Strongly semicontinuity in intuitionistic fuzzy specal topological spaces

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Abstract

The aim of this paper is generalize the concept of fuzzy strongly semicontinuous due to Bai shi zhong [2] to intuitionistic fuzzy specal topological space. We study the relation between strongly semicontinuous with semicontinuous and precontinuous. Some of properties are studied using these concepts .We investigate several characterization.

1. Introduction

After the introduction of fuzzy by Zadeh there have been number of generalization of this fundamental concept . The notion of intuitionistic fuzzy sets introduced by Atanassov is one among them . Using the notion of intuitionistic fuzzy sets Coker[3] introduced the notion of intuitionistic fuzzy topological spaces[4]. The concept is used to define intuitionistic fuzzy specal set by Coker [7] and intuitionistic fuzzy specal topological spaces are introduced.

In section 3 we introduced the concepts of strong semi-interior and strong semi-closure. We also establish some of their properties. In section 4 we introduce strongly semicontinuous, strongly semi-open function. At the same time we establish some of their characteristic properties and also discuss the relations between those and continuous(open),semicontinuous,(semi-open,pre-open functions).

preliminaries 2.

First we shall present the fundamental definitions

Definition 2.1 [8]

Let X be a nonempty set .An intuitionistic fuzzy special set A is an object having the form A = $\langle x, A_1, A_2 \rangle$, where A_1 and A_2 are subsets of X satisfying $A_1 \cap A_2 = \phi$ .The set A_1 is called the set of members of A , while A_2 is called the set of nonmembers of A.

Definition2.2 [2]

Let F:X→Y where X and Y are ITS.

1. If B =$\langle x, B_1, B_2 \rangle$ is an intuitionistic fuzzy special set in Y ,then the preimage of B under f denoted by f^{-1}(B) and defined $f^{-1}(B) = \langle x, f^{-1}(B_1), f^{-1}(B_2)\rangle$.
2. if A= $\langle x, A_1, A_2 \rangle$ is an intuitionistic fuzzy special set in X , then the image of A under f denoted by f(A)and defined by f $(A) = \langle y, f(A_1), \underline{f}(A_2)\rangle$, where $\underline{f}(A) = \left(f(A_2^c)\right)^c$.

Corollary2.3 [10]

Let A, A_i ($i \in J$) be an intuitionistic fuzzy special sets in X , B, B_j ($j \in K$) an intuitionistic fuzzy special sets in Y, and f: X→Y be function then

1. $A_1 \subseteq A_2 \to f(A_1) \subseteq f(A_2)$,
2. $B_1 \subseteq B_2 \to f^{-1}(B_1) \subseteq f^{-1}(B_2)$,
3. A ⊆ f^{-1} (f (A)) ,and if f is injective ,then A = f^{-1} (f (A)) ,
4. f(f^{-1}(B)⊆ B, and if f is surjective , then f(f^{-1}(B)) =B,
5. $f^{-1}(\cup B_j) = \cup f^{-1}(B_j)$,
6. $f^{-1}(\cap B_j) = \cap f^{-1}(B_j)$,
7. $f(\cup A_i) = \cup f(A_i)$,
8. $f(\cap A_i) \subseteq \cap f(A_i)$, if f is injecthve , then $f(\cap A_i) = \cap f(A_i)$,
9. $f^{-1}(\tilde{Y}) = \tilde{X}$, $f^{-1}(\tilde{\Phi}) = \tilde{\Phi}$,
10. if f is surjective $f(\tilde{X}) = \tilde{Y}$, $f^{-1}(\bar{B}) = \overline{f^{-1}(B)}$ .11

12. if f is surjective then $\overline{(f(A))} \subseteq f(\bar{A})$ and if furthermore f is injective , we have $\overline{(f(A))} = f(\bar{A})$.

Definition2.4

Let (X,T) be aspace and let A be an intuitionistic fuzzy special sat of X. Then A is called :

1- intuitionistic fuzzy special semi-open set (SOS, for short) iff A⊆cl(int(A))[1].
2- intuitionistic fuzzy special pre-open set (POS, for short) iff A⊆ int(cl (A))[1].
3- intuitionistic fuzzy special semi-closed set (SCS, for short) iff int(cl(A)) ⊆ A. [6].
4- intuitionistic fuzzy special pre-closed set (PCS, for short) iff cl(int (A)) ⊆ A. [5].

We denoted SO(X) to the family of intuitionistic fuzzy semi-open sets of a space (X,T)

5- intuitionistic fuzzy special strongly semi-open set of X iff there is open set B ∈ T such that B ⊆ A⊆int(cl(B)).
6- intuitionistic fuzzy special strongly semi-closed set of X iff there is closed set B ∈ X such that cl(int(B)) ⊆ A⊆ B.

Definition2.5[7]

An intuitionistic fuzzy special topology on a nonempty set X is family T of intuitionistic fuzzy special sets in X containing $\tilde{\Phi}$, $\tilde{X}$

And closed under finite infima and arbitrary suprema in this case

the pair (X, T) is called an intuitionistic fuzzy special topological space and any intuitionistic fuzzy special set in T known open set in X.

