THEORETICAL AND NUMERICAL ANALYSIS FOR BUCKLING OF ANTISYMMETRIC SIMPLY SUPPORTED LAMINATED PLATES UNDER UNIAXIAL LOADS

تحليل نظري وعددي للانبعاج في الصفائح غير المتناظرة الطبقية بسيطة الإسناد تحت الأحمال الأحادية المحور

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Abstract

The buckling behaviors of antisymmetrically angle ply laminated composite plate under uniaxial compression have been studied using eigenvalue buckling analysis. In this research, the effects of aspect ratio, fiber orientation, number of layers and material properties on the critical loads have been studied. Finite element model is carried out using ANSYS package version 5.4,the adopted element is 3-D with 8 nodes shell 99 composite one. A comparison study is made between the present analysis and the finite element method FEM and good agreement is obtained. It is found that in design of an antisymmetric laminate; the thickness is not only the element which must be determined, to satisfy the constraints of the problem, but in addition, the number of layers in the laminate and their relative fiber orientation must also be considered. The mathematical model has been implemented in a MAT LAP

Keywords. Critical buckling, Composite material, Antisymmetrically, Finite element analysis, Eigenvalue.

الخلاصة

تناول هذا البحث دراسة سلوك الانبعاج في صفيحة غير متناظرة تحت تأثير الانضغاط الأحادي المحور باستخدام تحليل eigenvalue للانبعاج وكذلك تمت دراسة تأثير الزيادة في طول الصفيحة,اتجاه الألياف ,عدد الطبقات وخواص المعدن على عملية الانبعاج الحرج.

حيث تضمن البحث دراسة الأنموذج الرياضي وتم تطبيقها باستخدام برنامج ANSYS 5.4 ثلاثي الأبعاد مع ثمانية عقد (8 node) و99 غلاف(shell 99) . تم إجراء دراسة مقارنة بين النتائج المستحصلة من الطريقتين وكان الاتفاق بينهما جيدا".

وتبين من البحث عند تصميم شريحة غير متناظرة فأن عامل سمك الصفيحة ليس الوحيد الذي يجب تحديد قيمته و أنما يجب أن نؤخذ بنظر الاعتبار عدد الطبقات لتلك الصفيحة واتجاه أليافها.

ومن الجديد بالذكر انه تم الاستعانة ببرنامج MATLAB لغرض تنفيذ المعادلات الرياضة للنموذج أعلاه.

1. Introduction

Thin-walled composite structure is used widely in civil, automotive, and aerospace engineering applications due to its light weight with high stiffness and strength as well as corrosion resistance. Buckling of a plate occurs when the in-plane compressive load gets large enough to cause a sudden lateral deflection of the plate. Initially a plate under compressive load undergoes only in-plane deformations, but as this compressive load gets large, the plate reaches its critical buckling load, the load at which a sudden lateral deflection of the plate takes place[1].

Austin [2] used ANSYS finite element software to determine the buckling load of symmetric laminated fiberglass reinforced plastic plates. It was found that the optimal fiber orientation of the mat layers was ± 45^o for simply supported edges only. Chainartin and Pairod, [3] studied the buckling behavior of rectangular and skew thin composite plates with different boundary

conditions. They used the Ritz method along with the proposed out-of-plane displacement functions. It was concluded that the proposed displacement functions can be used to solve the buckling problems panels with all combinations of boundary conditions. However, for the skew panels, the proposed functions can be used only with clamped-support.
Srinivas et al. [4] used the finite element to study the critical buckling factor of antisymmetric skew composite laminates. It was found that this factor increased with the skew angle and it is not appreciable in the case of laminate with large number of layers.
Liecheg Sun [5] used an analytical strip method for the analysis of stiffened and non-stiffened ant symmetric laminated composite plates. The outputs of this analysis are displacement, fundamental frequency and buckling load of cross-ply and angle-ply plates with different combination of boundary conditions. The results were compared with those published in the literature as well as ANSYS solution. Good agreement was obtained.
Michael [6] studied the effects of cutout on the buckling and post buckling behavior of rectangular plates made of advanced composite material. It was found that plates having cutout can buckle at loads higher than that of without cutout.
Analysis of plates buckling under in-plane loading involves solution of an eigenvalue problem as opposed to the boundary value problem of equilibrium analysis. The effect of skew angle and compression loading on the buckling load and mode are also investigated.
Seung et al. [7] derived the equations of motion from Hamilton's principle. They used Navier-solution to get closed form solutions of antisymmetric cross-ply and angle-ply laminates. The results were compared with three-dimensional elasticity theory. They concluded that the proposed theory is simple and accurate in solving both the static and buckling behaviors.
Karmonikova and Mamusic [8] used finite element method in program COSMOS/M in buckling analysis of rectangular laminated plate consisting of 4 layers distributed symmetrically and antisymmetrically.

This work deals with the analytical determination of the critical buckling load of various types of plates. The buckling load will be determined for plates with various laminations(2,4,6.8 plies) with different ply directions (α=0^o,40^o,70^o and 90^o). The results for the different lamination will be compared to ascertain the influences of twist coupling and bending extension coupling.

2. Buckling Equations for Laminated Plate

The differential equations governing the buckling behavior from a membrane prebaked state are [5,9]:

$$\left.\begin{aligned} \delta N_{x,x} + \delta N_{xy,y} &= 0 \\ \delta N_{xy,x} + \delta N_{y,y} &= 0 \\ \delta M_{x,xx} + 2\delta M_{xy,xy} + \delta M_{y,yy} + N_x \delta w_{,xx} + 2N_{xy}\,\delta w_{,xy} + N_y\,\delta w_{,yy} &= 0 \end{aligned}\right\} \quad (1)$$

Where terms δN_x----- δM_x are variations of forces and moments respectively from a membrane prebckling equilibrium state. The terms $\delta w, \delta u$ and δv are variations in displacement from the same flat perbuckled state .The buckling differential equations can be expressed in terms of the variations in displacement by substituting the variations in extensional strains and curvatures. The boundary conditions for buckling problems are applied only to the buckling deformations since the prebuckling deformations are assumed to be a membrane state [5].

