Engineering and Technology Journal
Journal homepage: engtechjournal.org

Improving the Stability and Critical Speed of Rotor Bearing System By Using External Damping Source

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Submitted: 13/06/2019 Accepted: 10/07/2019 Published: 25/03/2020

K E Y W O R D S

Rotor, Critical Speed, Ball Bearing, fluid Film Journal Bearing.

A B S T R A C T

Improving the essential factors of rotor bearing system are the aim of this study. In addition to the presence of two ball bearings to support the rotor, two additional fluid film journal bearings have been used as an external damping sources. The ANSYS Mechanical APDL 18.0 was used to model rotor bearing system with existence of two additional fluid film journal bearings. Matlab software has been used to achieve the analytical solution. The results clearly showed that the maximum response displacement strongly decreasing and the critical speed has been enhance by using an external damping source as well as the stability of rotor bearing system has been improved due to increasing damping of rotor system where the response displacement decreasing by 99% and critical speed increasing by 0.6% when using an external damping sources. The position of damping source has an important effect on the maximum response displacement and the more decreasing value of response displacement has been achieved when the damping source become as close as possible to the disk position.

How to cite this article: T.M. Hammza,N.R. Hmoad and E.N. Abas, “Improving the Stability and Critical Speed of Rotor Bearing System By Using External Damping Source,” Engineering and Technology Journal, Vol. 37, Part A, No. 03, pp. 319-324, 2019.
DOI: https://doi.org/10.30684/etj.v38i3A.344

1. Introduction

The critical speed of rotating machine is the rotor speed when the maximum response of rotor happens and this definition of critical speed is a perfectly practicable definition. The critical speed of rotating machines is very important factor where the rotor must be operate at speed less than or above the critical speed to prevent damage of rotating machine if it operate at the critical speed therefore if rotating machine designed to operate over critical speed it must be quickly running through it to minimizes both the maximum response and cycles number of high danger. The critical speed of rotor is depending on dynamic coefficients of bearings and mass of rotor therefore the bearings type which support the rotor have a major effect on the dynamic response and consequently critical speed of rotor. The maximum response of rotor bearing system can be find by analytical solution of equation of motion of rotor bearing system. To improve the rotor critical speed , the amplitude and location of maximum response must be changed by decreasing response displacement and shifting the location of maximum response to the right in the frequency- response diagram whereas the critical speed of rotor is the rotational speed at maximum response in most practical situations.

The researchers in the field of rotor dynamics have been studied a many parameters that effect on the critical speed and they tried to produce a new method to enhance those parameters and sequentially enhance the critical speed of rotating machines.[1], have been examined the stiffness influences on dynamic parameters of rotor bearing and foundation systems and they concluded that, the increasing of foundation size leads to rises the mode natural frequencies and expands the operating speed range. Vazquez et al.[2] have been studied the effects of bearings mounted on flexible supports on the dynamic response and critical speed and they concluded the cross coupling coefficients does not have a large effect on the dynamic response and critical speed. David et al.[3] study the effect of using roller bearings on the vibration amplitude of rotor and they concluded that damping will significantly decrease the rotor vibration amplitude. Kang et al Navarro et al.[4], Have been used an active scheme of unbalance compensation for a rotor-bearing system by apply two of control strategies together, and these strategies are rotor speed acceleration scheduling and dynamic bearing stiffness controller. And by using the previous strategies the unbalance response can be reduces when rotor running through the first critical speed. Zhang et al.[5], have been used ANSYS software to model the big tube compressor to determine the critical speed and find the parameters which effect on it. The result clearly showed that, moment due to impeller rotation is shifting the rotor system critical speed when the forward whirl occurs. Yao et al.[6], developed a finite element rotor model considering the gyroscopic effect and shear effect to study the dynamic behavior of two coupled rotor mounted on three bearings. The results showed that the unbalance response and critical speed of rotors are high sensitive to stiffness of coupling.. Eftekhari [7] , investigated the influence of dynamic coefficients of bearings on the rotor natural Frequencies. The final conclusions clear that the increasing of bearing damping leads to increases the first two critical speeds and decreases the last four critical speeds. Paul et al. [8] have been used finite element method to study the effect of lubricated oil viscosity on the critical speed of rotor mounted on fluid film journal bearings. The results showed that the critical speed increases with increasing in the oil grade. The aim of this research is to improve the important dynamic parameters of rotating parts in rotating machines whereas the decreasing of response displacement and increasing the critical speed are represent the main objective of researchers in the field of rotor dynamics science so that this study is try to provide an easy way to improve operation conditions of rotating machines.

