

## **Nonlinear Dynamic Modelling of Methanol-Water Continuous Distillation Column**

**النمذجة الديناميكية اللاخطية لعمود التقطير الميثانول – الماء المستمر**

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### **Abstract**

Methanol-water distillation column is an important unit operation in chemical processes. This research presents simple procedure of design and then dynamic model for binary continuous distillation column. This model is used to simulate the nonlinear dynamics behavior of the column, study product composition response and test effect of disturbance which takes place in feed. It plays an important role in controlling the column. The simulation results show that the effect of feed composition disturbances is greater than the effect of feed flow rate disturbances on the dynamic response behavior of product composition. The dynamic response of change in distillate composition is accompanied by dead time at different feed perturbations.

### **الخلاصة**

ان عمود تقطير الميثانول – الماء يعد وحدة عملية مهمة في العمليات الكيميائية لذلك تمت دراسته من خلال عرض الإجراءات البسيطة للتصميم وكذلك النموذج الديناميكي لعمود التقطير الثنائي المستمر الذي يستخدم لمحاكاة السلوكيات الديناميكية اللاخطية للعمود ودراسة استجابة تكوين المنتج واختبار تأثير الاضطرابات التي تحدث في التلقيم العمود حيث تلعب دور مهم في السيطرة على العمود. وان نتائج المحاكاة اظهرت تأثير اضطرابات تكوين تغذية أكبر من تأثير اضطرابات معدل تغذية التدفقي على سلوك الاستجابة الديناميكية لتكوين المنتج والوقت الضائع الذي يرافق الاستجابات الديناميكية للتغيير في تكوين نواتج التقطير عند الاضطرابات التغذية المختلفة.

### **1. Introduction**

Methanol-water distillation is a very common and important separation system, and also the most studied unit in term of control<sup>[1]</sup>. A distillation column is used to separate the Methanol-water mixture into its components by the application and removal of heat. It consumes a huge amount of energy in both heating and cooling operations. The controlled distillation column difficulties lie in their highly nonlinear characteristics consists their multiple inputs multiple outputs (MIMO) structure and the presence of severe disturbances during operation<sup>[2]</sup>. Luyben<sup>[3]</sup> has been shown the distillation column becomes the more nonlinear, that the products get the purer and because that the nonlinearity of distillation columns is well known.

Modelling of distillation columns is often constructed based on the physical properties of the system, such as the conservation of mass, energy and momentum. And depending on the accuracy of the assumptions at different models are ranging from simple to rigorous models<sup>[4]</sup>. Skogestad<sup>[5]</sup> presents comprehensive display of literature on dynamics and control of distillation columns up until 1991, which summarized the simplifications of the rigorous model since no references had been found on solving all the equations of the rigorous model. The simplifications are aimed to the vapor dynamics, to the energy balance and to the liquid flow dynamics. And also he recommends not neglecting liquid dynamics (i.e. not assuming constant liquid holdups) due to the fact that the initial response, an important factor in feedback control, is largely affected by the liquid holdups. Wittgens and Skogestad<sup>[6]</sup> emphasized on the initial response by use of carrying out an evaluation of dynamic models of distillation columns in order to find out that the most important parameters are the liquid holdup, the liquid hydraulic time constant and the vapor constant that represent the initial effect of a change in vapor flow on liquid flow. Abdulla et al.<sup>[2]</sup> have stated that the empirical modelling has been preferred in industry because of its simplicity compared to the nonlinear model;

and the current development focuses on hybrid models, which can exploit the advantages of both nonlinear model and empirical model.

Minh <sup>[7]</sup> introduces a calculation procedure for modelling and control simulation of a condensate distillation column based on the energy balance LV-configuration. The mathematical modelling simulation is an important part for the process dynamic analysis and the plant initial design. Truong et al. <sup>[8]</sup> review some techniques of modelling and simulation of distillation columns and focus on the dynamic behavior of the product compositions under feed disturbances. They get that the composition responses to disturbances are close to the response of a first order system and the response to change in feed composition has larger gain than the response to change in feed flow rate. Ahangar and Sadeghi <sup>[9]</sup> study the effect of variable changes such as methanol concentration of feed and feed flow rate on the performance of the controllers and the rate of purity of the product methanol flow and water in the bottom of the columns.

