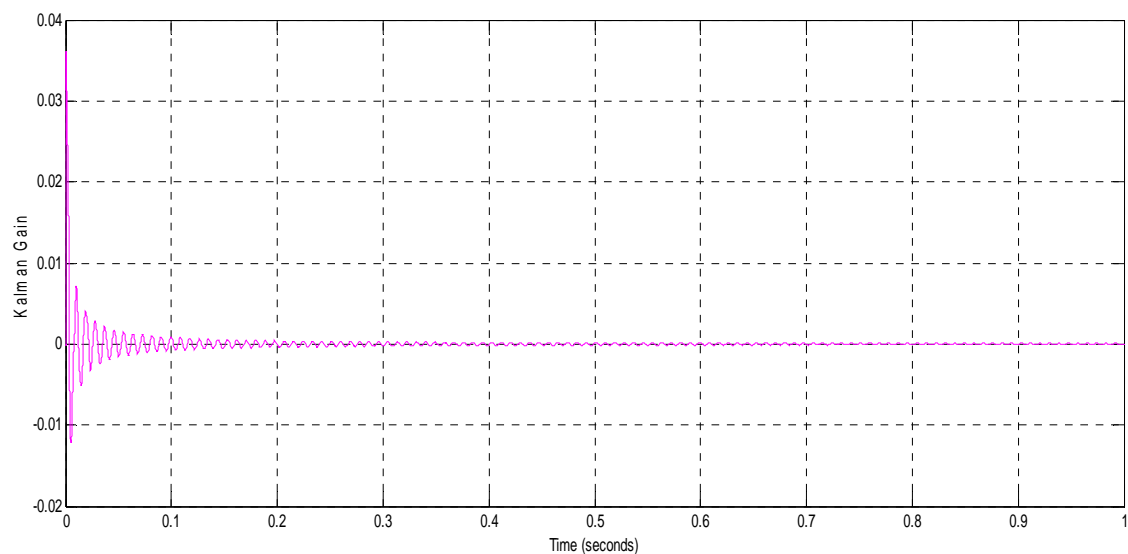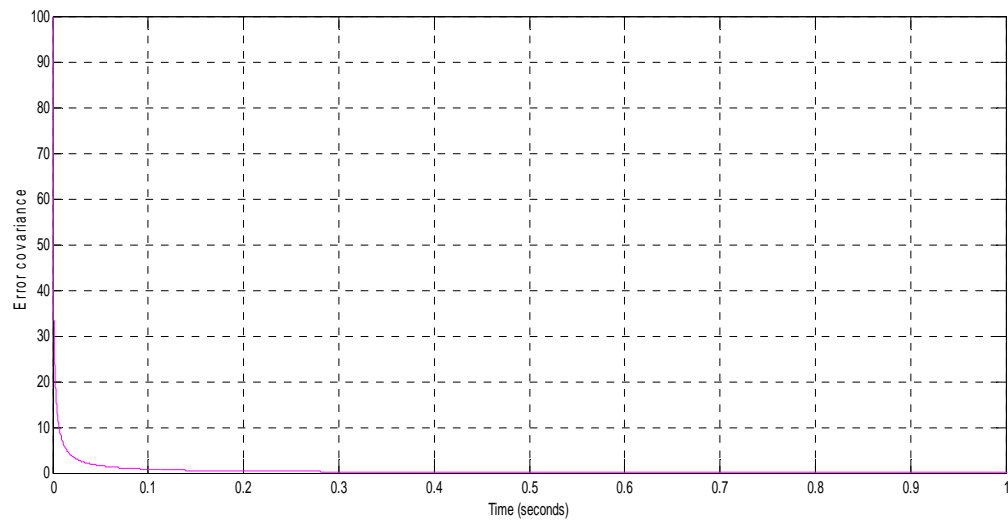

FIG. 12. THE SIMULATION RESULT OF THE KALMAN GAIN FOR THE SPEED.

The results of the error covariance indicates the accuracy of the Extended Kalman Filter as an estimator as we see the error decreases gradually   untile it becomes zero or a small value near to zero. If the covariance   is not as expexted this means that there is some thing wrong with the filter.

a)

b)

