تقدير مؤشرات الأداء لشعبة الأشعة في مستشفى البصرة التعليمي بأستعمال نموذج صف الانتظار (G/G/1) .[1]

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الباحثة ايات فاخر طاهر
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المستخلص :
يهدف البحث الى تقدير فترة انتظار المرضى المراجعين لشعبة الاشعة في مستشفى البصرة التعليمي بأستعمال نموذج صف الانتظار (G/G/1) بالاضافة الى تقدير مؤشرات الاداء لشعبة الاشعة بغية تقييم وتحسين اداء الخدمة المقدمة من شعبة الاشعة للمرضى . وقد تم التوصل الى أن التوزيع الملائم لفترات الوصول البينية تتمثل بتوزيع (ويبل) والخدمة بتوزيع (باريتو-الأسي) , وأخيراً تم الأستنتاج ان فترة انتظار المريض لحين حصوله على الخدمة بلغت (2.38) دقيقة وهي فترة جيدة مما يدل على سرعة الخدمة المقدمة من قبل شعبة الاشعة .

الكلمات المفتاحية : نماذج صف الأنتظار, معدل الوصول, معدل تقديم الخدمة , مقاييس أداء الأنموذج (G/G/1) .

[1] بحث مستل من رسالة الماجستير الموسومة ((توظيف توزيع (Exponential-Pareto Type I) المقترح في نمذجة صفوف الإنتظار مع التطبيق)) .

Estimating the performance indicators of the radiology department in Basra Teaching Hospital using the queue model (G/G/1) .

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Abstract :

The research aims to estimate the waiting period for patients attending the radiology department at Basra Teaching Hospital using the waiting queue model (G/G/1), in addition to estimating the performance indicators of the radiology department in order to evaluate and improve the performance of the service provided by the radiology department to patients. It has been concluded that the appropriate distribution of the inter-access periods is represented by the distribution of (Whipple) and the service by the distribution of (Pareto-exponential), and finally it was concluded that the waiting period for the patient until he received the service amounted to (2.38) minutes, which is a good period, which indicates the speed of the service provided by Radiology Division.

Keywords: queuing models, arrival rate, service delivery rate, model performance measures (G/G/1).

Introduction:

The historical origins of waiting lines go back to the German scientist A.K. Erlang in 1909 when the use of the manual telephone expanded, where he conducted his experiments on the problem of congestion in phone calls, as there is a delay in the service of the requesters of these calls due to the inability of the workers to meet the incoming orders at the required speed, as a result of the presence of fluctuation in the arrival of the service request according to the time periods, so it starts Increasing from the morning until noon, then starting to decrease, and therefore the scientist Erlang saw the possibility of modeling this phenomenon by studying the possibility of providing the service in appropriate time periods and on demand, and the application of this method did not extend to other phenomena until the end of World War II, when the use of the work that Erlang started it to include a number of problems related to waiting lines **(Salih, 2012, 13)**.

We always see the problem of waiting in our daily lives, and it is something that cannot be avoided, no matter how much effort we make, because there remains waiting in many services. Each of us wishes that he had not waited when dealing with any institution or department, as there are many institutions and departments in which there is waiting for customers to get On a specific service, and among those institutions are the health institutions that suffer from a large number of patients, especially in our dear country, which was exposed to several wars that caused the deterioration of the health status of citizens.

The study aims to apply the theory of waiting lines in the radiology department of Basra Teaching Hospital, based on the model (G/G/1), for the purpose of reducing waiting time and service time.

There are a number of studies that dealt with the use of queuing models in which the intervals of arrival and service follow a general distribution, including:

The two researchers (Jain & Dhakad) in 2003 derive a mathematical formula for the length of the queue based on the entropy function and the queue model (G/G/1).

