# GENERALIZATIONS HOMEOMORPHISM ON INTUITIONISTIC TOPOLOGICAL SPACES

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**Abstract**

In this paper we introduce several kinds of generalized homeomorphisms in ITS and related to each other. And we introduce also several kinds of weak generalized homeomorphisms and related to each other and with known ones.

## I- Preliminaries

Since we shall use the following definitions and some properties, we recall them in this section.[1], [14],17]

*a)* An intuitionistic set (IS, for short) in an intuitionistic topological space (X,T) ( ITS , for short) is called an intuitionistic Semi-open(resp. pre-open, semipre-open and presemi-open) If $A \subseteq cl(int(A))(\,resp. A \subseteq intcl(A),$ $A \subseteq clintcl(A)\ and\ A \subseteq intclint(A)$.[1]

An intuitionistic set F is in ITS (X,T) is called an intuitionistic semi-closed set (ISCS) (resp. pre-closed, semipre-closed and presemi-closed ) iff its complement $\bar{F}$ is semi-open (resp. pre-open, semipre-open and presemi-open)[1].

*b)* (1) The union of all semi-open (resp. pre-open, semipre-open and presemi-open) sets contained in A is called the semi-interior (sint) resp.( pre-interior (pint), semipre-interior (spint), presemi-interior (psint)) of A denoted by $sint(A)$, $pint(A)$, $spint(A)\ and\ psint(A)\ respectively.$
(2) the intersection of all semi-closed, pre-closed, semipre-closed and presemi-closed)sets containing A is called semi-closure( pre-closure , ,semipre-closure and presemi-closure) of A and denoted by $scl(A)$, $pcl(A)$, $spcl(A)\ and\ pscl(A)$ respectively . [5],[6].

*c)* Let f be a mapping from an ITS $(X,T)$ into ITS$(Y,\Psi)$, then f is called an intuitionistic open (pre-open, semipre-open and presemi-open) mapping if f(A) is an IOS [14] (resp. ISOS, IPOS, ISPOS, and IPSOS) in Y, for every intuitionistic open set A in X.

*d)* Let f be a mapping from an ITS $(X,T)$ into ITS$(Y,\Psi)$, then f is called an intuitionistic closed ,semi-closed (pre-closed, semipre-closed and presemi-closed) mapping if f(A) is an ICS (resp. ISCS, IPCS, ISPCS, and IPSCS) in Y, for every intuitionistic closed set A in X.

*e)* Let f be a mapping from an ITS $(X,T)$ into ITS$(Y,\Psi)$, then f is called an intuitionistic continuous (semi-continuous, pre-continuous, semipre-continuous and presemi-continuous) mapping if $f^{-1}(A)$ is an ICS (resp. ISCS, IPCS, ISPCS, and IPSCS) in X, for every intuitionistic closed set A in Y.[11,13,10].

*f)* Let $(X,T)$ be ITS and let $A = \langle x, A_1, A_2 \rangle$ be IS in X. A is called generalized-closed (resp. generalizedsemi-closed, pre semi-generalized-closed, generalized pre closed and generalized semipre-closed)sets (for short, g-closed, gs-closed, psg-closed, gp-closed and gsp-closed resp.) sets if $clA \subseteq$ [illegible] $\subseteq V, pcl \subseteq V\ and\ spcl \subseteq V)$ when ever $A \subseteq V$ and V is IOS

A semi generalized-closed (resp. generalized presemi-closed and pregeneralized-closed)sets (for short, sg-closed, gps-closed and pg-closed)sets if sc$lA \subseteq V(pscl \subseteq V, pcl \subseteq V, )$ when ever $A \subseteq V$ and V is ISOS(PSOS, and POS). A is said to be g-open (resp. gs-open, psg-open, gp-open, gsp-open, sg-open, gps-open, and pg-open) If the complement of A is g-closed ( (resp. gs-closed, psg-closed, gp-closed, gsp-closed, sg-closed, gps-closed, and pg-closed).[4,8,10,2]

*g)* Let $(X,T)$ and $(Y,\Psi)$ be two ITSs and let $f: X \to Y$ be a mapping, the f is said to be generalized-continuous (resp. semi generalized-continuous, generalized semi-continuous, generalized presemi-continuous, presemi generalized-continuous, generalized pre-continuous, pre generalized-continuous and generalized semipre-continuous) function (for short, g-cont, sg-cont, gs-cont, gps-cont, psg-cont, gp-cont, pg-cont, and gsp-cont) mapping if $f^{-1}(V)$ is g-closed(resp. sg-closed, gs-closed, gps-closed, psg-closed, gp-closed, pg-closed and gsp-closed) sets in X for every closed set V in Y [8,7].

*h)* Let $(X,T)$ and $(Y,\Psi)$ be two ITSs, and let $f: X \to Y$ be a bijective function. Then f is said to be semi-homeomorphism (resp. pre-homeomorphism, semipre-homeomorphism, presemi-homeomorphism) if f is semi-cont.( pre-continuous, semipre-continuous and presemi-continuous) and f is semi-closed(pre-closed, semipre-closed and presemi-closed [11,12].

*i)* Let $(X,T)$and $(Y,\Psi)$ be two ITS, and let $f: X \to Y$ be a mapping, then f is said to be g-closed mapping(resp. sg-closed, psg-closed,

* Extracted from M.SC. thesis

gps-closed, gsp-closed, gp-closed and pg-closed) function, if f(V) is g-closed(sg-closed, gs-closed, psg-closed, gps-closed, gsp-closed, gp-closed and pg-closed) sets in Y for every closed set V in X [16].

