Numerical solution for non-linear Murray equation using the operational matrices of the Haar wavelets method

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Abstract

The operational matrices of the Haar wavelets method is applied for finding numerical solution of non-linear Murray equation, we compared this numerical result with the exact solution for non-linear Murray equation. the accuracy of the obtained solutions is quite high even if the number of calculation points is small, by increasing the number of collocation points the error of the solution rapidly decreases as shown by solving an example.

1.Introduction:

Haar wavelets have become an increasingly popular tool in the computational sciences. They have had numerous applications in a wide rang of areas such as signal analysis, data compression and many others[6].

Using the operational matrix of an orthogonal function to perform integration for solving, identifying and optimizing a linear dynamic system has several advantages: (1) the method is computer oriented, thus solving higher order differential equation becomes a matter of dimension increasing; (2) the solution is a multi-resolution type and (3) the answer is convergent, even the size of increment is very large [6,7].

The main characteristic of the operational method is to convert a differential equation into an algebraic one, and the core is the operational matrix for integration. We start with the integral property of the basic orthonormal matrix, $\phi(t)$ by write the following approximation:

$$\underbrace{\int_0^t\int_0^t\int_0^t.....\int_0^t}_{k} \phi(t)(dt)^k \cong Q_\phi^k \phi(t) \quad ...(1)$$

where $\phi(t)=[\vec{\varphi}_0(t)\ \ \vec{\varphi}_1(t)\ \ \ldots\ \ \vec{\varphi}_{m-1}(t)]^T$ in which the elements $\vec{\varphi}_0(t),\vec{\varphi}_1(t),\ldots,\vec{\varphi}_{m-1}(t)$ are the discrete representation of the basis functions which are orthogonal on the interval [0,1) and Q_ϕ is the operational matrix for integration of $\phi(t)$ [6,7].

Cherniha R. M. (1997) [1] used the constructive method for obtaining exact solution of nonlinear Murray equation arising in mathematical biology. The method is based on the consideration of a fixed nonlinear partial differential equation together with additional generating condition in the form of a linear high-order ODE, he was obtained new exact solutions with help of this method, which are generalizations of the murray equations.

Wu and Chen (2003) [6] studied the numerical solution for partial differential equations of first order via operational matrices , they used the Haar wavelets in the solution with constant initial and boundary conditions.

Wu and Chen (2004) [7] studied the numerical solution for fractional calculus and the fractional differential equation by using the operational matrices of orthogonal functions. The fractional derivatives of the four typical functions and two classical fractional differential equations solved by the new method and they are compared the results with the exact solutions, they are found the solutions by this method is simple and computer oriented.

Lepik and Tamme (2007) [2] derived the solution of nonlinear Fredholm integral equations via the Haar wavelet method, they are find that the main benefits of the Haar wavelet method are sparse representation, fast transformation, and possibility of implementation of fast algorithms especially if matrix representation is used.

Lepik Uio (2007) [3] studied the application of the Haar wavelet transform to solve integral and differential equations, he demonstrated that the Haar wavelet method is a powerful tool for solving different types of integral equations and partial differential equations. The method with far less degrees of freedom and with smaller CPU time provides better solutions then classical ones.

Qasem A. F. (2008) [5] found the numerical solution for linear reaction diffusion system by using the Haar wavelets method, he transformed the linear partial differential system into a linear algebraic equations that can be solved by Gauss-Jordan method.

In this paper, we study the numerical solution for nonlinear Murray equation by the operational matrices of Haar wavelet method and we compare the results of this method with the exact solution.

2.Mathematical model:

We consider the nonlinear reaction-diffusion equations with convection term of the form [1]:

$$\frac{\partial u}{\partial t} = A(u)\frac{\partial^2 u}{\partial x^2} + B(u)\frac{\partial u}{\partial x} + C(u),\ 0 \le x < 1,\ 0 \le t < 1 \quad ...(2)$$

where $u(x,t)$ is an unknown function, A(u), B(u) and C(u) are arbitrary smooth functions. Equation (2) generalized a great number of the well-known nonlinear second-order evolution equations describing various processes in biology [1].

When $A(u)=1$, $B(u)=\lambda_1 u$ and $C(u)=\lambda_2 u-\lambda_3 u^2$ where λ_1,λ_2 and $\lambda_3 \in R$,

equation (2) becomes:

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + \lambda_1 u\frac{\partial u}{\partial x} + \lambda_2 u - \lambda_3 u^2,\quad 0 \le x < 1, 0 \le t < 1 \quad ...(3)$$

which is called the nonlinear Murray equation with initial condition:

$$u(x,0)=F(x),\ 0 \le x < 1 \quad ...(4)$$

and mixed boundary conditions:

$$u(0,t)=G(t),\quad 0 \le t < 1 \quad ...(5)$$

$$\frac{\partial u(0,t)}{\partial x} = I(t),\quad 0 \le t < 1 \quad ...(6)$$

such that F(x) is prescribed space-dependent and G(t), I(t) are prescribed time-dependent.

Cherniha, R. M. [1] is found the exact solution for (3) such that:

$$u(x,t)=\frac{\lambda_2+c_1\exp(\gamma^2 t+\gamma x)}{\lambda_3+c_0\exp(-\lambda_2 t)} \quad \ldots(7)$$

where $\gamma=\frac{\lambda_3}{\lambda_1},\ \lambda_1\neq 0$

c_0 is constant such that $\lambda_3+c_0\exp(-\lambda_2 t)\neq 0$ and c_1 is arbitrary constant .