3. Buckling of Simply Supported Laminated Plates

Let us consider the general case of laminated rectangular plate that is simply supported along edges $x = 0, x = a, y = 0, and\ y = b$ subjected to uniform in-plane force in the x-direction as shown in Fig.(1)

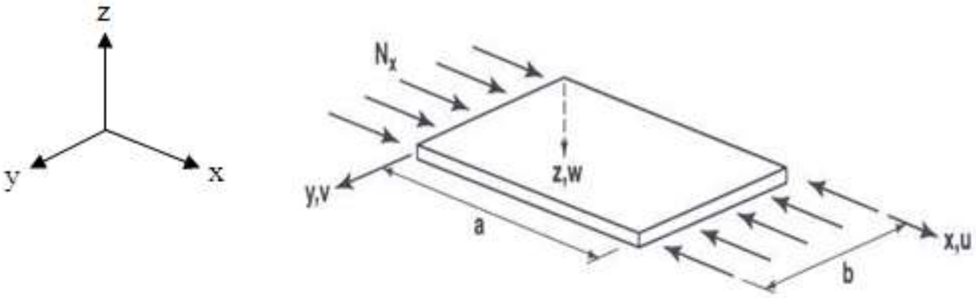


Fig. (1) Coordinates system and the applied force on a rectangular laminated plate.

4-Antisymmetric Angle Ply Laminates

Antisymmetric laminates are laminates that have pairs of layers of opposite orientations with the same material properties and thickness symmetrically loaded with respect to the middle surface, as shown in Fig.(2).

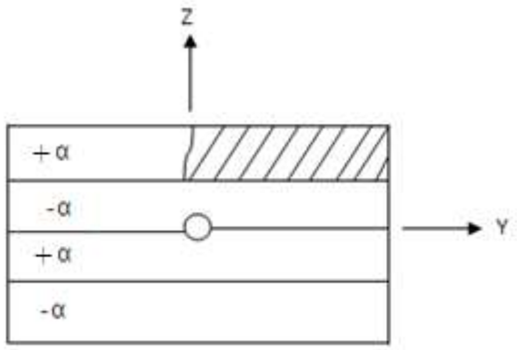


Fig (2) Antisymmetric angle-ply laminates

Antisymmetric laminates have $D_{16} = D_{26} = A_{16} = A_{26} = 0$ in the [D] and [A] matrices, therefore there is no bend-twist or shear-extension coupling, but there is bending- extension coupling. Antisymmetric angle-ply laminates, shown in Fig.(2) have extensional stiffness's A_{11}, A_{12}, A_{22}, and A_{66},coupling stiffness's B_{16}, B_{26} , and bending stiffness's D_{11}, D_{12}, D_{22}, and D_{66}, are given in appendix A. The coupled buckling differential equations are described by [5, 7, 8]:

$$\left.\begin{array}{l} A_{11}\delta u_{,xx} + A_{66}\delta u_{,yy} + (A_{12} + A_{66})\delta_{v,xy} - 3B_{16}\delta_{w,xxy} - B_{26}\delta_{w,yyy} = 0 \\ (A_{12} + A_{66})\delta u_{,xy} + A_{66}\delta_{v,xx} + A_{22}\delta_{v,yy} - B_{16}\delta_{w,xxx} - 3B_{26}\delta_{w,xyy} = 0 \\ D_{11}\delta w_{xx,xx} + 2(D_{12} + 2D_{66})\delta_{w,xxyy} + D_{22}\delta_{w,yyyy} \quad + B_{16}(3\delta_{u,xxy} + \\ \quad +\delta_{v,xxx}) + B_{26}\left(\delta_{u,yyy} + 3\delta_{v,xyy}\right) + N_x\delta_{,xx} = 0 \end{array}\right\} \quad (2)$$

To solve the problem for simply supported edges [7]:

$$\begin{array}{l} x = 0, a \\ \left.\begin{array}{ll} \delta w = 0, & \delta M_x = B_{16}\left(\delta v,_x + \delta u,_y\right) - D_{11}\delta w,_{xx} - D_{12}\delta w,_{yy} = 0 \\ \delta u = 0, & \delta N_{xy} = A_{66}\left(\delta v,_x + \delta u,_y\right) - B_{16}\delta w,_{xx} - B_{26}\delta w,_{yy} = 0 \end{array}\right\} \quad (3) \end{array}$$

$y = 0, b$

$$\left.\begin{aligned} \delta w = 0, \qquad & \delta M_y = B_{26}\left(\delta v,_x + \delta u,_y\right) - D_{12}\delta w,_{xx} - D_{22}\delta w,_{yy} = 0 \\ \delta v = 0, \qquad & \delta N_{xy} = A_{66}\left(\delta v,_x + \delta u,_y\right) - B_{16}\delta w,_{xx} - B_{26}\delta w,_{yy} = 0 \end{aligned}\right\} \quad (4)$$

It has been verified that the variations in deflections given by[8];

$$\left.\begin{aligned} \delta u &= u_0 \sin\frac{m\pi x}{a}\ \cos\frac{n\pi y}{b} \\ \delta v &= v_0 \cos\frac{m\pi x}{a}\ \sin\frac{n\pi y}{b} \\ \delta w &= w_0 \sin\frac{m\pi x}{a}\ \sin\frac{n\pi y}{b} \end{aligned}\right\} \quad (5)$$

