ON SOME INTUITIONISTIC FUZZY SEPARATION AXIOMS OF INTUITIONISTIC FUZZY SUPRATOPOLOGICAL SPACES

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Abstract

In this paper , we introduce a new some intuitionistic fuzzy separation axioms of intuitionistic fuzzy supratopological spaces $(IFSTS-T_\circ)$ and $(IFSTS-T_1)$ for short) , and study the intuitionistic supra- hereditary $(ISH$ for short) and intuitionistic supratopological property ($ISTP$ for short) .

1- Introduction :

let X be a nonempty set. An intuitionistic fuzzy set A (IFS for short) on X is an object having the form $A=\{\langle x,\mu_A(x),\gamma_A(x)\rangle : x\in X\}$ where $\mu_A : X\to I$ and $\gamma_A : X\to I$,denoted the membership function and the nonmembership function of A respectively , $I=[0,1]$, and satisfy $0<\mu_A(x)+\gamma_A(x)\le 1$ for each $x\in X$.

An IFS $A=\{\langle x,\mu_A(x),\gamma_A(x)\rangle : x\in X\}$ can be written in the form $A=\langle x,\mu_A,\gamma_A\rangle$, or simply $A=\langle \mu_A,\gamma_A\rangle$ [1].

Let $A=\{\langle x,\mu_A(x),\gamma_A(x)\rangle : x\in X\}$ and $B=\{\langle x,\mu_B(x),\gamma_B(x)\rangle : x\in X\}$ be $IFSs$ on X . Then :[2]

(a) $\overline{A}=\{\langle x,\gamma_A(x),\mu_A(x)\rangle : x\in X\}$ (the complement of A).

(b) $A\bigcap B=\{\langle x,\mu_A(x)\wedge\mu_B(x),\gamma_A(x)\vee\gamma_B(x)\rangle : x\in X\}$ (the meet of A and B).

(c) $A\bigcup B=\{\langle x,\mu_A(x)\vee\mu_B(x),\gamma_A(x)\wedge\gamma_B(x)\rangle : x\in X\}$ (the join of A and B).

(d) $A\subseteq B\Leftrightarrow \mu_A(x)\le\mu_B(x) and, \gamma_A(x)\ge\gamma_B(x)$ for each $x\in X$.

(e) $A=B\Leftrightarrow A\subseteq B$ and $B\subseteq A$.

(f) $\tilde{1}=\{\langle x,1,0\rangle : x\in X\}$, $\tilde{0}=\{\langle x,0,1\rangle : x\in X\}$

Afamily τ of $IFSs$ on X is called an intuitionistic fuzzy supratopology ($IFST$ for short) on X if $\tilde{0}\in\tau$, $\tilde{1}\in\tau$ and τ is closed under arbitrary union .Then we call the pair (X,τ) is an intuitionistic fuzzy supratopological space ($IFSTS$ for short). Each member of τ is called an intuitionistic fuzzy supraopen set ($IFSO$- for short) and the complement of intuitionistic fuzzy supraclosed sets ($IFSC$ - for short).

A is called a supraclosure ($s-cl(A)$ for short) if its equal the intersection of all intuitionistic fuzzy supraclosed sets containing A .

A is called suprainterior ($s-\text{int}(A)$ for short) if equal the union of all intuitionistic fuzzy supraopen sets contained in A.

Notice that the supraclosure is the smallest supraclosed sets and the suprainterior is the largest sets supraopen sets .

For example let $X=\{a,b\}$ and $A=\langle x,(0.3,0.4),(0.4,0.5)\rangle$, $B=\langle x,(0.4,0.2),(0.5,0.3)\rangle$ then $\tau=\{\tilde{0},\tilde{1},A,B,A\bigcup B\}$ is an $IFST$ on X .

Now let $IF(X)$ denoted the family of all intuitionistic fuzzy sets in X .

If β is an intuitionistic fuzzy supratopology on X then $\tau_\beta=\{T\in IF(X) : (\forall A)(A\in\beta\Rightarrow T\bigcap A\in\beta)\}$ is an intuitionistic fuzzy topology on X and $\tau_\beta\subseteq\beta$.

