Regional Asymptotic Approach for Dynamical Free Systems

Raheam A. Al-Saphory , Mohammed A. Al-Joubori
Department of Mathematics, College of Education, Tikrit University, Tikrit, Iraq.
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Abstract:
The aim of this paper is to develop an approach based on state asymptotic estimator. More precisely, we extend the notion of regional asymptotic observability as in ref. [1] to the case where the dynamical systems are uncontrolled (F-systems). For different sensors, we give the characterizations of regional asymptotic free observer in order that asymptotic free observability can be achieved. Furthermore, we show that, there exists a dynamical F-system for distributed diffusion F-system is not asymptotic F-observable in the usual sense, but it may be regional asymptotic F-observable.

Key words: Sensors, ω_{af}-observability, ω_{af}-detectability, ω_{af}-observer, dynamical F-systems.

1. Introduction.

In system theory, the asymptotic observability is related to the possibility to estimate the state from the knowledge of system dynamics and the output [2-5]. The notion of regional analysis was extended by El Jai *et al.* [6-7]. The study of this notion motivated by certain concrete-real problem, in thermic, mechanic, environment [8-11]. The concept of regional asymptotic analysis was introduced recently by Al-Saphory and El Jai in [1,12,13], consists in studying the behaviour of the system not in the entire domain Ω but only on particular region ω of the domain. The principle reason behind introducing this concept is that it provides a means to deal with some physical problems concern the determination of laminar flux conditions, developed in steady by vertical uniformly heated plate (Figure 1).

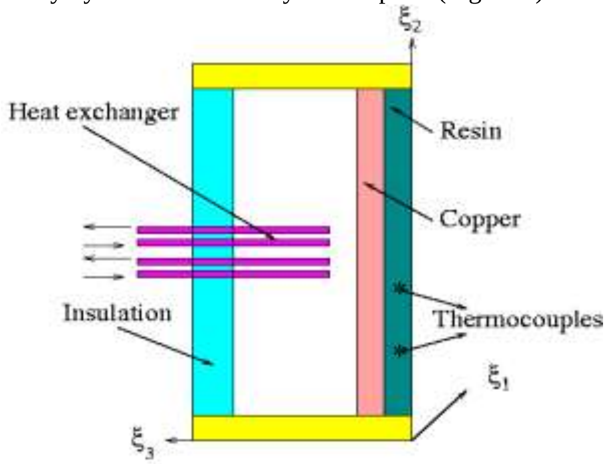

Figure 1: Profile of the active plate.

This asymptotic approach can be extended to find unknown boundary convective condition on the front face of the active plate, as in [9]. The reconstruction is based on knowledge of the dynamical F-system (regional asymptotical observer) and the measurement given by internal pointwise sensors (that by means by the thermocouples).

The paper is organized as follows. Section 2 devotes to the introduction of asymptotic regional approach. We give the formulation problem and preliminaries. We need some notions concern the asymptotical behaviour (ω_{af}-strategic sensor, ω_{af}-detectability, ω_{af}-observer). Section 3 related to the characterization notion of ω_{af}-observable by the use of strategic sensors. In last section, we illustrate applications with many situations of sensors locations.

Regional Asymptotic Approach

2.1 Considered systems

Let Ω be an open bounded subset of R^n, with boundary $\partial\Omega$ and $[0,T]$, $T>0$ be a time measurement interval. Suppose that ω be a connected non-empty given subregion of Ω. We denote $\Theta = \Omega \times (0,\infty)$ and $\prod = \partial\Omega \times (0,\infty)$. The considered F-systems is described by the following parabolic equations

$$\begin{cases} \dfrac{\partial x}{\partial t}(\xi,t) = Ax(\xi,t) & \Theta \\ x(\xi,0) = x_\circ(\xi) & \Omega \\ x(\eta,t) = 0 & \prod \end{cases} \quad (2.1)$$

where A is a second order linear differential operator, which generates a strongly continuous semi-group $(S_A(t))_{t\geq 0}$ on the Hilbert space $X = L^2(\Omega)$ and is self-adjoint with compact resolvent. The spaces X and O be separable Hilbert spaces where X is the state space and $O = L^2(0,\infty,R^q)$ is the observation space, where q are the numbers of sensors. Under the given assumption [16], the F-system (2.1) has a unique solution:

$$x(\xi,t) = S_A(t)x_\circ(\xi) \quad (2.2)$$

The problem is that, how to construct an approach which estimate asymptotically the current state in a given region ω (see figure 1).

