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Nonlinear Controller Design for a Mobile Manipulator Trajectory Tracking †

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Abstract – In this paper, a mobile manipulator consisting of four degrees of freedom (4-DOF) robotic manipulator mounted at the top of a mobile platform (front point) of a mobile robot with four differential drive wheels is presented. A mobile manipulator combines the dexterous manipulator capability offered by fixed-based manipulators and the mobility offered by the mobile robot. The work involves the modeling of mobile manipulator robot and using the partial feedback linearization approach. The central idea is to algebraically transform nonlinear systems dynamics into partially linear form, so that linear control techniques can be applied to control on the wheel mobile manipulator robot in order to track any trajectory such as an ellipse, circle….etc, without violating the non holonomic constraints. However, and in order to consider the uncertainty in system parameters and the effects of the external disturbances a nonlinear PID controller is proposed in this work. The results demonstrate a good ability of the designed nonlinear PID controller in regulating the mobile robot to track the desired path in the presence of the external disturbances and the uncertainty in system parameters

Keywords:– Mobile Manipulator Robot, Modeling, Partial Feedback Linearization, Nonlinear PID Controller.

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1. Introduction

A mobile manipulator is nowadays a widespread term to refer to robots built from a robotic arm mounted on a mobile platform. Such systems combine the advantages of mobile platforms and robotic arms and reduce their drawbacks. For instance, the mobile platform extends the arm workspace, whereas an arm offers much operational functionality. Applications for such systems are abound in nuclear reactor maintenance, construction, military, mining, agriculture and planetary exploration [1]. Although appeared very early in robotics history a host of issues related to mobile manipulators have mainly been studied in recent years [2]. These include dynamic and static stability, force development and application, control in the presence of base compliance, dynamic coupling issues particularly; motion planning has generally been treated within the framework of optimal control. Mobile manipulators can be built based on various mobile platform designs that differ by the drive mechanism employed. The most commonly available mobile platforms use either a differential drive or a car-like drive. The former employs two independently driven wheels with a common axis, and casters that add stability to the mobile platform. The later is a tricycle design in which the driving front wheel is also used for steering [3]. Therefore mobile manipulator is an open field for many important and original research problems.

This paper focuses on the trajectory tracking and control on the wheel mobile manipulator robot, where the four degrees of freedom manipulator **AL5A** type and the four driving wheel differential mobile robot A4WD1 type used in this work. For more details the reader can refer to [13].

2. Mobile Manipulator Robot Description

The mobile robot (A4WD1) under study is made up of a rigid cart equipped with non-deformable four wheels which driven by four DC motors, with four degrees of freedom manipulator (AL5A) mounted at the front point of the platform.

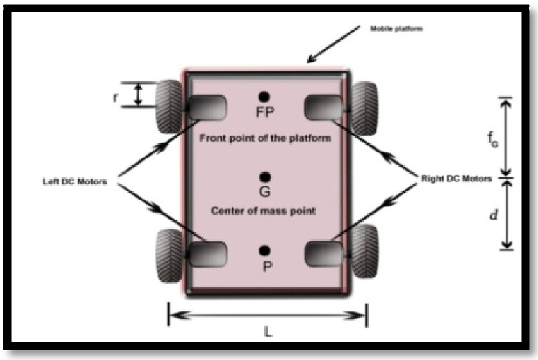

Figure 1 Mobile robot with four wheels differential drive

Figure 1 shows the mobile robot with four wheels differential drive; where r is radius of the mobile robot's wheel (cm), L is the distance between the two wheels of the robot (cm),f_G is the location of point FP with respect to point G (cm),d is the distance from P to G (cm), FP is the place of manipulator, G is the center of mass, and P is located in the middle of the axis that contacts the rear wheels of the mobile robot.

3. Kinematic Model of A4WD1 Mobile Robot

Kinematics is the study of the mathematics of motion (position and velocity) without considering the forces that affect the motion, robot kinematics describe robot movements [4]. This section analyzes the kinematics of 2-DOF differential drive vehicle shown in Fig. 1. The purpose of kinematics is to define the relationship between all known or measurable positions and velocities, and all quantities, which are computed by kinematics.

3.1. Position of A4WD1 Mobile Robot

In order to specify the position of the robot on the plane, a relationship between the world reference frame of the plane and the local reference frame of the robot must be established. The origin Oof the world reference frame is selected at an arbitrary point on the plane as shown in Fig. 2, the point G is the centre of mass of the mobile robot, and specified by coordinates x_G andy_G. The origin G of the local reference frame of the robot X_m,$Y_m$is at the centre of the robot. The position of point Pin the local reference frame is specified by coordinates x_P andy_P. The angular difference between the world and local reference frames is given byθ. The pose of the robot is described by a vector **q**, where:

$$\mathbf{q} = \begin{bmatrix} x_G \\ y_G \\ \theta \end{bmatrix} \tag{1}$$

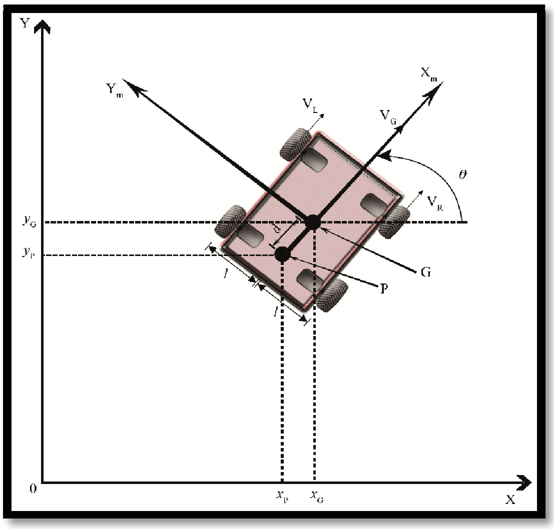

Figure 2 Posture coordinates of mobile robot type A4WD1.

