Combination Between Single Diagonal Implicit and Explicit Runge Kutta(SDIMEX-RK) Methods for solving stiff Differential equations

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Abstract
In this paper we constructed some single diagonal implicit -explicit Runge –Kutta methods SDIMEX-RK of orders ranging from 2 to 3 that may be used for solving stiff non linear systems of Odes. This method split in two methods, the first is single diagonal implicit Runge Kutta method which have $A(\alpha)$-stable, and the second is explicit Runge Kutta method .The no stiff term is treated by explicit Runge Kutta method while stiff term is treated by single diagonal implicit method. These methods also are suitable for solving chemical reactions problems which contain stiff and non-stiff terms. We have used Matlab programs for solving the problem .
Keywords: Split methods; IMEX-RK methods: SDIMEX-RK methods.

1.Introduction:
Many stiff system of differential equations can be written in the form $u' = f(u) + \frac{1}{\varepsilon} g(u)$ ...(1.1)

, $u = u(t) \in R^N$. f, $g : R^N \to R^N$ and $\varepsilon > 0$ is the stiffness parameters [5]. If f represents the non-stiff part of the linear system then Applying an explicit (EX) scheme and g represents the stiff part of the nonlinear system applying single diagonal implicit scheme (SDIM). Combining single diagonal implicit and explicit Runge Kutta scheme are knows as single diagonal implicit-Explicit Runge Kutta SDIMEX-RK scheme .Additive schemes were first proposed by Cooper and Safey [3], they derived additive Runge Kutta methods to solve a system of differential equations in the form $x' = J(t) + g(t, x)$, where the linear term use implicit A-stable Runge Kutta methods solve $g(t, x)$ simultaneously by using explicit Runge-Kutta methods. Recently Pareeschi and Russo [6] derived Implicit-Explicit Runge Kutta methods to solve stiff system. The explicit (EX) part is treated by a strong-stability–preserving (SSP) scheme, and the implicit part is treated by an L-stable implicit Runge Kutta (IM) scheme. Qing and Yong [7] studied schemes are applied to stiff reaction–diffusion equation, the diffusion term treated by Explicitly scheme, while the reaction term is treated by semi implicitly scheme. Thor Gjesdal [10] studied Implicit –Explicit schemes (IMEX) methods for the solving time dependent differential equation in the form $y' = f(y,t) + g(y,t)$, where f treaded by explicit method and g treated by implicit method. Also Spiteri [9] studied implicit- explicit Runge Kutta method to solve stiff partial differential equation defined by form $u' = f(u) + g(u)$.where f is highly nonlinear but non stiff, whereas g is linear stiff, then implicit scheme applying an implicit for g and an explicit for f . Hongyu and Zou [2] studied additive Runge–Kutta methods of orders ranging from 2 to 4 that may be used for solving some nonlinear systems of ODEs, especially for the temporal discrimination of some nonlinear systems of PDEs with constraints. Only linear ODEs or PDEs need to be solved at each time step with these new methods.

2. Preliminaries
(2.1)Definition [1]

An additive Runge- Kutta (ARK) method of s stages and N levels have the form

$$Y_{ni} = y_n + h\sum_{v=1}^{N} \sum_{j=1}^{s} a_{ij}^{[v]} f^{[v]}(t_n + c_i h, Y_{nj}), \text{ where}$$

$$y_{n+1} = y_n + h\sum_{v=1}^{N}\sum_{i=1}^{s} b_i^{[v]} f^{[v]}(t_n + c_i h, Y_{ni})$$

v= 1,2, …N , i = 1, 2, …s .

The coefficients of the above methods may be organized in the Butcher tableau

c	$A^{[1]}$	$A^{[2]}$	$\cdots$	$A^{[N]}$
	$b^{[1]T}$	$b^{[2]T}$	$\cdots$	$b^{[N]T}$

Where

$c = (c_1,...,c_s)^T$, $b^{[v]} = (b_1^{[v]},...,b_s^{[v]})$, $A^{[v]} = (a_{ij})_{i,j=1}^{s}$ $v = 1,2,...,N$

matrices coefficient vectors:

(2.2) Definition [6]

An Implicit –Explicit Runge Kutta (IMEX) scheme has the form

$$Y_i = y_0 + h\sum_{j=1}^{i-1} \tilde{a}_{ij} f(t_0 + \tilde{c}_j h, Y_j) + h\sum_{j=1}^{v} a_{ij} \frac{1}{\in} g(t_0 + c_j h, Y_j)$$

$$y_1 = y_0 + h\sum_{i=1}^{v} \tilde{w}_i f(t_0 + \tilde{c}_i h, Y_i) + h\sum_{i=1}^{v} w_i \frac{1}{\in} g(t_0 + c_i h, Y_i)$$