2. Equation of Motion

For a flexible rotor with central rigid disc mounted on two symmetrical ball bearings as shown in figure (1). The equation of motion can be driven as following;

$$m\ddot{x} + K(x - x_b) = mu\Omega^2 \cos \Omega t \qquad (1)$$
$$m\ddot{y} + K(y - y_b) = mu\Omega^2 \sin \Omega t$$

Where (x) and (y) are the center location of rotor disk and (x_b) and (y_b) are the location of the shaft center in the ball bearings and
($mu\Omega^2 \sin \Omega t$, $mu\Omega^2 \cos \Omega t$) are the unbalance forces.

$$K(x - x_b) = 2F_{jx} + 2F_{bx} \qquad (2)$$
$$K(y - y_b) = 2F_{jy} + 2F_{by}$$

Where K is shaft stiffness , F_{jx} , F_{jy} are the journal bearing reaction forces in x and y direction respectively, F_{bx}, F_{by} are the ball bearing reaction forces in x and y direction respectively. The fluid film journal bearings and ball bearings reaction forces are;

$$\begin{bmatrix} F_{jx} \\ F_{jy} \end{bmatrix} = \begin{bmatrix} K_{xx} & K_{xy} \\ K_{yx} & K_{yy} \end{bmatrix} \begin{Bmatrix} x_j \\ y_j \end{Bmatrix} + \begin{bmatrix} C_{xx} & C_{xy} \\ C_{yx} & C_{yy} \end{bmatrix} \begin{Bmatrix} \dot{x}_j \\ \dot{y}_j \end{Bmatrix} \qquad (3)$$

$$\begin{bmatrix} F_{bx} \\ F_{by} \end{bmatrix} = \begin{bmatrix} K_{bx} & 0 \\ 0 & K_{by} \end{bmatrix} \begin{Bmatrix} x_b \\ y_b \end{Bmatrix} + \begin{bmatrix} C_{bx} & 0 \\ 0 & C_{by} \end{bmatrix} \begin{Bmatrix} \dot{x}_b \\ \dot{y}_b \end{Bmatrix} \qquad (4)$$

Where dynamic coefficients of journal bearing and ball bearing are as shown in the figure (2)

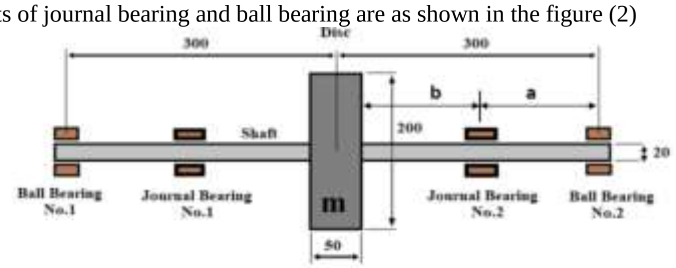

Figure 1: Flexible rotor mounted on two ball bearing with two additional journal

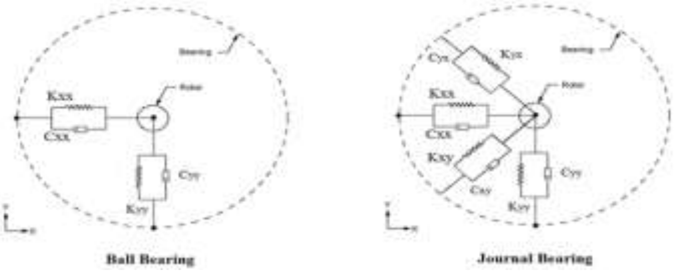

Figure 2: Physical representation of the ball and journal bearings dynamic coefficients

