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N-Micro topological spaces

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Abstract

The purpose of this paper is to study some definitions in general topology and generalized them to Micro topological space. These definitions are (Micro i-open and N-Micro open) set. The relations among these definitions with (Microopen, Micro i-open, N-Microopen, N-Micro semiopen, N- Microα- open,) set are studded, some new result are proved finally we define and study a new space namely N-Micro topological spaces.

Introduction:

In 1965, Njastad [1] introduced α- open in topological space and studied the relation between α- open and semi open set. Thivagar [2] introduced the concept of Nano topological space which has been known in terms of universe U subset boundary region and approximation through the utilization of equivalence relation on it. The concept has also been determined as Nano closure, Nano interior. Askander and Mohammed in 2018 [3] introduced i-openset. Chandrasekar [4] introduced the Micro topological space, he also studied the relations between Micro open set and Each of Micro-pre-open and Micro-semi open set. Amir A. and Beyda S. in 2019 [5] introduced ii-openset, iw-closed and iiw-closed sets. Nethaji, and Rajasekaran, 2019[6] introduced w-closed set in Nano topological space .Ibrahim, H. Z. 2020 [7] introduced Mic- α- open and Mic-β-open in Micro topological space .

In this paper definitions of Micro i-open and N-Micro topological space are studied and relations among (Mic- open, Micro i-open , Micro semiopen, Micro α- open, N-Micro open) sets are studied some new results are proved with example.

Basic definitions

Let's go we recall the definitions below which are useful in the sequel

Definition 1 [2]

"Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as" the "indiscernibility relation. Then U is divided into disjoint equivalence classes. Elements belonging to the same equivalence class are said to be indiscernible with one" another. "The pair (U,R) is said to be the approximation space. Let $X \subseteq U$".

1. "The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with" respect to. R and it is denoted by "$L_R(X)$. That is

   $L_R(X) = \cup_{x \in U}\{R(x) : R(x) \subseteq X\}$ where R(x) denotes the equivalence class determined by $x \in U$".

2. "The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and" it as denotes by $U_R(X)$. That is,

   $U_R(X) = \cup_{x \in U}\{R(x) : R(x) \cap X \neq \varphi$"$\}$

3. "The boundary region of X with respect to R is the set of all objects, which can be classified neither as X nor as not-X with respect to R and it is denoted by" $B_R(X)$. That is, $B_R(X)=U_R(X)-L_R(X)$.

Definition 2 [2]

Let's U is an "universe, R be an equivalence relation on U and"

$\tau_R(X) = \{\varphi, U, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq U$ pleasant the axioms below

1- U, $\varphi \in \tau_R(X)$
2- The union of the elements of any sub-collection of $\tau_R(X)$ is in $\tau_R(X)$
3- The intersection of the elements of any finite sub collection of $\tau_R(X)$ is in τ_R(X).

Then $\tau_R(X)$ be called the Nano topology on U with respect to X. The space (U,τ_R(X)) is the nano topological space.The elements of $\tau_R(X)$ are called nano open sets.

Definition 3 [4]

Let (U,τ_R(X)) be a Nano topological space. Then $\mu_R(X) = \cup \{N \cup (N^0 \cap \mu)\}$ is called Micro topology on U with respect to *X,* where N, $N^0 \in \tau_R(X)$ *and* $\mu \notin \tau_R(X)$, add: The triple $(U, \tau_R(X), \mu_R(X))$ is called micro topological space and the elements of the micro topology are called micro open sets and its complements are micro closed sets.

Definition 4[4]

The micro closure of a set B is denoted by $Mic - cl(B)$ and is defined as $Mic - cl(B) = \cap \{B : B \text{ is micro closed and } A \subseteq B\}$. The micro interior of a set B is denoted by $Mic - int(B)$ and is defined as $Mic - int(B) = \cup \{B : B \text{ is micro open and } A \supseteq B\}$.

Results and discussion

In this section, we present and study the concept of Micro-i-open “sets in a Micro topological space and we obtain some of its properties “.

