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On Artin Cokernel of The Group $D_n \times C_7$ When n is an Odd Number

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Abstract: The group of all Z-valued generalized characters of G over the group of induced unit characters from all cyclic subgroups of G, AC(G)= $\bar{R}$ (G)/T(G) forms a finite abelian group, called Artin Cokernel of G .The problem of finding the cyclic decomposition of Artin cokernel $AC(D_n \times C_7)$ has been considered in this paper when n is an odd number , we find that if n = $p_1^{\alpha_1} . p_1^{\alpha_2} .. p_m^{\alpha_m}$, where $p_1, p_2, \ldots, p_m$ are distinct primes and not equal to 2 , then :

$$AC(D_n \times C_7) = \bigoplus_{i=1}^{2((\alpha_1+1).(\alpha_2+1)\ldots(\alpha_m+1))-1} C_2$$

$$= \bigoplus_{i=1}^{2} AC(D_n) \oplus C_2$$

And we give the general form of Artin's characters table $Ar(D_n \times C_7)$ when n is an odd number.

Keyword: Group, cyclic group, Artin cokernel.

Introduction

For a finite group G the finite abelain factor group $\bar{R}$ (G)/T(G) is called Artin cokernel of G and denoted by AC(G) where $\bar{R}$ (G) denotes the abelian group generated by Z-valued characters of G under the operation of pointwise addition and T(G) is a normal subgroup of $\bar{R}$ (G) which is generated by Artin's characters. Permutation charcters induce from the principle charcters of cyclic Subgroups. A well-known theorem which is due to Artin asserated that T(G) has a finite index is, i.e [:T(G)] is finite .The exponent of AC(G) is called Artin exponent of G and denoted by A (G) .In 1968, Lam . T .Y [5] gave the definition of the group AC(G) and studied $AC(C_n)$ .In 1976, David .G [12] studied A(G) of arbitrary characters of cyclic subgroups. In 1996, Knwabuez .K [11] studied A(G) of p-groups. In 2000,H.R.Yassein [4] found AC(G) for the group $\bigoplus_{i=1}^{n} C_P$. In 2002, k.Sekieguchi [12] studied the irreducible Artin characters of p-group and in the Same year H.H.Abbass [10]found ≡*(Dn).

In 2006, Abid. A. S [6] found $Ar(C_n)$ when C_n is the cyclic group of order n. In 2007, Mirza .R .N [9] found in her thesis Artin cokernel of the dihedral group. In this paper we find the general form of $Ar(D_n \times C_7)$ and we study $AC(D_n \times C_7)$ of the non abelian group $D_n \times C_7$ when n is an odd number.

1.Basic Concepts and Notations:

In this section, we recall some basic concepts, about matrix representation, characters and Artin character which will be used in later section.

Definition (1.1): [1]

A matrix representation of a group G is a homomorphism T of G into GL $(n$,F), n is called the degree of matrix representation T .T is called a unit representation(principal) if T(g)=1, for all g∈G.

Definition (1.2):[2]

Let T be a matrix representation of a group G over the field F, the character χ of a matrix representation T is the mapping χ: G ⟶ F defined by χ (g)= Tr(T(g)) refers to the trace of the matrix T(g)(the sum of the elements diagonal of T(g)). The degree of T is called the degree of χ .

Definition (1.3):[3]

Let H be acyclic subgroup of G and let ∅ be a class function on H . The induced class function on G is given by:

$$\emptyset'(\mathrm{g}) = \frac{1}{|\mathrm{H}|}\sum_{\times\in G}\emptyset^{\circ}(xgx^{-1}), \forall \mathrm{g}\in \mathrm{G}$$

Where $\emptyset^{\circ}$ is defined by:

$$\emptyset^{\circ}(\mathrm{h}) = \begin{cases} \emptyset^{\circ}(h) & if \quad h\in H \\ 0 & if \quad h\in H \end{cases}$$

Theorem (1.4):[4]

Let H be a cyclic subgroup of G and $h_1, h_2, .., h_m$ are chosen representatives for Γ - conjugate classes, Then:

$$\emptyset'(\mathrm{g}) = \begin{cases} \frac{|\mathrm{C_G(g)}|}{|\mathrm{C_H(g)}|}\sum_{k=0}^{m}\emptyset^{\circ}(h_i) & if \quad h_i \in H\cap CL(g) \\ 0 & if \quad \mathrm{H}\cap CL(g) = \emptyset \end{cases}$$

Definition (1.5):[5]

Let G be a finite group, all characters of G induced from the principal character of cyclic subgroups of G is called Artin characters of G.