## II- **Generalization Homeomorphism In Intuitionistic Topological Spaces**

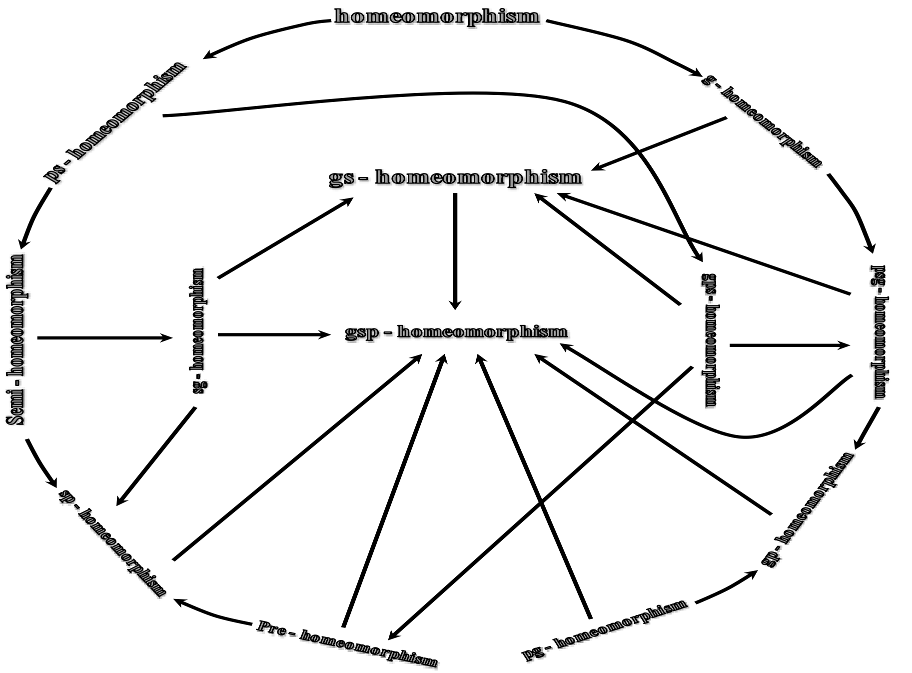
In this section is devoted to define Intuitionistic g - homeomorphism (resp.sg - homeomorphism, gs - homeomorphism, gps - homeomorphism, psg - homeomorphism, gp - homeomorphism, pg - homeomorphism and gsp-homeomorphism, and the relation among them by giving a diagram of implication illustrate these relation. And prove that gs - homeomorphism and gp-homeomorphism is independent notions.

**Definition 2.1**

Let $(X,T)$ and $(Y,\Psi)$ be two ITS's and let $f: X \to Y$ be a bijective mapping, then f is said to be a g-homeomorphism (resp. sg-homeomorphism, gs-homeomorphism, psg-homeomorphism, gps - homeomorphism, gsp-homeomorphism, gp - homeomorphism and pg-homeomorphism) if f both g-continuous (resp. sg-continuous, gs-continuous, psg-continuous, gps-continuous, gsp-continuous, gp-continuous and pg-continuous) and g-closed(sg-closed, gs-closed, psg-closed, gps-closed, gsp-closed, gp-closed and pg-closed) mapping [13].

**Proposition 2.2**

The following implications are true and not reversed.




We start to give counter examples to shows that the inverse of implications appears in the diagram above is not true in generals.

**Example 2.3**

Let $X = \{a,b,c\}, T = \{\widetilde{\emptyset}, \tilde{X}, A, B, C\}$ where $A = \langle x, \{b\}, \{a,c\}\rangle; B = \langle x, \{a\}, \{b\}\rangle, C = \langle x, \{a,b\}, \emptyset\rangle$. And let $Y = \{1,2,3\}, \Psi = \{\widetilde{\emptyset}, \tilde{Y}, D, E, F\}$ where $D = \langle y, \{2\}, \{3\}\rangle, E = \langle y, \{3\}, \{1,2\}\rangle, F = \langle y, \{2,3\}, \emptyset\rangle$. Define a mapping $: X \to Y \quad by \ f(a) = 1, f(b) = 3 \ and \ f(c) = 2$. We can see that:

1) f is sp-hom., but not hom. and not p-hom and not s-hom.
2) f is gp-hom., but not pg-hom.
3) f is not p-hom, not s-hom, not pg-hom but f is sp-hom., gp-hom and gsp-hom

**Example 2.4**

Let $X = \{a,b,c\}, T = \{\widetilde{\emptyset}, \tilde{X}, A, B\}$ where $A = \langle x, \{b\}, \{a,c\}\rangle, B = \langle x, \{a,b\}, \emptyset\rangle$.

And let $Y = \{1,2,3\}, \Psi = \{\widetilde{\emptyset}, \tilde{Y}, E, F\}$ where $E = \langle y, \{1\}, \{2\}\rangle, F = \langle y, \{1,3\}, \emptyset\rangle$. Define a mapping $: X \to Y \quad by \ f(a) = 1, f(b) = 3 \ and \ f(c) = 2$. We can see that:

f is pre-hom, but not ps-hom, and not hom.

**Example 2.5**

Let $X = \{a,b,c\}, T = \{\widetilde{\emptyset}, \tilde{X}, A, B, C\}$ where $A = \langle x, \{c\}, \{a,b\}\rangle; B = \langle x, \{a\}, \{b,c\}\rangle, C = \langle x, \{a,c\}, \{b\}\rangle$. And let $Y = \{1,2,3\}, \Psi = \{\widetilde{\emptyset}, \tilde{Y}, D, E, F\}$ where

$D = \langle y,\{2,3\},\{1\}\rangle, E = \langle y,\{1,3\},\{2\}\rangle, F = \langle y,\{3\},\{1,2\}\rangle$. Define a mapping $: X \to Y$ $by\ f(a) = 1, f(b) = 2\ and\ f(c) = 3$.
We can see that:
f is s-hom but not ps-hom.