From now the word space means an intuitionistic fuzzy special topological space.

Definition26 [2]

Let A be an intuitionistic fuzzy special set in a space (X, T). Then

1. sintA $=\cup$ { B : B$\subseteq$ A B$\in$SO(X) } is called the semi-interior of A .

2. sclA = $\cap$ { B : A$\subseteq$ B B $\in$ SC(X) } is called the semi-closure of A .

Definition2.7 [3]

An intuitionistic fuzzy special set A in a space (X, T) is called an intuitionistic fuzzy special preopen set if A $\subseteq$ intclA

The complement of an intuitionistic fuzzy special preopen set A is called an intuitionistic fuzzy special preclosed set in X.

Definition2.8 [5]

Let f: X $\rightarrow$ Y be a function from a space (X, T) into a space (Y,ψ) then :

1. f is continuous function if f^{-1}(B) is open set of X for each set B in Y.
2. f is semicontinuous if f^{-1}(B) is semi-open set in X ,for each open set B in Y .
3. f is precontinuous if f^{-1}(B) is pre-open set in X ,for each open set B in Y .
4. f is semi-open (semi-closed) function if f (A) is semi-open (semi-closed) set of Y ,for each open set A in X .
5. f is pre-open (pre-closed) function if f (A) is pre-open (pre-closed) set of Y ,for each open set A in X .

3- Intuitionistic fuzzy special strongly semi-interior and strongly semi-closure Definition3.1

Let A be an intuitionistic fuzzy special set in a space (X, T). Then.

1. SSint A $=\cup$ {B: B $\subseteq$ A, B is strongly semi-open set of X} is called intuitionistic fuzzy special strongly semi-interior of A.
2. SSCI = $\cap$ B: B $\subseteq$ A, B is strongly semi-closed set of X} is called intuitionistic fuzzy special strongly semi-closure of A

proposition3.2

Let A and B an intuitionistic fuzzy special set A in a space (X, T), then the following statement are valid

1- intA $\subseteq$ SSintA $\subseteq$ SintA $\subseteq$ A $\subseteq$ SclA $\subseteq$ SSclA $\subseteq$ clA

2- A $\subseteq$ B$\Rightarrow$ SSintA $\subseteq$ SSintB; SSclA $\subseteq$ SSclB

3- SSintX=X;SSclX=X , SSint ϕ = ϕ ; SScl ϕ = ϕ

4- A $\in$ SSO(X)iff SSintA=A

5- A $\in$ SSO(X) ff SSclA=A

6- SSint(SSintA)=SSintA, SScl(SSclA)=SSclA

7- SSint (A$\cap$B) $\subseteq$ SSintA $\cap$ SSintB and

8- SScl(AUB) $\subseteq$ SSclAU SSclB

Proof This is immediate from definition 3.1

Theorem3.3 Let Abe an intuitionistic fuzzy specal set of space (X, T) , then :

1- $\overline{\text{SSintA}}$ =SScl $\overline{\text{A}}$

2- $\overline{\text{SSintA}}$ =SSint $\overline{\text{A}}$

Proof

We prove only (1)

By [prop. 3.2] SSintA$\subseteq$ A and SSintA$\in$SSO(X) , then $\overline{\text{A}} \subseteq \overline{\text{SSintA}}$

And $\overline{\text{SSintA}}$ is strongly semi-closed set in X

Hance SScl $\overline{\text{A}} \subseteq \overline{\text{SSintA}}$ … … … (1)

Conversely

By [prop. 3.2] $\overline{\text{A}} \subseteq \overline{\text{SSintA}}$ and SSclA is strongly semi-closed set in X

Then $\overline{\text{SSintA}} \subseteq \overline{\text{A}}$ and $\overline{\text{SSintA}}$ $\in$ SSO(X)

So that

SScl $\overline{\text{A}}$ $\subseteq$ SSintA $\overline{\text{A}}$

Hance $\overline{\text{SSintA}}$ $\subseteq$ SScl $\overline{\text{A}}$ … … … (2)

From (1) and(2) we get

$\overline{\text{SSintA}}$ =SScl $\overline{\text{A}}$

In the sam way we prove 2

4-Intuitionistic fuzzy special strongly semicontinuous, strongly semi-open and strongly semi-closed functions Definition4.1 [1]

Let f: X $\rightarrow$ Y be a function from a space (X, T) into a space

(Y,ψ). The function f is called;

1. strongly semicontinuous if f^{-1}(B) is strongly semi-open set in X ,for each open set B in Y .
2. strongly semi-open function if f (A) is semi-open set in Y ,for each open set A in X .
3. strongly semi-closed function if f (A) is semi-closed set inY ,for each closed set A in X .