3. Review of the operational matrices and Haar wavelets:

The main benefit of operational matrix is to transfer the differential equations into the algebraic ones or the Lyaponov form. Then they can be solved in the computer oriented methods, that is much easy and time efficient.

The operational matrix Q_ϕ of an orthogonal matrix $\phi(t)$ can be expressed by:

$$[Q_\phi]=[\phi]\cdot[Q_B]\cdot[\phi]^{-1} \ldots(8)$$

where $[Q_B]$ is the operational matrix of the block pulse function:

$$Q_{B_m}=\frac{1}{m}\begin{bmatrix}1/2 & 1 & \ldots & \ldots & 1\\ 0 & 1/2 & 1 & \ldots & 1\\ 0 & \ldots & 1/2 & \ldots & 1\\ 0 & \ldots & 0 & 1/2 & 1\\ 0 & \ldots & \ldots & 0 & 1/2\end{bmatrix} \quad \ldots(9)$$

If the transform matrix $[\phi]$ is unitary ,that is $[\phi]^{-1}=[\phi]^T$, then the equation (8) can be rewritten as [7,8]:

$$[Q_\phi]=[\phi]\cdot[Q_B]\cdot[\phi]^T \quad \ldots(10)$$

The Haar functions are an orthogonal family of switched rectangular waveforms where amplitudes can differ from one function to another. They are defined in the interval [0,1) by [6,7]:

$$h_0(t)=\frac{1}{\sqrt{m}}$$

$$h_i(t)=\frac{1}{\sqrt{m}}\begin{cases}2^{\frac{J}{2}} & \frac{k-1}{2^J}\le t<\frac{k-1/2}{2^J}\\ -2^{\frac{J}{2}} & \frac{k-1/2}{2^J}\le t<\frac{k}{2^J}\\ 0 & otherwise \quad in \quad [0,1)\end{cases} \quad \ldots(11)$$

where i=0,1,2,…..,m-1, $m=2^\alpha$ and α is a positive integer. J and k represent the integer decomposition of the index i , i.e. $i=2^J+k-1$.

Theoretically, this set of functions is complete. $h_0(t)$ is called the scaling function and $h_1(t)$ the mother wavelet, such that from the mother wavelet $h_1(t)$, compression and translation are performed to obtain $h_2(t)$ and $h_3(t)$.

Any function $u(x,t)$ which is square integrable in the region $0\le t<1$ and $0\le x<1$ can be expanded into Haar series by [6]:

$$u(x,t)=\sum_{i=0}^{m-1}\sum_{J=0}^{m-1}c_{iJ}h_i(x)h_J(t) \quad \ldots(12)$$

where $c_{iJ}=\int_0^1 u(x,t)h_i(x)dx\cdot\int_0^1 u(x,t)h_J(t)dt\cdot$

The equation (12) can be written into the discrete form by:

$$u(x,t)=H^T(x)\cdot C\cdot H(t) \quad \ldots(13)$$

where

$$[C]=\begin{bmatrix}C_{0,0} & C_{0,1} & \ldots & \ldots & C_{0,m-1}\\ C_{1,0} & C_{1,1} & \ldots & \ldots & C_{1,m-1}\\ \vdots & \vdots & \ldots & \ldots & \vdots\\ \vdots & \vdots & \ldots & \ldots & \vdots\\ C_{m-1,0} & C_{m-1,1} & \ldots & \ldots & C_{m-1,m-1}\end{bmatrix}$$

is the coefficient matrix of $u(x,t)$ calculated by:

$$[C]=[H]\cdot[u]\cdot[H]^T \quad \ldots(14)$$

For deriving the operational matrix of Haar wavelets, we let $[\phi]=[H]$ in the equation (10), and obtain:

$$[Q_H]=[H]\cdot[Q_B]\cdot[H]^T \ldots(15)$$

where $[Q_H]$ is the operational matrix for integration of $[H]$.

For example, the operational matrix of the Haar wavelet in the case of m=4 is given by:

$$[Q_H]=[H]_{4*4}\cdot[Q_B]\cdot[H]^T_{4*4}$$

$$=\begin{bmatrix}\frac12 & \frac12 & \frac12 & \frac12\\ \frac12 & \frac12 & -\frac12 & -\frac12\\ \frac{1}{\sqrt2} & -\frac{1}{\sqrt2} & 0 & 0\\ 0 & 0 & \frac{1}{\sqrt2} & -\frac{1}{\sqrt2}\end{bmatrix}\cdot\frac14\begin{bmatrix}\frac12 & 1 & 1 & 1\\ 0 & \frac12 & 1 & 1\\ 0 & 0 & \frac12 & 1\\ 0 & 0 & 0 & \frac12\end{bmatrix}\cdot\begin{bmatrix}\frac12 & \frac12 & \frac12 & \frac12\\ \frac12 & \frac12 & -\frac12 & -\frac12\\ \frac{1}{\sqrt2} & -\frac{1}{\sqrt2} & 0 & 0\\ 0 & 0 & \frac{1}{\sqrt2} & -\frac{1}{\sqrt2}\end{bmatrix}^T$$

$$=\begin{bmatrix}0.5 & -0.25 & -0.0884 & -0.0884\\ 0.25 & 0 & -0.0884 & 0.0884\\ 0.0884 & 0.0884 & 0 & 0\\ 0.0884 & -0.0884 & 0 & 0\end{bmatrix}$$

4.Numerical solution:

We will use the operational matrices of the Haar wavelets to solve the nonlinear Murray equation (3) numerically.