Satisfy the boundary conditions and satisfy the governing differential equations exactly if [5,9]

$$N_x = (\frac{a}{m\pi})^2 \left[L_{33} + \frac{2L_{12}L_{23}L_{13} - L_{22}L_{13}^2 - L_{11}L_{23}^2}{L_{11}L_{22} - L_{12}^2}\right] \quad (6)$$

Where;

$L_{11} = A_{11}(\frac{m\pi}{a})^2 + A_{66}(\frac{n\pi}{b})^2$

$L_{12} = (A_{12} + A_{66})(\frac{m\pi}{a})(\frac{n\pi}{b})$

$L_{13} = -[3B_{16}(\frac{m\pi}{a})^2 + B_{26}\left(\frac{n\pi}{b})^2\right)](\frac{n\pi}{b})$

$L_{22} = A_{22}(\frac{n\pi}{b})^2 + A_{66}(\frac{m\pi}{b})^2$

$L_{23} = -[B_{16}(\frac{m\pi}{a})^2 + 3B_{26}(\frac{n\pi}{b})^2]\ (\frac{m\pi}{a})$

$L_{33} = D_{11}(\frac{m\pi}{a})^4 + 2(D_{12}+2D_{66})(\frac{m\pi}{a})^2(\frac{n\pi}{b})^2 + D_{22}(\frac{n\pi}{b})^4$

Since the buckling load is like the natural vibration frequency [8], the theoretical analysis presented in this research is called eigenvalue buckling analysis. A program was written in MAT LAP to solve the resulted equations.

5-Finite Element Method (FEM)

The finite element method (FEM) is a numerical procedure for solving physical problems governed by differential equation or an energy formulation, [10]. It has two characteristics that distinguish it from other domain numerical procedures, namely

1. The method is based upon an integral formulation to generate a system of algebraic equations.
2. The method uses continuous piecewise smooth functions for approximating the unknown quantities. It can be considered as the principal idea of FEM. That is the actual continuum or body, such as solid, liquid, or gas, is divided into a number of small sub-domains (finite elements) within which the solution can be interpolated by a continuous function. Then each element is discretized in a pointwise manner into a number of nodal points to simplify the satisfaction of the boundary conditions. Therefore the interpolated solution within each element can be expressed in terms of the nodal values and shape functions.

The assembly of the equations of all elements of the domain will yield a system of algebraic equations. The assembly procedure is based on the requirement of compatibility and equilibrium conditions at the element nodes. The resulting system of equations can be solved, after applying the boundary conditions, to give the values of the field variables at the nodal points and, hence, the solution within each element.

The finite element analysis is carried out using ANSYS 5.4 package. For the present study, finite element modeling of bulking of composite panels is analyzed. A 3-D model with 8-node shell element, shell 99 composite element, Fig. (3), is used for modeling the thin plate. This element can handle membrane and transverse shear effect [11]. It has six degrees of freedom at each node, translation in the x, y and z directions and rotation about x, y and z axes.

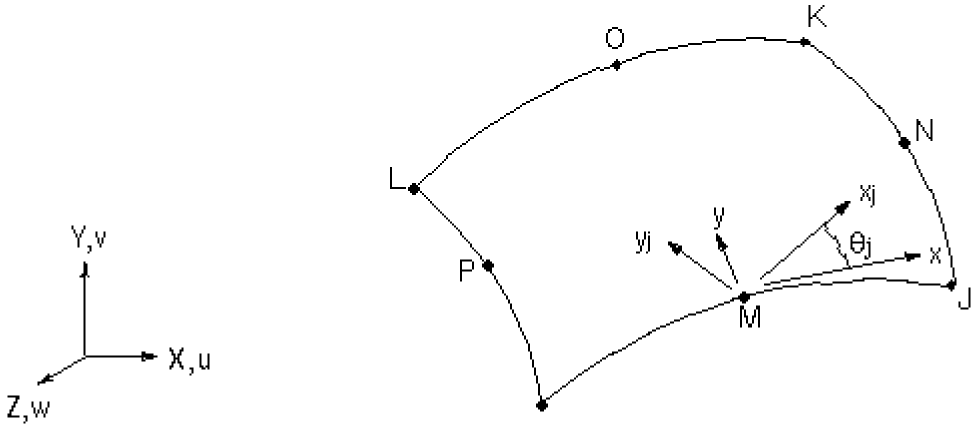


Fig. (3) Shell 99 structural elements [11]

6-Results and Discussion

The following mechanical properties and geometry of composite material were used in this research, to be analyzed:-

$E_1 = 100 * 10^9 \frac{\mathrm{N}}{m^2}, E_2 = 10 * 10^9 \frac{\mathrm{N}}{m^2}, V_{12} = 0.1,\ G_{12} = 5 * 10^9 \frac{\mathrm{N}}{m^2},\ t = 4mm,$
a=0.15 m, b=0.1 m, with different layer numbers [12].