Let (X,τ) be an intuitionistic fuzzy supratopological space then $\tau_1=\{\mu_A : A\in\tau\}$ is a fuzzy supratopological space on X [p] , and $\tau_2^C=\{\gamma_A : A\in\tau\}$ is the family of all fuzzy supraclosed sets of the fuzzy supratopological space $\tau_2=\{1-\gamma_A : A\in\tau\}$ on X .

Furthermore , in (X,τ_1) and (X,τ_2), the fuzzy sets μ_A and γ_A are fuzzy supraopen and fuzzy supraclosed sets . respectively . On the other hand , since τ_1 is a fuzzy supratopology on X , $T_{\tau_1}=\{P\in I^x : \mu_A\in\tau_1\Rightarrow P\wedge\mu_A\in\tau_1\}$ is a fuzzy topology on X and $T_{\tau_1}\subseteq\tau_1$ [2] .

Let (X,τ) and (Y,δ) be two intuitionistic fuzzy supratopological spaces , $A\in IF(X)$ and $B\in IF(Y)$. The product of A and B is defined by : $A\times B=\{\langle (x,y),\mu_A(x)\wedge\mu_B(y),\gamma_A(x)\vee\gamma_B(y)\rangle : (x,y)\in X\times Y\}$.

For example $X=\{a,b\}$ and $A=\langle x,(0.3,0.4),(0.4,0.5)\rangle$, $B=\langle x,(0.4,0.2),(0.5,0.3)\rangle$.

Now $\tau = \{\tilde{0}_X, \tilde{1}_X, A, B, A\bigcup B\}$ is an intuitionistic fuzzy supratopology on X . Secondly let $Y=\{a,b\}$ and $C=\langle y,(0.3,0.2),(0.6,0.5)\rangle$, $D=\langle y,(0.4,0.1),(0.5,0.3)\rangle$

Then $\delta = \{\tilde{0}_Y, \tilde{1}_Y, C, D, C\bigcup D\}$ is an intuitionistic fuzzy supratopology on Y . The product of these spaces is as follows:

$$\beta=\{\langle (x,y),\begin{pmatrix}0.3 & 0.3\\ 0.4 & 0.4\end{pmatrix},\begin{pmatrix}0.4 & 0.4\\ 0.5 & 0.5\end{pmatrix}\rangle,$$

$$\langle (x,y),\begin{pmatrix}0.4 & 0.4\\ 0.2 & 0.2\end{pmatrix},\begin{pmatrix}0.5 & 0.5\\ 0.3 & 0.3\end{pmatrix}\rangle,$$

$$\langle (x,y),\begin{pmatrix}0.4 & 0.4\\ 0.4 & 0.4\end{pmatrix},\begin{pmatrix}0.4 & 0.4\\ 0.3 & 0.3\end{pmatrix}\rangle,$$

$$\langle (x,y),\begin{pmatrix}0.3 & 0.2\\ 0.3 & 0.2\end{pmatrix},\begin{pmatrix}0.6 & 0.5\\ 0.6 & 0.5\end{pmatrix}\rangle,$$

$$\langle (x,y),\begin{pmatrix}0.4 & 0.1\\ 0.4 & 0.1\end{pmatrix},\begin{pmatrix}0.5 & 0.3\\ 0.5 & 0.3\end{pmatrix}\rangle,$$

$$\langle (x,y),\begin{pmatrix}0.4 & 0.2\\ 0.4 & 0.2\end{pmatrix},\begin{pmatrix}0.5 & 0.3\\ 0.5 & 0.3\end{pmatrix}\rangle,$$

$$\langle (x,y),\begin{pmatrix}0.4 & 0.3\\ 0.4 & 0.4\end{pmatrix},\begin{pmatrix}0.4 & 0.3\\ 0.5 & 0.3\end{pmatrix}\rangle,$$

$$\langle (x,y),\begin{pmatrix}0.4 & 0.4\\ 0.3 & 0.2\end{pmatrix},\begin{pmatrix}0.5 & 0.5\\ 0.3 & 0.3\end{pmatrix}\rangle,$$