The aim of this study are twofold: first, to present a theoretical calculation procedure of Methanol-water continuous distillation column design for simulation and analysis, and second, for the dynamics behavior: a nonlinear model is derived such that it best reflects the dynamics of the distillation process and used the LV- configuration model includes mass balance for the distillation column sections to verify the P regulatory level controller for a distillation process dealing with the influence of the feed disturbances.

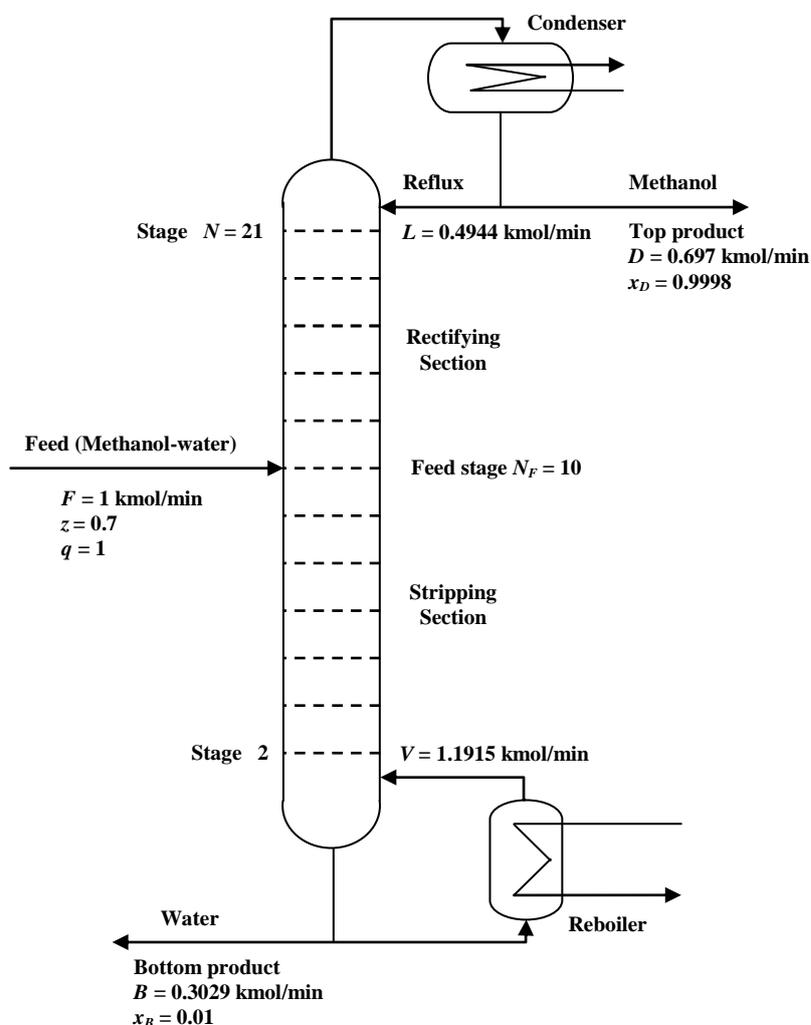


Figure 1. Methanol-water continuous distillation column

**2. Description of distillation process**

**2.1. Description of distillation column design**

Methanol-water continuous distillation column is shown in Figure 1. The column has  $N$  equilibrium stages, with the reboiler as stage number one. The methanol-water feed with total molar flow rate  $F$  (kmol/min) and mole fraction  $z$  enters at stage  $N_F$ . This stage divides the column into a rectifying section and a stripping section. Near the bottom of the column is a reboiler which provides energy to the column. The mixture is heated to form a flow of vapor  $V$  (kmol/min) rising up inside the column. In the stripping section, the less volatile component (water) is enriched while in the rectifying section the more volatile component (methanol) is enriched. The top product is condensed by the condenser from which there is a reflux flow  $L$  (kmol/min) back to the top of the column to enhance the purity of the distillate product outlet  $D$  (kmol/min) with mole fraction  $x_D$  and the bottom product  $B$  (kmol/min) with impurity mole fraction  $x_B$ .