Table 1. *Results of critical buckling load for laminated plate*

ply angle (Deg.)	*No. 0f layers*	*Critical buckling load* $*(10^6)$ *(N/m)*		*Percent difference %*
		Present study	*ANSYS*	
0°	2	1.104700	1.10100	0.3361
	4	0.954007	0.94020	1.4686
	6	0.911400	0.90340	0.8855
	8	0.901700	0.88300	2.1178
40°	2	3.57000	3.50100	1.9709
	4	1.75300	1.72120	1.8476
	6	1.41680	1.390010	1.9273
	8	1.29800	1.27100	2.1243
70°	2	3.83450	3.79540	1.0302
	4	1.81600	1.77060	2.5641
	6	1.40420	1.39870	4.7509
	8	1.31340	1.28709	2.0441
90°	2	1.27100	1.26201	0.7124
	4	0.92820	0.89310	3.9301
	6	0.91960	0.90010	2.1664
	8	0.90100	0.90101	-0.0011

The critical buckling load obtained by the analytical derived equation and finite element program is shown in table 1. It shows that the values predicted by the present research are higher than those calculated using FEM. However the maximum error is not exceeded 5% which is reasonable.

The relation between the coupling stiffness $\frac{B_{16}}{tA_{11}}$ and the ply's angle is illustrates in Fig. (4). It is obvious that the maximum value of B_{16} is when the number of layers equals two. This is due to the fact that B_{16} is inversely proportional to the number of layers. Also it can be seen that the coupling between bending and extension has its maximum value when the ply's angle equals 45 for any value of number layers. This value of the ply's angle will become 65^o in the case of B_{26} as shown in Fig. (5) for any value of number of layers.

The relation between the bending load and the modulus ratio is illustrated in Fig.(6).The buckling load is normalized by the buckling load of an orthotropic plate (i.e.$B_{ij} = 0$). It is clear that the variation of the buckling load will be very small beyond E_1/E_2=40. For rectangular plates, values of $G_{12/}E_2$and v_{12} are set fixed because their influence on the buckling load is small as compared with that of modulus ratio $E_{1/}E_2$.

The buckling load of composite plates with different fiber orientations was illustrated in Fig.(7). The graph also shows that there is remarkable difference in buckling load between double layered and multi layered for a fixed laminate thickness. This is due to the influence of bending – extension coupling, for a double layered plates, the effect of coupling is considerably high, however, for multilayered plates this effect is insignificant. The figure also shows that the buckling load increases with the increase in fiber orientation angle and the maximum buckling load occurs at α=45^o.

The critical buckling load are evaluated at different values of aspect ratio a/b , as shown in Fig.(8).The critical buckling load decreases continuously with increasing aspect ratio but the rate of decrease is not uniform. It is observed that the buckling load was minimum for aspect ratio (a/b)=1.8 and increases beyond this value. However, the rate of this decreases as the number of layers increases.

The buckling load decrease with the increase in the thickness ratio until to reach approximately constant value as shown in Fig.(9). It is also concluded that the buckling load decreases with the increase of the number of layers of the plate due to coupling effect.

The result of buckles mode shape for antisymmetric laminated plate, for different layers orientation is shown in Fig.(10), the result are obtained from ANSYS package at different angles of ply and number of layers.

7- Conclusion

From the foregoing investigations on the buckling load of antisymmetric laminated plate under uniaxial loads, the following conclusions can be withdrawn from the present research:-

1-The present analysis is very efficient tool to determine the buckling load. The values of the loads determined by the present analysis were compared with those obtained by the finite element method and good agreement was obtained. The obtained error percentage is not exceeded than 5%.

2-The largest values of the buckling load occur in the range of angle of ply direction of the laminate between 30^o-60^o for each size. For these laminates the fibers in the outermost plies are closely aligned with the compression diagonal of the plate.

3 - The maximum value occurs at α=45^o for each size. In this case, the fibers in the outermost plies are exactly aligned with the compression diagonal of the plate.

4- For antisymmetric laminate, the effect of coupling between bending and extension dies out rapidly as the number of layers increases, for fewer than six layers, the effect cannot be ignored.

5-The buckling load depends essentially on the modulus ratio $\frac{E_1}{E_2}$. As the modulus ratio decreases beyond the value of 40, the influence of coupling between bending and extension decreases slowly.