2.2 Regional strategic sensors

The purpose of this subsection is to give the characterization for sensors in order that the system (2.1) is regionally approximately observable in ω.

- Sensors are any couple $(\Omega_i, f_i)_{1\leq i\leq q}$ where Ω_i denote closed subsets of $\overline{\Omega}$, which is spatial supports of sensors and $f_i \in L^2(\Omega_i)$ define the spatial distributions of measurements on Ω_i.

The measurements may be given by q sensors $(\Omega_i, f_i)_{1\leq i\leq q}$ and then, the output functions are given by the form

$y(.,t)=\left[y_1(.,t), y_2(.,t),\dots y_q(.,t)\right]^{tr}$
where
$y_i(.,t)=<x(.,t), f_i(.)>_{L^2(\Omega_i)}$
and in the case of pointwise
$y_i(.,t)=<x(.,t),\delta_{b_i}(.)>_{L^2(\Omega)}$
Thus, these equations may be given the augmented output function of the F-system (2.1) by
$y(.,t)=Cx(.,t)$ (2.3)
The operator $C\in L(R^q, X)$, depend on the structures of sensors [14-15].

▪ According to the choice of the parameters Ω_i and f_i, we have various types of sensors. These sensors may be types of internal zones when $\Omega_i\subset\Omega$. The output function (2.3) can be written in the form
$y(.,t)=<x(.,t), f_i(.)>_{L^2(\Omega_i)}$ (2.4)

▪ Sensors may also be internal pointwises when $\Omega_i=\{b_i\}\subset\Omega$ and $f_i=\delta_{b_i}(x-b_i)$ where δ_{b_i} is Dirac mass concentrated in b_i with $i=1,\dots,q$. Then, the output function (2.3) can be given by the form
$y(.,t)=<x(.,t),\delta_{b_i}(.)>_{L^2(\Omega)}=x(b_i,t), \forall i=1,\dots,q$ (2.5)

▪ In the case, of internal pointwise sensors the operator C is unbounded and some precaution must be taken in [14-15].

▪ For $x(\xi,t)=S_A(t)x_\circ(\xi)$, defines the operator $Kh=CS_A(t)h$ by the form
$K: X\to O$
$h \to CS_A(.)h$
where is in the case of internal zone sensors is linear and bounded [17]. The adjoint operator K^* of K is defined by
$K^*y=\int_0^t S_A^*(s)\,C^*y(s)\,ds$

▪ For the region ω of the domain Ω, the restriction operator χ_ω is defined by
$\chi_\omega: L^2(\Omega)\to L^2(\omega)$
$x\to \chi_\omega x=x|_\omega$
where $x|_\omega$ is the restriction of x to ω.

Definition 2.1: An F-system (2.1) augmented with output function (2.3) is exactly ω-observable if:
$\operatorname{Im}\chi_\omega K^*=L^2(\omega)$

Definition 2.2: An F-system (2.1) augmented with output function (2.3) is approximately ω-observable if:
$\overline{\operatorname{Im}\chi_\omega K^*}=L^2(\omega)$

Definition 2.3: A sequence of sensors $(\Omega_i, f_i)_{1\le i\le q}$ is ω-strategic if the F-system (2.1)-(2.3) is approximately ω-observable [6].