**Example 2.6**

Let $X = \{a,b,c\}, T = \{\widetilde{\emptyset}, \tilde{X}, A\}$ where $A = \langle x,\{b\},\{a,c\}\rangle$;. And let $Y = \{1,2,3\}, \Psi = \{\widetilde{\emptyset}, \tilde{Y}, B\}$ where $B = \langle y,\{2\},\{1\}\rangle$ . Define a mapping f $: X \to Y$ $by\ f(a) = 3, f(b) = 1\ and\ f(c) = 2$.
We can see that:

1- f is gs-hom., but not sg-hom.
2- f is g-hom., but not hom.
3- f is gsp-hom., but not sg-hom.
4- f is sp-hom., but not sg-hom.
5- f is p-hom., but not gps-hom.
6- f is gs-hom., but not gps-hom.

**Example 2.7**

Let $X = \{a,b,c\}$ and $T = \{\widetilde{\emptyset}, \tilde{X}, A\}$ where$A = \langle x,\{a\},\{b,c\}\rangle$. and Let $Y = \{1,2,3\}$ and $\Psi = \{\widetilde{\emptyset}, \tilde{Y}, B\}$ where $B = \langle y,\{2\},\{1\}\rangle$, . Define a function $f: X \to Y, by\ f(a) = 1, f(b) = 3\ and\ f(c) = 2$. $SPOX = \{\widetilde{\emptyset}, \tilde{X}, A, C, D, E, F, G, H, I, J, L, M, O, P, Q, R, N\}$ =POX, where $C = \langle x,\{a\},\emptyset\rangle$, $D = \langle x,\{a\},\{b\}\rangle$, $E = \langle x,\{a\},\{c\}\rangle$,$F = \langle x,\{a,b\},\emptyset\rangle$, $G = \langle x,\{a,b\},\{c\}\rangle, H = \langle x,\{a,c\},\emptyset\rangle$,$I = \langle x,\{a,c\},\{b\}\rangle$,$J = \langle x,\{b\},\emptyset\rangle$, $L = \langle x,\emptyset,\{b,c\}\rangle$,$M = \langle x,\emptyset,\{b\}\rangle$, $O = \langle x,\{b\},\{c\}\rangle$, $P = \langle x,\emptyset,\{c\}\rangle, Q = \langle x,\{c\},\emptyset\rangle, R = \langle x,\{c\},\{b\}\rangle, N = \langle x,\{b,c\},\emptyset\rangle$.
We can see that f is gsp-hom and gs-hom, but f is not sp-hom

**Example 2.8**

Let $X = \{a,b,c\}, T = \{\widetilde{\emptyset}, \tilde{X}, A, B\}$ where $A = \langle x,\{a\},\{b\}\rangle; B = \langle x,\{a,c\},\emptyset\rangle$. And let $Y = \{1,2,3\},\ \Psi = \{\widetilde{\emptyset}, \tilde{Y}, C, D\}$ where C$= \langle y,\{2\},\{3\}\rangle, D = \langle y,\{2,3\},\emptyset\rangle$. Define a mapping f $: X \to Y$ $by\ f(a) = 3, f(b) = 1\ and\ f(c) = 2$. We can see that:
f is gsp-hom.and gp-hom but not gs-hom., and not psg-hom.

**Example 2.9**

Let $X = \{1,2,3\}, T = \{\widetilde{\emptyset}, \tilde{X}, A, B, C\}$ where $A = \langle x,\{2\},\{1,3\}\rangle; B = \langle x,\{1\},\{2\}\rangle, C = \langle x,\{1,2\},\emptyset\rangle$. And let $Y = \{a,b,c\}, \Psi = \{\widetilde{\emptyset}, \tilde{Y}, D, E, F\}$ where

$D = \langle y,\{a\},\{c\}\rangle, E = \langle y,\{c\},\{a,b\}\rangle, F = \langle y,\{a,c\},\emptyset\rangle$. Define a mapping $: X \to Y$ $by\ f(2) = a, f(1) = c\ and\ f(3) = b$.
We can see that:

1- f is gs-hom, but not g-hom and not psg-hom.
2- f is gsp-hom, but not gp-hom and not pg-hom and not p-hom.
3- f is sg-hom, but not s-hom.
4- f is gs-hom, but not gp-hom.

**Example 2.10**

Let $X = \{a,b,c\}, T = \{\widetilde{\emptyset}, \tilde{X}, A, B\}$ where $A = \langle x,\{c\},\{b\}\rangle; B = \langle x,\{a,c\},\emptyset\rangle$. And let $Y = \{1,2,3\}, \Psi = \{\widetilde{\emptyset}, \tilde{Y}, D, E\}$ where $D = \langle y,\{1,2\},\emptyset\rangle$, $E = \langle y,\{1\},\{3\}\rangle$,.Define a mapping f $: X \to Y$ $by\ f(a) = 1, f(b) = 3\ and\ f(c) = 2$.
We can see
that: f is psg-hom, but not gps-hom.

**Example 2.11**

Let $X = \{a,b,c\}, T = \{\widetilde{\emptyset}, \tilde{X}, A, B\}$ where $A = \langle x,\{c\},\{b\}\rangle; B = \langle x,\emptyset,\{b,c\}\rangle$. And let $Y = \{1,2,3\}, \Psi = \{\widetilde{\emptyset}, \tilde{Y}, D, F\}$ where $D = \langle y,\{3\},\{2\}\rangle, F = \langle y,\emptyset,\{2,3\}\rangle$. Define a mapping f$: X \to Y$ $by\ f(a) = 2, f(b) = 1\ and\ f(c) = 3$. We can see that: f is gps-hom. But not ps-hom.
In the following example shows that f is psg-hom, but not g-hom.

**Example 2.12**

Let $X = \{a,b,c\}, T = \{\widetilde{\emptyset}, \tilde{X}, A, B\}$ where $A = \langle x,\{c\},\{b\}\rangle; B = \langle x,\{a,c\},\emptyset\rangle$. And let $Y = \{1,2,3\}, \Psi = \{\widetilde{\emptyset}, \tilde{Y}, D, E\}$ where $D = \langle y,\{1\},\{3\}\rangle, E = \langle y,\{1\},\emptyset\rangle$. Define a mapping
f $: X \to Y$
$by\ f(a) = 2, f(b) = 3\ and\ f(c) = 1$.
We can see that: f is psg-hom.,but not g-hom.