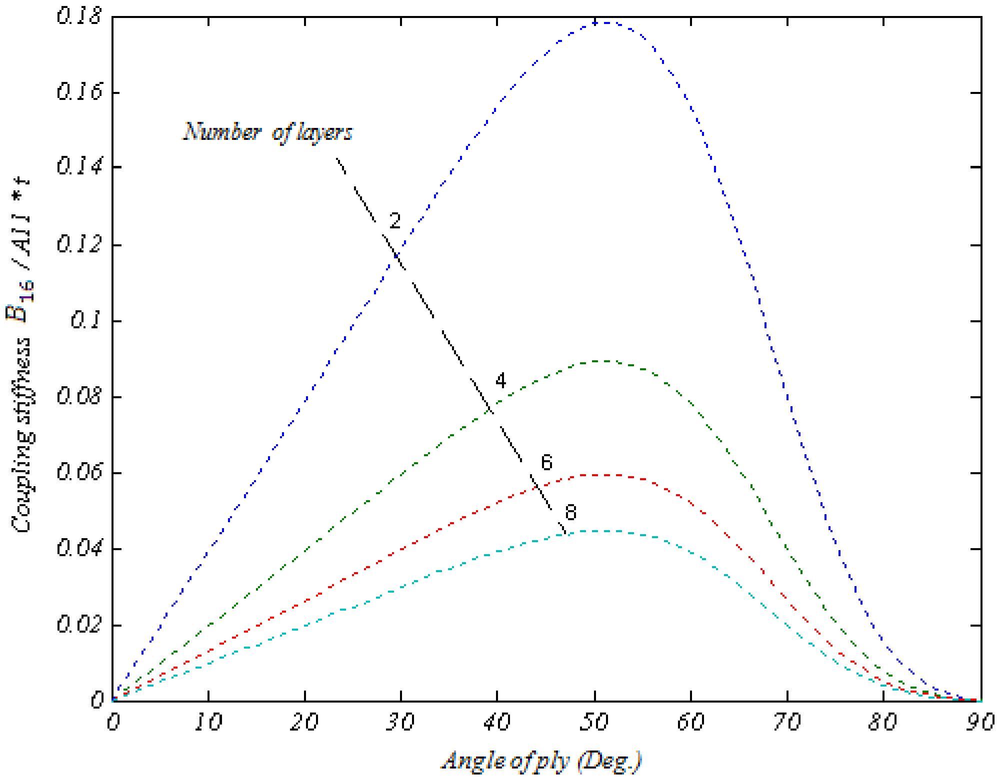


Fig.(4) Coupling stiffness for B_{16} for antisymmetric angle- ply laminate

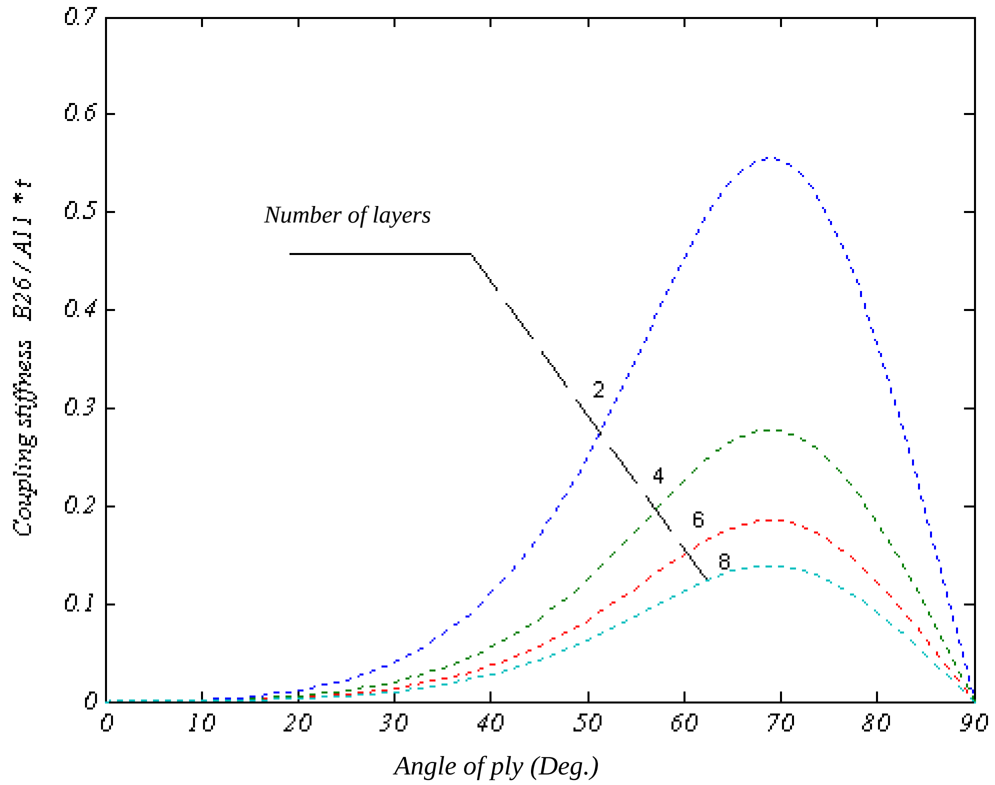


Fig.(5) Coupling stiffness for B_{26} for antisymmetric angle-ply laminate

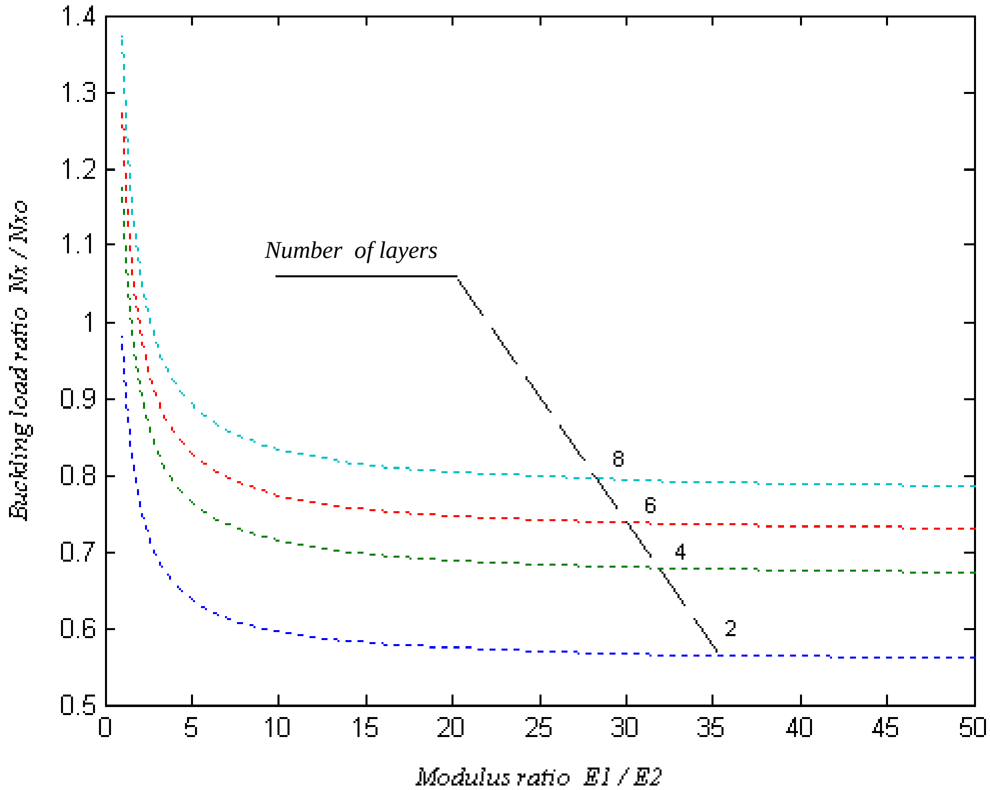


Fig.(6) Effect of modulus ratio on the critical buckling loads ratio at $\alpha = 45^{\circ}$