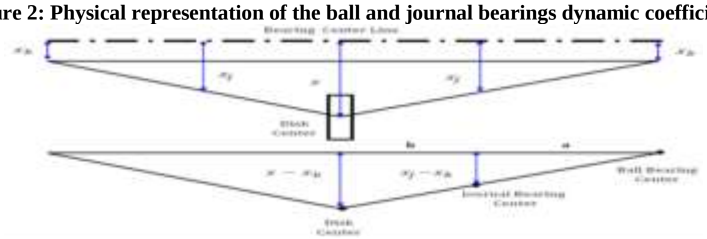

Figure 3: Bending of Rotor Mounted on Two Ball Bearings

From figure (3), the relation between displacement of disk (x), displacement of journal bearing (x_j) and displacement of ball bearing (x_b) can be written as following ;

$$x_j = c_1 x + c_2 x_b \qquad (5)$$
$$y_j = c_1 y + c_2 y_b$$

Where $c_1 = \frac{a}{a+b}$, $c_2 = \frac{b}{a+b}$

Substitute Eq. (5) in Eq. (3), get: $\begin{bmatrix} F_{jx} \\ F_{jy} \end{bmatrix} = \begin{bmatrix} K_{xx} & K_{xy} \\ K_{yx} & K_{yy} \end{bmatrix} \begin{Bmatrix} c_1 x + c_2 x_b \\ c_1 y + c_2 y_b \end{Bmatrix} + \begin{bmatrix} C_{xx} & C_{xy} \\ C_{yx} & C_{yy} \end{Bmatrix} \begin{Bmatrix} c_1 \dot{x} + c_2 \dot{x}_b \\ c_1 \dot{y} + c_2 \dot{y}_b \end{Bmatrix}$ (6)

3. Dynamic Response and Critical Speed

The rotor critical speed is the speed of rotor at which the response displacement of rotor due to the unbalance mass is maximum. Therefore firstly the unbalance response displacement of rotor must be determined. For rotor harmonic motion Eq. (4) and Eq.(6) become;

$$\begin{bmatrix} F_{jx} \\ F_{jy} \end{bmatrix} = \begin{bmatrix} K_{xx} & K_{xy} \\ K_{yx} & K_{yy} \end{bmatrix} \begin{Bmatrix} c_1 x + c_2 x_b \\ c_1 y + c_2 y_b \end{Bmatrix} + \begin{bmatrix} C_{xx} & C_{xy} \\ C_{yx} & C_{yy} \end{bmatrix} \begin{Bmatrix} c_1 i\Omega x + c_2 i\Omega x_b \\ c_1 i\Omega y + c_2 i\Omega y_b \end{Bmatrix} \quad (7)$$

$$\begin{bmatrix} F_{bx} \\ F_{by} \end{bmatrix} = \begin{bmatrix} K_{bx} & 0 \\ 0 & K_{by} \end{bmatrix} \begin{Bmatrix} x_b \\ y_b \end{Bmatrix} + \begin{bmatrix} C_{bx} & 0 \\ 0 & C_{by} \end{bmatrix} \begin{Bmatrix} i\Omega x_b \\ i\Omega y_b \end{Bmatrix} \quad (8)$$

Where

$\mathrm{x_j} = \mathrm{B}e^{i\Omega t}$, $\dot{\mathrm{x}}_\mathrm{j} = i\Omega \mathrm{B}e^{i\Omega t} = i\Omega \mathrm{x_j}$

$\mathrm{y_j} = \mathrm{D}e^{i\Omega t}$, $\dot{\mathrm{y}}_\mathrm{j} = i\Omega \mathrm{D}e^{i\Omega t} = i\Omega \mathrm{y_j}$

$\mathrm{X_b} = \mathrm{c}e^{i\Omega t}$, $\dot{\mathrm{x}}_\mathrm{b} = i\Omega \mathrm{c}e^{i\Omega t} = i\Omega \mathrm{x_b}$

$\mathrm{y_b} = \mathrm{H}e^{i\Omega t}$, $\dot{\mathrm{y}}_\mathrm{b} = i\Omega \mathrm{H}e^{i\Omega t} = i\Omega \mathrm{y_b}$

Substitute Eq. (7) and Eq.(8) in Eq. (2) to get:

$$x_b = \frac{(K_4K_6+K_1K_8)x-(K_4K_5+K_2K_8)y}{K_3K_8-K_4K_7} \quad (9)$$

$$y_b = \frac{(K_3K_5+K_2K_7)y-(K_3K_6+K_1K_7)x}{K_3K_8-K_4K_7}$$

Where:

$K_1 = K - 2c_1(K_{xx} - i\Omega C_{xx})$

$K_2 = 2c_1(K_{xy} + i\Omega C_{xy})$

$K_3 = K + 2c_2(K_{xx} + i\Omega C_{xx}) + 2(K_{bx} + i\Omega C_{bx})$ $K_4 = 2c_2(K_{xy} + i\Omega C_{xy})$