Definition (1.6):[4]

Artin characters of the finite group can be displayed in a table called Artin characters table of G which is denoted by Ar(G).

Proposition (1.7):[6]

The number of all distinct Artin characters on a group G is equal to the number of Γ - classes on G.

Definition (1.8):[1]

A rational valued character θ of G is a character whose values are in Z, which is θ (g) ∈ Z, for all g ∈G.

Definition (1.9):[6]

Let T(G) be the subgroup of $\bar{R}$ (G) generated by Artin characters.T(G) is a normal subgroup of $\bar{R}$ (G) .Then the factor abelian group $\bar{R}$(G)/T(G) is called Artin cokernel of G, denoted by AC(G).

Proposition (1.10):[6]

AC(G) is a finitely generated Z – module.

Theorem [Artin] (1.11):[7]

Every rational valued character of G can be written as a linear combination of Artin characters with rational coefficient .

2. The Factor Group AC(G):

In this section, we use some concepts in linear algebra to study the factor group AC(G). We will give the general form of Ar ($D_n \times C_7$) when n is an odd number. We shall study Ac(G) dihedral group D_n and ≡*(D_n) when n is an odd number.

Definition (2.1):[5]

Let T(G) be the subgroup of $\bar{R}$(G) generated by Artin characters .T(G) is a normal subgroup of$\bar{R}$(G), then the factor abelian group $\bar{R}$(G)/T(G) is called Artincokernel of G, denoted by AC(G).

Definition (2.2): [8]

Ak-th determinant divisor of M is the greatest common divisor (g.c.d)of all the k–minors of M. This is denoted by D_k(M).

Lemma (2.3): [8]

Let M , P and W be matrices with entries in the principal ideal domain R and p, W be invertible matrices , then :

D_k(P.M.W) = D_k(M) Modulo the group of units of R.

Theorem (2.4):[8]

Let M be an k×k matrix with entries in a principal ideal domain R , then there exits matrices P and W such that :

1 - P and W are invertible.

2 - P M W = D.

3 - D is a diagonal matrix.

4 -If we denote D_{jj} by d_i then there exists a natural number m ; 0 ≤ m ≤ k such that j > m implies d_j =0 and j ≤ m implies $d_j \neq 0$ and 1 ≤ j ≤mimplies $d_j | d_{j-1}$.

Definition (2.5):[8]

Let M be matrix with entries in a principal ideal domain R, equivalent to matrix D=diag {d_1 , d_2 , … ,d_m , 0, 0, … ,0} such that $d_j | d_{j-1}$ for 1 ≤ j<m ,we call D the invariant factor matrix of M and d_1, d_2 , … ,d_m the invariant factors of M.

Remark(2.6):

According to the Artin theorem (1.12) there exists an invertible matrix $M^{-1}(G)$ with entries in the set of rational numbers such that: $\equiv^*(G)= M^{-1}(G). Ar(G)$ and this implies, $M(G)=Ar(G).(\equiv^*(G))^{-1}$ M(G) is the matrix expressing the T(G) basis in terms of the $\bar{R}(G)$ basis.

By theorem (2.5) there exists two matrices P(G) and W(G) with a determinant ∓ 1 such that:

$P(G). M(G).W(G)=diag\{d_1, d_2, \dots, d_l\}= D(G)$ where $d_j = \pm D_j(G)|D_{i-1}(G)$ and l is the number of $\boldsymbol{\Gamma}$-classes.

Theorem (2.7):[4]

$$AC(G) = \bigoplus_{i=1}^{m} Z \text{ where } d_j = \pm D_j(G)|D_{i-1}(G)$$

where m is the number of all distinct Γ -classes.