2-Theoretical side

1-2 The concept of queuing theory

The theory of waiting lines in health institutions is defined as “one of the mathematical and probabilistic methods in operations research that helps decision-makers in health institutions in order to improve the quality of their services by addressing the problems of accumulation and formation of waiting lines for patients as a result of irregular arrival and the process of providing service to them, and that is According to different probability rules and distributions **(Ahlam, 2014: 48)**, it is often used when making business decisions about the amount and type of work required to provide the service, and it also works on analyzing probabilistic systems for clients and servers **(Al-Hassani, 2022: 36)**, and we can look at many Among the models that we encounter in our life as waiting queue models, as these models have two main elements: the service provider and the service requester.We also conclude that the waiting queue theory is an analysis tool that provides more important information than the studied model, and it is a theory based on describing the arrival rate and the time of submission. Service using appropriate distributions, and decisions related to queuing must be based on cost, which makes the total costs (service cost + waiting cost) as low as possible **(Mubarak, 2015: 12)**.

The application of the theory of waiting lines is useful in all fields of manufacturing and services, such as waiting for cars in front of the repair shop or the gas station, waiting for patients in front of the doctor's room, waiting for auditors in front of the service station, whether in banks, traffic directorates, real estate registration departments, etc., and waiting for trucks in front of warehouses for loading or Unloading **(Mahmood, 2021: 14-15)**, and the results of queuing models are often used in business decisions **(Shastrakar et al., 2016: 13)**, post offices and supermarkets **(Ali et al., 2018: 2)**, and the queuing method is widely used In the field of services, for example, restaurants, barbershops, and transportation **(Soumia and Mouchira, 2019: 8-9)**.

2-2- Characteristics of the mathematical model of the queuing system:

Before proceeding with any calculation for the purpose of solving the queue problem mathematically, the type of queue must be determined precisely according to its characteristics and according to the following symbolic series:

(a/b/c): (d/e/f) where:

a : Arrival time distribution.

b : Service time distribution .

c : No of Service Channels.

d : *System Capacity* .

e : Source of Population .

f : *Queue Discipline* .

symbols that can be used to express the basic characteristics of the queuing system, (1) shows the Table with details of each:

String code	the components	User code	the details
a	The average number of arrivals during a certain period of time	M D E_k GI	**Poisson** **Deterministic** **Erlang** **General**
b	Average number of customers served during a given time period	M D E_k G	**Poisson** **Deterministic** **Erlang** **General**
c	Number of service stations	1,2,.....	
d	Pattern(base) of service delivery	FCFS LCFS SIRO SWP	**First come first served** **last come first served** **random service system** **First-come-first-serve system**
e	Queue size	Finite Infinite	
f	The size of the community of customers requesting the service	∞ N	**Finite** **Infinite**

(Ali, 2010: 10),(Taha,2013:665-666)

3-2 Measures of Performance of Queuing System

There are many indicators that can be calculated by building queuing models to be a tool to distinguish the efficiency of queuing systems, which are built in the light of this and the standards.

Table (2) represents the symbols used to indicate the performance indicators of the queuing system.

code	Performance measures
λ	**Mean rate of arrival (The number of customers reached per unit of time) .**
μ	**Mean service rate (The number of customers served per unit of time by the service center).**
Ls	**Average Number customers in the system.**
L_q	**Average Number customers in the waiting line.**
W_s	**Average time a customer's spends in system (waiting and being served).**
Wq	**Average time a customer spends in waiting line.**
P_0	**It is possible that the system is down .**
P_n	**Probability of (n) customers in the system.**
ρ	**utility ratio .**

(Gosavi,2020: 2),(Khaskheli et al.,2021: 1359-1358)

<u>4-2 Littele's Formulas</u>

This law was developed by the scientist J. Little in the year (1961), where the law was applied to find the average number of connecting units, which was called Little's Law. Which states that the average number of customers in the system is equal to multiplying the rate of customer arrival by the average time spent by each unit in the system, and this theory is used for all queue models, regardless of the distribution of access, service, or the number of service centers (stations) . **(Ali, 2010: 21) ,(Sztrik,2010 : 18)**

$$N = \lambda t$$

Where λ is the rate of access ratio .

t: rate of time spent.

N: the number of customers in the system.