The column for separating a saturated liquid mixture of methanol ( $l$ ) and water ( $h$ ) is designed. The feed liquid fraction ( $q = 1$ ) describes the change in liquid and vapor flow rates at the feed stage into a distillate product with 99.98% methanol and a bottoms product with 99% water (mole fractions). The calculation procedure based on ideal mixtures and simplifying assumptions about constant molar flows and constant relative volatility  $\alpha$  when applying the simple methods presented in the article by Halvorsen and Skogestad [10].

Table 1. Data of mixture components [11].

Data	Methanol ( $l$ )	Water ( $h$ )
Boiling point °K	337.85	373.15
Heat of vaporization kJ/mol	35.28	40.83

Table 1 gives the data of mixture components [11]. The estimated relative volatility between components  $l$  and  $h$  at 1 atm based on the boiling points is defined as:

$$\alpha_{lh} \approx e^{\beta(T_{bh}-T_{bl})/\bar{T}_b} \text{ where } \beta = \Delta\bar{H}^{vap}/R\bar{T}_b \quad (1)$$

where  $\bar{T}_b = \sqrt{T_{bl}T_{bh}}$  is the geometric average boiling temperature, and  $\Delta\bar{H}^{vap}$  is the heat of vaporization which may use the geometric average also:

$$\Delta\bar{H}^{vap} = \sqrt{\Delta H_l^{vap}(T_{bl})\Delta H_h^{vap}(T_{bh})} \quad (2)$$

From the overall material balance we get product split:

$$D/F = (z - x_B)/(x_D - x_B) \quad (3)$$

The separation factor is:

$$S = x_D(1 - x_B)/[x_B(1 - x_D)] \quad (4)$$

The minimum number of stages required for the separation by Fenske's formula is:

$$N_{min} = \ln(S)/\ln(\alpha) \quad (5)$$

and the actual number of stages is selected as  $N = 2N_{min}$ . The corresponding optimal feed stage composition ( $x_F, y_F$ ) can be obtained by solving the following two equations:

$$\begin{aligned} z &= q x_F + (1 - q)y_F \\ x_F &= y_F/[\alpha - (\alpha - 1)y_F] \end{aligned} \quad (6)$$

For  $q = 1$  (liquid feed) we find ( $x_F = z$ ). Skogestad's approximate formula for the feed stage location gives:

$$N_T - N_B = \ln([(1 - y_F)/x_F][x_B/(1 - x_D)])/ \ln(\alpha) \quad (7)$$

where  $y_F$  and  $x_F$  at the feed stage are obtained as explained above. The optimal feed stage location counted from the bottom is then:

$$N_F = N_B + 1 = [N + 1 - (N_T - N_B)]/2 \quad (8)$$

where  $N$  is the actual number of stages in the column. The minimum energy usage for a liquid feed (assuming sharp separation) is:

$$V_{min}/F = (\alpha - 1)/1 + D/F \quad (9)$$

with the choice  $N = 2N_{min}$ , the actual energy usage ( $V$ ) is then typically about 10 % above the minimum ( $V_{min}$ ), i.e.  $V/F = 1.1(V_{min}/F)$ . These values are achieved at steady state case and

presented in Table 2. This concludes that the number of stages depends directly on the product purity, whereas for well-designed columns (with a sufficient number of stages) the energy usage is only weakly dependent on the product purity.

Table 2. Steady - state data for methanol-water distillation column.