Theorem(2.8):[9]

If n is an odd number such that $n = p_1^{\alpha_1} . p_1^{\alpha_2} .. p_m^{\alpha_m}$, where $p_1, p_2, \dots, p_m$ are distinct primes , then:

$$AC(D_n) = \bigoplus_{i=1}^{(\alpha_1+1).(\alpha_2+1)\dots(\alpha_m+1)-1} C_2$$

Proposition (2.9): [8]

The rational valued characters table of the cyclic group C_{p^s} of the rank S+1 where p is a prime number which is denoted by ($\equiv^*(C_{p^s})$), is given as follows:

Table (2.1)

$\boldsymbol{\Gamma}$ **- classes**	**[1]**	**[$r^{P^{s-1}}$]**	**[$r^{P^{s-2}}$]**	**[$r^{P^{s-3}}$]**	…	**[r^{P^2}]**	**[r^P]**	**[r]**
$\boldsymbol{\theta_1}$	**P^{s-1}(P-1)**	**- P^{s-1}**	**0**	**0**	…	**0**	**0**	**0**
$\boldsymbol{\theta_2}$	**P^{s-2}(P-1)**	**P^{s-2}(P-1)**	**- P^{s-2}**	**0**	…	**0**	**0**	**0**
$\boldsymbol{\theta_3}$	**P^{s-3}(P-1)**	**P^{s-3}(P-1)**	**P^{s-3}(P-1)**	**- P^{s-3}**	…	**0**	**0**	**0**
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
$\boldsymbol{\theta_{s-1}}$	**P(P-1)**	**P(P-1)**	**P(P-1)**	**P(P-1)**	…	**P(P-1)**	**-P**	**0**
$\boldsymbol{\theta_s}$	**P-1**	**P-1**	**P-1**	**P-1**	…	**P-1**	**P-1**	**-1**
$\boldsymbol{\theta_{s+1}}$	**1**	**1**	**1**	**1**	…	**1**	**1**	**1**

where its rank s+1 represents the number of all distinct Γ -classes.

Remark (2.10):[8]

If $n = p_1^{\alpha_1} . p_1^{\alpha_2} .. p_m^{\alpha_m}$, where $p_1, p_2, \dots, p_m$ are distinct primes, then:

$\equiv^*(C_n) = \equiv^*(C p_1^{\alpha_1}) \otimes \equiv^*(C p_1^{\alpha_2}) \otimes \dots \otimes \equiv^*(C p_m^{\alpha_m})$.

Definition (2.11):[7]

The dihedral group D_n is a certain non-abelian group of order2n .It is usually thought of as a group of transformations of the Euclidean plane of regular n-polygon consisting of rotations (about the origin) with the angle $2k\pi/n$,k=0,1,2,…,n-1 and reflections (across lines through the origin).In general we can write it as :

$D_n = \{ S^j r^k : 0 \le k \le n-1 , 0 \le j \le 1\}$

which has the following properties :

$r^n = 1$, $S^2 = 1$, $S r^k S^{-1} = r^{-k}$

Definition (2.12):

The group $D_n \times C_7$ is the direct product group $D_n \times C_7$,where C_7 is a cyclic group of order 7 consisting of elements $\{1, r', r^{2'}, r^{3'}, r^{4'}, r^{5'}, r^{6'}, r^{7'}\}$ with $(r')^7=1$. It is of order 14n.

Theorem(2.13):[10]

The rational valued characters table of D_n when n is an odd number is given as follows:

Table (2.2)

≡*(C_n)=

	Γ– classes of C_n	[S]
θ_1	≡*(C_n)	0
⋮		⋮
θ_{s-1}	1 1 1 ⋯ 1 1 1	0
θ_s		1
θ_{s+1}	1 1 1 ⋯ 1 1 1	-1

Where S is the number of Γ– classes of C_n.

Theorem(2.14):

The rational valued characters table of the group $D_n \times C_7$ when n is an odd number is given as follows:

≡*($D_n \times C_7$) = ≡*(D_n) ⊗ ≡*(C_7).

Theorem (2.15):[6]

The general form of Artin characters table of C_{p^s} when p is a prime number and s is positive integer is given by the lower Triangluer matrix:

Table (2.3)

Ar(C_{ps})=

Γ – classes	**[1]**	**[$r^{p^{s-1}}$]**	**[$r^{p^{s-2}}$]**	**[$r^{p^{s-3}}$]**	⋯	**[r]**
$\lvert CL_\alpha \rvert$	**1**	**1**	**1**	**1**	⋯	**1**
$\lvert C_{ps}(CL_\alpha)\rvert$	**P^s**	**P^s**	**P^s**	**P^s**	⋯	**P^s**
φ'_1	**P^s**	**0**	**0**	**0**	⋯	**0**
φ'_2	**P^{s-1}**	**P^{s-1}**	**0**	**0**	⋯	**0**
φ'_3	**P^{s-2}**	**P^{s-2}**	**P^{s-2}**	**0**	⋯	**0**
⋮	⋮	⋮	⋮	⋮	⋮	⋮
φ'_s	**P**	**P**	**P**	**P**	⋯	**0**
φ'_{s+1}	**1**	**1**	**1**	**1**	⋯	**1**