$K_5 = K - 2c_1(K_{yy} - i\Omega C_{yy})$

$K_6 = 2c_1(K_{yx} + i\Omega C_{yx})$

$K_7 = 2c_2(K_{yx} + i\Omega C_{yx})$

$K_8 = K + 2c_2(K_{yy} + i\Omega C_{yy}) + 2(K_{by} + i\Omega C_{by})$

Substitute Eq.(9) in Eq.(1) to eliminate x_b , y_b to obtain

$$m\ddot{x} + K_{11}x + K_{12}y = mu\Omega^2 \cos\Omega t \quad (10)$$

$$m\ddot{y} + K_{21}y + K_{22}x = mu\Omega^2 \sin\Omega t \quad (11)$$

Where :

$$K_{11} = \frac{K[(K_3K_8 - K_4K_7 - K_4K_6 - K_1K_8)]}{(K_3K_8 - K_4K_7)}$$

$$K_{12} = \frac{K[(K_4K_5 + K_2K_8)]}{(K_3K_8 - K_4K_7)}$$

$$K_{21} = \frac{K[(K_3K_8 - K_4K_7 - K_3K_5 - K_2K_7)]}{(K_3K_8 - K_4K_7)}$$

$$K_{22} = \frac{K[(K_3K_6 + K_1K_7)]}{(K_3K_8 - K_4K_7)}$$

The solution of Eq.(10) and Eq.(11) are as following;

$$x = Ae^{i\Omega t} + Be^{-i\Omega t} \ , \ y = Ce^{i\Omega t} + De^{-i\Omega t} \quad (12)$$

Substitute Eq. (12) into Eq. (10) and Eq.(11) with taking into consideration

$\cos(\Omega t) = 1/2\left(e^{i\Omega t} + e^{-i\Omega t}\right)$

$\sin(\Omega t) = 1/2i\left(e^{i\Omega t} - e^{-i\Omega t}\right)$ yields:

$$A = \frac{mu\Omega^2[i(K_{21} - m\Omega^2) - K_{12}]}{2i[(K_{11} - m\Omega^2)(K_{21} - m\Omega^2) - K_{12}K_{22}]} \quad (13.a)$$

$$B = \frac{mu\Omega^2[i(K_{21} - m\Omega^2) + K_{12}]}{2i[(K_{11} - m\Omega^2)(K_{21} - m\Omega^2) - K_{12}K_{22}]} \quad (13.b)$$

$$C = \frac{mu\Omega^2[K_{11} - m\Omega^2 - iK_{22}]}{2i[(K_{11} - m\Omega^2)(K_{21} - m\Omega^2) - K_{12}K_{22}]} \quad (14.a)$$

$$D = \frac{-mu\Omega^2[K_{11} - m\Omega^2 + iK_{22}]}{2i[(K_{11} - m\Omega^2)(K_{21} - m\Omega^2) - K_{12}K_{22}]} \quad (14.b)$$

The unbalance response of rotor can be written as follows,

$$r = x + iy = (A + iC)e^{i\Omega t} + (B + iD)e^{-i\Omega t} \quad (15)$$

Substitute Eq.(13) and Eq.(14) in the Eq. (12) then substitute the result in the Eq.(15) to obtain the harmonic response of rotor mounted on a symmetrical ball bearings with two additional journal bearings as follows.

$$r = r_f e^{i\Omega t} + r_b e^{-i\Omega t} \quad (16)$$

Where $r_f = \frac{mu\Omega^2[i(K_{11} + K_{21} - 2m\Omega^2) - K_{12} + K_{22}]}{2i[(K_{11} - m\Omega^2)(K_{21} - m\Omega^2) - K_{12}K_{22}]}$

$$r_b = \frac{mu\Omega^2[i(K_{21} - K_{11}) + K_{12} + K_{22}]}{2i[(K_{11} - m\Omega^2)(K_{21} - m\Omega^2) - K_{12}K_{22}]}$$

Where r_f and r_b are representing the forward and backward components of the rotor dynamic response, respectively

The maximum response is the major radii of the elliptical orbit of the rotor at disk

$$|r|_{maj} = |r_f| + |r_b \ , ||r|_{min} = |r_f| - |r_b| \quad (17)$$

The critical speed of the rotor is equal to rotor speed at maximum response, $|r|_{maj}$.