The coefficients of the above methods may be organized in the Butcher tableau

$\tilde{c}$	$\tilde{A}$
	$\tilde{w}^T$

c	A
	w^T

Where

$\tilde{c} = (\tilde{c}_1,...,\tilde{c}_n)^T$, $\tilde{w} = (\tilde{w}_1,...,\tilde{w}_n)^T$, $c = (c_1,...c_n)^T$, $w = (w_1,...w_n)^T$, and the coefficient matrix A is lower triangular while the coefficient matrix $\tilde{A}$ is strictly lower triangular, such that $\tilde{A} = (\tilde{a}_{ij})$, $\tilde{a}_{ij} = 0$ if $j \ge i$ and $A = (a_{ij})$, $a_{ij} = 0$, if $j > i$, $i, j = 1, 2,..s$

3. Single Diagonal Implicit - Explicit Runge Kutta methods
As we can easily see, the additive RK schemes of the general form are fully implicit, and will be very

expensive when applied it for solving ODEs [8]. Also since explicit schemes are simple to implement have poor stability limits. On the other hand, implicit schemes have better than stability limits but require the solution of implicit system of equations [4]. These schemes (IMEX) try to bring together both approaches and handle the stiff terms in Single Diagonal Implicit (SDIM) while leaving the rest of the terms to be integrated explicitly. The first coefficient matrix A is lower triangular while the second coefficient matrix B is strictly lower triangular method. A general s-stage single diagonal implicit explicit Runge -Kutta (SDIME-RK) methods integrates Equation (1.1) which have $A(\alpha)-$stable for values α where $0 \le \alpha \le k$, $0 < k < 1$ by simultaneously treating for g implicitly and f explicitly which described by the following Butcher tableau:

$$\begin{array}{c|cccc} a_1 & a_1 & 0 & 0 & 0 \\ a_2 + c_{21} & c_{21} & a_2 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & 0 \\ c_{s,1} + c_{s,2} + \ldots + a_s & c_{s,1} & c_{s,2} & \ldots & a_s \\ \hline & w_1 & w_2 & \ldots & w_s \end{array} \qquad \begin{array}{c|cccc} 0 & 0 & 0 & \cdots & 0 \\ b_{21} & b_{21} & 0 & \cdots & 0 \\ \vdots & \vdots & b_{32} & \cdots & 0 \\ b_{s,1} + b_{s,s-1} + \ldots + b_{s,s-n} & b_{s,1} & b_{s,s-1} & \ldots\ b_{s,s-n} & 0 \\ \hline & w_1 & w_2 & \cdots & w_s \end{array}$$

…(1.3)

where $A = (c_{ij})$, $B = (b_{ij})$. This method is equivalence to

$$\left.\begin{aligned} u^{n+1} &= u^n + \sum_{j=1}^{s} w_j k_j \\ k_i &= h\left\{ \frac{1}{\varepsilon} g\left(u^n + \sum_{j=1}^{i-1} c_{ij} k_j + a_i k_i \right) + f\left(u^n + \sum_{j=1}^{i-1} b_{ij} k_j \right) \right\} , \ i = 1,\ldots,s \end{aligned}\right\} \quad ..$$

…(2.3)

where h is the time –step size, a_i, b_{ij}, c_{ij}, and w_j are parameters to be determined by accuracy and stability requirements .

4. Conditions for A-stability [3]

A method (A, B_1) from type sime-implicit Runge Kutta which have s stages defined by

$$\begin{array}{cccc} 0 & 0 & \cdots & 1 \\ 0 & 0 & \cdots & 1 \\ \vdots & 0 & \cdots & 1 \\ 0 & 0 & \cdots & 1 \end{array} \qquad \begin{array}{cccccc} b_{11} & 0 & 0 & \cdots & 0 & c_1 \\ b_{21} & b_{22} & 0 & \cdots & 0 & c_2 \\ \vdots & \vdots & b_{33} & \cdots & & \vdots \\ b_{s1} & b_{s2} & b_{s3} & \ddots & b_{ss} & c_s \end{array}$$

to applying this method on test proble**m**

$x' = \lambda x$, $x(0) = 1$ **where** $\operatorname{Re}(\lambda) < 0$ **we get**

$$y_s^{(m)} = \frac{p(z)}{q(z)} y_s^{(m-1)} \ , \ z = \lambda h \ , \ m = 1,2,\cdots$$

Then $y_s^{(m)} = \left(\frac{p(z)}{q(z)} \right)^{(m)} y_s^{(0)}$, $m = 1,2,\cdots$