The integration of equation (13) with respect to the variable (t) yields [6]:

$$\int_0^t u(x,t)\,dt=\int_0^t H^T(x)\cdot Cu\cdot H(t)\,dt=H^T\cdot Cu\cdot\int_0^t H(t)\,dt$$

$$=[H]^T\cdot[Cu]\cdot[Q_H]\cdot[H] \quad \ldots(16)$$

Further integration of with respect to the variable (x) gives:

$$\int_0^x u(x,t)\,dx=\int_0^x H^T(x)\cdot Cu\cdot H(t)\,dx=\int_0^x H^T(x)\,dx\cdot Cu\cdot[H]$$

$$=[H]^T\cdot[Q_H]^T\cdot[Cu]\cdot[H] \quad \ldots(17)$$

The double integration of (u) with respect to the variables (x) and (t) gives:

$$\int_0^t\int_0^x u(x,t)\,dx\,dt=\int_0^t\int_0^x H^T(x)\cdot Cu\cdot H(t)\,dx\,dt$$

$$=\int_0^x H^T(x)\,dx\cdot Cu\cdot\int_0^t H(t)\,dt$$

$$=[H]^T\cdot[Q_H]^T\cdot[Cu]\cdot[Q_H]\cdot[H] \quad \ldots(18)$$

also

$$\int_0^t\int_0^x\int_0^x u(x,t)\,dx\,dx\,dt=\int_0^t\int_0^x\int_0^x H^T(x)\cdot Cu\cdot H(t)\,dx\,dx\,dt$$

$$=\int_0^x\int_0^x H^T(x)\,dx\,dx\cdot Cu\cdot\int_0^t H(t)\,dt$$

$$=[H]^T\cdot\left[{Q_H}^2\right]^T\cdot[Cu]\cdot[Q_H]\cdot[H]\ldots(19)$$

Now by integrating equation (3) with respect to (t), we get:

$$\int_0^t\frac{\partial u(x,t)}{\partial t}\,dt=\int_0^t\frac{\partial^2u(x,t)}{\partial x^2}\,dt+\lambda_1\int_0^t u(x,t)\frac{\partial u(x,t)}{\partial x}\,dt+\lambda_2\int_0^t u(x,t)\,dt-\lambda_3\int_0^t u^2(x,t)\,dt$$

…(20)

By using the initial condition (4), we get:

$$[u(x,t)-F(x)]=\int_0^t\frac{\partial^2u(x,t)}{\partial x^2}\,dt+\lambda_1\int_0^t u(x,t)\frac{\partial u(x,t)}{\partial x}\,dt+\lambda_2\int_0^t u(x,t)\,dt-\lambda_3\int_0^t u^2(x,t)\,dt$$

…(21)

Now, the double integration for equation (21) with respect to (x), gives:

$$\int_0^x\int_0^x u(x,t)\,dx\,dx-\int_0^x\int_0^x F(x)\,dx\,dx=\int_0^t\int_0^x\left[\frac{\partial u(x,t)}{\partial x}-\frac{\partial u(0,t)}{\partial x}\right]dx\,dt$$

$$+\lambda_1\int_0^t\int_0^x\left[\frac{u^2(x,t)}{2}\right]_0^x dx\,dt+\lambda_2\int_0^t\int_0^x\int_0^x u(x,t)\,dx\,dx\,dt$$

$$-\lambda_3\int_0^t\int_0^x\int_0^x u^2(x,t)\,dx\,dx\,dt\ \ldots(22)$$

by using the boundary condition (6), we get:

$$\int_0^x\int_0^x u(x,t)\,dx\,dx-\int_0^x\int_0^x F(x)\,dx\,dx=\int_0^t[u(x,t)-u(0,t)]\,dt-\int_0^t\int_0^x I(t)\,dx\,dt$$

$$+\lambda_1\int_0^t\int_0^x\left[\frac{u^2(x,t)}{2}-\frac{u^2(0,t)}{2}\right]dx\,dt+\lambda_2\int_0^t\int_0^x\int_0^x u(x,t)\,dx\,dx\,dt$$

$$-\lambda_3\int_0^t\int_0^x\int_0^x u^2(x,t)\,dx\,dx\,dt\quad\ldots(23)$$

by using the boundary condition (5) and by rearranging the equation (23), we get:

$$\int_0^x\int_0^x u(x,t)\,dx\,dx-\int_0^t u(x,t)\,dt-\frac{\lambda_1}{2}\int_0^t\int_0^x u^2(x,t)\,dx\,dt-\lambda_2\int_0^t\int_0^x\int_0^x u(x,t)\,dx\,dx\,dt$$

$$+\lambda_3\int_0^t\int_0^x\int_0^x u^2(x,t)\,dx\,dx\,dt=\int_0^x\int_0^x F(x)\,dx\,dx-\int_0^t G(t)\,dt-\int_0^t\int_0^x I(t)\,dx\,dt-\frac{\lambda_1}{2}\int_0^t\int_0^x G^2(t)\,dx\,dt$$