$$\langle (x,y),\begin{pmatrix}0.4 & 0.4\\ 0.4 & 0.2\end{pmatrix},\begin{pmatrix}0.5 & 0.3\\ 0.3 & 0.3\end{pmatrix}\rangle,$$

$$\langle (x,y),\begin{pmatrix}0.4 & 0.4\\ 0.4 & 0.4\end{pmatrix},\begin{pmatrix}0.4 & 0.3\\ 0.3 & 0.3\end{pmatrix}\rangle,$$

$$\langle (x,y),\begin{pmatrix}1 & 1\\ 1 & 1\end{pmatrix},\begin{pmatrix}0 & 0\\ 0 & 0\end{pmatrix}\rangle,$$

$$\langle (x,y),\begin{pmatrix}0 & 0\\ 0 & 0\end{pmatrix},\begin{pmatrix}1 & 1\\ 1 & 1\end{pmatrix}\rangle\} \ .$$

In general a product of two intuitionistic fuzzy supraopen sets is not necessarily supraopen . for example in the example above A and D are supraopen ,but

$$A\times D=\{\langle (x,y),\begin{pmatrix}0.3 & 0.1\\ 0.4 & 0.1\end{pmatrix},\begin{pmatrix}0.5 & 0.4\\ 0.5 & 0.5\end{pmatrix}\rangle \notin \beta$$

On the other hand the supraclosed sets with respect to $(X\times Y,\beta)$ are precisely the products $F\times K$ where F is supraclosed in (X,τ) and K supraclosed in (Y,δ). To see this we need only note that if $G=\overline{F}\in\tau$ and $H=\overline{K}\in\delta$ then $\overline{(G\times\tilde{1}_Y)\bigcup(\tilde{1}_X\times H)}=(\overline{G}\times\tilde{1}_Y)\bigcap(\tilde{1}_X\times\overline{H})=\overline{G}\times\overline{H}=F\times K$.

Let $f:(X,\tau)\to(Y,\delta)$ be a mapping between two intuitionistic fuzzy supratopological spaces . f is a fuzzy supracontinuous function if $f^{-1}(\delta)\subseteq\tau$ [3].

f is an intuitionistic fuzzy suprahomeomorphism ($IFSH$ for short) iff f , f^{-1} are supracontinous , one to one and onto .

Let (X,τ_X) be an intuitionistic fuzzy supratopological space , let $E\subseteq X$ then $\tau_E=\{E\bigcap F : F\in\tau_X\}$ is called an intuitionistic fuzzy relative supratopology .

2- $IFSTS-T_\circ$ - space induced by intuitionistic fuzzy supratopology .

Definition 2.1.

An intuitionistic fuzzy supratopological space (X,τ) is called $IFSTS-T_\circ$-space and denoted by($IFSTS-T_\circ$) if for any distinct pair of points x,y of X there exists one intuitionistic fuzzy supraopen set A in τ such that $x\in A$ but $y\notin A$ or $x\notin A$ and $y\in A$.

Example 2.2.

Let $X=\{a,b\}$ and $A=\langle x,(0.3,0.4),(0.4,0.5)\rangle$, $B=\langle x,(0.4,0.2),(0.5,0.3)\rangle$ it is clearly (X,τ) is a $IFSTS-T_\circ$-space , $\tau=\{\tilde{0},\tilde{1},A,B,A\bigcup B\}$.

Theorem 2.3.

An intuitionistic fuzzy supratopological space (X,τ) is $IFSTs-T_\circ$-space if and only if for each pair of distinct points x,y of X , $S-cl\{x\}\neq S-cl\{y\}$.

Proof:

Sufficiency . suppose that $x,y\in X$, $x\neq y$ and $S-cl\{x\}\neq S-cl\{y\}$

Let $w\in X$ such that $w\in S-cl\{x\}$ but $w\notin S-cl\{y\}$.

We claim that $x\notin S-cl\{y\}$. for if $x\in S-cl\{y\}$ then $S-cl\{x\}\subset S-cl\{y\}$. this contradiction the fact that $w\notin S-cl\{y\}$. consequently $x\in[S-cl\{y\}]^c$ to which y does not belong .