Corollary (2.16):[4]

Let n any positive integers and n = $p_1^{\alpha_1} . p_1^{\alpha_2} .. p_m^{\alpha_m}$, where $p_1, p_2, \ldots, p_m$ are distinct primes , then :

Ar(C_n) = Ar($C_{p_1^{\alpha_1}}$) ⊗ Ar($C_{p_2^{\alpha_2}}$)⊗ …… ⊗Ar($C_{p_m^{\alpha_m}}$)

Where⊗ is the tensor product.

Proposition (2.17):[6]

If p is a prime number and s is a positive integer , then M(Cp) is an upper triangular matrix with unite entries.

$$M(C_{ps}) = \begin{bmatrix} 111 & \cdots & 1 \\ 011 & \cdots & 1 \\ 001 & \cdots & 1 \\ \vdots\vdots\vdots & \ddots & \vdots \\ 000 & \cdots & 1 \end{bmatrix}$$

Which is (s+1) x (s+1) square matrix

Proposition (2.18):[2]

The general form of matrices P (C_{p^s})and W (C_{p^s}) are:

$$P(C_{p^s}) = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & \cdots & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 1 \end{bmatrix}$$

which is (s+1)×(s+1) square matrix and W (C_{p^s})= I_{s+1} where I_{s+1} is an identity matrix and D(Cp^s)=diag{1,1,… ,1}.

Remarks (2.19):

1- In general if n = $p_1^{\alpha_1} . p_1^{\alpha_2} ... p_m^{\alpha_m}$, such that $p_1, p_2, \ldots, p_m$ are distinct primes and any $\alpha_i$positive integers for all i = 1, 2, … , m ; then :

C_n=$C_{p_1^{\alpha_1}} \times C_{p_2^{\alpha_2}} \times \ldots \times C_{p_3^{\alpha_3}}$.

M (C_n) =M $(C_{p_1^{\alpha_1}})$ ⊗ M $(C_{p_2^{\alpha_2}})$ ⊗ … ⊗ M $(C_{p_3^{\alpha_3}})$.

So, we can write M (C_n) as:

$$M(C_n) = \begin{bmatrix} & & R(C_n) & & & \begin{matrix}1\\1\\\vdots\\1\end{matrix} \\ 0 & 0 & 0 & \cdots & 0 & 1 \end{bmatrix}$$

Where $R(C_n)$ is the matrix obtained by omitting the last row{0, 0,...,0, 1} and the last column {1,1,…,1} from the tensor product,

M $(C_{p_1^{\alpha_1}})$ ⊗ M $(C_{p_2^{\alpha_2}})$ ⊗…………… ⊗ M $(C_{p_m^{\alpha_m}})$ .M(C_n) is,

1- $(\alpha_1+1)(\alpha_2+1)\dots(\alpha_m+1)$ x $(\alpha_1+1)(\alpha_2+1)\dots(\alpha_m+1))$ square matrix.

2- P (C_n) = P $(C_{p_1^{\alpha_1}})$ ⊗ P $(C_{p_2^{\alpha_2}})$ ⊗ … ⊗ P $(C_{p_m^{\alpha_m}})$.

3-W (C_n) = W $(C_{p_1^{\alpha_1}})$ ⊗W $(C_{p_2^{\alpha_2}})$ ⊗… ⊗W $(C_{p_m^{\alpha_m}})$.

3. The Main Results

In this section we give the general form of Artin characters table of the group$D_n\times C_7$ and the cyclic decomposition of the factor group $AC(D_n\times C_7)$ when n is an odd number.

Theorem(3.1):

The Artin characters table of the group $D_n\times C_7$ when n is an odd number is given as follows :

$Ar(D_n\times C_7)$=

Table(3.1)

Γ– classes	[1,1']	[1,r']	Γ – classes of C_n X C_7					[S,1']	[S,r']
$\lvert CL_\alpha \rvert$	1	1	2	2	….	….	2	n	n
$\lvert C_{D_n x C_7}(CL_\alpha)\rvert$	14n	14n	7n	7n	….	….	7n	14	14
$\Phi_{(1,1)}$	2Ar(C_n)⊗ Ar(C_7)							0	0
$\Phi_{(1,2)}$								⋮	⋮
:								⋮	⋮
$\Phi(l,1)$								⋮	⋮
$\Phi(l,2)$								0	0
$\Phi(l+1,1)$	7n	0	0	….	….	….	0	7	0
$\Phi(l+1,2)$	7n	0	0	….	….	….	0	0	7

where l is the number of Γ -classes of C_n and C_7=< r'> .