…(24)

we transform equation (24) into the matrices forms by using equation (13), we get:

$$\int_0^x\int_0^x[H(x)]^T\cdot[Cu]\cdot[H(t)]\,dx\,dx-\int_0^t[H(x)]^T\cdot[Cu]\cdot[H(t)]\,dt$$

$$-\frac{\lambda_1}{2}\int_0^t\int_0^x[H(x)]^T\cdot\left[Cu^2\right]\cdot[H(t)]\,dx\,dt-\lambda_2\int_0^t\int_0^x\int_0^x[H(x)]^T\cdot[Cu]\cdot[H(t)]\,dx\,dx\,dt$$

$$+\lambda_3\int_0^t\int_0^x\int_0^x[H(x)]^T\cdot\left[Cu^2\right]\cdot[H(t)]\,dx\,dx\,dt=\int_0^x\int_0^x[H(x)]^T\cdot[J_1]\cdot[H(t)]\,dx\,dx$$

$$-\int_0^t[H(x)]^T\cdot[J_2]\cdot[H(t)]\,dt-\int_0^t\int_0^x[H(x)]^T\cdot[J_3]\cdot[H(t)]\,dx\,dt-\frac{\lambda_1}{2}\int_0^t\int_0^x[H(x)]^T\cdot[J_4]\cdot[H(t)]\,dx\,dt$$

….(25)

such that $\left[Cu^2\right]\neq[Cu]$, from equation (13) $\left[Cu^2\right]$ is :

$$\left[Cu^2\right]=[H]\cdot[u]^2\cdot[H]^T$$
$$=[H]\cdot\left[H^T\cdot Cu\cdot H\right]^2\cdot[H]^T\qquad\ldots(26)$$

and

$$[J_1]=[H]_{m\times m}\cdot\begin{bmatrix}F(x_1)&F(x_1)&\cdots&\cdots&F(x_1)\\F(x_2)&F(x_2)&\cdots&\cdots&F(x_2)\\\vdots&\vdots&\vdots&\vdots&\vdots\\\vdots&\vdots&\vdots&\vdots&\vdots\\F(x_m)&F(x_m)&\cdots&\cdots&F(x_m)\end{bmatrix}_{m\times m}\cdot[H]^T_{m\times m}$$

…(27)

where

$$x_i=\frac{1}{2m}+\frac{i-1}{m}\qquad i=1,2,3,\ldots\ldots\ldots$$

m is the dimension of the matrix.

$$[J_2]=[H]_{m\times m}\cdot\begin{bmatrix}G(t_1)&G(t_2)&\cdots&\cdots&G(t_m)\\G(t_1)&G(t_2)&\cdots&\cdots&G(t_m)\\\vdots&\vdots&\vdots&\vdots&\vdots\\\vdots&\vdots&\vdots&\vdots&\vdots\\G(t_1)&G(t_2)&\cdots&\cdots&G(t_m)\end{bmatrix}_{m\times m}\cdot[H]^T_{m\times m}\qquad(28)$$

and also

$$[J_3]=[H]_{m\times m}\cdot\begin{bmatrix}I(t_1)&I(t_2)&\cdots&\cdots&I(t_m)\\I(t_1)&I(t_2)&\cdots&\cdots&I(t_m)\\\vdots&\vdots&\vdots&\vdots&\vdots\\\vdots&\vdots&\vdots&\vdots&\vdots\\I(t_1)&I(t_2)&\cdots&\cdots&I(t_m)\end{bmatrix}_{m\times m}\cdot[H]^T_{m\times m}$$

…(29)

where

$$t_i=\frac{1}{2m}+\frac{i-1}{m}\qquad i=1,2,3,\ldots\ldots\ldots$$

$$[J_4]=[J_2]^2\ldots(30)$$

Now, by using the integrations (16),(17),(18) and (19), the equations (25) becomes:

$$[H]^T\cdot\left[Q_H^2\right]^T\cdot[Cu]\cdot[H]-[H]^T\cdot[Cu]\cdot[Q_H]\cdot[H]$$
$$-\frac{\lambda_1}{2}\cdot[H]^T\cdot[Q_H]^T\cdot\left[[H]\cdot\left([H]^T\cdot[Cu]\cdot[H]\right)^2\cdot[H]^T\right]\cdot[Q_H]\cdot[H]$$
$$-\lambda_2\cdot[H]^T\cdot\left[Q_H^2\right]^T\cdot[Cu]\cdot[Q_H]\cdot[H]$$
$$+\lambda_3\cdot[H]^T\cdot\left[Q_H^2\right]^T\cdot\left[[H]\cdot\left([H]^T\cdot[Cu]\cdot[H]\right)^2\cdot[H]^T\right]\cdot[Q_H]\cdot[H]$$
$$=[H]^T\cdot\left[Q_H^2\right]^T\cdot[J_1]\cdot[H]-[H]^T\cdot[J_2]\cdot[Q_H]\cdot[H]$$
$$-[H]^T\cdot[Q_H]^T\cdot[J_3]\cdot[Q_H]\cdot[H]$$
$$-\frac{\lambda_1}{2}\cdot[H]^T\cdot[Q_H]^T\cdot\left[[J_2]^2\right]\cdot[Q_H]\cdot[H]\ldots(31)$$

such that the dimension for all matrices are $m\times m$, $[H]$ is Haar wavelets matrix, $[Q_H]$ is the operational matrix of the Haar wavelet, $[Cu]$ is the coefficient matrix of u(x,t):

$$[Cu]=\begin{bmatrix}Cu_{0,0}&Cu_{0,1}&\cdots&\cdots&Cu_{0,m-1}\\Cu_{1,0}&Cu_{1,1}&\cdots&\cdots&Cu_{1,m-1}\\\vdots&\vdots&\vdots&\vdots&\vdots\\\vdots&\vdots&\vdots&\vdots&\vdots\\Cu_{m-1,0}&Cu_{m-1,1}&\cdots&\cdots&Cu_{m-1,m-1}\end{bmatrix}_{m\times m}$$