Definition 5

A subset B of a micro topological space $(U, \tau_R(X), \mu_R(X))$ is called

1- Micro semiopen set [2] if $B \subseteq Mic - cl(Mic - int(B))$
2- Micro$-\alpha-$open set [3] if $B \subseteq Mic - int(Mic - cl(Mic - int(B)))$

Definition 6

A subset B of a Micro topological space $(U, \tau_R(X), \mu_R(X))$ is called Micro-i-openset if ∃ a Micro openset G ∈ $\mu_R(X)$, such that

(1) G$\neq \varphi, U$
(2) B$\subseteq Mic - CL(B \cap G)$

The complement of the Micro-i-open set is Micro-i-closed set. Family of all Micro i-open sets denoted by Mic-i-o.

Remark: the set of all Micro i-open set is not topological space in fact is supra topology.

Theorem 1

The union of two Mic-i-open sets is also Mic-i-open set .

Proof:

Let A and B be two Mic-i-open sets .let G be any Mic-open set in $\left(U, \tau_R(X), \mu_R(X)\right)$ so that $A \cup B \subseteq G$, hence $A \subseteq G$ and $B \subseteq G$. since A and B are Mic-i-open sets then A $\subseteq MicCL(A \cap G)$ and B$\subseteq Mic - CL$(B∩G) ,we have(A ∪B)

$\subseteq \left(Mic - CL(A \cap G)\right) \cup \left(Mic - CL(B \cap G)\right)$

$=(Mic - CL(A \cup B) \cap G))$

Therefore, $A \cup B$ is Mic-i-open.

Remark 1

The intersection two Mic-i-open sets is not Mic-i-open set as shown from the following example.

Example 1

Let’s go “$U = \{a, b, c, d\}$, $U/R = \{\{a\}, \{c\}, \{b, d\}$, $X = \{b, d\} \subseteq U$”.

$\tau_R(X) =$ "$\{\varphi, U, \{b, d\}\}$" , when $\mu = \{a\}$ then there is

$\mu_R(X) =$ "$\{\varphi, U, \{b, d\}, \{a, b, d\}, \{a\}\}$" .

Mic-i-o

$=\{\varphi, U, \{a\}, \{b\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}\}$.

Then $\{b, c\} \cap \{c, d\} = \{c\}$ is not Mic-i-open set.

Definition 7

A subset B of a micro topological space $(U, \tau_R(X), \mu_R(X))$ is called N-Micro open set if there exist a Micro open set G ∈ $\mu_R(X)$, so that

1- G$\neq \varphi, U$
2- B$\subseteq Mic - CL(B \cap G)$
3- B$\subseteq Mic - int(B) \cup G$

The complete of N-Micro open set is N-Micro closed set, the family of all N-Micro Open

Sets is denote by $N\mu_R(X)$ or (N-Mic-o) and the triple $(U, \tau_R(X), N_{\mu R}(X))$ is Nmicro topological space.

Example 2

Let's go "$U = \{a, b, c, d\}$, $U/R = \{\{a\}, \{c\}, \{b, d\}$, $X = \{b, d\} \subseteq U$".

$\tau_R(X) =$ "$\{\varphi, U, \{b, d\}\}$" , when $\mu = \{a\}$ then there is

$\mu_R(X) = \{\varphi, U, \{b, d\}, \{a, b, d\}, \{a\}\}$.

Mic-i-o

=$\{\varphi, U, \{a\}, \{b\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}\}$.

N-Mic-o=$\{\varphi, U, \{a\}, \{b\}, \{d\}, \{a, b\}, \{a, d\}, \{b, d\}, \{a, b, d\}\}$.

Definition 8

A subset B of a N-Micro topological space $(U, \tau_R(X), N_{\mu_R}(X))$ is called

1- N-Micro semi open set if $B \subseteq NMic - cl(NMic - int(B))$

2- N-Micro alpha open set if $B \subseteq NMic - int(NMic - cl(NMic - int(B)))$, and the complement of N-Micro semi open set and N-Micro alpha open set respectively are N-Micro semi closed set and N-Micro alpha closed set .