3.2. Velocity of A4WD1 Mobile Robot

The velocity vector of the robot center of mass V_G where itsx_G,y_G components are given by:

$$\dot{x}_G = V_G \cos(\theta) \tag{2}$$
$$\dot{y}_G = V_G \sin(\theta) \tag{3}$$

Dividing (3) by (2), we obtain:

$$\theta = \tan^{-1}\frac{\dot{y}_G}{\dot{x}_G} \tag{4}$$

The linear velocity of the wheel symbolized by (V), having angular velocity ($\dot{\varphi}$), is expressed by;

$$V = \dot{\varphi}\, r \tag{5}$$

The velocity vector of the robot center of mass can be obtained by;

$$V_G = \frac{V_L + V_R}{2} \tag{6}$$

According to (6) the linear velocity for the left and the right wheels are in respective order;

$$V_L = \dot{\varphi}_L\, r \tag{7}$$
$$V_R = \dot{\varphi}_R\, r \tag{8}$$

The mobile robot body rotational velocity is:

$$\dot{\theta} = \omega = \frac{V_R - V_L}{L} = \frac{r}{L}\,(\dot{\varphi}_R - \dot{\varphi}_L) \tag{9}$$

The kinematic equations of the mobile robot are:

$$\dot{\mathbf{q}} = \begin{bmatrix} \dot{x}_G \\ \dot{y}_G \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \cos(\theta) & 0 \\ \sin(\theta) & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} V_G \\ \omega \end{bmatrix} \tag{10}$$

and,

$$\begin{bmatrix} V_G \\ \omega \end{bmatrix} = \begin{bmatrix} \frac{r}{2} & \frac{r}{2} \\ \frac{r}{L} & -\frac{r}{L} \end{bmatrix} \begin{bmatrix} \dot{\varphi}_R \\ \dot{\varphi}_L \end{bmatrix} \tag{11}$$

Combining (10) and (11) yields:

$$\begin{bmatrix}\dot{x}_G\\ \dot{y}_G\\ \dot{\theta}\end{bmatrix} = \begin{bmatrix}\frac{r}{2}\cos(\theta) & \frac{r}{2}\cos(\theta)\\ \frac{r}{2}\sin(\theta) & \frac{r}{2}\sin(\theta)\\ \frac{r}{L} & -\frac{r}{L}\end{bmatrix}\begin{bmatrix}\dot{\varphi}_R\\ \dot{\varphi}_L\end{bmatrix} \quad (12)$$

To construct viable system trajectories, the differential kinematics are required for point FP on the mobile platform. This point might be the base of arm or camera. On the other hand the system dynamics is minimum phase when designing the control law by considering the front point as the output [4]. It can be shown that this differential kinematics is described by [5]:

$$\begin{bmatrix}\dot{x}_F\\ \dot{y}_F\\ \dot{\theta}\end{bmatrix} = \frac{r}{2}\begin{bmatrix}\cos(\theta) - n_1 & \cos(\theta) + n_1\\ \sin(\theta) + n_2 & \sin(\theta) - n_2\\ \frac{2}{L} & -\frac{2}{L}\end{bmatrix}\begin{bmatrix}\dot{\varphi}_R\\ \dot{\varphi}_L\end{bmatrix} \quad (13)$$

where $n_1 = \frac{2\, f_G \sin(\theta)}{L}$, $n_2 = \frac{2\, f_G \cos(\theta)}{L}$

3.3. Kinematic Constraint of A4WD1 Mobile Robot

The mobile platform is subjected to three constraints. The first one is that the mobile robot cannot move in the lateral direction, which means that the platform must move in the direction of axis of symmetry. By multiplying (2) by $\sin(\theta)$ and (3) by $\cos(\theta)$, and subtract them results in;

$$\dot{y}_G\cos(\theta) - \dot{x}_G\sin(\theta) = 0 \quad (14)$$

Equation (14) can be expressed in terms of the coordinates (x_P, y_P) of the point P on the platform:

$$x_P = x_G - d\cos(\theta) \quad (15)$$
$$y_P = y_G - d\sin(\theta) \quad (16)$$

The velocity of point P which is located along the same axis of point G as:

$$\dot{x}_P = \dot{x}_G + d\dot{\theta}\sin(\theta) \quad (17)$$
$$\dot{y}_P = \dot{y}_G - d\dot{\theta}\cos(\theta) \quad (18)$$

Rewriting and arrangement of (17) and (18), we obtain:

$$\dot{x}_G = \dot{x}_P - d\dot{\theta}\sin(\theta) \quad (19)$$
$$\dot{y}_G = \dot{y}_P + d\dot{\theta}\cos(\theta) \quad (20)$$