4. Finite Element Analysis Using ANSYS

There are two finite element ANSYS model can be used to model the rotor bearing system, the first one is a 3-D Solid model and second one is a 1-D beam model [9]. Solid186 element used to model the disc and shaft of rotor in the state of 3-D solid model, while in the state of 1-D beam model, beam188 element was used to model the Rotor shaft and mass21 element used to model the disc, the ANSYS models of two methods appear as shown in Figure 4**,** as well Comi214 element used to model the ball and journal bearings. The stiffness and damping coefficients of ball bearing are independent of spin speed of rotor while the stiffness and damping of journal bearing are changing with change in the spin speed of rotor so that this must be noticeable in the ANSYS software. The dimensions of rotor two bearing system that used in present research are as shown in Figure 2 **,** and in Table 1**.**

Figure 4: ANSYS rotor model, a: 3-D, model using solid186, b: 1-D, model using beam188

Table 1: Rotor material and lubricant oil specifications

Shaft length mm	Shaft diam. mm)	Disc diam. mm	Disc thickness mm)	Modulus of Elasticity pa	Shaft and Disk Density Kg/m^3	Unbalance force Kg - m
600	20	200	50	2.1x10^{11}	7850	1x10^{-5}

5. Results and discussion

The goal of improve the critical speed of rotating machines can be achieved via shortening the rotor length or by increasing rotor shaft diameter but that leads to increase in the weight of rotor in case of increasing the rotor shaft diameter and the shortening of the rotor shaft is not always obtainable therefore the using of an external damping source may be give a valuable solution to improve the critical speed and stability of rotating parts. The using of an external damping source in the rotor bearing system not only enhance critical speed also decreasing the dynamic response amplitude and increasing the stability of rotor systems.

The dynamic coefficients of fluid film journal bearings are depending mainly on the bearing clearance, lubrication oil viscosity, bearing dimensions , rotor spin speed and applied load.

In this research the concern will be on the changing of bearing clearance and study their effect on the critical speed and dynamic response where increasing or decreasing in the bearing clearance will leads to increases or decreases the dynamic response and critical speed Because any change in the clearance of the journal bearing will result in a change in the dynamic coefficients and thus a change in the critical speed and dynamic response. The direct stiffness in x- direction (K_{xx}) decreases with increasing of bearing clearance while the direct stiffness in y- direction (K_{yy}) Approximately constant in spite of the increasing of clearance of the journal bearings. The cross couple stiffness (K_{xy} , K_{yx}) are have negative values (negative coefficients are shown as dashed lines), and one of them (K_{xy}) don't change with increasing of bearing clearance while the other (K_{yx}) sharply increasing until, (0.1) bearing clearance and then smoothly changing with increasing of bearing clearance as shown in Figure 5.a.

The direct damping coefficient in horizontal direction (C_{xx}) smoothly decreases with increasing of bearing clearance while the direct damping coefficient in vertical direction (C_{yy}) strongly decreases with increasing in the bearing clearance until (0.1) clearance and then smoothly decreases with increasing in the bearing clearance as shown in Figure 5.b. The dynamic response and critical speed of rotor supported by two ball bearing are obtained from Matlab and ANSYS results are shown in Figure 6. From the both results, it is noticed that the maximum response is happened at location of the disc and the critical speed is the rotor speed at maximum response and the comparisons between the results are presented in Table 2.

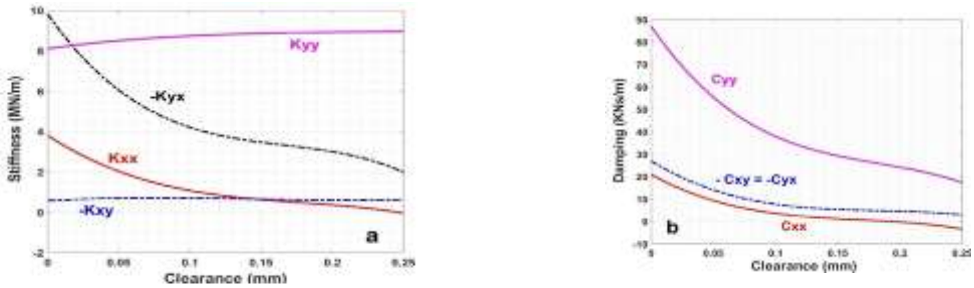

Figure 5: Effect of bearing clearance on the dynamic coefficients of journal bearing