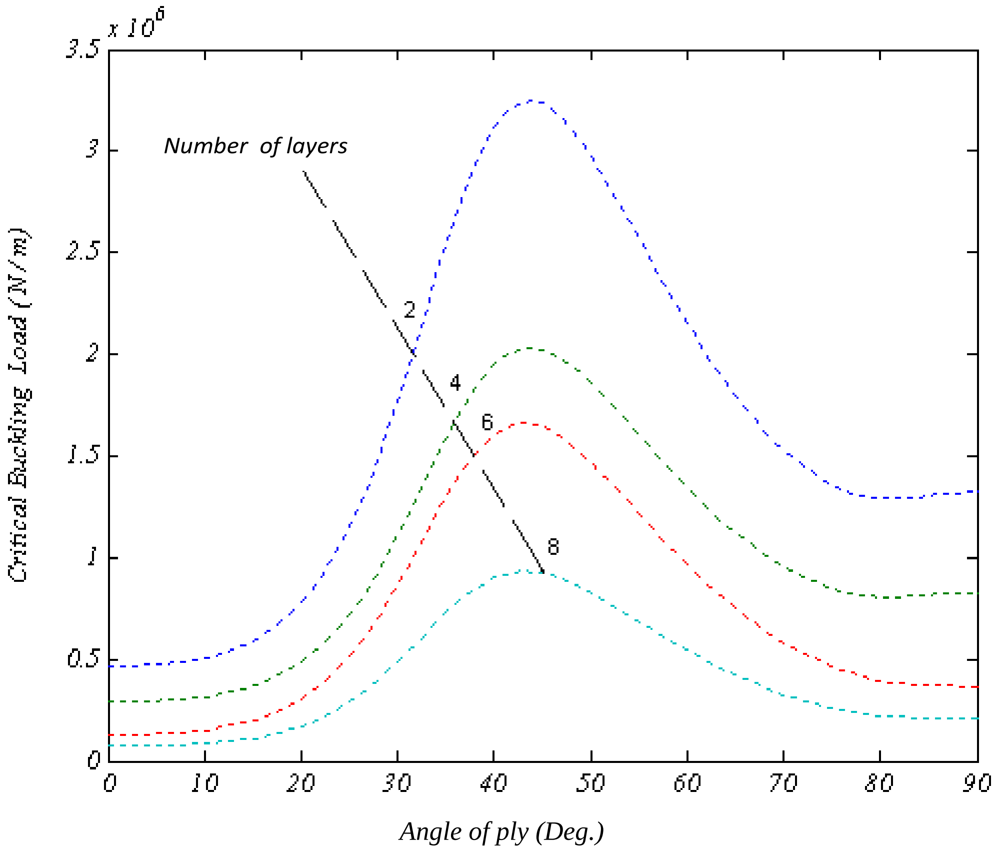


Fig.(7) Buckling loads for antisymmetric plate at different angle of ply laminated plate

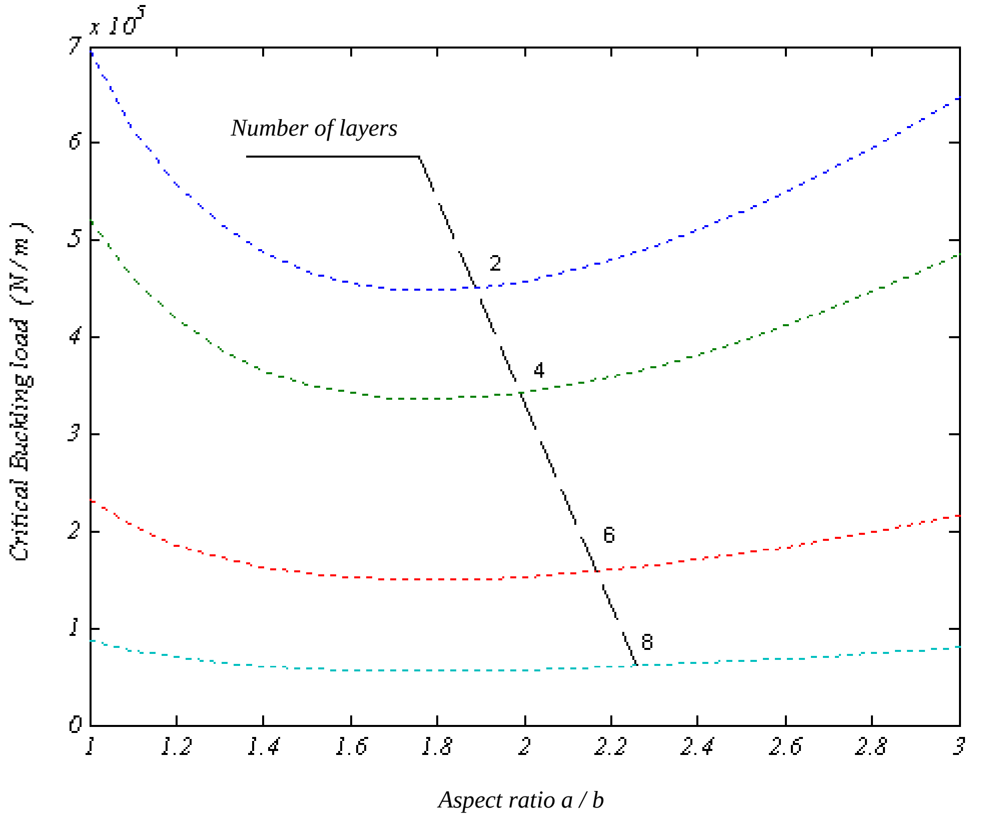


Fig.(8) Effect of aspect ratio on the critical buckling loads at $\alpha = 45^\circ$