Where p is a polynomial from degree (s-1) and q is polynomial from degree (s)

$$p(z) = 1 - \alpha_1 z + \alpha_2 z^2 - \cdots + (-1)^{s-1} \alpha_{s-1} z^{s-1}$$

$$q(z) = \beta_0 z - \beta_1 z + \beta_2 z^2 - \cdots + (-1)^{s-1} \beta_s z^s$$

We get

$$\alpha_1 = \beta_1 - 1$$

$$\alpha_2 = \beta_2 - \beta_1 + \frac{1}{2}$$

$$\alpha_3 = \beta_3 - \beta_2 + \frac{1}{2}\beta_1 - \frac{1}{6}$$

$$\vdots$$

$$\alpha_p = \beta_0 - \beta_{p-1} + \frac{1}{2}\beta_{p-2} - \cdots + (-1)^p \frac{1}{p!}$$

$$\beta_0 = 1$$

$$\beta_1 = a_1 + a_2 + \cdots + a_s$$

$$\beta_2 = a_1 a_2 + a_1 a_3 + \cdots + a_{s-1,s-1} a_s$$

$$\vdots$$

$$\beta_s = a_1 a_2 a_3 \cdots a_s$$

$$a_{rr} \ge 0 \quad , \quad r = 1,\cdots,s$$

$$\sum_{r=\left[\frac{\beta}{2}+1\right]}^{s} y^r \sum_{j=0}^{r}{}^{*} (-1)^{r+j} \left(\beta_{2r-j} \beta_j - \alpha_{2r-j} \alpha_j \right) \ge 0$$

$$\left.\begin{aligned} \beta_j &= \alpha_{j-1} = 0 \ ; \ j > s+1 \\ \beta_j &= \alpha_j = 0 \qquad j = 4,5,6,.. \end{aligned}\right\}.$$

Where r is the integral part $\beta/2 + 1$ and the asterisk * denoted that term $j = r$ is midpoint

5. Single diagonal implicit -explicit Runge –Kutta method from order =2 and stage = 2 defined by Butcher tableau:

$$\begin{array}{c|cc} a_1 & a_1 & 0 \\ c_{21} + a_2 & c_{21} & a_2 \\ \hline & w_1 & w_2 \end{array} \qquad \begin{array}{c|cc} 0 & 0 & 0 \\ b_{21} & b_{21} & 0 \\ \hline & w_1 & w_2 \end{array}$$

Since $C(\alpha)=(1-\alpha)A+\alpha B$ ….. (15)

$$C(\alpha)=\begin{pmatrix}(1-\alpha)a_1 & 0\\ (1-\alpha)c_{21}+\alpha b_{21} & (1-\alpha)a_2\end{pmatrix}$$

This method is equivalence to

$$k_1=h\{g(u^n+a_1k_1)+f(u^n)\}$$
$$k_2=h\{g(u^n+c_{21}k_1+a_2k_2)+f(u^n+b_{21}k_1)\}$$
$$u^{n+1}=u^n+w_1k_1+w_2k_2$$

It follows that the methods $(A,C(\alpha))$ is also of order 2 ,hence A-stable conditions gives the sufficient conditions for A-stability defined in (4). The choice $a_1=a_2=\gamma$. So that for a given γ the $(A,C(\alpha))$ method is A-stable for $0\le\alpha\le 1-\frac{1}{4\gamma}$. Now solving the order conditions defined by the following we obtain the family of single diagonal implicit-explicit Runge Kutta method in one parameters:

$$\left.\begin{array}{ll} w_1+w_2=1 & w_1+w_2=1\\ w_1a_1+w_2(c_{21}+a_2)=\frac{1}{2} & w_2b_{21}=\frac{1}{2}\end{array}\right\}\quad ...(2.5)$$

$$\begin{array}{c|cc}\gamma & \gamma & 0\\ 1-\gamma & 1-2\gamma & \gamma\\ \hline & \frac{1}{2} & \frac{1}{2}\end{array}\qquad \begin{array}{c|cc}0 & 0 & 0\\ 1 & 1 & 0\\ \hline & \frac{1}{2} & \frac{1}{2}\end{array}\quad\cdots(3.5)$$

Which are $A(\alpha)$-stable for $0\le\alpha\le 1-\frac{1}{4\gamma}$ three particular methods are given by the array

Let $\gamma=\frac{1}{4}$ we get SDIMEX-RK-(2,2,2)- $A(\alpha)-$stable

$$\begin{array}{c|cc}\gamma & \gamma & 0\\ 1-\gamma & 1-2\gamma & \gamma\\ \hline & \frac{1}{2} & \frac{1}{2}\end{array}\qquad \begin{array}{c|cc}0 & 0 & 0\\ 1 & 1 & 0\\ \hline & \frac{1}{2} & \frac{1}{2}\end{array}\quad[\cdots(4.5)]$$