**Example 2.13**

Let $X = \{a,b,c\}, T = \{\widetilde{\emptyset}, \tilde{X}, A, B\}$ where $A = \langle x,\{b\},\{a,c\}\rangle; B = \langle x,\{b\},\emptyset\rangle$. And let $Y = \{1,2,3\}, \Psi = \{\widetilde{\emptyset}, \tilde{Y}, D, E\}$ where

$D = \langle y, \{2\}, \{1,3\} \rangle, E = \langle y, \{1,2\}, \{3\} \rangle, F = \langle y, \{2,3\}, \emptyset \rangle$ Define a mapping f $: X \to Y$ by $f(a) = 1, f(b) = 2$ and $f(c) = 3$. We can see that: f is s-hom. and ps-hom, but not hom.

**Remark 2.14**

Every homeomorphism is ps-hom(resp.s-hom.,p-hom and sp-hom.) However the converse is not true as shown in Examples (2.4,2.3,).

**Remark 2.15**

The notion gp-hom and gs-hom. are independent Example 2.8. 2.9 show the case.

**Proposition 2.16**

Let $(X, T)$ and $(Y, \Psi)$ be two ITS , and let $f: X \to Y$ be a bijective mapping, then the following statement are equivalents.

1. $f^{-1}: Y \to X$ is g-continuous(resp. sg-continuous , gs -continuous, psg - continuous, gps - continuous, gsp -continuous, gp - continuous and pg-continuous).
2. $f$ $is$ g-open (resp. sg-open, gs - open, psg - open, gps -open, gsp-open,gp-open and pg-open).
3. $f$ $is$ g-closed( resp. sg-closed, gs-closed, psg-closed, gps-closed, gsp - closed, gp - closed and pg - closed).

**Proof**:

It follows directly from correspond definition of continuity, open and closed mapping.

The following result is a characterization to different kind of homeomorphisms which is direct from proposition 2.16.

**Proposition 2.17**

A bijective mapping $f: X \to Y$ g-hom.(resp. sg-hom., gs-hom., gsp-hom., psg-hom., gp-hom., and pg-hom.) if

1. f is g-continuous(resp. sg-continuous , gs-continuous, psg-continuous, gps-continuous, gsp-continuous, gp-continuous and pg-continuous).
2. $f$ $is$ g-closed ( resp. sg-closed, gs-closed, psg-closed, gps-closed, gsp-closed, gp-closed and pg-closed).

The proof is direct by using Proposition 2.16.

## II- WEAK GENERALIZATION HOMEOMORPHISM ON ITS

We introduce in this section some kinds of weak g-homeomorphism such as weak sg-homeomorphism, weak gs-homeomorphism, weak psg-homeomorphism, weak gps-homeomorphism and weak gp -homeomorphism and illustrate the relation among them by giving a diagram, as well as we give example for non true implications.

**Definition 3.1**

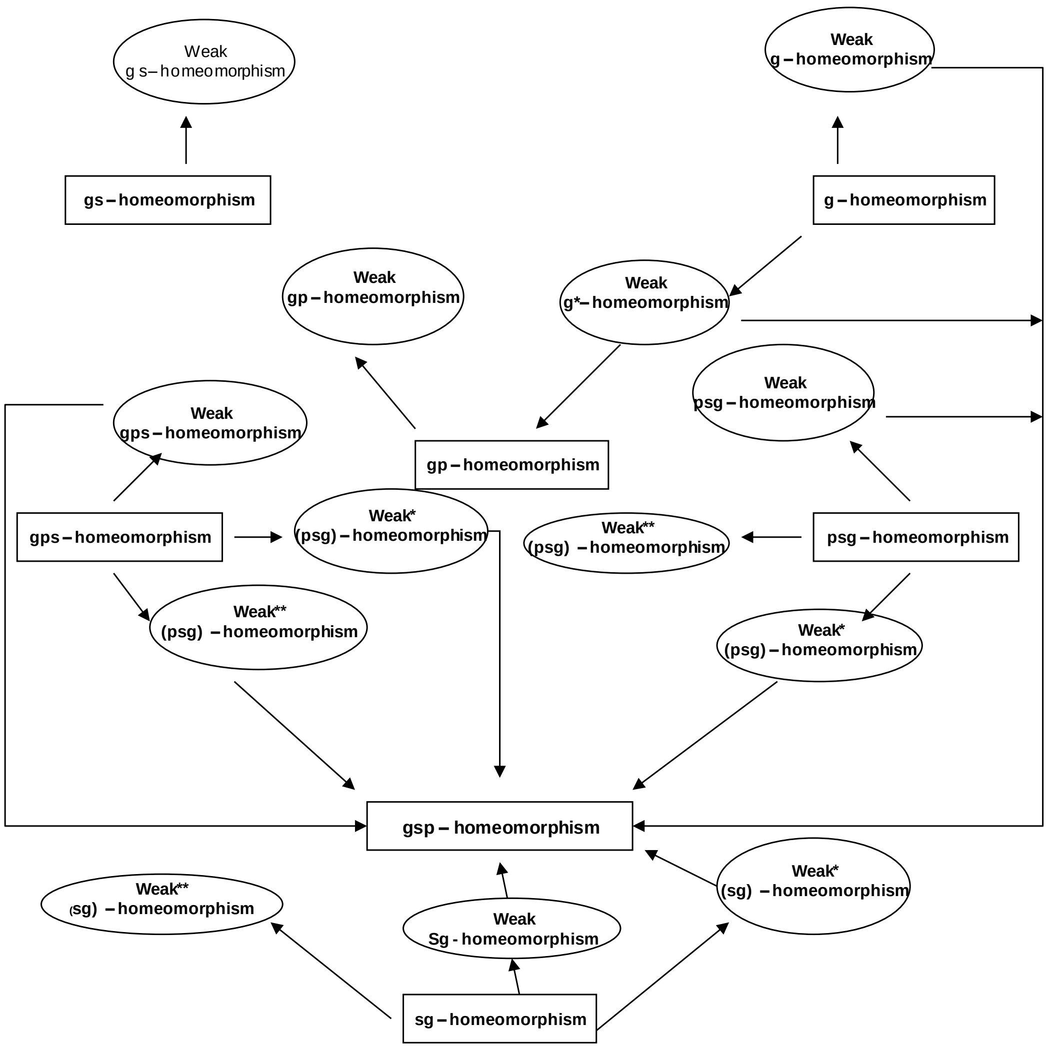
Let $(X, T) and (Y, \Psi)$ be two ITS, and let $f: X \to Y$ be a bijective function, then f is said to be