<u>Proof</u>:-By theorem(2.15)

Table (3.2)

Ar(C_7) =	Γ– classes	[1']	[r']
	$\lvert CL_\alpha\rvert$	1	1
	$\lvert C_7(CL_\alpha)\rvert$	7	7
	φ'_1	7	0
	φ'_2	1	1

Each cyclic subgroup of the group $D_n\times C_7$ is either a cyclic subgroup of C_nxC_7 or (S,r') or<(S,1')> .If H is a cyclic subgroup of C_nxC_7, then :

H=H_ix<1'> or H_ix<r'>= H_ixC_7 for all $1 \le i \le$ l where l is the number of Γ- classes of C_n

If H=H_ix<1'>and $x \in D_n \times C_7$

If $x \notin$ H then by theorem(1.4)

$\Phi_{(1,i)}(x)$=0 for all $0 \le i \le$ l [since H ∩ CL$(x) = \phi$]

If $x \in$ H then either x= (1,1') or ∃ s ,$0 < s <$n such that x= (r^7,1')

If x= (1,1'), then :

$\Phi_{(i,1)}(x) = \frac{|C_{D_n \times C_7}(x)|}{|C_{H(x)}|}.\varphi'(x)$ [since H ∩ CL(x)={ (1,1') }] ,

where φ is the principle character

$= \frac{14n}{|H_i|.\ |<1'>|}.1 = \frac{14n}{|H_i|} = 2.\ \frac{n}{|H_i|}.1.7 = 2\ \frac{|C_{C_n}(1)|}{|C_{H_i}(1)|}.\ \varphi(1).\ \varphi'(1')$

$= 2.\ \varphi_i(1).\ \varphi'(1')$

If x=(r^7 ,1') then

$\Phi_{(i,1)}(x) = \frac{|C_{D_n \times C_7}(x)|}{|C_{H(x)}|}.\sum_1^2 \varphi'(x)$ [since H∩ CL(x)={(r^s ,1'), (r^{-s} ,1') }]

$= \frac{7n}{|H_{i\times<1'>}|}.(1+1)$

$= \frac{7n}{|H_i|}.\ 2$

$= 2.\frac{n}{|H_{i(r^S)}|}.1.7 = 2\frac{|C_{C_n}(r^S)|}{|C_{H_i}(r^S)|}.\ \varphi(r^S).\ \varphi'(1') = 2.\varphi_i(r^S).\ \varphi_1'(1')$

If H=H_ix<r'>= H_ixC_7

let $x \in D_n \times C_7$

if $x \notin$ H then

$\Phi_{(i,2)}(x)$ =0 for all $1 \le i \le$ l [since H ∩ CL$(x) = \phi$]

If x ∈ H then either g=(1,1') or x=(1,r') or ∃ s ,$0 < s < n$ such that x= (r^s,r')

If x=(1,1')

$\Phi_{(i,2)} = \frac{|C_{D_n \times C_{7(x)}}|}{|C_{H(x)}|}.\varphi(x)$ [since H∩ CL(x)={ (1,1') }]

$= \frac{14}{|H_{i\times C_7}|} = \frac{14n}{2|H_i|} = \frac{7n}{|H_i|} = 2\ \frac{|C_{C_n}(1)|}{|C_{H_i}(1)|}.\varphi(1) = 7.\varphi_i(1).\ \varphi_2'(1)$

If x=(1,r') then

$\Phi_{(i,2)}(x) = \frac{|C_{D_n \times C_{7(x)}}|}{|C_{H(x)}|}.\varphi(x)$ [since H∩ CL(x)={ (1,r') }]

$= \frac{14}{|H_{i\times C_7}|}$

$= \frac{14n}{2|H_i|} = \frac{7n}{|H_i|} = 7\frac{|C_{C_n}(1)|}{|C_{H_i}(1)|} = 7.\varphi_i(1).\ \varphi_2'(r')$