The concept of ω-strategic has been extended to the regional boundary case as in [18]. Assume that the set (φ_{n_j}) of eigenfunctions of $L^2(\Omega)$ orthonormal in $L^2(\omega)$ associated with eigenvalues λ_n of multiplicity r_n and suppose that the F-system (2.1) has J unstable modes. Then we have the following result:

Proposition 2.4 Suppose that $\sup r_n=r<\infty$. Then suite of sensors $(\Omega_i, f_i)_{1\le i\le q}$ is ω-strategic if and only if:
1. $q\ge r$
2. rank $G_n=r_n,\quad \forall n,\ n=1,\dots, J$ where

$$G_n=\begin{bmatrix} <\varphi_{n_1}, f_1(.)>_{L^2(\Omega_1)},\dots,<\varphi_{n_{r_n}}, f_1(.)>_{L^2(\Omega_1)} \\ \vdots \\ <\varphi_{n_1}, f_q(.)>_{L^2(\Omega_q)},\dots,<\varphi_{n_{r_n}}, f_q(.)>_{L^2(\Omega_q)} \end{bmatrix}$$

with $J=1,\dots,r_n$.

Proof: The proof of this proposition is similar to the rank condition in [17], the main difference is that the rank condition
rank $G_n=r_n,\quad \forall n$
For the proposition 2.1. need only hold for rank $G_n=r_n,\quad \forall n,\ n=1,\dots, J$. In the case where the sensors are pointwise, then, we have

$$G_n=\begin{bmatrix} <\varphi_{n_1},\delta_1(.)>_{L^2(\Omega)},\dots,<\varphi_{n_{r_n}},\delta_1(.)>_{L^2(\Omega)} \\ \vdots \\ <\varphi_{n_1},\delta_q(.)>_{L^2(\Omega)},\dots,<\varphi_{n_{r_n}},\delta_q(.)>_{L^2(\Omega)} \end{bmatrix}$$

2.3 Regional asymptotical approach behaviour
Regional asymptotic F-observability characterization needs some notions which are related to the asymptotical behaviour (stability, detectability and observer). The concept of asymptotical approach has been extended recently by Al- Saphory as in [13]. In this subsection, we need to extend these results to F-system.

Definition 2.5: A semi-group is regionally asymptotically free stable in $L^2(\omega)$ (or ω_{af}-stable) if for every initial state $x_\circ(.)\in L^2(\Omega)$, the solution of the dynamical F-system (2.1) converges asymptotically to zero when $t\to\infty$.

Definition 2.6: The F-system (2.1) is said to be asymptotically free stable on ω (or ω_{af}-stable), if the operator A generates a semi-group which is asymptotically free stable in $L^2(\omega)$. It is easy to see that the F-system (2.1) is ω_{af}-stable if and only if, for some positive constants M_ω and α_ω such that:

$$\left\| \chi_{\omega_{af}} S_A(.)\right\|_{L(L^2(\Omega), L^2(\omega))}\le M_{\omega_{af}}\, e^{-\alpha_{\omega_{af}}t}\quad t\ge 0 \qquad (2.6)$$

If $(S_A(t))_{t\ge 0}$ is ω_{af}-stable, then for all $x_\circ(.)\in L^2(\Omega)$, the solution of F-system (2.1) satisfies

$$\|x(t)\|_{L^2(\omega))}=\|\chi_\omega S_A(.)x_\circ\|_{L(L^2(\Omega), L^2(\omega))}\le M_{\omega_{af}}\, e^{-\alpha_{\omega_{af}}t}\|x_\circ\|_{L^2(\Omega)}$$

and then
$\lim_{t\to\infty}\|x(t)\|_{L^2(\omega)}=0$

Definition 2.7: The F-system (2.1) augmented with then output function (2.3) is said to be asymptotically free detectable on ω (or ω_{af}-detectable) if there exists an operator $H_{\omega_{af}} : R^q \to L^2(\omega)$ such that $(A - H_{\omega_{af}} C)$ generates a strongly continuous semi-group $\left(S_{H_{\omega_{af}}}(t)\right)_{t\geq 0}$ which is ω_{af}-stable.