From (14), (19), and (20) we obtain the first constraint:

$$\dot{y}_P\cos(\theta) - \dot{x}_P\sin(\theta) - d\dot{\theta} = 0 \quad (21)$$

The other two constraints are that the four driving wheels roll without slip. This implies that;

$$\dot{x}_P\cos(\theta) + \dot{y}_P\sin(\theta) + l\dot{\theta} = \dot{\varphi}_R\, r \quad (22)$$
$$\dot{x}_P\cos(\theta) + \dot{y}_P\sin(\theta) - l\dot{\theta} = \dot{\varphi}_L\, r \quad (23)$$

whereφ_R and φ_L are angular positions of the right wheel and the left wheel respectively, and$l$is the distance between the driving wheels and point P as shown in Fig. 2($l = \frac{L}{2}$) . The coordinate vector of mobile robot at point P becomes:

$$\mathbf{q} = [x_P, y_P, \theta, \varphi_R, \varphi_L]^T = [q_1, q_2, q_3, q_4, q_5]^T \quad (24)$$

While at point G the coordinate vector of the mobile robot is:

$$\mathbf{q} = [x_G, y_G, \theta, \varphi_R, \varphi_L]^T = [q_1, q_2, q_3, q_4, q_5]^T \quad (25)$$

Hence, the three constrains can be written in Pfaffian form as follows [6], [7] and [8]:

$$\mathbf{A}(q).\dot{\mathbf{q}} = 0 \quad (26)$$

$$\mathbf{A}(q) = \begin{bmatrix}-\sin(\theta) & \cos(\theta) & -d & 0 & 0\\ -\cos(\theta) & -\sin(\theta) & -l & r & 0\\ -\cos(\theta) & -\sin(\theta) & l & 0 & r\end{bmatrix} \quad (27)$$

or

$$\mathbf{A}(\mathrm{q}) = \begin{bmatrix} a_{11} & a_{12} & a_{13}a_{14}a_{15} \\ a_{21} & a_{22} & a_{23}a_{24}a_{25} \\ a_{31} & a_{32} & a_{33}a_{34}a_{35} \end{bmatrix} \quad (28)$$

Equations (22) and (23) are the nonholonomic constraint and (21) is the holonomic constraint [7].

4. Equation of Motion of A4WD1 Mobile Robot

For accurate mobile robot system description, it is necessary to generate an appropriate dynamic model. The dynamic model allows the consideration of such properties as: mass, inertia, friction forces, centrifugal force, torque, etc. Mobile robot dynamical model is very important in cases when mobile robot velocities generate forces whose influence cannot be neglected during movement, so those appropriate control strategies and trajectories need to include mobile robot dynamic properties [9]. Dynamics motion equation can also be derived using Euler-Lagrange. According to (29),$L(q,\dot{q})$ stands for difference of kinetic$K$ and potential$V$ energy. Under assumption that mobile robot moves only on a plane surface, potential energy of robot is zero ($V = 0$), and we have to find only the kinetic energy of the mobile robot.

$$L(q,\dot{q}) = K(q,\dot{q}) - V(q) = K(q,\dot{q}) \quad (29)$$

The equations of motion of non-holonomically constrained systems are governed by [6,7]:

$$\mathbf{M}(\mathrm{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathrm{q},\dot{\mathrm{q}}) + \mathbf{G}(\mathrm{q}) = \mathbf{E}(\mathrm{q})\tau + \mathbf{A}^{\mathrm{T}}(\mathrm{q})\,\lambda \quad (30)$$

Where $\mathbf{M}$ (q) is the n x n dimensional positive definite inertia and mass matrix,$\mathbf{C}(\mathrm{q},\dot{\mathrm{q}})$ represents the n vector of centripetal and coriol is torques,$\mathbf{G}(\mathrm{q})$ is the gravitational force vector,τ is the r-dimensional vector of actuator force/torque, **E**(q) is the n x r dimensional matrix mapping the actuator space into the generalized coordinate space,λ is an m-dimensional vector of Lagrange multiplierswith$m$ represents the number of nonholinomic constraints and $\mathbf{A}(\mathrm{q})$ is as defined in (27). The Lagrange equations of motion for the nonholonomic mobile robot system are governed by [10]:

$$\frac{d}{dx}\left(\frac{\partial K}{\partial \dot{q}_i}\right) - \frac{\partial K}{\partial q_i} = Q_i - a_{1i}\lambda_1 - a_{2i}\lambda_2,$$
$$i = 1,..,4 \quad (31)$$

Where $\mathrm{Q_i}$ is the Lagrange force, $\mathrm{a_{ij}}$, $\mathrm{j} = 1,2$ is given by (28), and λ_1 , λ_2 are the Lagrange multipliers. To obtain the holonomic constraint, we subtract (22) from (23),which leads to:

$$2l\dot{\theta} = r(\dot{\varphi}_R - \dot{\varphi}_L) \quad (32)$$

Let constant$c = \frac{\mathrm{r}}{2l}$, and take the integral for both sides of (32) and properly choosing the initial condition of θ,φ_R , and φ_L we can get:

$$\theta = c(\varphi_R - \varphi_L) \quad (33)$$

This is clearly a holonomic constraint. Thus θ may be eliminated from the coordinate vector, where the coordinate vector **q**becomes:

$$\mathbf{q} = [x_{\mathrm{G}}, y_{\mathrm{G}}, \varphi_{\mathrm{R}}, \varphi_{\mathrm{L}}]^{\mathrm{T}} = [\mathrm{q}_1, \mathrm{q}_2, \mathrm{q}_3, \mathrm{q}_4]^{\mathrm{T}} \quad (34)$$

By adding (22) and (23) we obtain:

$$\dot{x}_{\mathrm{P}}\cos(\theta) + \dot{y}_{\mathrm{P}}\sin(\theta) = c(\dot{\varphi}_{\mathrm{R}} + \dot{\varphi}_{\mathrm{L}}) \quad (35)$$