If x=(r^s,r') then

$\Phi_{(i,2)}(x) = \frac{|C_{D_n \times C_7}(x)|}{|C_{H(x)}|}.\sum_1^2 \varphi'(x)$ [since H∩ CL(x)={(r^s ,r'), (r^{-s} ,r') }]

$= \frac{7n}{|H_{i\times C_7}|}(1+1) = \frac{14n}{2|H_i|} = \frac{7n}{|H_i|} = 2\ \frac{|C_{C_n}(r^S)|}{|C_{H_i}(r^S)|}.\varphi(r^S).\varphi_2'(r') = 7.\varphi_i(r^S).\ \varphi_2'(r')$

If H=<(S,1')> = { (1,1') , (S,1')} then:

$\Phi_{(l+1,1)}((1,1')) = \frac{|C_{D_n \times C_{7(1,1')}}|}{|C_{H(S,1')}|}.\varphi(x) = \frac{14n}{2} = 7n$

$\Phi_{(l+1,1)}((1,1')) = \frac{\left|C_{D_n \times C_{7(1,1')}}\right|}{|C_{H(S,1')}|} .\varphi\ (x)$[since H∩ CL$(S,1')$ =$\{(S,1')\}$]

$= \frac{14}{2} = 7$

Otherwise

$\Phi_{(l+1,1)}\ (x) = 0$ for all $x \in D_n \times C_7$,[since $x \notin$ H]

If H=<(S,r')> = { (1,1') , (S,r') }

$\Phi_{(l+1,2)}((1,1')) = \frac{\left|C_{D_n \times C_{7(1,1')}}\right|}{|C_{H(1,1')}|} .\varphi\ (1,1')$ [since H∩ CL((1,1'))= { (1,1') }]

$= \frac{14n}{2} .1 = 7n$

$\Phi_{(l+1,2)}((S,r')) = \frac{\left|C_{D_n \times C_{7(S,r')}}\right|}{|C_{H(S,r')}|} .\varphi\ (S,r') = \frac{14}{2} .1 = 7$

Otherwise $\Phi_{(l+1,2)}\ (x) = 0$ for all $x \in D_n \times C_7$ since H ∩ CL$(x) = \phi$ ■

Proposition (3.2):

If n = $p_1^{\alpha_1} .p_1^{\alpha_2} ..p_m^{\alpha_m}$ where $p_1, p_2, \ldots, p_m$ are distinct primes and $p_i \neq 2$ for all $1 \leq i \leq m$ and α_i any positive integers, then:

$$M(D_{nxC7}) = \begin{bmatrix} 2R(C_n) \times M(C_7) & & & & & 1 & 1 & 1 & 1 \\ & & & & & 1 & 0 & 1 & 0 \\ & & & & & 1 & 1 & 1 & 1 \\ & & & & & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & \cdots & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & \cdots & \cdots & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & \cdots & \cdots & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & \cdots & \cdots & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

which is 2[(α_1+1).(α_2+1)…(α_m+1) +1]x2 [(α_1+1).(α_2+1)…(α_m+1) +1] square matrix .

Proof:

By theorem(3.1) we obtain the Artin characters table Ar($D_{n \times c_7}$) and from theorem(1.11) we find the rational valued characters table

≡*($D_{n \times c_7}$).

Thus by the definition of M(G) we can find the matrix M($D_{n \times c_7}$) :

M ($D_n \times C_7$)=Ar($D_n \times C_7$).(≡*($D_n \times C_7$))$^{-1}$=

$$\begin{bmatrix} 2 & 2 & 2 & 12 & \cdots & \cdots & \cdots & \cdots & 1 & 1 & 1 & 1 \\ 0 & 2 & 0 & 2 & \cdots & \cdots & \cdots & \cdots & 1 & 0 & 1 & 0 \\ 0 & 0 & 2 & 2 & \cdots & \cdots & \cdots & \cdots & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 2 & \cdots & \cdots & \cdots & \cdots & 1 & 0 & 1 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & & & & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & & \ddots & & & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & & & \ddots & & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & & & & \ddots & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & \cdots & \cdots & \cdots & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & \cdots & \cdots & \cdots & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & \cdots & \cdots & \cdots & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & \cdots & \cdots & \cdots & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2R(C_n)\otimes M(C_7) & & & & & 1 & 1 & 1 & 1 \\ & & & & & 1 & 0 & 1 & 0 \\ & & & & & 1 & 1 & 1 & 1 \\ & & & & & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & \cdots & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & \cdots & \cdots & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & \cdots & \cdots & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & \cdots & \cdots & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

which is **2[($\boldsymbol{\alpha_1}$+1).($\boldsymbol{\alpha_2}$+1)… +1]x2 [($\boldsymbol{\alpha_1}$+1).($\boldsymbol{\alpha_2}$+1)… +1]**square matrix .