1. Weak generalized-homeomorphism (for short wg-hom.) if f is gs-cont. and g-closed function.
2. Weak$^*$ generalized - homeomorphism (for short w$^*$g hom.) if f is psg-cont. and g-closed function.
3. Weak presemi generalized - homeomorphism (for short wpsg-hom.) if f gp-continuous and psg-closed function.
4. Weak$^*$presemi generalized - homeomorphism (for short w$^*$psg-hom) if f gs-continuous and psg-closed function.
5. Weak$^{**}$presemi generalized homeomorphism (for short w** psg-hom) if f gsp-continuous and psg-closed function.
6. Weak generalized semi-homeomorphism (for short wgs-hom if f is gsp-cont. and gs-closed function.
7. Weak generalized pre-homeomorphism (for short wgp-hom if f is gsp-cont. and gp-closed function.
8. Weak semi generalized-homeomorphism (for short wsg-hom if f is sp-cont. and sg-closed function.
9. Weak$^*$ semi generalized-homeomorphism (for short w$^*$ sg-hom if f is gs-cont. and sg-closed function.
10. Weak$^{**}$ semigeneralized-homeomorphism (for short w$^{**}$ sg-hom if f is gsp-cont. and sg-closed function.
11. Weak generalized presemi-homeomorphism (for short wgps-hom if f is gs-cont. and gps-closed function.
12. Weak$^*$generalized presemi - homeomorphism (for short w$^*$gps-hom if f is psg-cont. and gps-closed function.
13. Weak$^{**}$generalized presemi-homeomorphism (for short w$^{**}$ gps-hom if f is p-cont. and gps-closed function.

The following proposition gives relations among all kind of homeomorphism appears in Definition 3.1 which can be proved directly from proposition 2.2.

**Proposition 3.2**

The following implication are valid and cannot be reversed




We start with example shows that; there exists a function f such that:

1. $f$ is gsp-hom. , but not w** gps-hom. 2-$f$ is gsp-hom., but not wsg-hom.

**Example 3.3**

Let $X=\{a,b,c\}$ and $T=\{\widetilde{\emptyset},\tilde{X},A\}$ where $A=\langle x,\{a\},\{b,c\}\rangle$.

and Let $Y=\{1,2,3\}$ and $\Psi=\{\widetilde{\emptyset},\tilde{Y},B\}$ where $B=\langle y,\{2\},\{1\}\rangle$, . Define a function $f\colon X\to Y, by\ f(a)=1, f(b)=3\ and\ f(c)=2$.

$SPOX=\{\widetilde{\emptyset},\tilde{X},A,C,D,E,F,G,H,I,J,L,M,O,P,Q,R,N\}$ =POX,

$PSOX=\{\widetilde{\emptyset},\tilde{X},A,C,D,E,F,G,H,I\}=SOX$ .

where

$C=\langle x,\{a\},\emptyset\rangle$, $D=\langle x,\{a\},\{b\}\rangle$, $E=\langle x,\{a\},\{c\}\rangle$, $F=\langle x,\{a,b\},\emptyset\rangle$, $G=\langle x,\{a,b\},\{c\}\rangle$, $H=\langle x,\{a,c\},\emptyset\rangle$, $I=\langle x,\{a,c\},\{b\}\rangle$, $J=\langle x,\{b\},\emptyset\rangle$, $L=\langle x,\emptyset,\{b,c\}\rangle$, $M=\langle x,\emptyset,\{b\}\rangle$, $O=\langle x,\{b\},\{c\}\rangle$, $P=\langle x,\emptyset,\{c\}\rangle$, $Q=\langle x,\{c\},\emptyset\rangle$, $R=\langle x,\{c\},\{b\}\rangle$, $N=\langle x,\{b,c\},\emptyset\rangle$.

$$POY=\Psi\cup\{\mathrm{Z_i}\}_{\mathrm{i=1}}^{18}=SPOY$$

$Z_1 = \langle y, \{2\}, \emptyset \rangle, Z_2 = \langle y, \{2\}, \{1,3\} \rangle, Z_3 = \langle y, \{2\}, \{3\} \rangle, Z_4 = \langle y, \{1,2\}, \emptyset \rangle, Z_5 = \langle y, \{1,2\}, \{3\} \rangle, Z_6 = \langle y, \{2,3\}, \emptyset \rangle, Z_7 = \langle y, \{2,3\}, \{1\} \rangle, Z_8 = \langle y, \{1\}, \emptyset \rangle, Z_9 = \langle y, \{1\}, \{3\} \rangle, Z_{10} = \langle y, \{1,3\}, \emptyset \rangle, Z_{11} = \langle y, \{1,3\}, \{2\} \rangle, Z_{12} = \langle y, \{3\}, \emptyset \rangle, Z_{13} = \langle y, \{3\}, \{1\} \rangle, Z_{14} = \langle y, \{3\}, \{2\} \rangle, Z_{15} = \langle y, \emptyset, \{1\} \rangle, Z_{16} = \langle y, \emptyset, \{3\} \rangle\ Z_{17} = \langle y, \{3\}, \{1,2\} \rangle \ and\ Z_{18} = \langle y, \emptyset, \{1,3\} \rangle.$

where

SOY=$\left\{\widetilde{\emptyset}, \tilde{Y}, B, Z_1, Z_4, Z_6, Z_7\right\} = PSOY$.We can see that f is gsp-hom, but f is not w**gps-hom. and not wsg-hom.