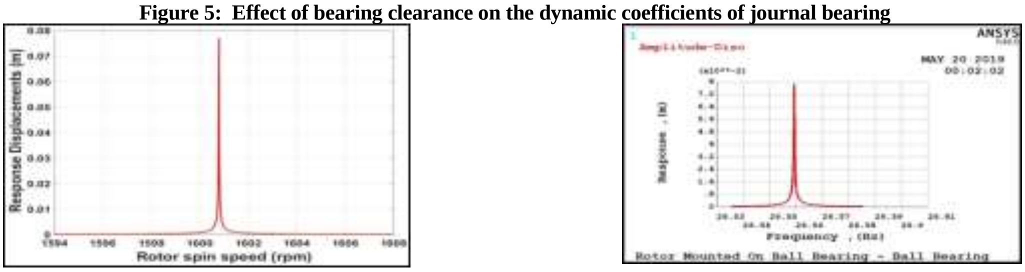

Figure 6: Dynamic response of rotor mounted on ball - ball bearings

Table 2: Maximum Response Displacement and critical speed of rotor ball bearings system

Rotor Dynamic Parameters	Matlab	ANSYS	Ratio
Maximum Response Displacement, m	0.077	0.078	0.984
Critical Speed, rpm	1601	1594	0.995

The values of maximum response displacement and critical speed that identified from Matlab and ANSYS are very identical to each other.

The dynamic response displacement mainly depends on the dynamic coefficients so that the changing in the stiffness and damping of bearings which supported the rotor surly will lead to change the dynamic response and critical speed. The dynamic response of rotor mounted on ball bearing is commonly high because the stiffness of ball bearing is high and its damping is very low. The response displacement of rotor can be decreasing as well as increasing the critical speed by using the journal bearings as an external damping sources as shown in Figure 7, and Table 3.

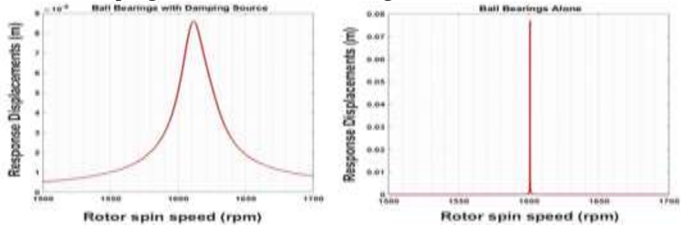

Figure 7: Effect of the external damping source on the response displacement of rotor

Table 3: Critical speed and maximum response displacement Comparison

	Ball bearings alone	Ball bearings with damping source
Critical speed rpm	1600	1610
Response displacement (10^{-3}) mm	77	0.086

The clearance of journal bearings that used as an external damping source has an effect on the response displacement and critical speed where the clearance is one of the main parameters which used to calculate the stiffness and damping coefficients of journal bearing. The eccentricity ratio of the journal bearings ($\varepsilon = \frac{e}{c}$, where e is the journal displacement and c is the radial clearance), when they used as an external damping source is equal to the ratio of ball bearing displacement and radial clearance $\left(\varepsilon = \frac{x_b}{c}\right)$, because the displacement of shaft part inside journal bearing will equal to the ball bearing displacement because the rotor mounted on the ball bearings only and the role of the journal bearings are damping sources and don't bear the rotor. The effect of bearing clearance on the critical speed and dynamic response of rotor can be seen in Figure 8.

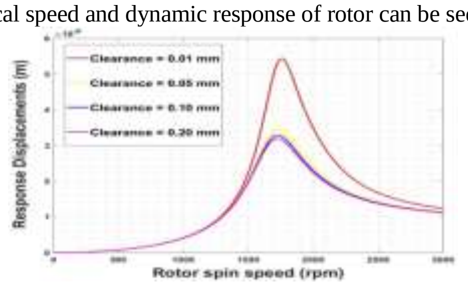

Figure 8: Dynamic response of rotor with different journal bearing clearances

Where the increasing of bearing clearance leads to strongly decrease the response displacement until bearing clearance equal to 0.05 and over that approximately there is no change in the response displacement while the critical speed slightly decreasing with the increasing of bearing clearances as shown in Figure 9,and Table 4, that because, the increasing of bearing clearance mean generally decreasing of dynamic coefficients of bearing (see Figure 5.)