Proposition (3.3):

If n = $p_1^{\alpha_1}.p_1^{\alpha_2}\ldots\ldots p_m^{\alpha_m}$ such that g.c.d(P_i,P_j)=1 and $P_i \neq 2$ are prime numbers and α_i any positive integers, then:

$$P(D_{nxC7})= \begin{bmatrix} & & & & & & & 0 & 0 \\ & & & & & & & 0 & 0 \\ & & & & & & & \vdots & \vdots \\ & P(C_n)\otimes P(C_7) & & & & & & 0 & 0 \\ & & & & & & & -1 & -1 \\ & & & & & & & 0 & 0 \\ 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

and

$$W(D_n\times C_7)= \begin{bmatrix} & & & & & & 0 & 0 & 0 & 0 \\ & & & & & & 0 & 0 & 0 & 0 \\ & I_k & & & & & \vdots & \vdots & \vdots & \vdots \\ & & & & & & \vdots & \vdots & \vdots & \vdots \\ & & & & & & 0 & 0 & 0 & 0 \\ & & & & & & 0 & 0 & 0 & 0 \\ -1 & -1 & \cdots & \cdots & -1 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & \cdots & \cdots & 1 & 0 & -1 & 0 & 1 & 0 \\ 1 & 1 & \cdots & \cdots & 1 & 1 & -1 & 1 & 0 & 0 \\ 0 & 0 & \cdots & \cdots & 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

Where K = **2[**($\boldsymbol{\alpha_1}$+1).($\boldsymbol{\alpha_2}$+1).($\boldsymbol{\alpha_3}$+1)…($\boldsymbol{\alpha_m}$+1) -1**]x2 [**($\boldsymbol{\alpha_1}$+1).($\boldsymbol{\alpha_2}$+1).($\boldsymbol{\alpha_3}$+1)…($\boldsymbol{\alpha_m}$+1) -1**]**

They are **2[**($\boldsymbol{\alpha_1}$+1).($\boldsymbol{\alpha_2}$+1)…($\boldsymbol{\alpha_m}$+1)+1**]x2 [**($\boldsymbol{\alpha_1}$+1).($\boldsymbol{\alpha_2}$+1)…($\boldsymbol{\alpha_m}$+1) +1**]** square matrix .

Proof :

By using theorem(2.5) and taking the form M($D_n\times C_7$) from proposition(3.2) and the above forms of P($D_n\times C_7$) and W($D_n\times C_7$) then we have

$$P(D_n\times C_7).M(D_n\times C_7).W(D_n\times C_7)= \begin{bmatrix} 2 & 0 & 0 & 0 & \cdots & \cdots & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & \cdots & \cdots & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & \cdots & \cdots & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & \cdots & \cdots & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & & \ddots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & \cdots & -2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \cdots & \cdots & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & \cdots & \cdots & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & \cdots & \cdots & 0 & 0 & 0 & 1 \end{bmatrix}$$

D($D_n\times C_7$)=diag{2,2,2,…..,-2,1,1,1}

Which is **2[**($\boldsymbol{\alpha_1}$+1).($\boldsymbol{\alpha_2}$+1)…($\boldsymbol{\alpha_m}$+1)+1**]x2 [**($\boldsymbol{\alpha_1}$+1).($\boldsymbol{\alpha_2}$+1)…($\boldsymbol{\alpha_m}$+1) +1**]**square matrix .

Theorem (3.4):

If n = $p_1^{\alpha_1}.p_1^{\alpha_2}......p_m^{\alpha_m}$ where $p_1,p_2,\ldots,p_m$ are distinct prime numbers such that $P_i \neq 2$ and α_i any positive integers for all $i, 1 \leq i \leq m$, then the cyclic decomposition AC($D_{n\times c_7}$) is :

$$AC(D_{n\times c_7}) = \bigoplus_{i=1}^{2((\alpha_1+1).(\alpha_2+1)\ldots(\alpha_m+1))-1} C_2$$

$$AC(D_{n\times c_7}) = \bigoplus_{i=1}^{2} AC(D_n) \oplus C_2$$

Proof :