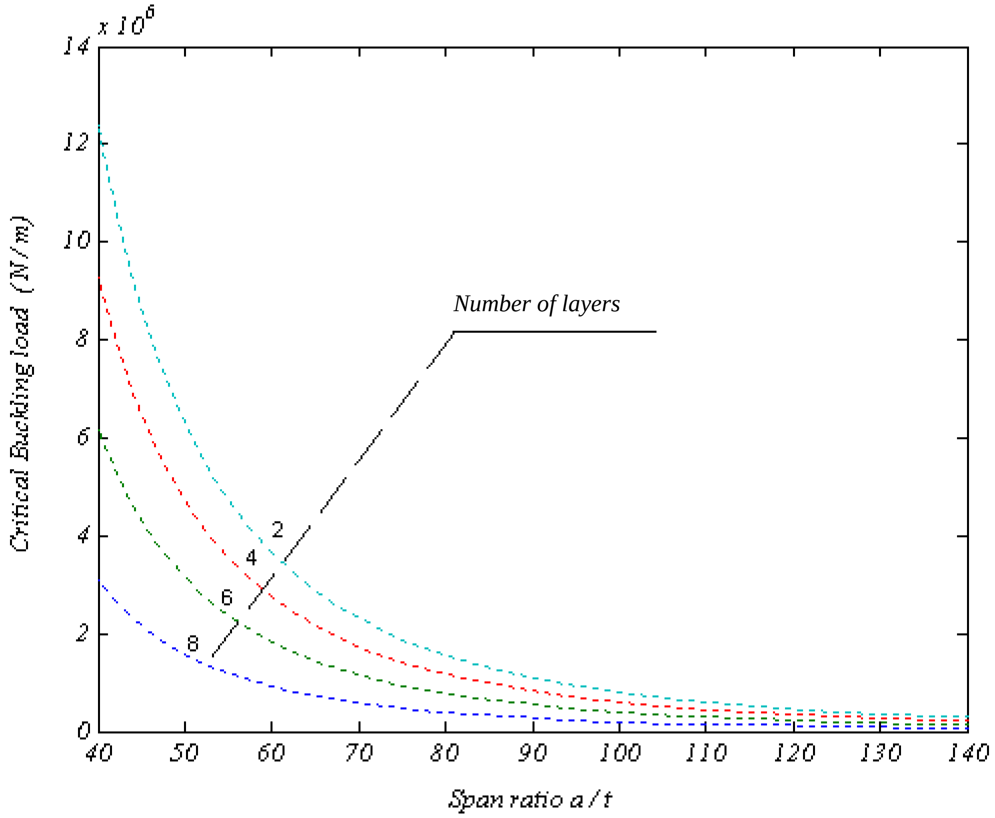
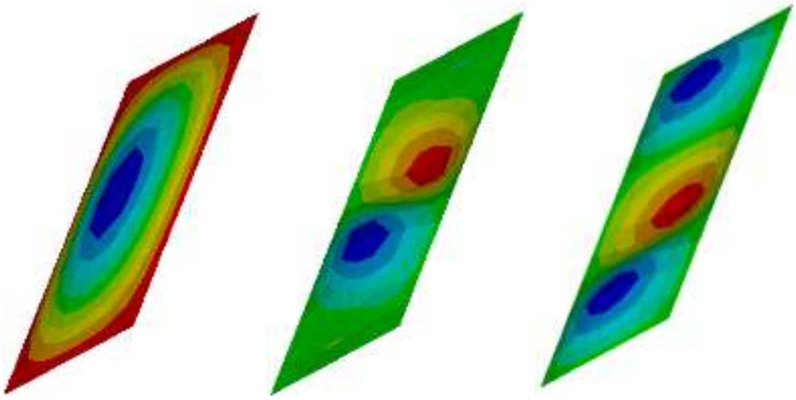
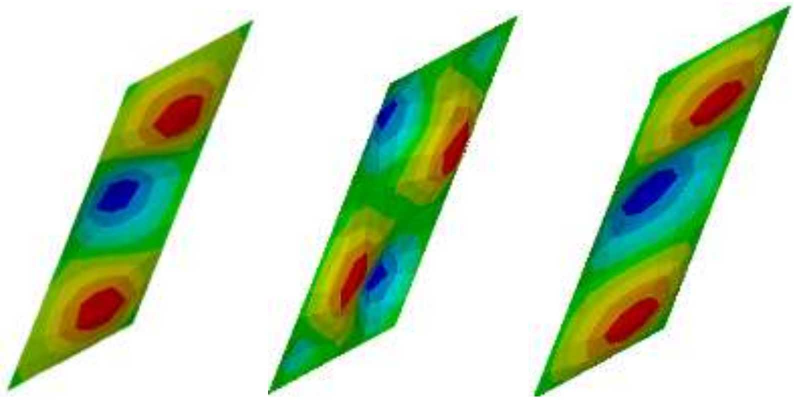