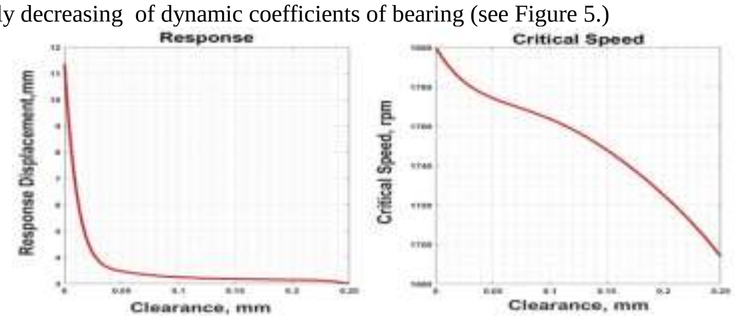

Figure 9: Effect of journal bearing clearance on the critical speed and response

Table 4: Effect of journal bearing clearance on the critical speed and response

Clearance mm	Critical Speed rpm	Maximum Response displacement (10^{-3}) mm
0.01	1760	5.4299
0.05	1735	3.818
0.10	1730	3.2836
0.20	1725	3.1878

The location of two journal bearings which used as an external damping must be as close as possible to ball bearings because the rotor shaft in the real rotor have no empty space which can be used to put the journal bearings where the rotor shaft always carries blades or fans. The location of journal bearing has a strongly effect on the response displacement and slightly effect on the critical speed and of course when put journal bearings as close as to the disc the response displacement and critical speed will more enhance as shown in Figure 10 , the response displacement becomes less as the bearing position is close to the disc and the critical speed (speed at maximum response) increases also when bearing position becomes more close to the disc but this state can't be achieved in a realistic situation but nevertheless , the use of a journal bearings as an external damping sources adjacent to ball bearings gives good results. The response displacement and critical speed for the case of existence damping sources have been calculated analytically by using Matlab and the results verified with ANSYS as shown in Figure 11. From results of maximum response and critical speed, it is noticed that the maximum response displacement is take placed at the disk position and the comparisons of response displacement and critical speed are listed in Table 5.

Table 5: Maximum response and critical speed comparison of rotor

Rotor dynamic parameter	Ansys	Matlab	Ratio Ansys/Matlab
Critical speed rpm	1633	1650	0.989
Response (10^{-2}) mm	4.7	3.25	1.44

The ANSYS and Matlab results showed an acceptable agreement and of course the ANSYS results are more accurate than Matlab where ANSYS software depends on finite element method, whenever the analytical results clearly showed it can be used to calculate the dynamic response and critical speed of rotor mounted on ball bearings with an external damping sources.

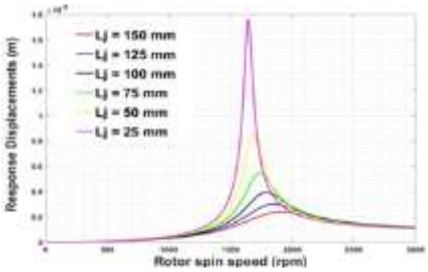

Figure 10: Effect of journal bearing position on the response displacement (L_j, is distance from shaft ends to journal bearings positions)

Figure 11: Response of rotor Mounted on ball bearings with an external damping source

6. Conclusion

The using of an external damping source to reduce the response displacement will leads to enhance stability of the rotor system because the rotor system becomes more stable when decreasing maximum response and vice versa that mean the using of an external damping source not only enhance the critical speed and decreasing response displacement also improve stability of rotor bearing system, from the results of this research the following remarks can be concluded;

1. The maximum response displacement strongly decreasing by using an external damping source.
2. The critical speed slightly increasing by using an external damping source.
3. The stability of rotor bearing system has been improved by using an external damping source due to decreasing the maximum response displacement.
4. The position of damping source has an important effect on the maximum response displacement and when the position becomes more close to the disc position the response displacement strongly decreasing and that mean the rotor bearing system becomes more stable.
5. The bearing small clearance strongly influence on the maximum response displacement and slightly effect on the critical speed, while high bearing clearance has little effect on the response displacement and slightly effect on the critical speed.

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