Let $\gamma=\frac{3}{4}$ we get SDIMEX-RK-(2,2,2)- $A(\alpha)-$stable

$$\begin{array}{c|cc}\gamma & \gamma & 0\\ 1-\gamma & 1-2\gamma & \gamma\\ \hline & \frac{1}{2} & \frac{1}{2}\end{array}\qquad \begin{array}{c|cc}0 & 0 & 0\\ 1 & 1 & 0\\ \hline & \frac{1}{2} & \frac{1}{2}\end{array}\quad ...(5.5)$$

$$\cdots(1.5.6)$$

Let $\gamma=\frac{3+\sqrt{3}}{6}$ we get SDIEX-RK -(2,2,2)- $A(\alpha)-$stable

$$\begin{array}{c|cc}\gamma & \gamma & 0\\ 1-\gamma & 1-2\gamma & \gamma\\ \hline & \frac{1}{2} & \frac{1}{2}\end{array}\qquad \begin{array}{c|cc}0 & 0 & 0\\ 1 & 1 & 0\\ \hline & \frac{1}{2} & \frac{1}{2}\end{array}$$

…(6.5)

The method (4.5) is $A(\alpha)$-stable for $\alpha=0$, the method (5.5) is $A(\alpha)$- stable for $0\le\alpha\le\frac{2}{3}$, and the method (6.5) is $A(\alpha)$-stable for $\alpha=0$

6. Single diagonal implicit -explicit Runge –Kutta method from order =3 and stages = 3 defined by the following Butcher tableau:

$$\begin{array}{c|ccc}a_1 & a_1 & 0 & 0\\ c_{21}+a_1 & c_{21} & a_2 & 0\\ c_{31}+c_{32}+a_3 & c_{31} & c_{32} & a_3\\ \hline & w_1 & w_2 & w_3\end{array}\qquad \begin{array}{c|ccc}0 & 0 & 0 & 0\\ b_{21} & b_{21} & 0 & 0\\ b_{31}+b_{32} & b_{31} & b_{32} & 0\\ \hline & w_1 & w_2 & w_3\end{array}\quad\cdots(1.6)$$

since $C(\alpha)=(1-\alpha)A+\alpha B$ we get

$$B(\alpha)=\begin{pmatrix} (1-\alpha)a_1 & 0 & 0 \\ c_{21}(1-\alpha)+b_{31}\alpha & (1-\alpha)a_2 & 0 \\ c_{31}(1-\alpha)+b_{31}\alpha & c_{32}(1-\alpha)b_{32}\alpha & (1-\alpha)a_3 \end{pmatrix}$$

This method is equivalence to

$$k_1=h\{g(u^n+a_1k_1)+f(u^n)\}$$

$$k_2=h\{g(u^n+c_{21}k_1+a_2k_2)+f(u^n+b_{21}k_1)\}$$

$$k_3=h\{g(u^n+c_{31}k_1+c_{32}k_2+a_3k_3)+f(u^n+b_{31}k_1+b_{32}k_2)\}$$

$$u^{n+1}=u^n+w_1k_1+w_2k_2+w_3k_3$$

It follows that the methods $(A,C(\alpha))$ is also of order 3 ,hence A-stable conditions gives the sufficient conditions for A-stability defined in (4). The choice $a_1=a_2=a_3=\gamma$. So that for a given γ the $(A,C(\alpha))$ method is A-stable. Now solving the order conditions defined by the following we obtain the family of single diagonal implicit - explicit Runge Kutta methods :

$$\left.\begin{array}{lcl} w_1+w_2+w_3=1 & ; & w_1+w_2+w_3=1 \\ w_1(a_1)+w_2(a_2+c_{21})+w_3(a_3+c_{31}+c_{32})=\frac{1}{2} & ; & w_2b_{21}+w_3(b_{31}+b_{32})=\frac{1}{2} \\ w_1a_1^2+w_2(c_{21}+a_2)^2+w_3(c_{31}+c_{32}+a_3)^2=\frac{1}{3} & ; & w_2(b_{21})^2+w_3(b_{31}+b_{32})^2=\frac{1}{3} \\ w_1a_1^2+w_2(a_2^2+a_2c_{21}+a_1c_{21})+w_3(a_1c_{31}+a_2c_{32}+c_{21}c_{32}+a_{31}c_{31}+a_3c_{32}+a_3^2)=\frac{1}{6} & ; & w_3b_{21}b_{32}=\frac{1}{6} \end{array}\right\}$$