by multiplying $[H]^T$ to the right hand side and $[H]$ to the left hand side of each term in equations (31), we get:

$$\left[Q_H^2\right]^T\cdot[Cu]-[Cu]\cdot[Q_H]-\frac{\lambda_1}{2}\cdot[Q_H]^T\cdot\left[[H]\cdot\left([H]^T\cdot[Cu]\cdot[H]\right)^2\cdot[H]^T\right]\cdot[Q_H]$$
$$-\lambda_2\cdot\left[Q_H^2\right]^T\cdot[Cu]\cdot[Q_H]+\lambda_3\cdot\left[Q_H^2\right]^T\cdot\left[[H]\cdot\left([H]^T\cdot[Cu]\cdot[H]\right)^2\cdot[H]^T\right]\cdot[Q_H]$$

$$=\left[Q_H^2\right]^T\cdot[J_1]-[J_2]\cdot[Q_H]-[Q_H]^T\cdot[J_3]\cdot[Q_H]-\frac{\lambda_1}{2}\cdot[Q_H]^T\cdot\left[[J_2]^2\right]\cdot[Q_H]$$

…(32)

To find the coefficient matrix [Cu] which have 2^m of the elements respectively, we solve the equation (32) which given nonlinear system of the equations such that the variables number are 2^m and we will can be solved this nonlinear system by Newton-Raphson method, after this we find the matrix solution [u] by using the equation (13) that is:

$$[u]=[H]^T\cdot[Cu]\cdot[H]$$

5.Numerical results:

In this section, we have solved equation (32) with the initial condition:

$$u(x,0)=F(x)=\frac{\lambda_2+c_1\exp(\gamma x)}{\lambda_3+c_0}$$

and mixed boundary conditions:

$$u(0,t)=G(t)=\frac{\lambda_2+c_1\exp(\gamma^2 t)}{\lambda_3+c_0\exp(-\lambda_2 t)}$$

$$\frac{\partial u(0,t)}{\partial x}=I(t)=\frac{c_1\gamma\ \exp(\gamma^2 t)}{\lambda_3+c_0\exp(-\lambda_2 t)}$$

with $c_0=1, c_1=1, \gamma=1\ \ ,\lambda_1=1, \lambda_2=1\ and\ \lambda_3=1$.

When m=4 then, from the equation (11),we get:

$$[H]=\begin{bmatrix}\frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2}\\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2}\\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & 0\\ 0 & 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}}\end{bmatrix}$$

from the equation (15), we get:

$$[Q_H]=\begin{bmatrix}0.5 & -0.25 & -0.08838835 & -0.08838835\\ 0.25 & 0 & -0.08838835 & 0.08838835\\ 0.08838835 & 0.08838835 & 0 & 0\\ 0.08838835 & -0.08838835 & 0 & 0\end{bmatrix}$$

from the equations (27),(28) and (29), we get:

$$[J_1]=[H]\cdot\begin{bmatrix}\frac{1+\exp(1/8)}{2} & \frac{1+\exp(1/8)}{2} & \frac{1+\exp(1/8)}{2} & \frac{1+\exp(1/8)}{2}\\ \frac{1+\exp(3/8)}{2} & \frac{1+\exp(3/8)}{2} & \frac{1+\exp(3/8)}{2} & \frac{1+\exp(3/8)}{2}\\ \frac{1+\exp(5/8)}{2} & \frac{1+\exp(5/8)}{2} & \frac{1+\exp(5/8)}{2} & \frac{1+\exp(5/8)}{2}\\ \frac{1+\exp(7/8)}{2} & \frac{1+\exp(7/8)}{2} & \frac{1+\exp(7/8)}{2} & \frac{1+\exp(7/8)}{2}\end{bmatrix}\cdot[H]^T$$

where $x_i=\frac{1}{2m}+\frac{i-1}{m}$ then

$x_1=1/8, x_2=3/8, x_3=5/8, x_4=7/8$.

$$[J_2]=[H]\cdot\begin{bmatrix}\frac{1+\exp(1/8)}{1+\exp(-1/8)} & \frac{1+\exp(3/8)}{1+\exp(-3/8)} & \frac{1+\exp(5/8)}{1+\exp(-5/8)} & \frac{1+\exp(7/8)}{1+\exp(-7/8)}\\ \frac{1+\exp(1/8)}{1+\exp(-1/8)} & \frac{1+\exp(3/8)}{1+\exp(-3/8)} & \frac{1+\exp(5/8)}{1+\exp(-5/8)} & \frac{1+\exp(7/8)}{1+\exp(-7/8)}\\ \frac{1+\exp(1/8)}{1+\exp(-1/8)} & \frac{1+\exp(3/8)}{1+\exp(-3/8)} & \frac{1+\exp(5/8)}{1+\exp(-5/8)} & \frac{1+\exp(7/8)}{1+\exp(-7/8)}\\ \frac{1+\exp(1/8)}{1+\exp(-1/8)} & \frac{1+\exp(3/8)}{1+\exp(-3/8)} & \frac{1+\exp(5/8)}{1+\exp(-5/8)} & \frac{1+\exp(7/8)}{1+\exp(-7/8)}\end{bmatrix}\cdot[H]^T$$

where $t_i=\frac{1}{2m}+\frac{i-1}{m}$ then

$t_1=1/8, t_2=3/8, t_3=5/8, t_4=7/8$.