$N$	$N_{tot}$	$N_F$	$z$	$q$	$\alpha$	$D/F$	$V/F$	$x_D$	$x_B$	$M_i$	$\tau_i$	$\lambda_i$
21	22	10	0.7	1	3.6	0.697	1.192	0.9998	0.01	0.5	0.063	0

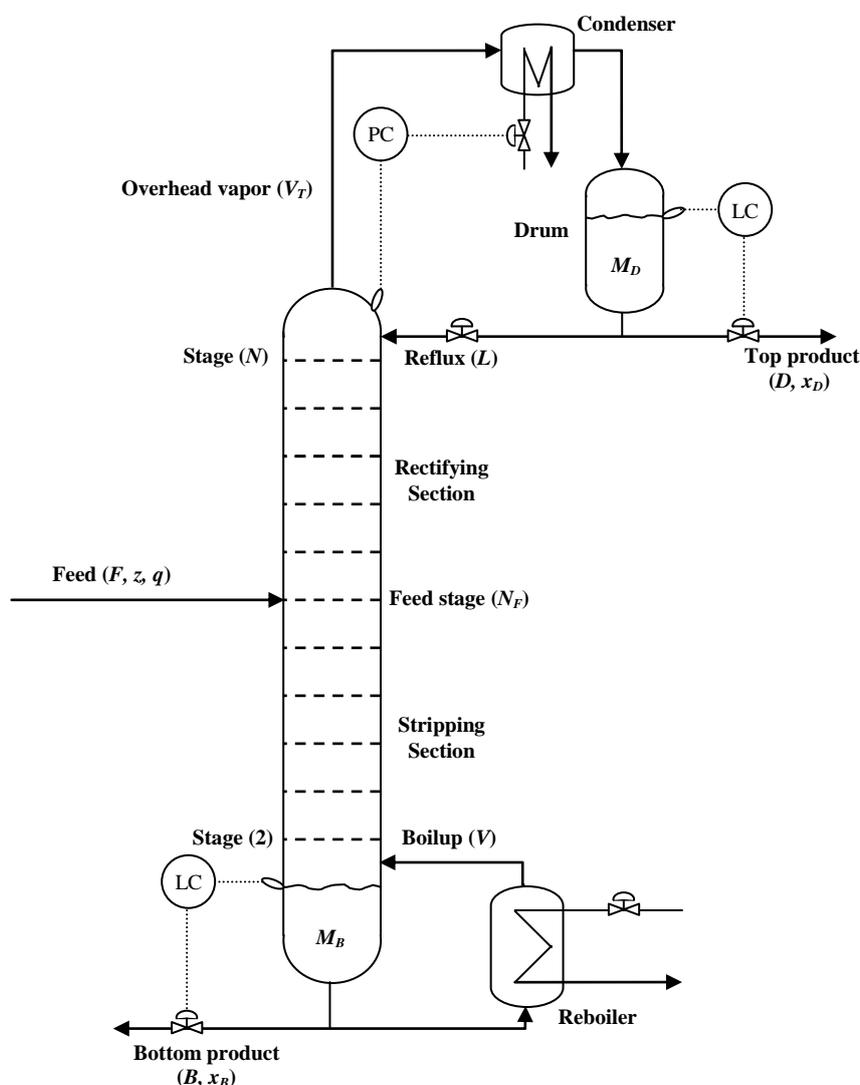


Figure 2. Control of continuous distillation column with LV-configuration.

## 2.2.LV-configuration

The LV-configuration consists using reflux  $L$  and boilup  $V$ , which consider the conventional choice of controlling pressure with cooling  $V_T$ , top level with distillate flow  $D$ , and bottom level with bottom flow  $B$ , this gives rise to the energy balance structure, shown in Figure 2. The LV-configuration can be considered as the standard control structure for a dual composition control distillation. It was given this name because the reflux  $L$  and boilup  $V$  were the remaining independent variables to be used for composition control. In this control structure the liquid flow rate  $L$  and the vapor flow rate  $V$  are used as the control inputs. The objective of the controller is to maintain the product outputs of the purity  $x_D$  of the overhead distillate and the impurity  $x_B$  of the bottom despite the disturbance in the feed flow  $F$  and the feed mole fraction  $z$  [7] [12].