The resultant two nonholonomic constraints are:

$$\dot{x}_{\mathrm{G}}\sin(\theta) - \dot{y}_{\mathrm{G}}\cos(\theta) = 0 \quad (36)$$
$$\dot{x}_{\mathrm{P}}\cos(\theta) + \dot{y}_{\mathrm{P}}\sin(\theta) = c(\dot{\varphi}_{\mathrm{R}} + \dot{\varphi}_{\mathrm{L}}) \quad (37)$$

Also $\mathbf{A}(\mathrm{q})$((27)) becomes:

$$\mathbf{A}(q) = \begin{bmatrix} -\sin(\theta) & \cos(\theta) & 0 & 0 \\ -\cos(\theta) & -\sin(\theta) & c & c \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} a_{13} a_{14} \\ a_{21} & a_{22} a_{23} \; a_{24} \end{bmatrix} \quad (38)$$

To apply the Lagrange formulation, the total kinetic energy of the mobile base and the right and left wheels are [11]:

$$K = \frac{1}{2} m(\dot{x}_G^2 + \dot{y}_G^2) + m_c c(\dot{\varphi}_R - \dot{\varphi}_L)(\dot{y}_G cos(\theta) - \dot{x}_G sin(\theta)) + \frac{1}{2} I_\omega (\dot{\varphi}_R^2 + \dot{\varphi}_L^2) + \frac{1}{2} I c^2 (\dot{\varphi}_R - \dot{\varphi}_L)^2 \quad (39)$$

where

$$m = m_c + 4\, m_\omega + m_m$$
$$I = I_c + 4\, I_\omega$$

I is the moment of inertia of entire mobile robot with respect to the center of mass about vertical axis (kg.cm²), I_c is the moment of inertia of platform without the driving wheel and motor rotor about vertical axis through G (kg.cm²),I_ω is the moment of inertia of each wheel and the motor rotor about the wheel axis (kg.cm²),m is the mass of entire mobile robot (kg),m_c is the mass of the platform without the driving wheels and the rotors of DC motors (kg),m_ω is the mass of each driving wheel plus the rotor of DC motor (kg),m_m is the mass of the manipulator at FP (kg).Substituting the total kinetic energy (39) into (31), and substitute θ instead of $c(\varphi_R - \varphi_L)$ according (33) we obtain:

$$m\ddot{x}_G - m_c(\ddot{\theta} sin\theta + \dot{\theta}^2 cos\theta) - \lambda_1 sin\theta - \lambda_2 cos\theta = 0 \quad (40)$$

$$m\ddot{y}_G + m_c(\ddot{\theta} cos\theta - \dot{\theta}^2 sin\theta) + \lambda_1 cos\theta - \lambda_2 sin\theta = 0 \quad (41)$$

$$m_c c(\ddot{y}_G cos\theta - \ddot{x}_G sin\theta) + (Ic^2 + I_\omega)\ddot{\theta}_R - Ic^2 \ddot{\theta}_L - \tau_R + c\lambda_2 = 0 \quad (42)$$

$$-m_c c(\ddot{y}_G cos\theta - \ddot{x}_G sin\theta) - Ic^2 \ddot{\theta}_R + (Ic^2 + I_\omega)\ddot{\theta}_L - \tau_L + c\lambda_2 = 0 \quad (43)$$

These five equations of motion can be written in matrix form as:

$$\mathbf{M}(q)\ddot{q} + \mathbf{C}(q,\dot{q}) = \mathbf{E}(q)\tau - \mathbf{A}^T(q)\lambda \quad (44)$$

This equation is a form of (30) which ignores the gravity element. The matrices $\mathbf{M}(q)\ddot{q}$, $\mathbf{C}(q,\dot{q})$, and $\mathbf{E}(q)$ are given by:

$$\mathbf{M}(q) = \begin{bmatrix} m & 0 & -m_c c sin\theta & m_c c sin\theta \\ 0 & m & m_c c cos\theta & -m_c c cos\theta \\ -m_c c sin\theta & m_c c cos\theta & Ic^2 + I_\omega & -Ic^2 \\ m_c c sin\theta & -m_c c cos\theta & -Ic^2 & Ic^2 + I_\omega \end{bmatrix}$$

$$\mathbf{C}(q,\dot{q}) = \begin{bmatrix} -m_c \dot{\theta}^2 cos\theta \\ -m_c \dot{\theta}^2 sin\theta \\ 0 \\ 0 \end{bmatrix}, \tau = \begin{bmatrix} \tau_R \\ \tau_L \end{bmatrix},$$

$$\lambda = \begin{bmatrix} \lambda_1 \\ \lambda_2 \end{bmatrix} \quad \text{and} \quad \mathbf{E}(q) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

The robot parameters are given in Table 1, where they were taken from the manufacture data [13] except I_c and I_ω where they were calculated.

Table 1. Robot Parameters

Parameter	values	Units
L	34	cm
r	6	cm
m_c	0.575	kg
m_ω	0.332	kg
m_m	0.595	kg
I_ω	4.15	kg−cm²
I_c	6.609	kg−cm²

For more details about manipulator robot (4-DOF) dynamics and kinematics (forward & inverse) the reader may refer to [10].

5. Partial Feedback Linearization

Feedback linearization is an approach to nonlinear control design that has attracted lots of research in recent years. The central idea is to algebraically transform nonlinear systems dynamics into (fully or partly) linear ones, so that linear control techniques can be applied. The basic idea of simplifying the form of a system by choosing a different state representation is not completely unfamiliar; rather it is similar to the choice of reference frames or coordinate systems in mechanics. The applications of feedback linearization include helicopters, high-performance aircraft, industrial robots, biomedical devices, and vehicle control [11] and [12].