$$[J_3]=[H]\cdot\begin{bmatrix}\frac{\exp(1/8)}{1+\exp(-1/8)} & \frac{\exp(3/8)}{1+\exp(-3/8)} & \frac{\exp(5/8)}{1+\exp(-5/8)} & \frac{\exp(7/8)}{1+\exp(-7/8)}\\ \frac{\exp(1/8)}{1+\exp(-1/8)} & \frac{\exp(3/8)}{1+\exp(-3/8)} & \frac{\exp(5/8)}{1+\exp(-5/8)} & \frac{\exp(7/8)}{1+\exp(-7/8)}\\ \frac{\exp(1/8)}{1+\exp(-1/8)} & \frac{\exp(3/8)}{1+\exp(-3/8)} & \frac{\exp(5/8)}{1+\exp(-5/8)} & \frac{\exp(7/8)}{1+\exp(-7/8)}\\ \frac{\exp(1/8)}{1+\exp(-1/8)} & \frac{\exp(3/8)}{1+\exp(-3/8)} & \frac{\exp(5/8)}{1+\exp(-5/8)} & \frac{\exp(7/8)}{1+\exp(-7/8)}\end{bmatrix}\cdot[H]^T$$

$$[J_4]=[J_2]^2$$

Now, by substitute the matrices $[Q_H],[J_1],[J_2]\,and\,[J_3]$ in the system (32) we get the nonlinear system consist of 16 equations and 16 variables represents the matrix element [Cu] and by solving this system by Newton-Raphson method, we obtain:

$$[Cu]=\begin{bmatrix}10.10183769 & -2.75857293 & -0.75219935 & -1.31428609\\ -1.89521262 & 0.63010358 & 0.17830194 & 0.34581088\\ -0.50987624 & 0.16954517 & 0.04533763 & 0.08340445\\ -0.85421023 & 0.28412063 & 0.08412725 & 0.17254757\end{bmatrix}$$

Now, by using the equation (13), we get:

$$[u]=[H]^T\cdot[Cu]\cdot[H]$$

$$=\begin{bmatrix}1.21897916 & 1.57944828 & 2.04285639 & 2.64426733\\ 1.41429195 & 1.86543632 & 2.43987543 & 3.20809527\\ 1.66361808 & 2.23745460 & 2.94331184 & 3.94463010\\ 1.98260505 & 2.72469607 & 3.57568574 & 4.92209913\end{bmatrix}$$

Table (1). A comparison between the operational matrix of the Haar wavelets method with exact solution for nonlinear Murray equation with: m=4 $c_0=1, c_1=1, \gamma=1\ \ ,\lambda_1=1, \lambda_2=1\ and\ \lambda_3=1$.

The value of (x)	The value of (t)	The numerical solution of u(x,t)	The exact solution of u(x,t)	The absolute error
0.125	0.125	1.21897916	1.21329571	0.00568345
0.125	0.375	1.57944828	1.56980863	0.00963965
0.125	0.625	2.04285639	2.03027312	0.01258327
0.125	0.875	2.64426733	2.62430764	0.01995968
0.375	0.125	1.41429195	1.40702557	0.00726638
0.375	0.375	1.86543632	1.84734180	0.01809451
0.375	0.625	2.43987543	2.42192096	0.01795447
0.375	0.875	3.20809527	3.16921683	0.03887843
0.625	0.125	1.66361808	1.65577963	0.00783845
0.625	0.375	2.23745460	2.20370145	0.03375315
0.625	0.625	2.94331184	2.92480673	0.01850511
0.625	0.875	3.94463010	3.86889407	0.07573602
0.875	0.125	1.98260505	1.97518616	0.00741889
0.875	0.375	2.72469607	2.66127629	0.06341977
0.875	0.625	3.57568574	3.57052484	0.00516090
0.875	0.875	4.92209913	4.76729744	0.15480169

Now when m=8 then, from the equation (11),we get:

$$H=\begin{bmatrix} 0.3536 & 0.3536 & 0.3536 & 0.3536 & 0.3536 & 0.3536 & 0.3536 & 0.3536 \\ 0.3536 & 0.3536 & 0.3536 & 0.3536 & -0.3536 & -0.3536 & -0.3536 & -0.3536 \\ 0.5000 & 0.5000 & -0.5000 & -0.5000 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.5000 & 0.5000 & -0.5000 & -0.5000 \\ 0.7071 & -0.7071 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.7071 & -0.7071 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.7071 & -0.7071 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.7071 & -0.7071 \end{bmatrix}$$

$$cu=\begin{bmatrix} 20.0712 & -5.4431 & -1.4417 & -2.4719 & -0.4421 & -0.5818 & -0.7618 & -0.9933 \\ -3.6959 & 1.2140 & 0.3056 & 0.5627 & 0.0880 & 0.1227 & 0.1677 & 0.2256 \\ -1.0056 & 0.3304 & 0.0842 & 0.1539 & 0.0252 & 0.0346 & 0.0468 & 0.0625 \\ -1.6547 & 0.5448 & 0.1388 & 0.2537 & 0.0416 & 0.0571 & 0.0772 & 0.1030 \\ -0.3160 & 0.1027 & 0.0262 & 0.0478 & 0.0078 & 0.0108 & 0.0145 & 0.0194 \\ -0.4043 & 0.1319 & 0.0336 & 0.0614 & 0.0101 & 0.0138 & 0.0187 & 0.0249 \\ -0.5177 & 0.1693 & 0.0431 & 0.0789 & 0.0129 & 0.0177 & 0.0240 & 0.0320 \\ -0.6633 & 0.2174 & 0.0554 & 0.1013 & 0.0166 & 0.0228 & 0.0308 & 0.0411 \end{bmatrix}$$