Theorem 2

The family Sof all N-Mic-open set is a topological space.

Proof:

Let $\{B_\alpha : \alpha \in I\}$ be a family N-Mic-open set to prove $\{B_\alpha : \alpha \in I\}$ is a topology .

(1) from Defintion 7(2) N-Mic-open set $\varphi \in \{B_\alpha : \alpha \in I\}$
also from Defintion 7(3) B⊆Mic-int$(B) \cup G$ take B=U then $U \in \{B_\alpha : \alpha \in I\}$.

(2) let $A_i \in \{B_\alpha : \alpha \in I, i \in \alpha\}$ from De Morgan's lows we get
$(\cup_i A_i)^c = \cap_i A_i^c$. Since $A_i \in \{B_\alpha : \alpha \in I\}$ then
$A_i^c \in \{B_\alpha : \alpha \in I\}$ Impels that $\cap_i A_i^c \in \{B_\alpha : \alpha \in I\}$.Hence $\cup_i A_i \in \mu\{B_\alpha : \alpha \in I\}$.

(3) let $A_i \in \{B_\alpha : \alpha \in I, i \in \alpha\}, (1 \le i \le n)$ then $(\cap_{i=1}^n A_i)^c = \cup_{i=1}^n A_i^c$ Impels that $\cap_{i=1}^n \in \{B_\alpha : \alpha \in I\}$.Hence $\{B_\alpha : \alpha \in I\}$ is topological space .

Theorem 3

Each Micro alpha open sets is Micro-i-open set.

Proof:

Let B be Micro alpha openset, and G a Mic-open set, $G \neq \varphi, U$ so

$B \subseteq Mic - int(\ Mic - CL(Mic - int(B)))$

$\subseteq\ Mic - CL(Mic - int(B)))$

Since $\exists\ G$ a Mic-open set then

$Mic - int(B) \subseteq G$ and $Mic - int(B) \subseteq B$ Then

$B \subseteq\ Mic - CL(B \cap G)$. Hence

Hence B is Mic-i-open set.

Theorem 4

Each Micro semi open set is Micro-i-openset.

Proof:

Let B is Micro –semi-open set, and G a Mic-open set, $G \neq \varphi, U$ so

B $\subseteq Mic - CL\big(Mic - int(B)\big)$

Since $\exists\ G$ a Mic-open set then

$Mic - int(B) \subseteq G$ and $Mic - int(B) \subseteq B$ Then

$B \subseteq\ Mic - CL(B \cap G)$. Hence B is Mic-i-open set.

Theorem 5[7]
Each Micro-α-openset is Micro semi open.
The converse of the Theorem 3, the Theorem 4 and the Theorem 5 is not true through the example below.

Example 3
In examples 1, than B={ c, d } Is micro-i- open but is not Mic-$semi$ –open and Mic- α –open, B = {a, c} Mic-semi –open but is not Mic- α –open.

Theorem 6
Each N-Micro open isMicro-i-open set.

Proof:
Direct from definitions 6, 7.
The converse of the Theorem 6 is not true through the example below.

Example 4
In examples 2, The B={c, d} is micro-i- open but is not N-Micro open.

Theorem 7
Each Micro open set is N-Micro open.

Proof:
Let G is Micro openset in $(U, \tau_R(X), \mu_R(X))$ and G$\neq \varphi, U$. Since than $G \subseteq MicCL(G \cap G)$ $= MicCL(G)$ impels that G be Micro-i-open. Also $G \subseteq Mic - int(G) \cup G$ .Hens' G is N-Micro open.
The converse of the theorem 7 need not be true through the example below.