$$\begin{array}{c|ccc} \frac{1}{3} & \frac{1}{3} & 0 & 0 \\ \frac{1}{9} & \frac{-2}{3} & \frac{1}{3} & 0 \\ \frac{7}{9} & \frac{-1}{18} & \frac{1}{2} & \frac{1}{3} \\ \hline & \frac{1}{4} & \frac{1}{4} & \frac{1}{2} \end{array} \qquad \begin{array}{c|ccc} 0 & 0 & 0 & 0 \\ \frac{2}{3} & \frac{2}{3} & 0 & 0 \\ \frac{2}{3} & \frac{1}{6} & \frac{1}{2} & 0 \\ \hline & \frac{1}{4} & \frac{1}{4} & \frac{1}{2} \end{array} \quad \cdots(2.6)$$

$$\begin{array}{c|ccc} \frac{1}{3} & \frac{1}{3} & 0 & 0 \\ 1 & \frac{2}{3} & \frac{1}{3} & 0 \\ \frac{1}{3} & \frac{-1}{3} & \frac{1}{3} & \frac{1}{3} \\ \hline & \frac{1}{4} & \frac{1}{4} & \frac{1}{2} \end{array} \qquad \begin{array}{c|ccc} 0 & 0 & 0 & 0 \\ \frac{2}{3} & \frac{2}{3} & 0 & 0 \\ \frac{2}{3} & \frac{1}{6} & \frac{1}{2} & 0 \\ \hline & \frac{1}{4} & \frac{1}{4} & \frac{1}{2} \end{array} \quad \cdots(3.6)$$

$$\begin{array}{c|ccc} \frac{1}{3} & \frac{1}{3} & 0 & 0 \\ \frac{1}{9} & \frac{-2}{9} & \frac{1}{3} & 0 \\ \frac{7}{9} & \frac{-1}{18} & \frac{1}{2} & \frac{1}{3} \\ \hline & \frac{1}{4} & \frac{1}{4} & \frac{1}{2} \end{array} \qquad \begin{array}{c|ccc} 0 & 0 & 0 & 0 \\ \frac{2}{3} & \frac{2}{3} & 0 & 0 \\ \frac{5}{9} & \frac{2}{9} & \frac{1}{3} & 0 \\ \hline & \frac{1}{8} & \frac{1}{8} & \frac{3}{4} \end{array} \quad \cdots(4.6)$$

$$\begin{array}{c|ccc} \frac{1}{3} & \frac{1}{3} & 0 & 0 \\ 1 & \frac{2}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{-1}{3} & \frac{1}{3} & \frac{1}{3} \\ \hline & \frac{1}{4} & \frac{1}{4} & \frac{1}{2} \end{array} \quad 0 \quad \begin{array}{c|ccc} 0 & 0 & 0 & 0 \\ \frac{2}{3} & \frac{2}{3} & 0 & 0 \\ \frac{5}{9} & \frac{2}{9} & \frac{1}{3} & 0 \\ \hline & \frac{1}{8} & \frac{1}{8} & \frac{3}{4} \end{array} \cdots(5.6)$$

These methods (2.6), (3.6), (4.6), and (5.6) are $A(\alpha)$-stable for $\alpha = 0$

7. Test Problem: consider this problem from Pareschi and Russo problems

$$x'(t) = -y(t) \qquad x(0) = \frac{\pi}{2}$$

$$y'(t) = x(t) + \frac{1}{\varepsilon}(\sin(x(t)) - y(t)) \qquad y(0) = 1$$

The main idea is to split the right hand-side into

$$x'(t) = -y(t)$$
$$y'(t) = x(t)$$

which represent the non stiff part of the system then Appling an Explicit (EX) scheme and

$$x'(t) = 0$$
$$y'(t) = \frac{1}{\varepsilon}(\sin(x(t)) - y(t))$$

represents the stiff part of the system applying the single diagonal implicit scheme(SDIM)

Remark (1):

Method $-(s, \beta, p)$ where s represent implicit stage, β is explicit stage and p is order condition the method in left side is signal diagonal implicit method while the method in right side is explicit method this method is called Signal Diagonal Implicit Explicit Runge –Kutta methods from order p and stages s, β and is donated by SDIMEX-RK-(s, β, p).