As the mathematician John D.C. Little proved in 1961 that there is a set of general relations linking performance indicators (measures) and the possibility of using these relations

(L_q , L_s , W_q , W_s) for all queue models are as follows:

$$L_s = L_q + \rho \qquad \ldots (1)$$

$$L_s = \lambda W_s$$

$$L_q = L_s - \rho$$

$$L_q = \lambda W_q \quad \text{... (2)}$$

$$W_s = W_q + \frac{1}{\mu}$$

$$W_s = \frac{L_s}{\lambda} \quad \text{... (3)}$$

$$W_q = W_s - \frac{1}{\mu}$$

$$W_q = \frac{L_q}{\lambda} \quad \text{... (4)}$$

$$\rho = \frac{\lambda}{\mu} \quad \text{... (5)}$$

$$\lambda = \frac{1}{E(\acute{X})} \quad \text{... (6)}$$

$$\mu = \frac{1}{E(X)} \quad \text{... (7)}$$

As E(X): represents the expectation of service times.

$E(\acute{X})$: represents the expected inter-arrival times. **(Gosavi, 2020: 2**

5-2 Queuing Model (G/G/1): (GD/∞/∞) :

This model is considered a special case of the (G/G/C) model in that it has the same assumptions except for owning one service channel , The mathematical equation for the expected number of customers in the queue (L_q) is defined by equation (8): **(Ali, 2010: 21), (Gosavi, 2020:4-5)**

$$L_q = \frac{\rho^2(1 + C_s{}^2)(C_a{}^2 + \rho^2 C_s{}^2)}{2(1-\rho)(1 + \rho^2 C_s{}^2)} \quad \text{... (8)}$$

The mathematical formulas $(C_s{}^2, C_a{}^2)$ are defined as follows:

$$C_a{}^2 = \frac{\sigma_a{}^2}{(1/\lambda)^2} \quad ; \quad C_s{}^2 = \frac{\sigma_s{}^2}{(1/\mu)^2}$$

Since $C_s{}^2$: is a coefficient of service time variation.

$C_a{}^2$ Arriving time variance parameter .

And by substituting $(C_s{}^2, C_a{}^2)$ into equation (92-2), we get:

$$L_q = \frac{\rho^2\left(1+\frac{\sigma_s{}^2}{(1/\mu)^2}\right)\left(\frac{\sigma_a{}^2}{(1/\lambda)^2}+\rho^2\frac{\sigma_s{}^2}{(1/\mu)^2}\right)}{2(1-\rho)\left(1+\rho^2\frac{\sigma_s{}^2}{(1/\mu)^2}\right)} \quad \dots(9)$$

In this system, it is required that $\frac{\rho}{c} < 1$and by substituting equation (9) into equations (1), (3), (4) we obtain the remaining performance indicators in the queuing system (G/G/1) as follows :

$$L_s = L_q + \rho$$

$$L_s = \frac{\rho^2\left(1+\frac{\sigma_s{}^2}{(1/\mu)^2}\right)\left(\frac{\sigma_a{}^2}{(1/\lambda)^2}+\rho^2\frac{\sigma_s{}^2}{(1/\mu)^2}\right)}{2(1-\rho)\left(1+\rho^2\frac{\sigma_s{}^2}{(1/\mu)^2}\right)} + \rho \quad \dots(10)$$

$$W_q = \frac{L_q}{\lambda}$$

$$W_q = \frac{\frac{\rho^2\left(1+\frac{\sigma_s{}^2}{(1/\mu)^2}\right)\left(\frac{\sigma_a{}^2}{(1/\lambda)^2}+\rho^2\frac{\sigma_s{}^2}{(1/\mu)^2}\right)}{2(1-\rho)\left(1+\rho^2\frac{\sigma_s{}^2}{(1/\mu)^2}\right)}}{\lambda} \quad \dots(11)$$

$$W_s = W_q + \frac{1}{\mu}$$

$$W_s = \frac{\dfrac{\rho^2\left(1+\frac{\sigma_s{}^2}{(^1/_\mu)^2}\right)\left(\frac{\sigma_a{}^2}{(^1/_\lambda)^2}+\rho^2\frac{\sigma_s{}^2}{(^1/_\mu)^2}\right)}{2(1-\rho)\left(1+\rho^2\frac{\sigma_s{}^2}{(^1/_\mu)^2}\right)}}{\lambda} + \frac{1}{\mu} \qquad \dots(12)$$