Backing to (44) we see that the number of degree of freedom is greater than the number of actuators (four degree of freedom and two actuators). Accordingly the system is classified as under actuator mechanical system. One can design a controller for this type of system is by partially linearizing it with respect to desired outputs. It is also named as input-output linearization [14]. Here it is required that the number of outputs is equal to the actuator numbers and also the system are minimum phase with respect to these outputs (refer to reference [15] for more details about the minimum phase concept).

In order to partially linearize the system dynamics as given by (45), we need first to write the system dynamics independent of Lagrange multipliers λ and secondly assign two outputs for input-output linearization.

6. Input-Output Linearization

As a first step to input output linearization we will represent the motion (44) and the constraint (26) independent of λ in state space by properly choosing a state vector. To do so, we define a 4×2 dimensional matrix $\boldsymbol{S}(q)$ such that $\boldsymbol{A}(q)\boldsymbol{S}(q) = 0$ [11]. It is straightforward to verify that the following matrix has the required property:

$$\mathbf{S}(\mathrm{q}) = \begin{bmatrix} cl\cos\theta & cl\cos\theta \\ cl\sin\theta & cl\sin\theta \\ 1 & 0 \\ 0 & 1 \end{bmatrix} = [S_1 \quad S_2] \qquad (45)$$

where S_1 and S_2 are two linearly independent column matrices. From the constrain (26),$\dot{\mathbf{q}}$ is in the null-space of$\mathbf{A}(\mathrm{q})$i.e., $\dot{\mathbf{q}} \in span\{S_1, S_2\}$. Hence it is possible to express $\dot{\mathbf{q}}$ as a linear combination of $S_1$and S_2, that is,

$$\dot{\mathbf{q}} = \mathbf{S}(\mathrm{q})\boldsymbol{\delta} = \delta_1 S_1 + \delta_2 S_2 \qquad (46)$$

Owing to the choice of $\mathbf{S}(\mathrm{q})$ matrix, we have:

$$\boldsymbol{\delta} = \begin{bmatrix} \delta_1 \\ \delta_2 \end{bmatrix} = \begin{bmatrix} \dot{\varphi_R} \\ \dot{\varphi}_L \end{bmatrix} \qquad (47)$$

Differentiating (46), and substituting the expression for $\ddot{\mathbf{q}}$ into (44), and by multiplying both sides by$\mathbf{S}^{\mathrm{T}}(\mathrm{q})$, we obtain:

$$\mathbf{S}^{\mathrm{T}}(\mathrm{q}) * \mathbf{M}(\mathrm{q})\big(\mathbf{S}(\mathrm{q}) * \dot{\boldsymbol{\delta}} + \dot{\mathbf{S}}(\mathrm{q}) * \boldsymbol{\delta}\big) + \mathbf{S}^{\mathrm{T}}(\mathrm{q}) * \mathbf{C}(\mathrm{q}, \dot{\mathrm{q}}) = \tau \qquad (48)$$

Where $\mathbf{S}^{\mathrm{T}}(\mathrm{q}) * \mathbf{A}^{\mathrm{T}}(\mathrm{q}) = 0$ is used to eliminate Lagrange multiplierλ and $\mathbf{S}^{\mathrm{T}}(\mathrm{q}) * \mathbf{E}(\mathrm{q}) = \mathbf{I}_{2\times2}$ (the 2×2 identity matrix).

Now by using the state-space vector $\boldsymbol{x} = [\mathrm{q}^{\mathrm{T}}\delta^{\mathrm{T}}]^{\mathrm{T}}$=$[x_G y_G \varphi_R \varphi_L \dot{\varphi}_R \dot{\varphi}_L]^{\mathrm{T}}$, we will be able to represent the constraint and motion equations of mobile platform in state space form, that is;

$$\dot{\boldsymbol{x}} = \begin{bmatrix} \dot{q} \\ \dot{\boldsymbol{\delta}} \end{bmatrix} = \begin{bmatrix} \mathbf{S}(\mathrm{q}) * \boldsymbol{\delta} \\ \boldsymbol{\rho} \end{bmatrix} + \begin{bmatrix} \mathbf{0} \\ (\mathbf{S}^{\mathrm{T}}(\mathrm{q}) * \mathbf{M}(\mathrm{q}) * \mathbf{S}(\mathrm{q}))^{-1} \end{bmatrix} \tau \quad (49)$$

where

$$\boldsymbol{\rho} = (\mathbf{S}^{\mathrm{T}}(\mathrm{q})\mathbf{M}(\mathrm{q})\mathbf{S}(\mathrm{q}))^{-1} * (-\mathbf{S}^{\mathrm{T}}(\mathrm{q})\mathbf{M}(\mathrm{q})\dot{\mathbf{S}}(\mathrm{q})\boldsymbol{\delta} - \mathbf{S}^{\mathrm{T}}(\mathrm{q})\mathbf{C}(\mathrm{q},\dot{\mathrm{q}})) \quad (50)$$

The coordinates of the reference point to be the output equation which represents the head of mobile robot is**:**

$$\boldsymbol{y} = \boldsymbol{h}(\mathrm{q}) = [x_F y_F]^{\mathrm{T}} = [x_G + \mathrm{f_G}\cos(\theta) y_G + \mathrm{f_G}\sin(\theta)]^{\mathrm{T}} \quad (51)$$

It has been proved in reference [11] that the system dynamics (49) is minimum phase when the output is taken as in (51). This satisfies the second requirement for input-output linearization described above.