Definition 2.8: Consider the F-system (2.1)-(2.3) together with the dynamical F-system

$$\begin{cases} \frac{\partial z}{\partial t}(\xi,t) = F_{\omega_{af}} x(\xi,t) + H_{\omega_{af}} y(t) & \Theta \\ z(\xi,0) = z_{\circ}(\xi) & \Omega \\ z(\eta,t) = 0 & \Pi \end{cases}$$

(2.7)

where $F_{\omega_{af}}$ generates a strongly continuous semi-group $\left(S_{F_{\omega_{af}}}(t)\right)_{t\geq 0}$ which is stable on Hilbert space Z and $H_{\omega_{af}} \in L(R^q, Z)$. The F-system (2.7) defines an ω_{af}-estimator for $\chi_{\omega} Tx(\xi,t)$ if:

(1) $\lim_{t\to\infty} \| z(.,t) - \chi_{\omega} Tx(.,t) \|_{L(L(\Omega), L^2(\omega))} = 0$

(2) $\chi_{\omega} T$ maps $D(A)$ in $D(F)$ where $z(\xi,t)$ is the solution of the F-system (2.7).

Definition 2.9: The F-system (2.7) specifies an ω_{af}-observer for the F-system (2.1)-(2.3) if the following conditions hold:

(1) There exists $M_{\omega_{af}} \in L(R^q, L^2(\omega))$ and $N_{\omega_{af}} \in L(L^2(\omega))$ such that $M_{\omega_{af}} C + N_{\omega_{af}} \chi_{\omega} T = I_{\omega_{af}}$

(2) $\chi_{\omega} TA + F_{\omega_{af}} \chi_{\omega} T = H_{\omega_{af}} C$

(3) The F-system (2.7) defines an ω_{af}-observer.

Definition 2.10: The system (2.7) is said to be ω_{af}-observer for the F-system (2.1)-(2.3) if $X = Z$ and $\chi_{\omega} T = I_{\omega}$. in this case, we have $F_{\omega_{af}} = A - H_{\omega_{af}} C$. Then the dynamical F-system (2.7) becomes

$$\begin{cases} \frac{\partial z}{\partial t}(\xi,t) = Az(\xi,t) - H_{\omega_{af}}(Cz(\xi,t) - y(.,t)) & \Theta \\ z(\xi,0) = 0 & \Omega \\ z(\eta,t) = 0 & \Pi \end{cases}$$

(2.8)

Definition 2.11: The F-system (2.1)-(2.3) is ω_{af}-observable, if there exists a dynamical F-system which is ω_{af}-observer, for the original F-system.

Now, the approach which is observed the current state $x(\xi,t)$ asymptotically is given by the following section:

3. Regional state reconstruction method

In this section, we give an approach which allow to construct an ω_{af}-estimator of $x(\xi,t)$. This method avoids the consideration of initial state [7], it enables to observe asymptotically the current state in ω without needing the effect of the initial state of the considered F-system .

Theorem 3.1: Suppose that the sequence of sensors $(D_i, f_i)_{1\leq i\leq q}$ is ω-strategic and the spectrum of A contain J eigenvalues with non-negative real parts. Then, the F-system (2.1) augmented with the output function (2.2) is ω_{af}-observable by the following dynamical F-system

$$\begin{cases} \frac{\partial z}{\partial t}(\xi,t) = Az(\xi,t) - H_{\omega_{af}} C(z(\xi,t) - y(.,t)) & \Theta \\ z(\xi,0) = z_{\circ}(\xi) & \Omega \\ z(\eta,t) = 0 & \Pi \end{cases}$$

(2.9)

Proof: The proof is limited to the case of zone sensors in the following steps:

<u>**Step 1**</u>. Under the assumptions of subsection 2.1, the F-system (2.1) can be decomposed on two parts, unstable and stable. The state vector may be given by $x(\xi,t) = \left[x_1(\xi,t) + x_2(\xi,t) \right]^{tr}$ where $x_1(\xi,t)$ is the state component of the unstable part of the F-system (2.1), may be written in the form

$$\begin{cases} \frac{\partial x_1}{\partial t}(\xi,t) = A_1 x_1(\xi,t) & \Theta \\ x_1(\xi,0) = x_{\circ 1}(\xi) & \Omega \\ x_1(\eta,t) = 0 & \Pi \end{cases}$$