In the next example we show that; there is a function f such that:

1) f is gsp-hom, but not wg-hom.
2) f is wgs-hom., but not ngs-hom.
3) f isgp-hom., but not W*-hom.
4) f is gsp-hom,. butnot W*-hom.

**Example 3.4**

Let $X = \{a, b, c\}$ and $T = \left\{\widetilde{\emptyset}, \tilde{X}, A, B\right\}$ where$A = \langle x, \{a\}, \{b\} \rangle\ and\ B = \langle x, \{a, c\}, \emptyset \rangle$,

and Let $Y = \{1,2,3\}$ and $\Psi = \left\{\widetilde{\emptyset}, \tilde{Y}, C, D\right\}$ where $C = \langle y, \{2\}, \{3\} \rangle, D = \langle y, \{2,3\}, \emptyset \rangle$ .

Define a function $f: X \to Y, by\ f(a) = 3, f(b) = 1\ and\ f(c) = 2$.

We can see that f is gs-hom., gp-hom., and gsp-hom. Since $\bar{C}$ is closed in Y and $f^{-1}(\bar{C})$ is not gs-closed and not psg-closed in X. So f is not gs-hom. and not w*g-hom. and not wg-hom.

We show in this example that there exist a function f such that f is gsp-hom. But not wpsg-hom.

**Example 3.5**

Let $X = \{1,2,3\}$ and $T = \left\{\widetilde{\emptyset}, \tilde{X}, A, B, C\right\}$ where$A = \langle x, \{2\}, \{1,3\} \rangle, B = \langle x, \{1\}, \{2\} \rangle, C = \langle x, \{1,2\}, \emptyset \rangle$. And Let $Y = \{a, b, c\}$ and $\Psi = \left\{\widetilde{\emptyset}, \tilde{Y}, D, E, F\right\}$ where $D = \langle y, \{a\}, \{c\} \rangle, E = \langle x, \{c\}, \{a, b\} \rangle\ and\ F = \langle x, \{a, c\}, \phi \rangle$ Define a function $f: X \to Y, by\ f(2) = a, f(3) = b\ and\ f(1) = c$.

$POX = \mathrm{T} \cup \{N_i\}_{i=1}^{15}$, where $N_1 = \langle x, \{2\}, \emptyset \rangle, N_2 = \langle x, \emptyset, \{2,3\} \rangle, N_3 = \langle x, \{2\}, \{3\} \rangle, N_4 = \langle y, \{1\}, \emptyset \rangle, N_5 = \langle y, \emptyset, \{1,3\} \rangle, N_6 = \langle x, \{1\}, \{3\} \rangle, N_7 = \langle x, \{1\}, \{2,3\} \rangle, N_8 = \langle x, \{1,2\}, \{3\rangle, N_9 = \langle x, \{1,3\}, \emptyset \rangle, N_{10} = \langle x, \{2,3\}, \emptyset \rangle, N_{11} = \langle x, \{2,3\}, \{1\} \rangle, N_{12} = \langle x, \{3\}, \emptyset \rangle, N_{13} = \langle x, \{3\}, \{1\} \rangle, N_{14} = \langle x, \emptyset, \{2\} \rangle, N_{15} = \langle x, \emptyset, \{3\} \rangle. SOX = \mathrm{T} \cup \{N_i\}_{i=16}^{17}$where $N_{16} = \langle x, \{2\}, \{1\} \rangle, N_{17} = \langle x, \{1,3\}, \{2\} \rangle$ and $SPOX = \mathrm{T} \cup \{N_i\}_{i=1}^{20}$ where $N_{18} = \langle x, \{3\}, \{2\} \rangle, N_{19} = \langle x, \{3\}, \{1,2\} \rangle$, $N_{20} = \langle x, \emptyset, \{1\} \rangle$

$$SOY = \Psi \cup \{Z_1, Z_2\}\ where\ Z_1 = \langle y, \{c\}, \{a\} \rangle, Z_2 = \langle y\{a, b\}, \{c\} \rangle .$$

$SPOY = \Psi \cup \{\mathrm{Z_i}\}_{\mathrm{i=1}}^{20}$ where

$$Z_3 = \langle y, \{b, c\}, \{a\} \rangle, Z_4 = \langle y, \{a\}, \{b\} \rangle, Z_5 = \langle y, \{a\}, \emptyset \rangle, Z_6 = \langle y, \{a\}, \{b, c\} \rangle, Z_7 = \langle y, \{a, b\}, \emptyset \rangle, Z_8 = \langle y, \{c\}, \{b\} \rangle, Z_9 = \langle y, \{b, c\}, \emptyset \rangle, Z_{10} = \langle y, \{a, c\}, \{b\} \rangle, Z_{11} = \langle y, \{c\}, \emptyset \rangle, Z_{12} = \langle y, \{b\}, \emptyset \rangle, Z_{13} = \langle y, \{b\}, \{a\} \rangle, Z_{14} = \langle y, \emptyset, \{b\} \rangle, Z_{15} = \langle y, \emptyset, \{c\} \rangle, Z_{16} = \langle y, \emptyset, \{a, b\} \rangle\ Z_{17} = \langle y, \emptyset, \{b, c\} \rangle , Z_{18} = \langle y, \{b\}, \{c\} \rangle, Z_{19} = \langle y, \{b\}, \{a, c\} \rangle, Z_{20} = \langle y, \emptyset, \{a\} \rangle.$$

POY$= \Psi \cup \left\{Z_j\right\}_{j \in J}$ where

J={3,4,5,6,7,9,10,12,13,14,15,16,17}

It is easily to satisfy that f is gsp-hom. But f is not wpsg-hom.