Table 8: Results solve of problem 7 by using SDIMEX-RK(2,2,2) defined by (6.5)

Epsilon ε	Value of relative error for method SDIMEX-RK-(2,2,2)- $A[\alpha]$ h		Value of relative error for method SDIMEX-RK-(3,3,3)- $A[\alpha]$ h/2		Value of convergence for method (SDIMEX-RK)-(3,3,3)- $A[\alpha]$	
	EX	EY	EX	EY	CX	CY
1.00000	0.03200	0.05650	0.01710	0.02990	0.90107	0.92116
0.79433	0.02920	0.06120	0.01570	0.03230	0.89486	0.92239
0.63096	0.02570	0.06480	0.01390	0.03420	0.89074	0.92414
0.50119	0.02200	0.06860	0.01190	0.03580	0.89004	0.92475
0. 39811	0.01840	0.07080	0.00244	0.03780	0.89164	0.92420
0.07943	0.004420	0.08830	0.00198	0.04560	0.86469	0.88835
0.05012	0.003580	0.08830	0.00161	0.04800	0.85417	0.87842
0.06310	0.002900	0.08970	0.00132	0.04920	0.84339	0.86787
0.05012	0.002340	0.09090	0.00243	0.05020	0.83319	0.85706
0.03981	0.004420	0.08670	0.00198	0.04690	0.82449	0.84636
0.07943	0.003580	0.03580	0.00161	0.08800	0.86473	0.79264
0.06310	0.003580	0.02900	0.000300	0.04900	0.90096	0.78855
0.00794	0.000549	0.09550	0.000244	0.05530	0.95101	0.78517
0.00631	0.000456	0.09600	0.000199	0.05560	1.01667	0.78240
0.00501	0.000384	0.09620	0.000162	0.05580	1.09861	1.71907
0.00398	0.000328	0.09650	0.000133	0.05600	1.71907	0.77390
0.00316	0.000285	0.09680	0.0000552	0.05660	1.79192	0.77329
0.00079	0.000182	0.09680	0.0000511	0.05660	1.84914	0.77280
0.00063	0.000177	0.09680	0.0000481	0.05670	1.89254	0.77241
0.00050	0.000173	0.09680	0.0000459	0.05680	1.99692	0.77119
0.00040	0.000161	0.09680	0.0000410	0.05680	2.00037	0.77104
0.00008	0.000164	0.09690	0.0000408	0.05680	2.00037	0.77113
0.00006	0.000163	0.09690	0.0000408	0.05680	2.00898	0.77095
0.00002	0.000162	0.09690	0.0000403	0.05680	2.00950	0.77092
0.00001	0.000162	0.09690	0.0000403	0.05680	2.00990	0.77092

Table 9: results solve of problem 7 by using SDIMEX-RK(3,3,3) defined by (4.6)

Epsilon ε	Value of relative error for method SDIMEX-RK-(3,3,3)- $A[\alpha]$ h		Value of relative error for method SDIMEX-RK-(3,3,3)- $A[\alpha]$ h/2		Value of convergence for method (SDIMEX-RK)-(3,3,3)- $A[\alpha]$	
	EX	EY	EX	EY	CX	CY
1.00000	0.00137	0.002002	0.000696	0.00103	0.97539	0.97373
0.79433	0.00137	0.00226	0.000702	0.00115	0.96817	0.97224
0.63096	0.00132	0.00247	0.000680	0.00126	0.96270	0.97292
0.50119	0.00124	0.00268	0.000637	0.00136	0.96057	0.97603
0. 39811	0.00115	0.00289	0.000588	0.00146	0.96344	0.98091
0.07943	0.00115	0.00444	0.000493	0.00210	1.02412	1.02566
0.05012	0.00100	0.00473	0.000498	0.00217	1.02883	1.03211
0.06310	0.00120	0.00486	0.000504	0.00224	1.03256	1.03820
0.05012	0.00103	0.00498	0.000510	0.00229	1.03524	1.04366
0.03981	0.00104	0.000509	0.000516	0.00235	1.03600	1.04818
0.07943	0.00106	0.00542	0.000522	0.00235	1.03680	1.05149
0.06310	0.00107	0.00547	0.000529	0.00240	1.03544	1.05338
0.00794	0.00108	0.00553	0.000553	0.00245	1.01826	1.04510
0.00631	0.00112	0.00561	0.000558	0.00263	1.01272	1.04123
0.00501	0.00133	0.00572	0.000562	0.00266	1.00746	1.04086
0.00398	0.00133	0.00711	0.000565	0.00269	1.00276	1.047
0.00316	0.00133	0.00740	0.000588	0.00271	0.99877	1.06451
0.00079	0.00133	0.00769	0.000574	0.00273	0.98803	1.26694
0.00063	0.00133	0.00797	0.000575	0.00295	0.98754	1.29235
0.00050	0.00114	0.00929	0.000575	0.00302	0.98719	1.31228
0.00040	0.00114	0.00939	0.000575	0.00310	0.98694	1.3271
0.00008	0.00114	0.00970	0.000575	0.00318	0.98610	1.33825
0.00006	0.00114	0.00975	0.000575	0.00371	0.98611	1.32632
0.00002	0.00114	0.00975	0.000575	0.00371	0.98596	1.32242
0.00001	0.00114	0.00975	0.000575	0.00390	0.98593	1.32242