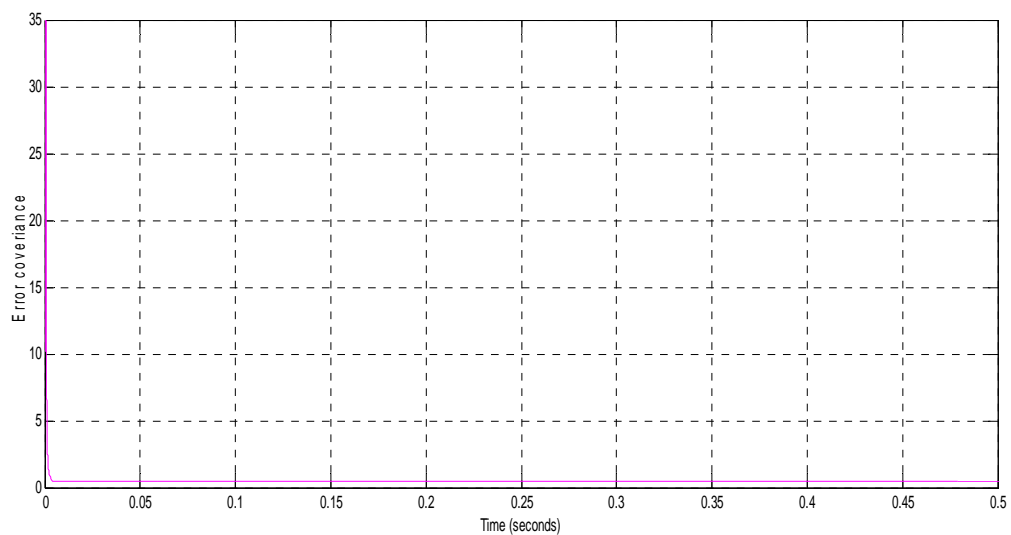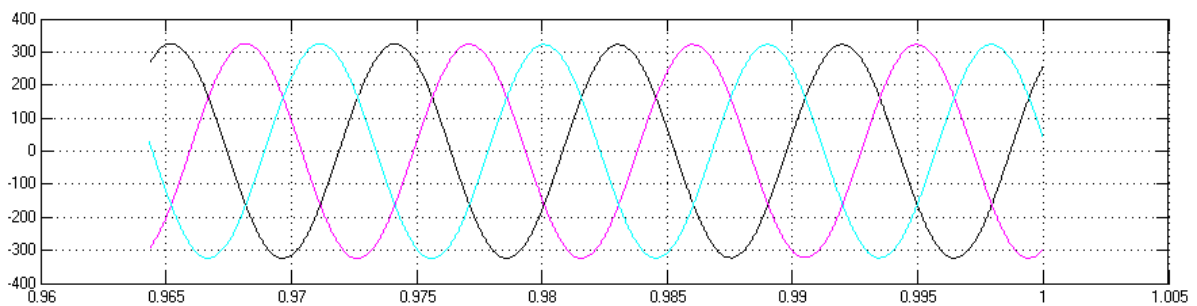
FIG.13. THE SIMULATION RESULT OF THE ERROR COVARIANCE
a) THE SPEED ERROR COVARIANCE.
b) THE POSITION ERROR COVARIANCE.

a)

b)
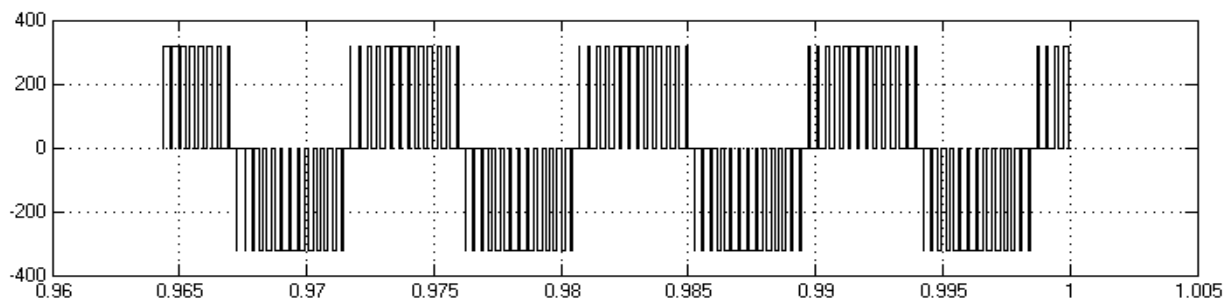
c)