We shown in this example that there is a function f such that:

1. f is wg-hom and not g-hom . f is w*g-hom and not g-hom.

**Example 3.6**

Let $X = \{a, b, c\}$ and $T = \{\widetilde{\emptyset}, \tilde{X}, A, B\}$ where $A = \langle x, \{c\}, \{b\}\rangle, B = \langle x, \{a, c\}, \emptyset\rangle$. And Let $Y = \{1,2,3\}$ and $\Psi = \{\widetilde{\emptyset}, \tilde{Y}, C, D\}$ where $C = \langle y, \{1\}, \{3\}\rangle, D = \langle y, \{1\}, \emptyset\rangle$, Define a function $f: X \to Y, by\ f(a) = 2, f(b) = 3\ and\ f(c) = 1$. We can see that f is wg-hom. and w*g-hom And since $K = f^{-1}(\bar{D}) = \langle x, \emptyset, \{c\}\rangle$ is not g-closed so f is not g-hom.

In the next example we show that there exists a function f such that:

1. $f$ is wpsg-hom. and not psg-hom
2. $f$ is w*psg-hom, and not psg-hom.
3. $f$ is a w**psg-hom and not psg-hom.
4. $f$ is w**gps-hom and not gps-hom.

**Example 3.7**

Let $X = \{a, b, c\}$ and $T = \{\widetilde{\emptyset}, \tilde{X}, A, B, C\}$ where $A = \langle x, \{b\}, \{a, c\}\rangle, B = \langle x, \{a\}, \{b\}\rangle\ and\ C = \langle x, \{a, b\}, \emptyset\rangle$

And Let $Y = \{1,2,3\}$ and $\Psi = \{\widetilde{\emptyset}, \tilde{Y}, D, E\}$ where $D = \langle y, \{1\}, \{2,3\}\rangle, E = \langle y, \{2,3\}, \{1\}\rangle$ . Define a function $f: X \to Y, by\ f(a) = 1, f(b) = 2\ and\ f(c) = 3$.

$POX = \{\widetilde{\emptyset}, \tilde{X}, A, B, C, G, H, I, J, K, L, M, N, S, O, R, V, U, Z, P\}$

where $G = \langle x, \{a, b\}, \{c\}\rangle, H = \langle x, \{b\}, \emptyset\rangle$, $I = \langle x, \{b, c\}, \emptyset\rangle$, $J = \langle x, \{b\}, \{c\}\rangle$, $K = \langle x, \{b, c\}, \{a\}\rangle$, $L = \langle x, \{a\}, \emptyset\rangle$, U$= \langle x, \emptyset, \{c\}\rangle$, $N = \langle x, \{a\}, \{b, c\}\rangle$, $S = \langle x, \{a, c\}, \emptyset\rangle$, $M = \langle x, \{a\}, \{c\}\rangle, O = \langle x, \{c\}, \emptyset\rangle$, $P = \langle x, \emptyset, \{c\}\rangle, Q = \langle x, \{c\}, \emptyset\rangle, R = \langle x, \{c\}, \{a\}\rangle$, $V = \langle x, \emptyset, \{b\}\rangle$. Z$= \langle x, \emptyset, \{b, c\}\rangle$, $P = \langle x, \emptyset, \emptyset\rangle$

$PSOX = \{\widetilde{\emptyset}, \tilde{X}, A, B, C\}$. $PSOY = \Psi$

$f$ is wpsg-hom ,(w* psg-hom, w** psg-hom, and w** gps-hom. and $f$ is not psg-hom and not gps-hom.

In the next example we show that there is a function f such that $f$ is wgps-hom., but not gps-hom. and there is a function f is w*gps-hom and not gps-hom.

**Example 3.8**

Let $X = \{a, b, c\}$ and $T = \{\widetilde{\emptyset}, \tilde{X}, A, B\}$ where $A = \langle x, \{a\}, \{b\}\rangle, B = \langle x, \{a, b\}, \emptyset\rangle$. And Let $Y = \{1,2,3\}$ and $\Psi = \{\widetilde{\emptyset}, \tilde{Y}, C, D, E\}$ where $C = \langle y, \{2\}, \{3\}\rangle, D = \langle y, \{3\}, \{1,2\}\rangle$, $and\ E = \langle y, \{2,3\}, \emptyset\rangle$. Define a function $f: X \to Y, by\ f(a) = 2, f(b) = 3\ and\ f(c) = 1$. $PSOX = \{\widetilde{\emptyset}, \tilde{X}, A, B, M, G, H\}$, where $G = \langle x, \{a, c\}, \emptyset\rangle, H = \langle x, \{a, c\}, \{b\}\rangle$ and M=$\langle x, \{a\}, \emptyset\rangle$

$PSOY = \Psi$. f is wgps-hom and w* gps-hom. and f is not gps-hom

In the following example we show that there exists a function f which is:

1. $f$ is wgp-hom. and not gp-hom.
2. $f$ is gsp-hom. and not w*psg-hom.
3. $f$ is gsp-hom. and not w*sg-hom.
4. $f$ is gsp-hom. and not wgps-hom.
5. $f$ is gsp-hom and not w*gps-hom.

**Example 3.9**

Let $X = \{a, b, c\}$ and $T = \{\widetilde{\emptyset}, \tilde{X}, A, B, C, D\}$ where $A = \langle x, \{a\}, \{b\}\rangle, B = \langle x, \{a, b\}, \emptyset\rangle\ and\ C = \langle x, \{b\}, \{c\}\rangle\ and\ D = \langle x, \emptyset, \{b, c\}\rangle$ And Let $Y = \{1,2,3\}$ and $\Psi = \{\widetilde{\emptyset}, \tilde{Y}, E, F, G\}$ where $E = \langle y, \{3\}, \{2\}\rangle, F = \langle y, \{2\}, \{1,3\}\rangle\ and\ G = \langle y, \{2,3\}, \emptyset\rangle$.