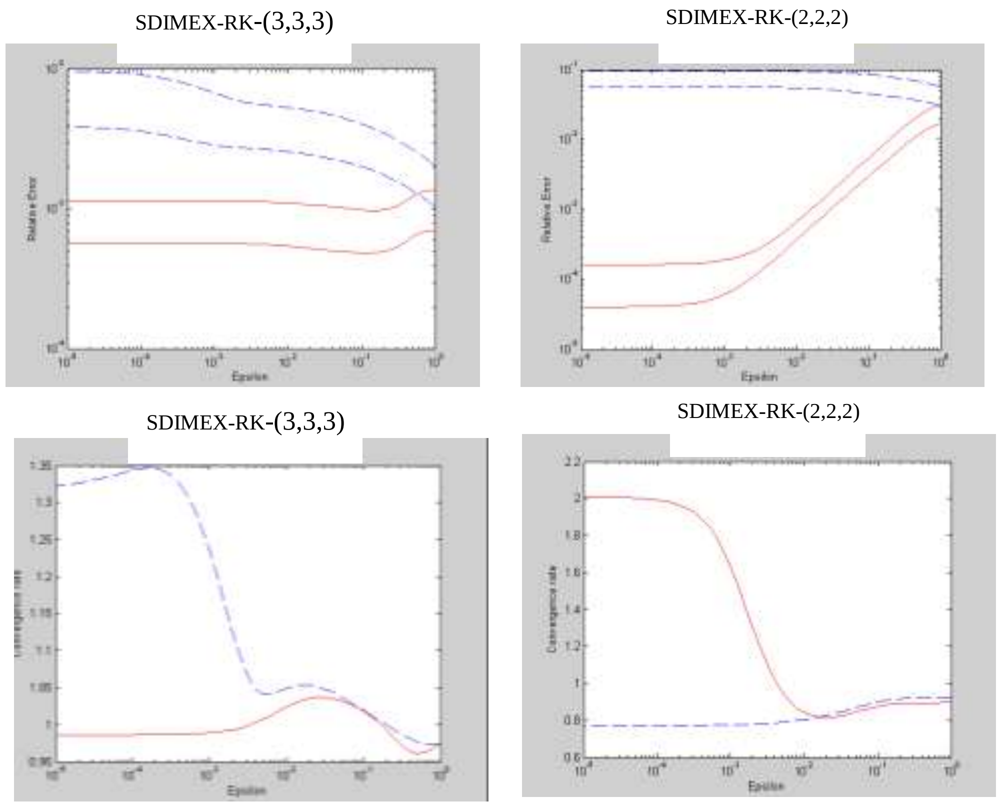
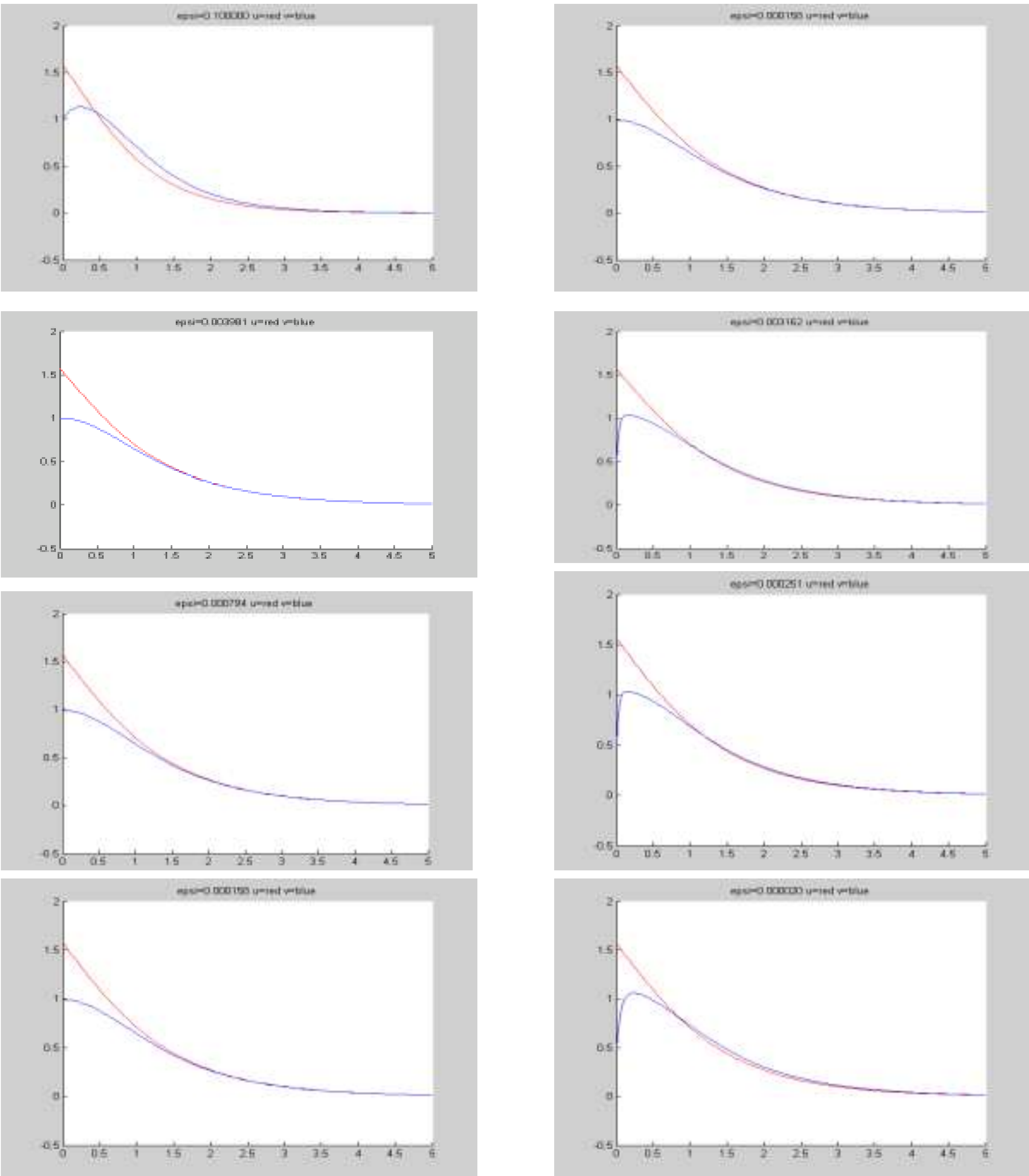