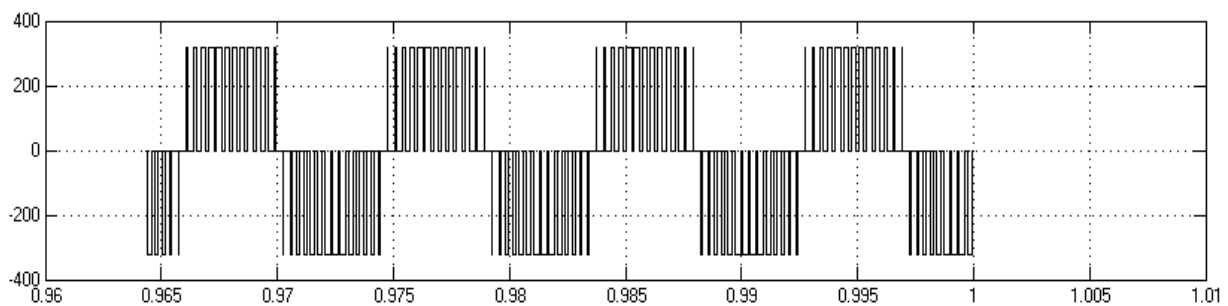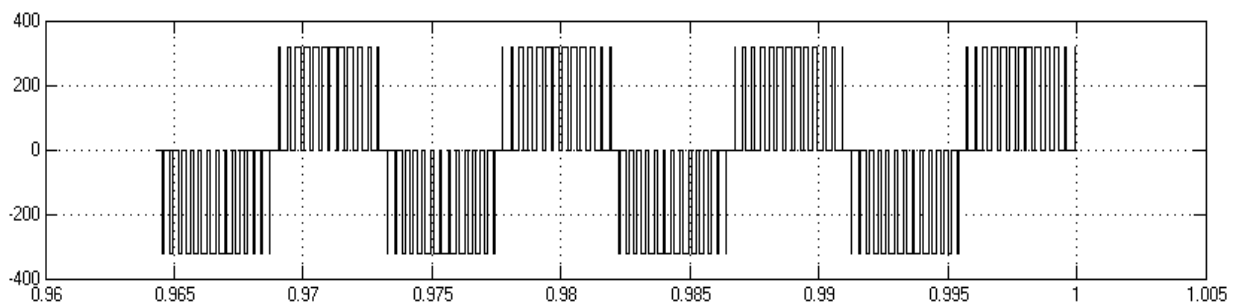**d)**

FIG. 14. SIMULATION WAVEFORMS OF SVPWM INVERTER.
a) THE THREE PHASE VOLTAGE (Va, Vb, Vc).
b) INVERTER OUTPUT V_{iAB} BEFORE FILTERING.
c) INVERTER OUTPUT V_{iBC} BEFORE FILTERING.
d) INVERTER OUTPUT V_{iCA} BEFORE FILTERING.

VII.   CONCLUSION

The performance of the sensorless speed estimation of the PMSM using FOC strategy with an EKF was studied. When the load torque applied at time t = 0.3s the speed is disturbed and then it becomes stable, this is due to the nature of the motor that runs with synchronous speed N_s only. The estimated speed $\widehat{\omega}_r$ by the EKF by using measurements of the motor currents only is compared with reference speed ω_r^* the resulting speed error (e) is fed into FLC with seven triangular membership functions to produce the frequency required for the PMSM. The PWM voltage source inverter generates the three phase voltages with a phase shift of 120° and 320 volt amplitude. The simulation results showed the feasibility of the filter for the estimation of the PMSM state vector also its robustness against the noise through reducing the error to minimum values and this results the great compatibility between the estimated and the actual states. Several simulation results have been presented such as the kalman gain and the error covariance for the speed and position to show the effectiveness of the Estimator the most important thing is knowing the values of Q and R whenever they were accurate this indicates the filter effectiveness.

NOMENCLATURE

symbol	Definition	Units
F	Friction coefficient	None
f	frequency	Hertz
i_d, i_q	d-q axis stator currents in the rotor reference frame	Amperes
i_α, i_β	α- β axis stator currents in the stationary reference frame	Amperes
J	moment of inertia	Kg $*$ m^2
L_d, L_q	d- q axis inductances	henry
L_s	Stator winding self-inductance	henry
N_p	Number of pole pairs	none
P	state error covariance matrix	none
Q	the covariance matrix of system noise	none
R	the covariance matrix of the observation noise	none
R_s	Stator resistance	ohms
T_e	Electromagnetic torque	newton meters
T_m	Motor load torque	newton meters
U_a, U_b, U_c	Phase a, b and c rotor voltages	volts
U_d, U_q	d- q axis voltages in the rotor reference frame	volts
U_α, U_β	α- β axis stator voltages in the stationary reference	volts
ψ_d, ψ_q	The d-q axis flux-linkages	webers
ψ_f	Induced flux	webers
ω_r	The mechanical angular speed	radians per second
ω_e	The stator electrical angular speed	radians per second
θ_e	Angular position	radians

Acronyms

Acronyms	Definition
EKF	Extended Kalman Filter
FLC	Fuzzy Logic Control
FOC	Field Oriented Control
PMSM	Permanent Magnet Synchronous Motor
PWM	Pulse Width Modulation
SVPWM	Space-Vector Pulse Width Modulation

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