Define a function $f: X \to Y, by\ f(a) = 3, f(b) = 1\ and\ f(c) = 2$.

$f$ is not (gp-hom., w*sg-hom., w*psg-hom., wgps-hom and w*gps-hom.) since $\bar{F}$ is closed set in Y, but its inverse under f is not (gs-closed, gp-closed and psg-closed). However f is wgp-hom. and gsp-hom.

The last example shows that; there is a function f such that:

1- is wsg-hom. and not sg-hom.
2- is w*sg-hom and not sg-hom.
3- is w**sg-hom. and not sg-hom.

**Example 3.10**

Let $X = \{a, b, c\}$ and $T = \{\widetilde{\emptyset}, \tilde{X}, A, B\}$ where $A = \langle x, \{c\}, \{a, b\}\rangle and B = \langle x, \{b, c\}, \{a\}\rangle$ . And Let $Y = \{1,2,3\}$ and $\Psi = \{\widetilde{\emptyset}, \tilde{Y}, C\}$ where $C = \langle y, \emptyset, \{3\}\rangle$, . Define a function $f: X \to Y, by\ f(a) = 3, f(b) = 2\ and\ f(c) = 1$.$SPOX = T \cup$

$\{K_i\}_{i=1}^{17}$ where $K_1 = \langle x, \{c\}, \emptyset \rangle, K_2 = \langle x, \{c\}, \{a\} \rangle, K_3 = \langle x, \{c\}, \{b\} \rangle, K_4 = \langle x, \{a, c\}, \emptyset \rangle, K_5 = \langle x, \{a, c\}, \{b\} \rangle, K_6 = \langle x, \{b, c\}, \emptyset \rangle$, $K_7 = \langle x, \{a\}, \emptyset \rangle, K_8 = \langle x, \{a\}, \{b\} \rangle, K_9 = \langle x, \{a, b\}, \emptyset \rangle, K_{10} = \langle x, \{a, b\}, \{c\} \rangle, K_{11} = \langle x, \{c\}, \{a\} \rangle, K_{12} = \langle x, \{c\}, \{b\} \rangle$,
$K_{13} = \langle x, \{c\}, \{a, b\} \rangle,\ K_{14} = \langle x, \emptyset, \emptyset \rangle, K_{15} = \langle x, \emptyset, \{a\} \rangle,\ K_{16} = \langle x, \emptyset, \{a, b\} \rangle,\ K_{17} = \langle x, \emptyset, \{b\} \rangle$. $SOX = T \cup \{K_i\}_{i=1}^{6}$

SOY=$\{\tilde{\emptyset}, \tilde{Y}, C, R\}$ where $R = \langle y, \emptyset, \emptyset \rangle$. We can see that:

$f$ is wsg-hom. and not sg-hom.$f$ is w*sg-hom and not sg-hom.$and$ $f$ is w**sg-hom. and not sg-hom.

## References


[1]. Andvijevc,D. " on b-open sets" m. bechnk 48(1996), pp.54-64.

[2]. Caldas, M. and Jafari, S. " weak and strong form of $\beta -$irresoluteeness" Th. A. J for Sci. and E. , Vol.31, No1A (2006).

[3]. Cammarroto, F. and Noiri, T. " Almost irresolute functions" Indian J. pure appl. Math. 20 N0b.5 (1989), pp472-482.

[4]. Cao, J., Ganster, M. and Reilly, I. " Sub-maximality, Extremal disconnectedness and generalized closed sets" Houston, J of Math. Vol.24, No. 4 (1998), pp681-688.

[5]. Cuev, M.C. " Weak and strong forms of irresolute maps" I. J. Math. Sci. Vol. 23, No. 4 (2000), pp 253-259.

[6]. **Cueva, M.C. and Saraf, R.K. "preserving fuzzy sg-closed sets" Conservando coniuntos fuzzy sg-cerrados Vol.20, No. (2001), pp 127-138.

[7]. Dontcher, J and Maki, H. "On $\theta -$ generalized closed sets" I.J. Math. Sci. Vol. 22 , No. 2 (1999), PP 239-249.

[8]. Gnanambal, Y. " On generalized closed sets in topological spaces" Indian J. pure appl. Math. 28 no. (1997), pp351-360.

[9]. Jafari, S. and Noiri, T. "On almost pre continuous function" I. J. Math. Sci. Vol. 24, No. 3 (2000), pp. 193-201.

[10]. Jean, J.K., jun, Y.B., and Park, J.H. "Intuitionistic fuzzy alpha-continuity and Intuitionistic fuzzy pre-continuity" I. J. Math. Sci. Vol. 19, (2005), pp. 3091-3101.

[11]. Lee , J. P. " On semi- homeomorphism" I. J. Math. Sci. Vol. 13 No.1, (1990), pp. 129-134.

[12]. Nagar, BM. P. and Arya, S. P. " Semi-topological properties" I. J. of Math., And math. Sci. Vol. 15, No. 2(1992), pp. 267-272.

[13]. Ozcelik, A. Z. and Narli, S. " Decomposition of homeomorphism on topological spaces" I. J. of Math. Sci. Vol. 1,No.1 (2007), pp. 72-75.

[14]. Yaseen,Y. J. and Raouf. A.G. " On generalized closed sets and generalized continuity on ITS" To appear.

[15]. Yaseen,Y. J. and Raouf. A.G. "Strongly generalized continuous functions and generalized Irresolute function on ITS" To appear.

# التشاكل المعمم على الفضاءات التبولوجية الحدسية

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**الملخص**

يهدف هذا البحث الى تقديم انواع مختلفة من التشاكلات التبولوجية في الفضاءات التبولوجية الحدسية ودراسة علاقتها ببعضها البعض والانواع المعروفة من التشاكلات التبولوجية وقدمنا كذلك اشكال ضعيفة من التشاكلات ودرسنا علاقتها ببعضها والانواع المعروفة الاخرى.