$$u=\begin{bmatrix} 1.0994 & 1.2507 & 1.4190 & 1.6133 & 1.8279 & 2.0773 & 2.3567 & 2.6769 \\ 1.1797 & 1.3467 & 1.5334 & 1.7492 & 1.9889 & 2.2674 & 2.5804 & 2.9395 \\ 1.2671 & 1.4519 & 1.6594 & 1.8996 & 2.1677 & 2.4792 & 2.8304 & 3.2335 \\ 1.3695 & 1.5744 & 1.8056 & 2.0734 & 2.3737 & 2.7225 & 3.1170 & 3.5700 \\ 1.4852 & 1.7079 & 1.9673 & 2.2614 & 2.6042 & 2.9904 & 3.4353 & 3.9398 \\ 1.6159 & 1.8645 & 2.1543 & 2.4839 & 2.8680 & 3.3022 & 3.8026 & 4.3712 \\ 1.7605 & 2.0384 & 2.3626 & 2.7324 & 3.1633 & 3.6518 & 4.2152 & 4.8563 \\ 1.9277 & 2.2388 & 2.6019 & 3.0173 & 3.5013 & 4.0514 & 4.6860 & 5.4094 \end{bmatrix}$$

Table (2). A comparison between the operational matrix of the Haar wavelets method with exact solution for nonlinear Murray equation with: m=8 $c_0=1, c_1=1, \gamma=1 \;, \lambda_1=1, \lambda_2=1 \; and \; \lambda_3=1$ **.**

The value of (x)	The value of (t)	The numerical solution of u(x,t)	The exact solution of u(x,t)	The absolute error
0.0625	0.0625	1.09937606	1.09989383	0.00051777
0.0625	0.1875	1.25074607	1.24876384	0.00198223
0.0625	0.3125	1.41899616	1.41774616	0.00125000
0.0625	0.4375	1.61328036	1.60953036	0.00375000
0.0625	0.5625	1.82789323	1.82716100	0.00073223
0.0625	0.6875	2.07731696	2.07408472	0.00323223
0.0625	0.8125	2.35670372	2.35420372	0.00250000
0.0625	0.9375	2.67693647	2.67193647	0.00500000
0.1875	0.0625	1.17967123	1.17768899	0.00198223
0.1875	0.1875	1.34671970	1.34223747	0.00448223
0.1875	0.3125	1.53337426	1.52962426	0.00375000
0.1875	0.4375	1.74917743	1.74292743	0.00625000
0.1875	0.5625	1.98885723	1.98562410	0.00323223
0.1875	0.6875	2.26737974	2.26164751	0.00573223
0.1875	0.8125	2.58043810	2.57543810	0.00500000
0.1875	0.9375	2.93952095	2.93202095	0.00750000
0.3125	0.0625	1.26709247	1.26584247	0.00125000
0.3125	0.1875	1.45190697	1.44815697	0.00375000
0.3125	0.3125	1.65941651	1.65639875	0.00301777
0.3125	0.4375	1.89960388	1.89408611	0.00551777
0.3125	0.5625	2.16768823	2.16518823	0.00250000
0.3125	0.6875	2.47918399	2.47418399	0.00500000
0.3125	0.8125	2.83039725	2.82612949	0.00426777
0.3125	0.9375	3.23350304	3.22673527	0.00676777
0.4375	0.0625	1.36948344	1.36573344	0.00375000
0.4375	0.1875	1.57442949	1.56817949	0.00625000
0.4375	0.3125	1.80557083	1.80005306	0.00551777
0.4375	0.4375	2.07338910	2.06537134	0.00801777
0.4375	0.5625	2.37366003	2.36866003	0.00500000
0.4375	0.6875	2.72251937	2.71501937	0.00750000
0.4375	0.8125	3.11696782	3.11020005	0.00676777
0.4375	0.9375	3.56995811	3.56069035	0.00926777
0.5625	0.0625	1.48517474	1.47892474	0.00625000
0.5625	0.1875	1.70793283	1.70418283	0.00375000

0.5625	0.3125	1.96731696	1.96283473	0.00448223
0.5625	0.4375	2.26144516	2.25946292	0.00198223
0.5625	0.5625	2.60422378	2.59922378	0.00500000
0.5625	0.6875	2.99042162	2.98792162	0.00250000
0.5625	0.8125	3.43532640	3.43209417	0.00323223
0.5625	0.9375	3.93984326	3.93911103	0.00073223
0.6875	0.0625	1.61593729	1.60718729	0.00875000
0.6875	0.1875	1.86454480	1.85829480	0.00625000
0.6875	0.3125	2.15427276	2.14729052	0.00698223
0.6875	0.4375	2.48387974	2.47939750	0.00448223
0.6875	0.5625	2.86798674	2.86048674	0.00750000
0.6875	0.6875	3.30216037	3.29716037	0.00500000
0.6875	0.8125	3.80258022	3.79684799	0.00573223
0.6875	0.9375	4.37115007	4.36791784	0.00323223
0.8125	0.0625	1.76054556	1.75252779	0.00801777
0.8125	0.1875	2.03844430	2.03292654	0.00551777
0.8125	0.3125	2.36255632	2.35630632	0.00625000
0.8125	0.4375	2.73236604	2.72861604	0.00375000
0.8125	0.5625	3.16330423	3.15653646	0.00676777
0.8125	0.6875	3.65184155	3.64757379	0.00426777
0.8125	0.8125	4.21516822	4.21016822	0.00500000
0.8125	0.9375	4.85631961	4.85381961	0.00250000
0.9375	0.0625	1.92773793	1.91922016	0.00851777
0.9375	0.1875	2.23882799	2.23081022	0.00801777
0.9375	0.3125	2.60190225	2.59315225	0.00875000
0.9375	0.4375	3.01726763	3.01101763	0.00625000
0.9375	0.5625	3.50127250	3.49200474	0.00926777
0.9375	0.6875	4.05141198	4.04464421	0.00676777
0.9375	0.8125	4.68602139	4.67852139	0.00750000
0.9375	0.9375	5.40941845	5.40441845	0.00500000

