On Lyapunov- Schmidt Reduction to Reduce Differential Algebraic Equation

Eman A. Mansour

Southern Technical University, Al- Nassiriya Technical Institute, Thi-Qar, Iraq

Corresponding author iman.an73@yahoo.com

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Abstract: In this paper , we transformation singularity differential – Algebraic equation to an ordinary differential equations by use Lyapunov– Schmidt reduction and constructing Lyapunov functions depending on Reiss and Geiss method.

Keywords: Reiss and Geiss method, singularity, Lyapunov– Schmidt reduction.

1. Introduction

The word singularity is used pervasively in mathmatics. Even within a given research field, this term often has different meanings. Differential –algebriac equation theory is not an exception; the expression singular system has been used to mean at least differential algebriac system, or higher index differential algebriac system. Roughly speakings singular differential–algebriac equations or more precisely, singular points of differential– algebriac equations will be locally defined as those for which the assumptions supporting an index notion conditions in [1, 2].

The importance of studying singularity comes since if the system of equations is regular at all its points then the system will be just an explicit an ordinary differential equations system [3] (an index zero differential– algebriac equation), hence there should be some singular points, such problems arise naturally in a variety of applications, e.g., electrical networks, constraint mechanical systems of rigid bodies, chemical reaction kinetics.

2. Lyapunov– Schmidt Reduction [4]

We discuss the case when the semi– explicit differential algebraic equation formula is singular.

Let

$$X^{\circ} = f(x,y) \qquad (1)$$
$$0 = g(x,y)$$

where $f, g: R^n \times R^m \to R^n \times R^m$ has a critical point at (0, 0) and the condition.

$$\text{rank } D_y\, g(x,y) = m-1 \qquad (2)$$

holds. Let $D_y\, g(x,y) = B$ then $rank\ B = m-1$

Choose complements vectors spaces (H) and (N) to Kernal B and range B respectively. Then:

$$\text{Ker B} \oplus \text{H} = R^m \qquad (3)$$
$$\text{N} \oplus \text{range B} = R^m \qquad (4)$$

From (3) we conclude that dim H=m–1, and from (4) that dim N=1. Define aprojection $E: R^m \to range\ B$ and the complementary projection

$I - E : R^m \to N$

Such that expanded to an equivalent paris of equation:

$$X^{\circ} = f(x,y)$$
$$E\, g(x,y) = 0 \qquad (5)$$

and

$$X^{\circ} = f(x,y)$$
$$(I-E)\, g(x,y) = 0 \qquad (6)$$

Because of this splitting any vector $y \in R^m$ can be decomposed in the form y=v$\oplus$w. where $v \in Ker\ B$ and $\in H$. Then (5) can be written as:

$X^{\circ} = f(x, v ⊕ w)$

$E g(x, v ⊕ w) = 0$ (7)

The second part of the equation (7) can be considered as a map:

$\emptyset : R^n \times Ker\ B \times H \times R^v \to range\ B$

where

$\emptyset(x, v, w) = E g(x, v ⊕ w)$

Since E acts as the identity map on range B then we have:

$$\left(\frac{\partial E g(x, v ⊕ w)}{\partial w}\right)_{(0,0)} = EB = B$$

and since

$B : H \to range\ B$

Has a full rank at (0, 0), it follows from the implicit function theorem [5] that the second equation of (5) can be solved uniquely for w near (0, 0) i. e.,

w =W (x, y) where:

$W: R^n \times Ker\ B \times R^v$ satisfies

$E g(x, v ⊕ w(x, v, \lambda) = 0), w(0, 0, 0) = 0$
(8)

From the second equation of (7) and from differential– algebraic equation (1) we get the reduce differential– algebraic equations.

$X^{\circ} = F(x, y)$ (9)

$0 = G(x, v)$

Where $F, G : R^n \times Ker\ B \to R^n \times N$ are defined by

$G(x, v) = (I - E)\, g\left(x, v + W(x, v)\right)$
(10)

$F(x, v) = f\left(x, v + W(x, v)\right)$

3. Construction of Lyapunove Functions for Reduce Differential– Algebraic Equation

In this section, we will construction Lyapunov functions by use Reiss and Geiss method on reduce differential– algebraic equation.

3.1. Reiss and Geiss method [6]:

Reiss and Geiss in 1963 proposed a method which is applicable to the system

$X^{\circ} = F(x)$

This method is a simple application of integration by parts to the problem of constructing a V function.