Now to derive the input-output dynamics we differentiate (51) as follows:

$$\dot{\boldsymbol{y}} = \frac{\partial \boldsymbol{h}(\mathrm{q})}{\partial \mathrm{q}} \dot{\mathbf{q}} = J_h * (\mathbf{S}(\mathrm{q}) * \boldsymbol{\delta}) = (J_h * \mathbf{S}(\mathrm{q}))\boldsymbol{\delta} = \emptyset(\mathrm{q}) * \boldsymbol{\delta} \quad (52)$$

Where J_h is the Jacobian matrix**,** and $\emptyset(\mathrm{q})$ is called the decoupling matrix that described by:

$$\emptyset = (J_h * \mathbf{S}(\mathrm{q})) = \begin{bmatrix} cl\cos(\theta) - c\,\mathrm{f_G}\sin(\theta) & cl\cos(\theta) + c\,\mathrm{f_G}\sin(\theta) \\ cl\sin(\theta) - c\,\mathrm{f_G}\cos(\theta) & cl\sin(\theta) + c\,\mathrm{f_G}\cos(\theta) \end{bmatrix} \quad (53)$$

Then, differentiate (46) again to get;

$$\ddot{\boldsymbol{y}} = \dot{\emptyset}(\mathrm{q}) * \boldsymbol{\delta} + \emptyset(\mathrm{q}) * \dot{\boldsymbol{\delta}} \quad (54)$$

To complete the input-output linearization, the following input transformation is utilized;

$$v = \dot{\emptyset}(\mathrm{q}) * \boldsymbol{\delta} + \emptyset(\mathrm{q}) * \dot{\boldsymbol{\delta}} = \dot{\emptyset}(\mathrm{q}) * \boldsymbol{\delta} + \emptyset(\mathrm{q})\{\boldsymbol{\rho} + (\mathbf{S}^{\mathrm{T}}(\mathrm{q}) * \mathbf{M}(\mathrm{q}) * \mathbf{S}(\mathrm{q}))^{-1}\tau\} \quad (55)$$

Let $\dot{\boldsymbol{\delta}} = u$, yields

$$v = \dot{\emptyset}(\mathrm{q}) * \boldsymbol{\delta} + \emptyset(\mathrm{q})u \quad (56)$$

Hence**,**

$$u = \emptyset(\mathrm{q})^{-1}(v - \dot{\emptyset}(\mathrm{q}) * \boldsymbol{\delta}) \quad (57)$$

and

$$\ddot{\boldsymbol{y}} = v \quad (58)$$

Consequently (49) becomes:

$$\dot{\boldsymbol{x}} = \begin{bmatrix} \mathbf{S}(\mathrm{q}) * \boldsymbol{\delta} \\ \mathbf{0} \end{bmatrix} + \begin{bmatrix} \mathbf{0} \\ \mathbf{I} \end{bmatrix} u \quad (59)$$

where **I** is 2×2 identity matrix. Also the actuators τ in terms of u is:

$$\tau = \mathbf{S}^{\mathrm{T}}(\mathrm{q})[\mathbf{M}(\mathrm{q})\dot{\mathbf{S}}(\mathrm{q})\boldsymbol{\delta} + \mathbf{C}(\mathrm{q},\dot{\mathrm{q}})] + \left(\mathbf{S}^{\mathrm{T}}(\mathrm{q})\mathbf{M}(\mathrm{q})\mathbf{S}(\mathrm{q})\right)u \quad (60)$$

Finally it is more convenient to rewrite (58) in terms of the error function; i.e., if we define the error $e = \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} \in \mathcal{R}^2$ as $e = y - y_d = \begin{bmatrix} x_F - y_{d1} \\ y_F - y_{d2} \end{bmatrix}$, then (58) becomes:

$$\ddot{e} = v - \ddot{y}_d \quad (61)$$

where y_d is the desired path. The task now is to design a control law that will asymptotically regulate the error dynamics in the above equation to the origin. The controller design details are presented in the following section.

7. The Proposed Control Law and Simulation Results

In the present work a nonlinear PID controller is proposed

$$v = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} \ddot{y}_{d1} - k_{p1}e_1 - k_{d2}\dot{e}_1 - k_{i1}\int_0^t \tan^{-1}(\gamma_1 e_1)\,dt \\ \ddot{y}_{d2} - k_{p2}e_2 - k_{d2}\dot{e}_2 - k_{i2}\int_0^t \tan^{-1}(\gamma_2 e_2)\,dt \end{bmatrix} \quad (62)$$

This control law is a PD plus nonlinear integral controller. Due to the nonlinear integral term, the controller is robust with respect to the disturbances and the uncertainty in system model. If the disturbances are of non-vanishing type the controller will be able only to attenuate their effects [14] and keep the output near the desired path. The disturbances and the uncertainty in system model can be represented in (61) as follows:

$$\ddot{e} = v - \ddot{y}_d + \eta(t) \quad (63)$$

Where $\eta(t)$ is the perturbation term that represents the effect of the disturbances and the uncertainty in system model in the input-output dynamics. It is assumed bounded at least locally, i.e.,

$$|\eta(t)| < \eta_o \quad (64)$$

In the following figures, the front point (FP) is represented by blue color, and the center of mass point (G) is represented by magenta color, and in all cases the ellipsoid figure is the desired trajectory (in (61) the desired trajectory is represented by y_d). The ellipsoid trajectory is shown in Fig. 3 where the $x-axis$ and $y-axis$ are measured in meters.