Example 5
Let $U = \{i, j, k, l, m\}$, and $U/R = \{\{i\}, \{j, k, l\}, \{m\}\}$, $X = \{j, k\} \subseteq U$.Then
$\tau_R(X) = \{\varphi, U, \{j, k, l\}\}$, when $\mu = \{i\}$ then
$\mu_R(X) = \{\varphi, U, \{i\}, \{i, j, k, l\}, \{j, k, l"\}\}$.
N-mic-o= $\{\varphi, U, \{i\}, \{j\}, \{k\}, \{l\}, \{i, j\}, \{i, k\}, \{i, l\}, \{j, k\}, \{j, l\}, \{k, l\}, \{i, j, k\},$
$\{i, j, l\}, \{j, k, l\}, \{i, k, l\}, \{i, j, k, l\}\}$
Then B= $\{i, j\}$ is N-Micro open but is not Micro open set.

Theorem 8

Each Micro open set is N-Micro semi open.

Proof:

Let B is Micro open then

$B \subseteq Mic - int(Mic - int(B)$. Since each Micro open set is N-Microopen then

B$\subseteq NMic - int(NMic - int(B)$, since

$NMic - int(NMic - int(B)$

$\subseteq NMic - CL(NMic - int(B)$ Impels that

$B \subseteq NMic - CL(NMic - int(B)$.Hens B is N-Micro semi open set.

Theorem 9

Each Micro open is N-Micro alpha open set.

Proof:

Let B is Micro open then

$B \subseteq Mic-int(Mic-int(B)$. Since each Micro open set is N-Microopen then

B$\subseteq NMic-int(NMic-int(B)$, since

$NMic-int(NMic-int(B) \subseteq NMic-int(NMic-CL(NMic-int(B)$

Impels that $B \subseteq \; NMic-int(NMic-CL(NMic-int(B)$.Henc

B is N-Micro alpha open set.

Theorem 10

Each N-Micro alpha open is N-Micro semi open .

Proof:

Let B is N- Micro alpha open then

$B \subseteq NMic-int(NMic-cl(NMic-int(B))$ .It obvious that

$B \subseteq NMic-int(NMic-cl(NMic-int(B))$

$\subseteq NMic-cl(NMic-int(B))$ .Hence

$B \subseteq NMic-cl(NMic-int(B))$.

The converse of the Theorem 8, Theorem 9 and Theorem 10 is not true through the example below.

Example 6

In examples 1, The B= {a, b} is N-Micro-α- open and N-Micro semi open but is not Micro open set. And The B= {a, c} is N-Micro semi open but not N-Micro-α- open set.

Remark

From the following diagram figure (1) show the relations of N-Mic-open set with other micro sets.

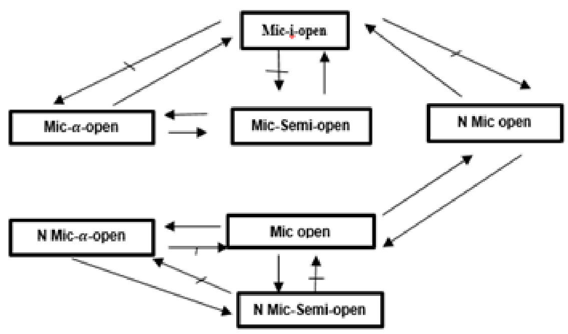

Fig.1: The relations of N-Mic-open set with other micro sets.

Continuous Function in N-Micro Topological Space

Definition 9

A mapping $f: (U, \tau_R(X), N\mu_R(X)) \to (V, \tau_R(Y), N\mu_R(Y))$ is called N- Microcontinuous if $f^{-1}(G)$ is N-Mic-open in $(U, \tau_R(X),\ N_{\mu_R}(X))$,for each N-Mic-open set G in $(V, \tau_R(Y), N_{\mu_R}(Y))$.

Example 7

Lets' $U = \{a, b, c, d\}, U/R = \{\{a\}, \{b, d\}, \{c\}\}, X = \{b, d\} \subseteq U$.

$\tau_R(X) = \{\varphi, U, \{b, d\}\}, \mu = \{a\}. Than$

$\mu_R(X) = \{\varphi, U, \{a\}, \{b, d\}, \{a, b, d\}\}$.