6.Conclusion:

In this paper, we solved the non-linear Murray equation by using the operational matrices of the Haar wavelets method. This method is very convenient for solving the boundary value problems, since the boundary conditions are taken care for automatically. It transforms the nonlinear partial differential equation into a nonlinear algebraic equation by using matrices representation, which is implied to smooth solution. All these matrices in this numerical solution are represented in all time steps. We solved this nonlinear algebraic equations by using Newton-Raphson method and we compared the numerical solution with the exact solution of the Murray equation. We obtained that this method is simple in the computation and we note that the accuracy of the obtained solution is quite high even if the number of calculation points is small. This circumstance follows from table (1) and figures (1) and (2) such that the number of grid points (the dimensions of the matrices) are 4*4. By increasing the number of collocation points the error of the solution rapidly decreases as shown in the table (2) and figures (3) and (4) such that the number of grid points are 8*8.

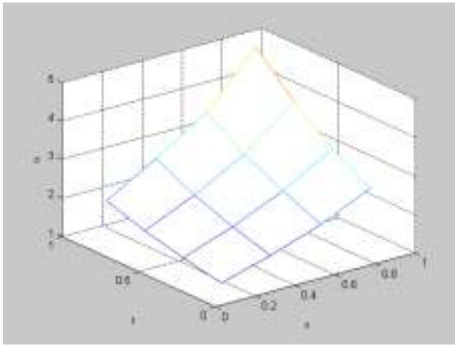

Figure (1). An illustration the numerical solution for nonlinear Murray equation u(x,t) with: m=4 $c_0 = 1, c_1 = 1, \gamma = 1$, $\lambda_1 = 1, \lambda_2 = 1$ *and* $\lambda_3 = 1$

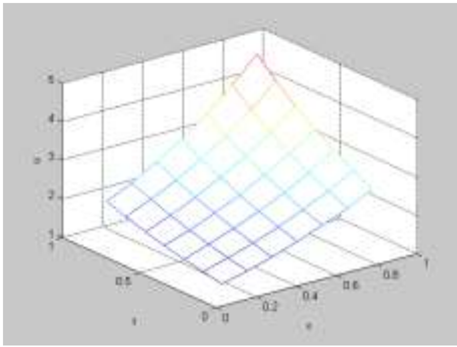

Figure (2). An illustration the exact solution for nonlinear Murray equation u(x,t) with: m=4 $c_0 = 1, c_1 = 1, \gamma = 1$, $\lambda_1 = 1, \lambda_2 = 1$ *and* $\lambda_3 = 1$.

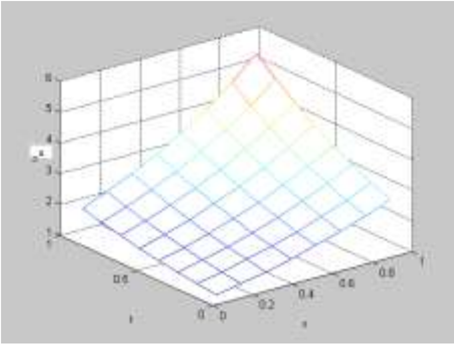

Figure (3). An illustration the numerical solution for nonlinear Murray equation u(x,t) with: m=8

$c_0 = 1, c_1 = 1, \gamma = 1 \quad , \lambda_1 = 1, \lambda_2 = 1 \text{ and } \lambda_3 = 1.$

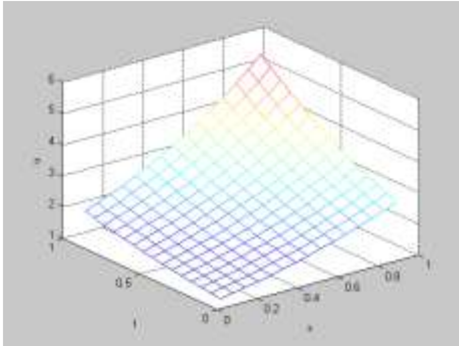

Figure (4). An illustration the exact solution for nonlinear Murray equation u(x,t) with: m=8

$c_0 = 1, c_1 = 1, \gamma = 1 \quad , \lambda_1 = 1, \lambda_2 = 1 \text{ and } \lambda_3 = 1$

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الحل العددي لمعادلة Murray اللاخطية باستخدام طريقة مصفوفات العوامل لموجات Haar القصيرة

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الملخص

تم تطبيق طريقة مصفوفات العوامل لموجات Haar القصيرة لإيجاد الحل العددي لمعادلة Murray اللاخطية وتم مقارنة هذه النتائج العددية مع الحل المضبوط لمعادلة Murray اللاخطية. إن دقة الحلول التي حصلنا عليها عالية حتى إذا كانت عدد نقاط الشبكة المحسوبة صغيرة وكلما زادت عدد نقاط الشبكة المحسوبة فان الدقة تزداد والخطأ يتناقص وقد تم توضيح ذلك من خلال حل مثال.