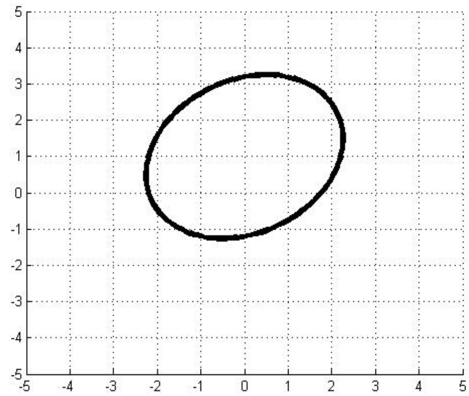

Figure.3.the ellipsoid trajectory in x, y plane

The controller parameters k_p, k_i, k_d and γ are selected for the case where the perturbation term equal to zero ($\eta(t) = 0$) and according to the following tests;

Test 1:
$k_p = 1$, $k_i = 1$, $k_d = 1$ and $\gamma = 1$

Figure 4 shows the simulation result for the front point (FP) and the center of mass point (G) trajectory which response under the proposed controller in (62) and when they starting at the desired trajectory.

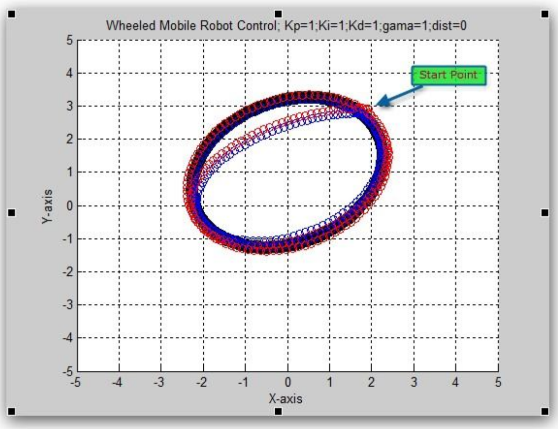

Figure 4 Robot tracking with$k_p = 1$, $k_i = 1$, $k_d = 1$ and $\gamma = 1$

The performance of the controller is well clarified in Fig. 5. where the error betwwen x , y and thier desired trijectory are eleminated after 9 $sec.$.

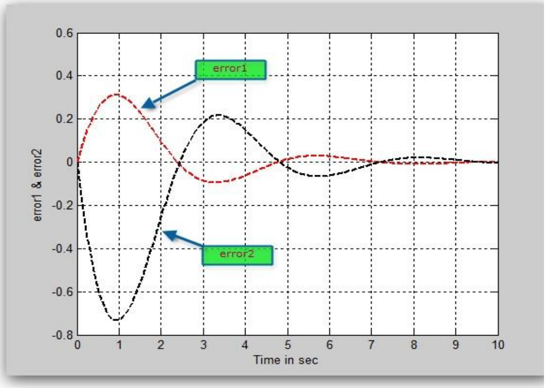

Figure.5.The errors $e_1 = (x_F - y_{d1})$ and $e_2 = (y_F - y_{d2})$ with$k_p = 1,\ k_i = 1,\ k_d = 1$ and $\gamma = 1$

To improve the response, the next two tests are made in order to select the controller parameters with better performance as follows;

Test 2:

$\boldsymbol{k_p = 6,\ k_i = 2,\ k_d = 3}$ **and** $\boldsymbol{\gamma = 3}$

Figure 6 plots the FP and G points which track the ellipsoid trajectory while in Fig. 7 the error is plotted for x_F and y_F from their desired values. Figure 7 shows the error is die out after3 $sec.$.

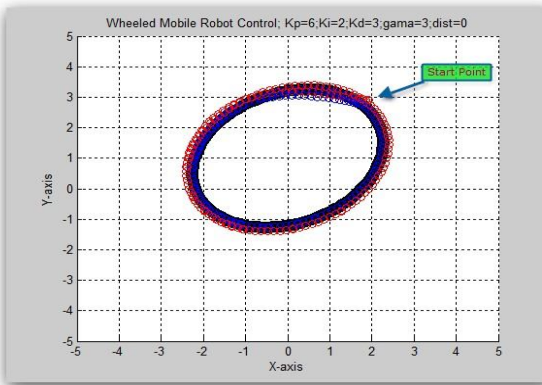

Figure 6 Robot tracking with$k_p = 6,\ k_i = 2,$ $k_d = 3$ and $\gamma = 3$

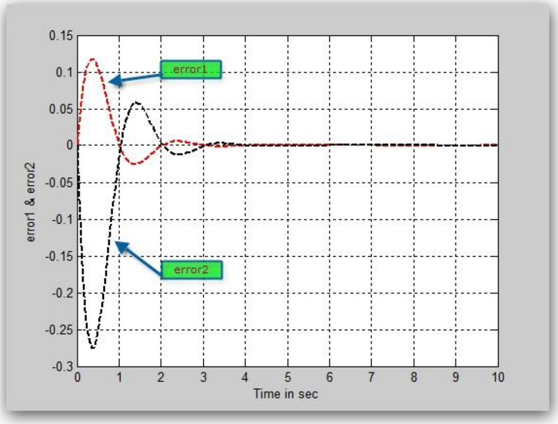

Figure 7 The errors $e_1 = (x_F - y_{d1})$ and $e_2 = (y_F - y_{d2})$ with$k_p = 6,\ k_i = 2,\ k_d = 3$ and $\gamma = 3$

Test 3:

$\boldsymbol{k_p = 2,\ k_i = 3,\ k_d = 5}$ **and** $\boldsymbol{\gamma = 2}$

Further improvement is gained in this test as depicted in Figs. 8 and 9 where the error is eliminated after 2 $sec.$ only. The controller parameters used in this test is used also in the following test where the perturbation term existed and bounded by 3 ($|\eta(t)| < 3$).