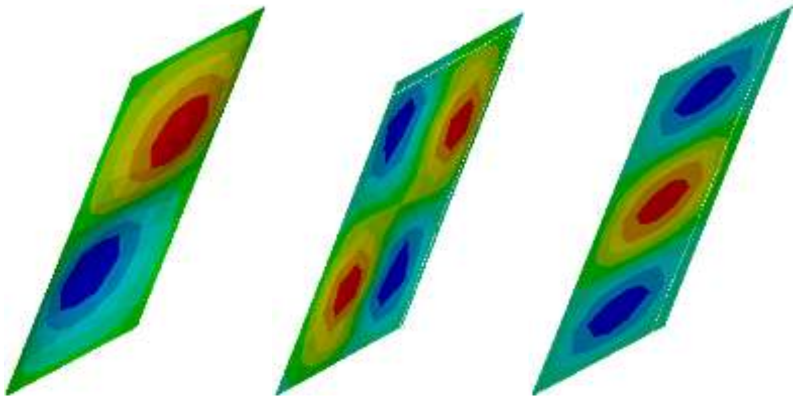
Fig.(9) Effect of length to thickness ratio of the pale on the critical buckling loads



$2-layer\ at\ \alpha = 0^0$ m=1, n=1 | $2-layer\ at\ \alpha = 30^0$ m=2, n=1 | $4-layer\ at\ \alpha = 90^0$ m=3, n=1



$4-layer\ at\ \alpha = 30^0$ m=3, n=1 | $4-layer\ at\ \alpha = 60^0$ m=2, n=2 | $6-layer\ at\ \alpha = 0^0$ m=3, n=1



$6-layer\ at\ \alpha = 30^0$ m=2, n=1 | $8-layer\ at\ \alpha = 30^0$ m=2, n=2 | $8-layer\ at\ \alpha = 60^0$ m=3, n=1

Fig.(10) Buckling modes shapes for antisymmetric laminates at different angle of ply and number of layer

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List of symbols

A_{ij} Extensional stiffness coefficients $[N/m]$
a, b Plate dimensions in x-direction and y-direction respectively[m]
B_{ij} Bending- extension coupling stiffness coefficients [N]
D_{ij} Bending stiffness coefficients $[N.m]$
E_1 Young's modulus in directions-1 [N/m^2]
E_2 Young's modulus in directions-2 [N/m^2]
G_{12} Shear modulus in the 1-2 plane [N/m^2]
N_x Critical buckling load [N/m]
$N_{x,0}$ Buckling load at (Bij=0) [N/m]
$N_{x,x}$ First order differential of buckling load [N/m]
$N_{x,xx}$ Second order differential of buckling load [N/m]
n,m Number of half waves in transverse and longitudinal directions respectively
t Thickness of ply [m]
u, v, w Displacements of the plate along x, y and z direction, respectively [m]
$u_{0,}v_{0,}w_0$ Variations in deformation edge in the direction of loading [m]
υ_{12} Poison's ratio [dimensionless]
α Ply direction [deg.]

Appendix A

Where [Q] is the reduced stiffness matrix. The components of the reduced stiffness matrix are defined of the in-plane mechanical properties of the laminate and are[2,3,7];

$$Q_{11} = \frac{E_1}{1-\nu_{12}\nu_{21}}$$

$$Q_{12} = \frac{\nu_{12}E_2}{1-\nu_{12}\nu_{21}} = \frac{\nu_{21}E_1}{1-\nu_{12}\nu_{21}}$$

$$Q_{22} = \frac{E_2}{1-\nu_{12}\nu_{21}}$$

$$Q_{22} = G_{12}$$

Where $[\bar{Q}]$ the transformed reduced stiffness matrix, and the components of the transformed reduced stiffness matrix are;

$$\bar{Q}_{11} = Q_{11}cos^4\alpha + 2(Q_{12} + 2Q_{66})sin^2\alpha\ cos^2\alpha + Q_{22}sin^4\alpha$$

$$\bar{Q}_{12} = (Q_{11} + Q_{22} - 4Q_{66})sin^2\alpha\ cos^2\alpha + Q_{12}(sin^4\alpha + sin^4\alpha)$$

$$\bar{Q}_{22} = Q_{11}sin^4\alpha + 2(Q_{12} + 2Q_{66})sin^2\alpha\ cos^2\alpha + Q_{22}cos^4\alpha$$

$$\bar{Q}_{16} = (Q_{11} - Q_{12} - 2Q_{66})sin\alpha\ cos^3\alpha + (Q_{12} - Q_{22} + 2Q_{66})sin^3\alpha cos\alpha$$

$$\bar{Q}_{26} = (Q_{11} - Q_{12} - 2Q_{66)}sin^3\alpha\ cos\alpha + (Q_{12} - Q_{22} + 2Q_{66})sin\alpha\ cos^3\alpha$$

$$\bar{Q}_{66} = (Q_{11} + Q_{22} - 2Q_{66} - 2Q_{66})sin^2\alpha\ cos^2\alpha + Q_{66}(sin^4\alpha + sin^4\alpha)$$

Extensional stiffness coefficients $A_{ij} = \sum_{k=1}^{N}(\bar{Q}_{ij})_k t_k$

Coupling stiffness coefficients $B_{ij} = \sum_{k=1}^{N}(\bar{Q}_{ij})_k t_k \bar{z}_k$

Bending stiffness coefficients $D_{ij} = \sum_{k=1}^{N}(\bar{Q}_{ij})_k (t_k \bar{z}_k^2 + \frac{t_k^3}{12})$

Where,

$\bar{z}_k$ is the distance between midplane of plate and the K^{th} of laminate surface.