Since V°need only to be semidefinite, the simplist form that one can choose is the square of a state variable $V^o = -x_n^2$. If this choice does not prove to be suitable then one may choose $V_1^{\circ} = -X_{n-1}^2$ and V° as follows:

$\dot{V} = \dot{V}_{\circ} + \propto \dot{V}_1 = -(X_n^2 + \propto X_{n-1}^2)$,

where (α) is arbitrary constant.

4. A New Method for Construction of Lyapunov Function

Now, we introduce the new method for construction of Lyapunov function for non-singularity differential algebraic equations:

Consider non singularity differential algebraic equation, non-singularity differential algebraic equation Now consider below equation:

$X_1^{\circ} = -x_1 + x_2 - x_2^3$

$X_2^{\circ} = x_1 - 4x_2$

Assuming that $\dot{V}_{\circ} = -x_2^2$, which is equivalent to $(-\dot{V}_{\circ} = x_2^2)$ and by integration an using the first equation, we get:

$$-V_{\circ} = \int_0^t x_2^2\, dt$$

$$= \int_0^t x_2(\dot{x_1} + x_1 + x_2^3)dt$$

$$= \int_0^t x_2 \dot{x_1} dt + \int_0^t x_1 x_2\, dt + \int_0^t x_2^4\, dt$$

Hence, integrating the first integration by parts, getting.

$$-V_{\circ} = x_1 x_2 - \int_0^t x_1^2\, dt + 4\int_0^t x_1 x_2\, dt + \int_0^t x_1 x_2\, dt + \int_0^t x_2^4\, dt$$

$$= x_1 x_2 - \int_0^t x_1^2\, dt + 4\int_0^t x_1(x_1^{\circ} + x_1 + x_2^3)\, dt + \int_0^t (x_2^{\circ} + 4x_2)x_2\, dt + \int_0^t x_2^4\, dt$$

$$= x_1 x_2 - \int_0^t x_1^2\, dt + 4\int_0^t x_1 \overset{\circ}{x}_1\, dt + 4\int_0^t x_1^2\, dt + 4\int_0^t x_1 x_2^3\, dt + \int_0^t (x_2 \overset{\circ}{x}_2\, dt) + 4\int_0^t x_2^2\, dt + \int_0^t x_2^4\, dt$$

$$= x_1 x_2 - \int_0^t x_1^2\, dt + 2x_1^2 + 4\int_0^t x_1^2\, dt + 4\int_0^t (\overset{\circ}{x}_2 + 4x_2)\, x_2^3\, dt + \frac{x_2^2}{2} + 4\int_0^t x_2^2\, dt + \int_0^t x_2^4\, dt$$

$$= x_1 x_2 - \int_0^t x_1^2\, dt + 2x_1^2 + 4\int_0^t x_1^2\, dt + x_2^4 + 16\int_0^t x_2^4\, dt + \frac{x_2^2}{2} + 4\int_0^t x_2^2\, dt + \int_0^t x_2^4\, dt$$

$$= 2x_1^2 + x_1 x_2 + \frac{x_2^2}{2} + x_2^4 - \int_0^t x_1^2\, dt + 4\int_0^t x_1^2\, dt + 16\int_0^t x_2^4\, dt + 4\int_0^t x_2^2\, dt + \int_0^t x_2^4\, dt$$

Hence:

$$-V_\circ = 2x_1^2 + x_1 x_2 + \frac{x_2^2}{2} + x_2^4 + 3\int_0^t x_1^2\, dt + 17\int_0^t x_2^4\, dt + 4\int_0^t x_2^4\, dt$$

Let:

$$V = -V_\circ - \left\{ 3\int_0^t x_1^2\, dt + 17\int_0^t x_2^4\, dt + 4\int_0^t x_2^4\, dt \right\}$$

$$= 2x_1^2 + x_1 x_2 + \frac{x_2^2}{2} + x_2^4$$

and:

$$\dot{V} = -\dot{V}_\circ\ 3x_1^2 - 17x_2^4 - 4x_2^2$$
$$= x_2^2 - 3x_1^2 - 17x_2^4 - 4x_2^2$$

Then:

$$\dot{V} = -3x_1^2 - 3x_2^2 - 17x_2^4 < 0 \text{ for } (x_1, x_2) \neq 0$$

Therefore, from V and $\dot{V}$ above it follows that the zero solution is asymptotically stable.

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