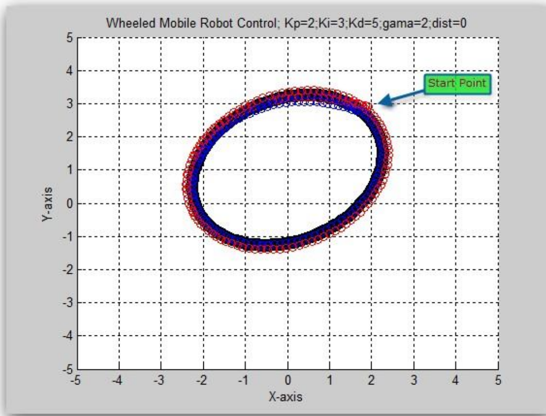

Figure 8 Robot tracking with$k_p = 2,\ k_i = 3,$ $k_d = 5$ and $\gamma = 2$

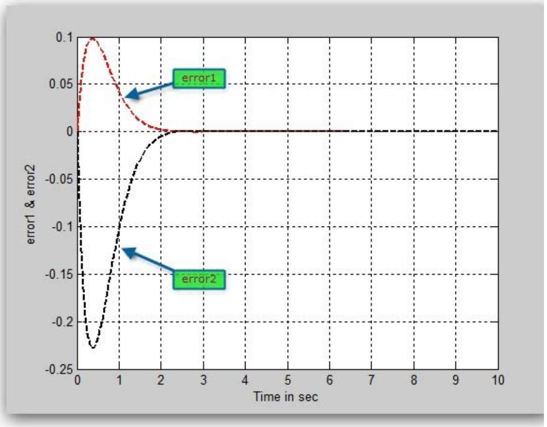

Figure 9 The errors $e_1 = (x_F - y_{d1})$ and $e_2 = (y_F - y_{d2})$ with$k_p = 2,\ k_i = 3,\ k_d = 5$ and $\gamma = 2$

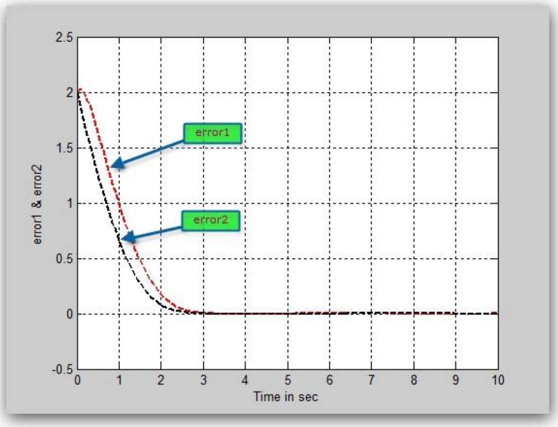

Figure 11 The errors $e_1 = (x_F - y_{d1})$ and $e_2 = (y_F - y_{d2})$ with$k_p = 2,\ k_i = 3,\ k_d = 5k_d = 5$, $\gamma = 2$ and $|\eta(t)| < 3$

Test 4:

$k_p = 2,\ k_i = 3,\ k_d = 5,\quad \gamma = 2$ and $|\eta(t)| < 3$

In this test the performance of the proposed controller with nonlinear integral element is examined for the case of existence of a bounded perturbation term and for initial condition not lies on the desired. The result is shown in Fig. 10 and with good error decay as plotted in Fig. 11. The results also show the ability of the proposed nonlinear controller in attenuating the effect of the perturbations within 3 sec..

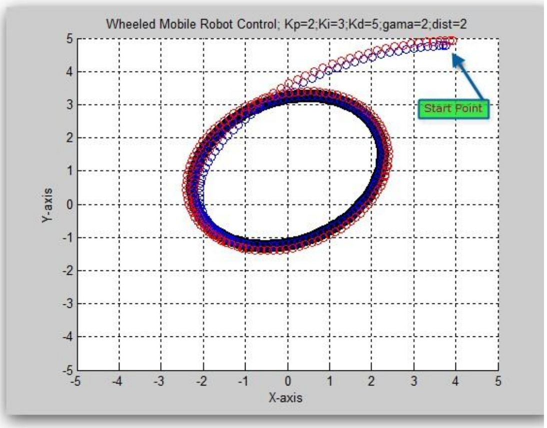

Figure10 Robot tracking with$k_p = 2,\ k_i = 3,$ $k_d = 5$, $\gamma = 2$ and $|\eta(t)| < 3$

8. Conclusions

Using the partial feedback linearization the model for a mobile robot with four differential drive wheels is derived. The two outputs selected in this work are the x and y position of the front point where the mobile robot is minimum phase. To force the front point to track the desired trajectory, a nonlinear PID controller with nonlinear integral element is designed. The results prove the ability of the proposed controller in regulating the front point to track an ellipsoid reference trajectory for different initial condition and with and without the presence of a perturbation term. This also reveals that the proposed controller is robust with respect the uncertainty in system parameters and for bounded external disturbances.

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