(2.10)

and $x_2(\xi,t)$ is the component state of the part of the F-system (2.1) given by

$$\begin{cases} \frac{\partial x_2}{\partial t}(\xi,t) = A_2 x_2(\xi,t) & \Theta \\ x_2(\xi,0) = x_{\circ 2}(\xi) & \Omega \\ x_2(\eta,t) = 0 & \Pi \end{cases}$$

(2.11)

The operator A_1 is represented by matrix of order ($\sum_{n=1}^{J} r_n, \sum_{n=1}^{J} r_n$) given by

$$A_1 = diag\left[\lambda_1,..., \lambda_1, \lambda_2,..., \lambda_2,..., \lambda_j,..., \lambda_j\right]$$

Step 2. Since the sequence suite of sensors $(D_i, f_i)_{1\le i\le q}$ is ω-strategic for the unstable part of the F-system (2.1). The sub F-system (2.10) is approximately ω-observable [6], and since it is of finite dimensional, then it is exactly ω-observable [3]. Therefore it is ω_{af}-detectable, and hence there exists an operator $H^1_{\omega_{af}}$ such that $A_1 - H^1_{\omega_{af}}C$ which satisfies the following:

$\exists M^1_{\omega_{af}}, \alpha^1_{\omega_{af}} > 0$ such that $\left\| e^{(A_1 - H^1_{\omega_{af}}C)t} \right\| \le M^1_{\omega_{af}} e^{-\alpha^1_{af}t}$

and then, we have $\|x_1(.,t)\|_{L^2(\omega)} \le M^1_{\omega_{af}} e^{-\alpha^1_{af}t} \|x_\circ\|_{L^2(\Omega)}$

Since the semi-group generated by the operator A_2 is ω_{af}-stable, then there exists $M^2_{\omega_{af}}, \alpha^2_{\omega_{af}} > 0$ such that

$$\|x_2(.,t)\|_{L^2(\omega)} \le M^1_{\omega_{af}} e^{-\alpha^1_{\omega_{af}}} \|x_{\circ 2}(.)\|_{L^2(\Omega)}$$

and therefore $\|x(\xi,t)\|_{L^2(\omega)} \to 0$ when $t \to \infty$.

Finally, the F-system (2.1)-(2.3) is ω_{af} - detectable.

Step 3. Let $e(\xi,t) = x(\xi,t) - z(\xi,t)$ where $z(\xi,t)$ solution of the F-system (2.9). Driving the above equation and using equation (2.1) and (2.9), we obtain

$$\frac{\partial e}{\partial t}(\xi,t) = \frac{\partial x}{\partial t}(\xi,t) - \frac{\partial z}{\partial t}(\xi,t)$$
$$= Ax(\xi,t) - Az(\xi,t) + H_{\omega_{af}}C(z(\xi,t) - x(.,t))$$
$$= (A - H_{\omega_{af}}C)e(\xi,t)$$

Since the F-system (2.1)-(2.2) is ω_{af}-detectable, there exists an operator $H_{\omega_{af}} \in L(R^q, L^2(\omega))$, such that the operator $(A - H_{\omega_{af}}C)$ generates asymptotically regionally stable, strongly continuous semi-group $\left(S_{H_{\omega_{af}}}(t)\right)_{t\ge 0}$ on $L^2(\omega)$ which is satisfied the following relations:

$\exists M_{\omega_{af}}, \alpha_{\omega_{af}} > 0$ such that

$$\left\| \chi_\omega S_{H_{\omega_{af}}}(t) \right\|_{L(L^2(\Omega), L^2(\omega))} \le M_{\omega_{af}} e^{-\alpha_{\omega_{af}}t}$$

Finally, we have

$$\| e(.,t)\|_{L(L(\Omega), L^2(\omega)} \le \left\| \chi_\omega S_{H_{\omega_{af}}}(t) \right\|_{L(L(\Omega), L^2(\omega))} \| e_\circ(.)\| \le M_{\omega_{af}} e^{-\alpha_{\omega_{af}}t} \| e_\circ(.)\|$$

with $e_\circ(.) = x_\circ(.) - z_\circ(.)$ and therefore $e(\xi,t)$ converges asymptotically to zero as $t \to \infty$. Thus the dynamical F-system (2.9) is observes asymptotically the regional state $x(\xi,t)$ of the F-system original F-system and (2.1)-(2.3) is ω_{af}-observable.