$N_{\mu_R}(X) = \{\varphi, U, \{a\}, \{b\}, \{d\}, \{a, b\}, \{a, d\}, \{b, d\}, \{a, b, d\}\}$.

Let $V = \{1, 2, 3, 4\}, V/R = \{\{2\}, \{3, 4"\}, \{1\}\}, Y = \{2,3\} \subseteq V$.

$\tau_R(Y) = \{\varphi, V, \{2\}, \{2,3,4\}, \{3, 4\}\}\}, \mu = \{3\}$.

$\mu_R(Y) = \{\varphi, V, \{2\}, \{3, 4\}, \{2, 3, 4\}, \{3\}, \{2,3\}\}$.

$N_{\mu_R}(Y) = \{\varphi, V, \{2\}, \{3\}, \{4\}, \{2,3\}, \{2,4\}, \{3,4\}, \{2,3,4\}\}$. $f: U \to V$ is a mapping defined as f(c)=1 f(d)=2, f(a)=3, f(b)=4.

Then N-Micro open sets in U are $\{a\}, \{b\}, \{d\}, \{a, b\}, \{a, b, d\}, \{a, d\}, \{b, d\}$ and N-Micro opensets in V are $\{2\}, \{3\}, \{4\}, \{2,3\}, \{2,4\}, \{3,4\}, \{2,3,4\}$ .This implies that for every N-Micro open set G in V, $f^{-1}(G)$ is N-Mic-open in U .Then there is f N-Microcontinuous mapping.

Defintion 10

A mapping $f: (U, \tau_R(X), N_{\mu_R}(X)) \to (V, \tau_R(Y), N_{\mu_R}(Y))$ is called N- Microcontinuous at a point $x \in U$ if for every N-Mic-open set S containing $f(x)$ in V, $\exists$ a N-Mic-openset G contaning x in U so that$f(G) \subset S$.

The Profs of the following results are similar to those in Micro continuous functions.

Theorem 11

A mapping $f: (U, \tau_R(X), N_{\mu_R}(X)) \to (V, \tau_R(Y), N_{\mu_R}(Y))$ is called N- Micro continuous iff N-Mic-continuous at each point of U

Theorem 12

A mapping $f: (U, \tau_R(X), N_{\mu_R}(X)) \to (V, \tau_R(Y), N_{\mu_R}(Y))$ is called N- Micro continuous iff $f^{-1}(S)$ isN-Mic-closed in U , for each closed setin S .

Theorem 13

Let $(U, \tau_R(X), N_{\mu_R}(X), (V, \tau_R(Y), N_{\mu_R}(Y)$) and $(W, \tau_R(W), N_{\mu_R}(W))$ are three N- Micro topological space if $f: U \to V$, $g: V \to W$are N-Mic-continuous then $gof: U \to W$ Is N- Mic-contnuous.

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الفضاء التوبولوجي الميكروي من النمط N

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الكلمات المفتاحية:

المجموعة الميكروية المفتوحة من النمط N، المجموعة الميكروية المفتوحة من النمط i، المجموعه الميكروية المفتوحة من النمطα، المجموعة الميكروية شبه المفتوحة.

الخلاصة:

الغرض من هذا البحث هو دراسة بعض التعريفات في التوبولوجيا العامة وتعميمها الى الفضاءات التوبولوجية الميكروية وهذه التعريفات هي المجموعة المفتوحة الميكروية من النمط i، المجموعة المفتوحة الميكروية من النمط N ، والعلاقة بين هذه التعريفات مع المجموعة الميكروية المفتوحة من النمط i، المجموعة المفتوحة الميكروية من النمط N ,المجموعة الميكروية شبه المفتوحة من النمطi ، المجموعة المفتوحة الميكروية من النمط α. درسنا بعض النتائج الجديدة وبرهنها واخيرا تولد لدينا فضاء توبولوجي جديد اسمينه الفضاء التوبولوجي الميكروي من النمط N.