Theorem4.2

Let f: X $\rightarrow$ Y be a function from a space (X, T) into a space

(Y,ψ) then the following are equivalent:

1. f is strongly semicontinuous.
2. f^{-1}(B) is strongly semi-closed set in X ,for each closed set B of Y .
3. f (SSclA) $\subseteq$ clA ,for each set A of X
4. SScl(f^{-1}(B)) $\subseteq f^{-1}$(clB) for each set B of Y.
5. f^{-1}(intB) $\subseteq$ SSint(f^{-1}(B)) for each set Bof Y

Theorem4.3

Let f: X $\rightarrow$ Y be a function from a space (X, T) into a space (Y,ψ) where is one-one and onto , then f is strongly semicontinuous function iff (f(A)) $\subseteq$ (SSintA) for each set A of X

Proof

Let f be strongly semicontinuous and A be any open set of X , then (int f^{-1}(A)) is strongly semi- open set in X. By theorem 4.2 and fact that f is one –one , we have f^{-1}(intf (A)) $\subseteq$ SSint f^{-1}f(A))=SSintA again since f is onto , we have intf(A) $\subseteq$f f^{-1} (intA) $\subseteq$ (SSintA)

Conversely

Let B open set in Y , then B =intB By hypothesis

f (SSint(f^{-1}(B))$\supseteq$ int (f f^{-1}(B)=intB=B

This implies that

(f^{-1} (B))= f^{-1}f (SSint(f^{-1}(B)) $\supseteq f^{-1}$ (B)

Hance

f^{-1} (B)=SSint(f^{-1}(B)), i.e f^{-1}(B) is strongly semi-open set of X
Thus
F is strongly semicontinuous.

Theorem4.4

Let f: X → Y be a function from a space (X, T) into a space
(Y,ψ) then the following are equivalent:

1. f is strongly semicontinuous
2. f(intA) ⊆ SSint(f(A)) for each set A of X.
3. int(f^{-1}(B)) ⊆ f^{-1}(SSintB) for each setB of Y.

Proof (1) ⇔ (2) ⇔ (3) . analogous to the proof of theorem 4.2

Theorem4.5

Let f: X → Y be a function from a space (X, T) into a space
(Y,ψ) where is one-one and onto , then f is strongly semi-closed function iff f^{-1}(SSclB) ⊆ cl(f^{-1}(B)) for each set B of Y. Proof analogous to the proof of theorem 4.3

Theorem4.6

For the function f: X → Y from a space (X, T) into a space
(Y,ψ) then the following statements are valid

1- if f is continous then is stongly semicontinuous then f is semicontinuous.
2- if f is open (closed) then f is stongly semi-open(stongly semi-closed).
3- If f is strongly semicontinuous then f is precontinuous.
4- If f is stongly semi-open (stongly semi-closed) then f is pre-open (pre-closed).
5- If f is stongly semi-open (stongly semi-closed) then f is semi-open (semi-closed).

Example 4.7

Let X ={1,2,3}, T=< $\tilde{\Phi}$,$\tilde{X}$,A,B } , where A= <x,{1},{2,3}>, B=<x,{1,2}, $\tilde{\Phi}$ > and Y= {a,b,c}, ψ =< $\tilde{\Phi}$,$\tilde{Y}$,C} where C=<y,{a,c},{b} >
Defined a function f: X → Y as follows f(1)=a, f(2)=b,f(3)=c
f is not continuous since f^{-1}(C) =<x,{1,2,3} , {2} > ∉T
But f is strongly semicontinuous since intclint f^{-1}(C)=intclA=X
And f^{-1}(C)⊆X , so f is precontinuous where intcl f^{-1}(C)=X
From example 4.7 we conclude that:

1- f is not continuous but f is precontinuous.
2- f is not continuous but f is strongly semicontinuous.
3- f is not continuous but f is semicontinuous.

Example 4.8

Let X ={1,2,3}, T=< $\tilde{A}$, $\tilde{X}$, ϕ }, where A= <x, {2}, {1,3}>, And Y= {a,b,c}, ψ =< $\tilde{\Phi}$,$\tilde{Y}$,B} where B=<y,{a },{c} > Defined a function f: X → Y as follows f(1)=a,f(2)=b,f(3)=c
f is not continuous since f^{-1}(B) =<x,{1} , {3} > ∉T
also f is not semicontinuous since intclint f^{-1}(B)= ϕ
And f^{-1} (B) ⊄ ϕ
But f is precontinuous since intcl f^{-1}(B)=X and f^{-1}(B)⊆X

Example 4.9

Let X ={1,2,3}, T=< $\tilde{A}$, $\tilde{B}$, $\tilde{X}$, ϕ } , where A= <x,{1},{2,3}>,
B= <x,{2},{1}>, C=<x,{1,2}, ϕ >
And Y= {a,b,c,d}, ψ =< $\tilde{\Phi}$,$\tilde{Y}$,D } where D=<y,{a ,b},{c,d} > Defined a function f: X → Y as follows f(1)=a,f(2)=b, f(3)=c
f^{-1} (D) =<x,{2,3} , {1} > ∉ T, so f is not continuous but is semicontinuous since clint f^{-1}(D)=clB=A

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