Remark 3.2: We can deduce that:

1. An F-system which is exactly ω-observable, is ω_{af}-observable.

2. An F-system which is asymptotically observable is ω_{af}-observable.

Example 3.3: Consider the F-system:

$$\begin{cases} \frac{\partial x}{\partial t}(\xi,t) = \Delta x(\xi,t) + x(\xi,t) & \Theta \\ x(\xi,0) = x_\circ(\xi) & \Omega \\ z(\eta,t) = 0 & \Pi \end{cases} \quad (2.12)$$

augmented with the output function

$$y(t) = \int_\Omega x(\xi,t)\,\delta(\xi - b_i)\,d\xi \quad (2.13)$$

where $\Omega = (0,1)$ and $b_i \in \Omega$ are the location of sensors (b_i, δ_{b_i}). The operator $A = (\Delta + 1)$ generates a strongly continuous semi-group $\left(S_A(t)\right)_{t\ge 0}$ on the Hilbert space $L^2(\omega)$ [16]. Consider the dynamical F-system

$$\begin{cases} \frac{\partial z}{\partial t}(\xi,t) = \Delta z(\xi,t) + z(\xi,t) - HC\left(z(\xi,t) - x(\xi,t)\right) & (0,1), t > 0 \\ z(\xi,0) = z_\circ(\xi) & (0,1) \\ z(0,t) = z(1,t) = 0 & t > 0 \end{cases}$$

(2.14)

where $H \in L(R^q, Z)$, Z is the Hilbert space and $C: Z \to R^q$ is linear operator. If $b_i \in Q$, then the sensors (b_i, δ_{b_i}) is not strategic for the unstable sub F-system of (2.12) [2] and therefore the free system (2.12)-(2.13) is not asymptotically detectable in Ω [15]. Then the dynamical F-system (3.14) is not free observer and then (2.12)-(3.13) is not asymptotically observable [16].

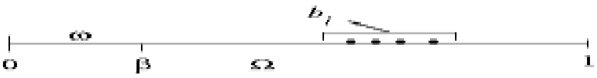

Figure 2: The domain Ω, the subregion ω and the location sensors b_i.

Now, we consider the region $\omega = [0, \beta] \subset (0,1)$ and the dynamical F-system

$$\begin{cases} \frac{\partial z}{\partial t}(\xi,t) = \Delta z(\xi,t) + z(\xi,t) - H_\omega C\left(z(\xi,t) - x(\xi,t)\right) & (0,1), t > 0 \\ z(\xi,0) = z_\circ(\xi) & (0,1) \\ z(0,t) = z(1,t) = 0 & t > 0 \end{cases}$$

(2.15)

where $H_{\omega_{af}} \in L(R^q, L^2(\omega))$. If $\frac{b_i}{\beta} \notin$ Q, then the sensors (b_i, δ_{b_i}) is ω-strategic for the unstable free sub F-system of (2.12) [8], and then the F-system (2.12)-(2.13) is ω_{af}-detectable. Therefore the F-system (2.12)-(2.13) is ω_{af}-observable by ω_{af}-observer [13].