Necessity let (X,τ) be an $IFSTS-T_\circ$-space and $x,y\in X, x\neq y,\exists$ intuitionistic fuzzy supraopen set A $\ni x\in A$ or $y\in A$ then A^C is an intuitionistic fuzzy supraclosed set which $x\in A$ and $y\in A^C$. Since $S-cl\{y\}$ is the smallest intuitionistic fuzzy supraclosed set containing y [because $S-cl(E)=E\cup S-\text{int}(S-cl(E))$] , $S-cl\{y\}\subset A^C$, and therefore $x\notin S-cl\{y\}$. hence $S-cl\{x\}\neq S-cl\{y\}$.

Example 2.4.

Let $X=\{a,b\}$ and $A=\langle x,(0.3,0.4),(0.4,0.5)\rangle$, $B=\langle x,(0.4,0.2),(0.5,0.3)\rangle$, $\tau=\{\tilde{0},\tilde{1},A,B,A\bigcup B\}$ and $S-cl(A)\neq S-cl(B)$ then (X,τ) is a $IFSTs-T_\circ$-space .

Definition 2.5.

Let (X,τ_X) be a $IFSTS$, E be a subset of X , then classes τ_E of all intersections of E with τ -intuitionistic fuzzy supra open subsets of X belong to τ is a topology on E it is called relative intuitionistic fuzzy (intuitionistic fuzzy supra-subspace).

Example 2-6:

Let $X=\{a,b\}$ and $E=\langle x,(0.3,0.3),(0.3,0.3)\rangle$, $F=\langle x,(0.4,0.4),(0.4,0.4)\rangle$ then $\tau_X=\{\tilde{0},\tilde{1},E,F,E\bigcup F\}$ is an $IFST$ on X , $\tau_E=\{\tilde{0},\tilde{1},E\}$ and (E,τ_E) is called relative intuitionistic fuzzy suprasubtopological space

Definition 2.7.

Let (X,τ_X) be any intuitionistic fuzzy supratopological space if p is any property in X , then we called p is intuitionistic supra- hereditary (ISH for short))if its appear in a relative intuitionistic fuzzy supratopological space .

Theorem 2.8.

Let (X,τ) be any $IFSTS-T_\circ$-space , then the relative intuitionistic fuzzy supratopological space (E,τ_E) is $IFSTS-T_\circ$-space .

Proof:

Since (X,τ) be the intuitionistic fuzzy supratopology space , $E\subseteq X$, let $e_1,e_2\in E$, and $e_1\neq e_2$

To show (E,τ_E) is $IFSTS-T_\circ$-space , then $e_1,e_2\in X$, there exist an intuitionistic fuzzy supraopen set $A\subseteq X$, such that A containing one of e_1,e_2 but not both , new we have $e_1\in E$ and $e_1\in A$ then $e_1\in E\cap A$ or $e_2\in E$ and $e_2\in A$ then $e_2\in E\cap A$

is $IFSTS-T_\circ$-space .

Definition 2.9.

Let $f:(X,\tau_X)\to(Y,\delta_Y)$ be an intuitionistic fuzzy suprahomeomorphism ($IFSH$ for short) , let p any property in X we say that p is intuitionistic fuzzy topological property if p is appear in Y .

Theorem 2.10.

The property of $IFSTS-T_\circ$ is S - intuitionistic fuzzy topological property .

Proof:

Let $(X,\tau_X),(Y,\delta_Y)$ be an intuitionistic fuzzy supratopological spaces , $f:(X,\tau_X)\to(Y,\delta_Y)$ is ($IFSH$) , let $y_1,y_2\in Y$ and $y_1\neq y_2$ since f is objective , $\exists x_1\neq x_2$ such that $y_1=f(x_1),y_2=f(x_2)$ since (X,τ_X) is $IFSTS-T_\circ$-space , $\exists$ one supraopen set A such that $x_1\in A,x_2\notin A$ or $x_1\notin A,x_2\in A$ and function intuitionistic fuzzy supraopen then $f(x_1)\in f(A),\forall x_1\in f(A),x_1\in A$ hence (Y,δ_Y) is $IFSTS-T_\circ$-space .