Figure 10: Relative and convergence rate for problem (7) by using SDIMEX-RK(2,2,2) the right side and SDIMEX-RK(3,3,3) the left side defined by (8.4) and (4.6)



Figures (11): Numerical solution for problem (7) by using SDIMEX-RK (2,2,2) the right side and the SDIME-RK (3,3,3) the left side defined by (6.4) and (4.6) for $\varepsilon = 10^{-1}, 10^{-2}, 10^{-3}, 10^{-4}$

12. Conclusion

The main aim of this paper are constructed single diagonal implicit explicit Runge –Kutta method are Special class of Additive Runge-Kutta method .Additive Runge-Kutta method for system of initial value problems have useful in many applications. A method of this type is characterized by a pair of methods (A,B) where the method A is single diagonal implicit which have A-stable and the method B is explicit .We conclusion a single diagonal implicit explicit Runge – Kutta method with equal non-zero $(a_1 \neq 0)$ diagonal elements in a gives a computational advantage over many implementation schemes for single diagonal implicit -explicit Runge Kutta and diagonal implicit-explicit Runge Kutta methods for which the modified Newton or any iteration method is used. The test results and figures are show that these schemes are stable and accurate for the calculation.

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الدمج بين طريقتي رانج – كوتا الصريحة والطريقة الضمنية القطرية لحل المعادلات التفاضلية الصلبة

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الملخص

في هذا البحث تم إنشاء بعض طرائق رانج–كوتا الصريحة الضمنية االقطرية المفردة لرتب تتراوح من ٢ إلى ٣ والتي يمكن أن تستخدم لحل بعض النظم الغير خطية الصلبة للمعادلات التفاضلية الاعتيادية. وهذه الطريقة تتجزأ الى طريقتين، الأولى هي طريقة رانج –كوتا الضمنية القطرية المفردة ذات الاستقرارية – $A(\alpha)$ والثانية طريقة رانج–كوتا الصريحة .الحد الغير صلب يعالج من قبل طريقة رانج–كوتا الصريحة بينما الحد الصلب يعالج من قبل طريقة رانج –كوتا الصريحة الضمنية القطرية المفردة. وهذه الطرائق مناسبة لحل المسائل التفاعلات الكيميائية التي تحتوي على حدود صلبة وغير صلبة. وتم استخدام برنامج بنظام (Matlab) في حل هذه المسالة.