4. Application to Sensors Locations

In this section, we present an application of the above results to a two-dimensional F-system defined on $\Omega=(0,1)\times(0,1)$ by the form

$$\begin{cases}\dfrac{\partial x}{\partial t}(\xi_1,\xi_2,t)=\Delta x(\xi_1,\xi_2 t) & \Theta \\ x(\xi_1,\xi_2,0)=x_{\circ}(\xi_1,\xi_2) & \Omega \\ x(\eta_1,\eta_2,t)=0 & \Pi\end{cases} \quad (4.1)$$

together with output function by (2.4), (2.5). Let $\omega=(\alpha_1,\beta_1)\times(\alpha_2,\beta_2)$ be the considered region is subset of $(0,1)\times(0,1)$. In this case, the eigenfunctions of F-system (4.1) are given by

$$\varphi_{ij}(\xi_1,\xi_2)=\frac{2}{\sqrt{(\beta_1-\alpha_1)(\beta_2-\alpha_2)}}\sin i\pi(\frac{\xi_1-\alpha_1}{\beta_1-\alpha_1})\sin j\pi(\frac{\xi_2-\alpha_2}{\beta_2-\alpha_2})$$

(4.2)

associated with eigenvalues

$$\lambda_{ij}=-\left(\frac{i^2}{(\beta_1-\alpha_1)^2}+\frac{j^2}{(\beta_2-\alpha_2)^2}\right) \quad (4.3)$$

The following results give information on the location of internal zone or pointwise ω-strategic sensors.

4.1 Internal zone sensor

Consider the F-system (4.1)-(2.3) where the sensor supports Ω_1 are located in Ω. The output (2.2) can be written by the form

$$y(t)=<x(.,t),f_1(.)>_{L^2(\Omega_1)}=\int_{\Omega_1}x(\xi_1,\xi_2,t)f_1(\xi_1,\xi_2)d\xi_1 d\xi_2$$

(4.4)

where $\Omega_1\subset\Omega$ is location of zone sensor and $f_1\in L^2(\Omega_1)$. In this case of (figure 3), the eigenfunctions and the eigenvalues

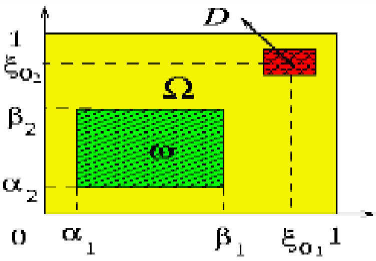

Figure 3: Domain Ω, subregion ω and location Ω_1 of internal zone sensor.

are given by (4.2) and (4.3). However, if we suppose that

$$\frac{(\beta_1-\alpha_1)^2}{(\beta_2-\alpha_2)^2}\notin Q \quad (4.5)$$

Then $r=1$ and one sensor may be sufficient to achieve ω_{af}-observability [19]. In this case the dynamical F-system (2.9) is given by

$$\begin{cases}\dfrac{\partial z}{\partial t}(\xi_1,\xi_2,t)=\Delta z(\xi_1,\xi_2,t)-H_{\omega_{af}}<x(.,t),f_i(.)>-Cz(\xi,t) & \Theta \\ z(\xi_1,\xi_2,0)=z_{\circ}(\xi_1,\xi_2) & \Omega \\ z(\eta_1,\eta_2,t)=0 & \Pi\end{cases}$$

(4.6)

Let the measurement support is rectangular with

$\Omega_1=[\xi_1-l_1,\xi_1+l_2]\times[\xi_2-l_2,\xi_2+l_2]\in\Omega$

then, we have the following result:

Corollary 4.1: If f_{1_1} is symmetric about $\xi_1=\xi_{\circ_1}$ and f_{1_2} is symmetric about $\xi_2=\xi_{\circ_2}$, then the F-system (4.1)-(4.4) is ω_{af}-observable by the dynamical F-system (4.6) if $(\xi_{\circ1}-\alpha_1)/(\beta_1-\alpha_1)$ and $(\xi_{\circ2}-\alpha_2)/(\beta_2-\alpha_2)\notin Q$.