Theorem 2.11.

A two intuitionistic fuzzy supratopological spaces $(X,\tau_X),(Y,\delta_Y)$ are $IFSTS-T_\circ$-spaces if and only if $X\times Y$ is a $IFSTS-T_\circ$-space .

Proof:

Sufficiency . let X,Y are $IFSTS-T_\circ$-spaces , if $(x_1,y_1)\neq(x_2,y_2)$ then either $x_1\neq x_2$ or $y_1\neq y_2$.

We claim $x_1\neq x_2$, since X is $IFSTS-T_\circ$-space , there exists one intuitionistic fuzzy supraopen set A such that $x_1\in A,x_2\notin A$ or $x_1\notin A,x_2\in A$, now intuitionistic fuzzy supraopen set $A\times Y\in X\times Y$, $(x_1,y_1)\in A\times Y$ or also $(x_2,y_2)\in A\times X$ then $X\times Y$ is a $IFSTS-T_\circ$-space .

Necessity . let $X\times Y$ is a $IFSTS-T_\circ$-space , to show that X is $IFSTS-T_\circ$-space , take $x_1\neq x_2\in X,\exists$ two points $(x_1,y),(x_2,y)$ by definition of product , since $x_1\neq x_2$ then $(x_1,y)\neq(x_2,y)$, since $X\times Y$ is $IFSTS-T_\circ$-space ,$\exists$ one intuitionistic fuzzy supraopen set $A\in X\times Y$ such that $(x_1,y)\in A$, $(x_2,y)\notin A$ or $(x_2,y)\in A$, $(x_1,y)\notin A$, now there exist two intuitionistic fuzzy supraopen sets A_1,A_2 such that $A_1\times A_2\subset A$, $x_1\in A_1, y\in A_2$ or $x_2\in A_1, y\in A_2$, since $(x_1,y)\in A$, $(x_2,y)\notin A$ therefore $x_1\in A_1,x_2\notin A_1$ and $x_2\in A_1,x_1\notin A_1$ then X is $IFSTS-T_\circ$- space , simarly Y is $IFSTS-T_\circ$- space .

3 - $IFSTs-T_1$- space induced by intuitionistic fuzzy supratopology .

Definition 3.1.

An intuitionistic fuzzy supratopological space (X,τ_X) is called intuitionistic fuzzy supra T_1-space and denoted by $IFSTS-T_1$-space if for any distinct pair of points x,y of X there exists two intuitionistic fuzzy supraopen sets A, B in τ_X such that $x\in A$, $x\notin B$ and $y\in B$, $y\notin A$.

Remark 3.2.

Every $IFSTS-T_1$-spaces is $IFSTS-T_\circ$-spaces but the converse is not true .

Example 3.3.

Let $X=\{a,b\}$ and

$E=\langle x,(0.3,0.3),(0.3,0.3)\rangle$, $\tau=\{\tilde{0},\tilde{1},E\}$ it is clearly (X,τ) is $IFSTS-T_\circ$-space but not $IFSTS-T_1$-space .

Theorem 3.4.

Let (X,τ) be any $IFSTS-T_1$-space , then the relative intuitionistic fuzzy supratopological space (E,τ_E) is $IFSTS-T_1$.

Proof:

An intuitionistic fuzzy supratopological space (X,τ) be $IFSTS-T_1$, let $e_1,e_2\in X$, $\exists$ two intuitionistic fuzzy supraopen sets A, B , let $E\subseteq X$, $e_1,e_2\in E$ and $e_1\neq e_2$ such that $e_1\in A$ and $e_1\in E$ then $e_1\in E\cap A$ and $e_2\in B$ and $e_2\in E$ then $e_2\in E\cap B$

Hence (E,τ_E) is $IFSTs-T_1$.

Theorem 3.5.

The property of $IFSTS-T_1$-space is topological property.