From proposition (3.3) we have

P($D_{n\times c_7}$) .M($D_{n\times c_7}$) .W($D_{n\times c_7}$) =diag{2,2,2,…,-2,1,1,1}= {$d_1,d_2,\ldots\ldots\ldots$,

{$d_{2((\alpha_1+1).(\alpha_2+1).(\alpha_3+1)\ldots(\alpha_m+1))-1}$, $d_{2((\alpha_1+1).(\alpha_2+1).(\alpha_3+1)\ldots(\alpha_m+1))-1}$,$d_{2((\alpha_1+1).(\alpha_2+1).(\alpha_3+1)\ldots(\alpha_m+1))}$, $d_{2((\alpha_1+1).(\alpha_2+1).(\alpha_3+1)\ldots(\alpha_m+1))+1}d_{2((\alpha_1+1).(\alpha_2+1).(\alpha_3+1)\ldots(\alpha_m+1))+2}$ }.

By theorem (2.8) we get

$$AC(D_{n\times c_7}) = \bigoplus_{i=1}^{2((\alpha_1+1).(\alpha_2+1)\ldots(\alpha_m+1))-1} C_{d_i}$$

$$= \bigoplus_{i=1}^{2((\alpha_1+1).(\alpha_2+1)\ldots(\alpha_m+1))-1} C_2$$

From theorem(2.9) we have :

$$AC(D_{n\times c_7}) = \bigoplus_{i=1}^{2} AC(D_n) \oplus C_2$$

Example (3.6):

To find the cyclic decomposition of the groups AC($D_{12167\times C_7}$)) ,AC($D_{11692487\times C_7}$)) and AC($D_{222157253\times C_7}$)) .

We can use above theorem :

1-AC($D_{12167\times C_7}$) =AC($23^3\times C_7$)= $\bigoplus_{i=1}^{2(3+1)-1} C_2 = \bigoplus_{i=1}^{7} C_2 = \bigoplus_{i=1}^{2} AC(D_{23^3}) \oplus C_2$.

2-AC($D_{11692487\times C_7}$) =AC($D_{23^3.31^2}\times c_7$)= $\bigoplus_{i=1}^{2((3+1).(2+1))-1} C_2 = \bigoplus_{i=1}^{2} C_2$

$= \bigoplus_{i=1}^{2} AC(D_{23^3.31^2}) \oplus C_2$

3-AC($D_{222157253\times C_7}$) =AC($D_{23^3.31^2.19}\times c_7$)= $\bigoplus_{i=1}^{2((3+1).(2+1).(1+1))-1} C2$ 15

$= \bigoplus_{i=1}^{47} C_2 = \bigoplus_{i=1}^{2} AC(D_{23^3.31^2.19}) \oplus C_2$

References

[1] Culirits . C and.Reiner . I ,"Methods of Representation Theory with Appcation to Finite Groups and Order", John wily & sons, New york, 1981.

[2] A. M, Basheer "Representation Theory of Finite Groups" ,AIMS, South Africa , 2006.

[3] J Moori, "Finite Groups and Representation Theory" ,University of Kawzulu–Natal , 2006 .

[4] H. R Yassien ,"On ArtinCokernel of Finite Group", M.Sc. Thesis, Babylon niversity, 2000.

[5] T .Y Lam,"Artin Exponent of Finite Groups", Columbia University, New York,1968.

[6] A. S, Abid. "Artin's Characters Table of Dihedral Group for Odd Number", MSc.Thesis, university of kufa,2006.

[7] J. P Serre, "Linear Representation of Finite Groups", Springer- Verlage,1977.

[8] M . S Kirdar .M . S, "The Factor Group of The Z- Valued Class Function Modulo the Group of The Generalized Characters ",Ph. D. Thesis, University of Birmingham, 1982.

[9] Mirza . R. N, "On ArtinCokernel of Dihedral Group Dn When n is An Odd Number", M.Sc. thesis, University of Kufa ,2007.

[10] H .H Abass, "On Rational of Finite Group $D_n$when n is an odd "Journal Babylon University,vol7,No-3,2002.

[11] Knwabusz . K, "Some Definitions of Artin's Exponent of Finite Group", USA.National foundation Math.GR.1996.

[12] David .G, "Artin Exponent of Arbitrary Characters of Cyclic Subgroups", Journal of Algebra,61,pp.58-76,1976.