4.2 Internal pointwise sensor

Let us consider the case of pointwise sensor located inside of Ω. The F-system (4.1) is augmented with the following output function:

$$y(t)=<x(.,t),\delta_b(.)>_{L^2(\Omega)}=x(b,t) \quad (4.7)$$

where $b=(b_1,b_2)$ is the location of pointwise sensor as defined in (figure 4)

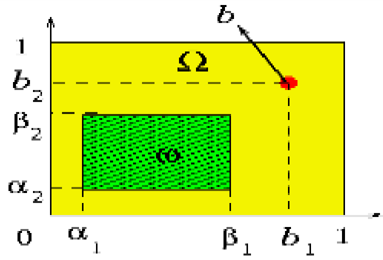

Figure 4: Rectangular domain, and location b of internal pointwise sensor.

If $(\beta_1-\alpha_1)/(\beta_2-\alpha_2)\notin Q$ then $m=1$ and one sensor (b,δ_b) may be sufficient for ω_{af}-observability. Then, the dynamical F-system is given by

$$\begin{cases}\dfrac{\partial z}{\partial t}(\xi_1,\xi_2,t)=\Delta z(\xi_1,\xi_2,t)+H_{\omega_{af}}\left(x(b_1,b_2,t)-y(t)\right) & \Theta \\ z(\xi_1,\xi_2,0)=z_{\circ}(\xi_1,\xi_2) & \Omega \\ z(\eta_1,\eta_2,t)=0 & \Pi\end{cases}$$

(4.8)

Thus, we obtain,

Corollary 4.2: The free system (4.1)-(4.7) is not ω_{af}-observable by the dynamical F-system (4.8), if $(b_1-\alpha_1)/(\beta_1-\alpha_1)$ and $(b_2-\alpha_2)/(\beta_2-\alpha_2)\in Q$.

4.3 Internal filament sensor

Consider the case where the observation on the curve $\sigma=\mathrm{Im}(\gamma)$ with $\gamma\in C^1(0,1)$ (see figure 5), then we have,

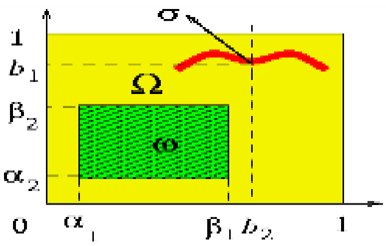

Figure 5: Rectangular domain, and location σ of internal filament sensors.

Corollary 4.3: If the observation recovered by filament sensor (σ, δ_σ) such that is symmetric with respect to the line $\xi = b$. The F-system (4.1)-(4.7) is not ω_{af}-observable by (4.8) if $(b_1 - \alpha_1)/(\beta_1 - \alpha_1)$ and $(b_2 - \alpha_2)/(\beta_2 - \alpha_2) \in Q$.

Remark 4.4: These results can be extended to the following:

1. Case of Neumann or mixed boundary conditions [2-3].
2. Case of disc domain $\Omega = (D,1)$ and $\omega = (0, r_\omega)$ where $\omega \subset \Omega$ and $0 < r_\omega < 1$ [1].
3. Case of boundary sensors where $C \notin L(X, R^q)$, we refer to see [14-15].

5. Conclusion

The concept developed in this paper is related to the regional asymptotical F-observability in connection with the strategic sensors. It permits us to avoid some "bad" sensors locations. Various interesting results concerning the choice of sensors structure are given and illustrated in specific situations. Many questions still opened. This is the case for example, the problem of finding the optimal sensor location ensuring such an objective.

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الملخص

الهدف من هذا البحث هو تطوير طريقه تعتمد على مفهوم المخمن التقاربي للحاله. بشكل ادق توسيع القابليه على المشاهده التقاربيه المنطقيه كما في المصدر [1]، عندما تكون المنظومات الحركيه غير مسيطر عليها (منظومات حره). وعليه لانواع مختلفه من المجسات نعطي مميزات مفهوم المشاهد الحر التقاربي المنطقي، لاجل انجاز القابليه على المشاهده الحره التقاربيه لتلك المنظومات. علاوة على ذلك برهنا انه توجد منظومه حره توزيعيه حراريه ليست قابله للمشاهده الحره التقاربيه في المعنى العام، ولكن قابله للمشاهده الحره التقاربيه المنطقيه.