6-2 The probability distributions used in the research:

The intervals of arrival and service periods for customers are characterized by continuity and therefore will form a continuous random variable that has a continuous probability distribution. Among these distributions are:

1-6-2 Weibull distribution

The random variable X follows the Weibull probability distribution, and the probability density function is defined by equation (13). **(Sumaya and Mushaira, 2019: 8-9),(Mjely,2019:12)**

$$f(x) = \lambda\alpha x^{\alpha-1}e^{-\lambda x^{\alpha}} \;\; ; \;\; x \geq 0 \, ; \, \lambda, \alpha > 0 \qquad \dots(13)$$

$$\text{Mean :} \quad E(X) = \frac{\Gamma(1+\frac{1}{\alpha})}{\lambda^{\frac{1}{\alpha}}}$$

$$Variance : \quad V(X) = \frac{\Gamma\left(1+\frac{2}{\alpha}\right) - \left(\Gamma\Gamma(1+\frac{1}{\alpha})\right)^2}{\lambda^{\frac{2}{\alpha}}}$$

Since λ : Scale Parameter .

α : Shape Parameter .

2-6-2 (Pareto-Exponential) Distribution

The random variable X follows the Pareto-exponential probability distribution, and its probability density function is defined by equation (13) below .

$$g_{\text{nlte-pa}}(x_i, \theta, \alpha, k) = \frac{\frac{\alpha k^{\alpha}}{x^{\alpha+1}}}{e^{\theta(1-\left(\frac{k}{x}\right)^{\alpha})}}\left[1+\theta(\frac{k}{x})^{\alpha}\right]; x \geq k; \theta, k, \alpha > 0$$

$$\dots(14)$$

And by substituting r = 1 , r = 2 in equation (15) we get the first eccentric moment (which represents the expectation) and the second first eccentric moment.

$$\acute{\mu}_r = E(X^r) = \left[\frac{k^r}{\theta^{j+1}}\sum_{j=0}^{\infty}\binom{j+\frac{r}{\alpha}-1}{\frac{r}{\alpha}-1}\gamma(j+1,y) + \frac{k^r}{\theta^j}\sum_{j=0}^{\infty}\binom{j+\frac{r}{\alpha}}{\frac{r}{\alpha}}\gamma(j+1,y)\right] \quad \ldots(15)$$

After substituting the moments into the equation below, we get the variance of the (Pareto-exponential) distribution.

$$V(x) = E(X^2) - (E(X))^2 = \acute{\mu}_2 - \acute{\mu}_1{}^2 \quad \ldots(16)$$

Since:

θ: represents the parameter of the exponential family.

k, α : represent the parameters of the basis (Pareto) distribution.

<u>3- The applied aspect (performance indicators for the Radiology Division in the Teaching Hospital (Al-Jamhouri)</u>

<u>3-1 Data analysis</u>

The method of Maximum Likelihood Estimation (MLE) was adopted in estimating the parameters of the distributions (NLTE-Pa, Weibull) shown in Table (5) and applied to the real data for the service periods and inter-arrival of the Radiology Division in Teaching Hospital / Basra, which are summarized in Tables (3), (4).

Table (3) represents the service periods (measured in minutes) for the Radiology Department at the Teaching Hospital (Al-Jumhuri) / Basra.

1	1	1	2	1	1	1	4	2	2
1	3	2	1	1	1	1	1	1	1
1	1	1	1	3	1	1	2	1	3
1	2	3	1	1	2	1	1	1	3
2	2	1	1	2	1	2	2	3	1
1	1	1	2	1	3	1	1	2	1
2	2	1	1	2	1	1	1	1	3
2	1	1	3	1	1	2	1	1	1
1	1	1	1	1	1	2	1	1	1
1	2	2	1	1	1	1	1	1	1
2	2	1	1	2	3	1	1	1	1
2	1	2	1	2	1	1	1	1	2
1	1	2	2	1	1	2	1		

Source: Prepared by the researcher

Table (4) Data of the study sample for the inter-arrive periods.