Proof:

Let $(X,\tau_X),(Y,\delta_Y)$ be an intuitionistic fuzzy supratopological spaces , $f:(X,\tau_X)\to(Y,\delta_Y)$ be an intuitionistic fuzzy suprahomeomorphism ($IFSH$ for short) , to show (Y,δ_Y) is $IFSTS-T_1$.

let $y_1,y_2\in Y$ and $y_1\neq y_2$,since f is onto, $\exists x_1,x_2\in X$ such that $y_1=f(x_1),y_2=f(x_2)$, since f is one-one , $y_1\neq y_2$ there exists $x_1\neq x_2$,

since (X,τ) is $IFSTS-T_1$, $\exists$ two intuitionistic fuzzy supraopen sets A, B such that $x_1\in A,x_2\notin A$ and $x_1\notin B,x_2\in B$.

Now function intuitionistic fuzzy supraopen then $f(x_1)\in f(A),\forall x_1\in f(A),x_1\in A$ and $f(x_2)\in f(B),\forall x_2\in f(B),x_2\in B$ hence (Y,δ_Y) is $IFSTS-T_1$-space .

Theorem 3.6.

Let $(X,\tau_X),(Y,\delta_Y)$ are $IFSTS-T_1$-spaces if and only if $X\times Y$ is a $IFSTS-T_1$-space .

Proof:

Sufficiency . suppose that X,Y are $IFSTS-T_1$-spaces , if $(x_1,y_1)\neq(x_2,y_2)$ then either $x_1\neq x_2$ or $y_1\neq y_2$.

we claim $x_1\neq x_2$, since (X,τ_X) is $IFSTS-T_1$-space , $\exists$ two intuitionistic fuzzy supraopen sets A, B such that $x_1\in A,x_2\notin A$ and $x_1\notin B,x_2\in B$, now $A\times Y,B\times Y$ two intuitionistic fuzzy supraopen sets in $X\times Y$, $(x_1,y_1)\in A\times X$ but $(x_1,y_1)\notin B\times Y$ also $(x_1,y_1)\in B\times Y$ but $(x_1,y_1)\notin B\times Y$ then $X\times Y$ is $IFSTS-T_1$-space.

Necessity . let $X\times Y$ is $IFSTS-T_1$-space to prove X is $IFSTS-T_1$-space

take $x_1,x_2\in X$, $x_1\neq x_2$,$\exists$ $(x_1,y),(x_2,y)\in XXY$, since $x_1\neq x_2$ then $(x_1,y)\neq(x_2,y)$, since $X\times Y$ is $IFSTS-T_1$-space ,$\exists$ two supraopen sets $A,B\in XXY$ such that $(x_1,y)\in A$,$(x_2,y)\notin A$ and $(x_2,y)\in B$, $(x_1,y)\notin B$. now there exist two intuitionistic fuzzy supra open sets A_1 , A_2 such that $A_1\times A_2\subset A$, and $x_1\in A_1$, $y\in A_2$, since $(x_1,y)\in A$, $(x_2,y)\notin A$ and $(x_2,y)\in B$, $(x_1,y)\notin B$ then $x_1\in A_1$, $x_2\notin A_1$ and $x_2\in B_1$, $x_1\notin B_1$ then (X,τ_X) is $IFSTS-T_1$-space .Simarly (Y,δ_Y) is $IFSTS-T_1$- space .

Remark 3.7.

Because the intersections is very difficult in intuitionistic fuzzy sets , the $IFSTS-T_2$-space is non satisfied in general but satisfies in the case intuitionistic fuzzy special supratopological spaces

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حول بعض بديهيات الفصل الضبابية الحدسية في الفضاءات التبولوجية الضبابية الحدسية

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الملخص

في هذا البحث قدمت تعريفا لبعض بديهيات الفصل فضاء− سبرا ضبابي حدسي− T_o، فضاء − سبرا ضبابي حدسي− T_1 وفضاء سبرا ضبابي حدسي − T_2 في الحالة الخاصة فقط ($(IFSTS - T_\circ)$، $(IFSTS - T_1)$ و $(IFSTS - T_2)$ باختصار) ودرست الصفات السبرا ضبابية حدسية الوراثية والصفات السبرا ضبابية حدسية التبولوجية على هذه البديهيات