2	**5**	**4**	**3**	**2**	**4**	**3**	**5**	**5**	**10**
4	3	3	3	3	6	8	2	6	3
4	4	4	7	3	6	2	2	1	7
1	4	1	2	3	2	5	9	1	4
15	3	1	5	2	7	1	5	7	4
1	8	2	1	5	8	1	6	2	1
11	9	2	6						

Source: Prepared by the researcher

As for Table (5) below, it shows the results of the appropriate distributions for the intervals of arrive and service.

Table (5) represents the inter-arrival and service data.

periods	distributions	parameters	expectation	variance
inter-arrival	Weibull	$\hat{\alpha} = 2.0101$ $\hat{\theta} = 0.0577$ $\hat{k} = 1.0002$	1.9528	**0.6725**
service	Pareto-Exponential	$\hat{\alpha} = 1.5833$ $\hat{\beta} = 4.7081$	4.2251	**7.4521**

Source: Prepared by the researcher

3-2 Estimating the performance indicators of the Radiology Division

After demonstrating the suitability of the service periods for the Pareto-exponential distribution and the interfacial arrival periods for the Weibull distribution, we are now working on estimating the performance indicators for the Radiology Division based on the waiting queue theory and the following agencies:

➢ **Determine the characteristics of the queue model**

After determining the appropriate probability distribution for the number of auditors and the probability distribution for their periods of service, the following characteristics can be established:

- The arrival distribution (a) follows the Weibull distribution (G) with an expectation of (4.2253) and a variance of (7.4521).
- The service distribution (b) follows the distribution of the Pareto-exponential (NLTE-Pa) , whose parameters were equal to ($\hat{\alpha} = 2.0101$, $\hat{\theta} = 0.0577$, $\hat{k} = 1.0002$), with an expectation of (1.9528) and a variance of (0.6725).
- The number of service centers (C=1), the service delivery pattern (FCFS), the queue size is unlimited (∞), and the community size is unlimited (∞).

Accordingly, we find that the appropriate queue model for the radiology department in the teaching hospital (Al-Jumhuri) is: (G/G/1): (GD/∞/∞) .

➢ Measurement of performance indicators for the Radiology Division

A program was prepared using (Mathematica) program to find performance indicators for the queuing system (G/G/1): (GD/∞/∞) based on equations (9), (10), (11), (12) and after feeding the program With the required data represented by the arrive rate and the service rate, we get the following:

Table (6): Performance indicators for the queuing system (G/G/1): (GD/∞/∞)

Pointers	Estimation
ρ	0.4622
L_q	0.1025
L_s	0.5647
W_q	0.4330
W_s	2.3857

Source: Prepared by the researcher

We note from Table (6) that the average waiting time in the waiting line for the patient amounted to 0.4330 minutes, and the waiting period for the patient was equal to (2.3857) minutes , and we find that this period is short. To wait, which leads to the conclusion that food production is good and Convenient , and does not require the addition of other devices .

4-Conclusion:

1- The G/G/1 model is considered more flexible than the M/M/1 model because it can be used in the case that the intervals between access and service have any of the probability distributions.

2- Through the results of the applied side:

a- The probability distribution of the inter-arrival periods are represented by the Weibull distribution, while the service periods are (NLTE-Pa) Pareto exponential distribution.

b- The performance indicators showed the quality of the performance of the Radiology Division of Basra Teaching Hospital (Al-Jumhuri), as the average waiting period for the patient until he received the service was approximately two minutes, which is a good result, and therefore there is no need to add another device to the Radiology Division.

5-Recommendations

Based on what has been reached, we summarize the recommendations as follows :

1- There must be an integrated database that contains all patients' information from the moment they return to the hospital until they are discharged, including the patient's arrival period, the period of beginning of obtaining the service, and the period of his departure.

2- Recommending the need to increase scientific studies in the health sector that would improve the quality of health services provided to patients.

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