### 3. Dynamic modelling of distillation column

The methanol-water distillation column is designed with  $N=21$  trays. The feed is placed at tray 10. Table 2 summaries the initial calculated data for column. The model is simplified under assumptions in Skogestad<sup>[5]</sup>; Binary mixture, equilibrium and perfect mixing on all stages, pressure inside the column constant by use of controlling the cooling water, total condenser, constant relative volatility, constant molar flows, no vapor holdup on each tray, linear liquid dynamics and liquid holdup  $M_i$  on all trays constant.

The mathematical dynamic model was developed and presented below based on a model by Truong et.al.<sup>[8]</sup>. It was described by a large number of differential equations. This yields  $(N+1)^{th}$  order model with one ordinary differential equation on each tray ( $i=1, N+1$ ). These equations are valid for a tray without feed, side draw and external heating or cooling and can be formulated for the material balance of light component on each tray (composition dynamics) by the following equations:

Total condenser ( $i=N+1$ ):

$$M_D \dot{x}_i = V_{i-1} y_{i-1} - L_i x_i - D x_i \quad (10)$$

Each tray ( $i=N_F+2$  to  $N$ ):

$$M_i \dot{x}_i = L_{i+1} x_{i+1} + V_{i-1} y_{i-1} - L_i x_i - V_i y_i \quad (11)$$

Above feed location tray ( $i=N_F+1$ ):

$$M_i \dot{x}_i = L_{i+1} x_{i+1} + V_{i-1} y_{i-1} - L_i x_i - V_i y_i + F_V y_F \quad (12)$$

Below feed location tray ( $i=N_F$ ):

$$M_i \dot{x}_i = L_{i+1} x_{i+1} + V_{i-1} y_{i-1} - L_i x_i - V_i y_i + F_L x_F \quad (13)$$

Each tray ( $i=2$  to  $N_F-1$ ):

$$M_i \dot{x}_i = L_{i+1} x_{i+1} + V_{i-1} y_{i-1} - L_i x_i - V_i y_i \quad (14)$$

Reboiler ( $i=1$ ):

$$M_B \dot{x}_i = L_{i+1} x_{i+1} - V_i y_i - B x_i \quad (15)$$

Here  $M_D$ ,  $M_i$  and  $M_B$  are the liquid holdup on condenser, tray  $i$  and reboiler respectively.  $L_i$  and  $V_i$  are the liquid flow rate and vapor flow rate that go out from tray  $i$ . where  $x_i$  and  $y_i$  are the compositions of the light component on tray  $i$  at liquid and vapor phases respectively. Constant relative volatility throughout the column and the vapor-liquid equilibrium relation on each tray ( $i=1, N$ ) can be expressed as:

$$y_i = \alpha x_i / [1 + (\alpha - 1)x_i] \quad (16)$$

But at total condenser is:

$$y_D = x_{N+1} \quad (17)$$

Although the model is simplified, the representation of the distillation system is still nonlinear due to the vapor-liquid equilibrium relationship between  $y_i$  and  $x_i$  in equation (16). Flow rates assuming constant molar flows above feed ( $i > N_F$ ):

$$L_i = L, \quad V_i = V + F_V \quad (18)$$

where  $F_V$  is vapor feed flow rate, flow rates below feed ( $i \leq N_F$ ):

$$L_i = L + F_L, \quad V_i = V \quad (19)$$

$$F_L = qF, \quad F_V = F - F_L \quad (20)$$

where  $F_L$  is liquid feed flow rate, in which condenser and reboiler holdup constant:

$$D = V_N - L = V + F_V - L \quad (21)$$

$$B = L_2 - V_1 = L + F_L - V \quad (22)$$

Compositions  $x_F$  and  $y_F$  in the liquid and vapor phase of the feed are obtained by solving the flash equations:

$$FZ = F_L x_F + F_V y_F \quad (23)$$

$$y_F = \alpha x_F / [1 + (\alpha - 1)x_F] \quad (24)$$

The simplified model for liquid flow dynamics with negligible vapor holdup are decoupled from the dynamic composition, which the formula for the liquid holdup on tray ( $i=N_F+1$  to  $N$ ) is:

$$L_i = L_0 + (M_i - M_{0i})/\tau + (V_{i-1} - V_{0T})\lambda \quad (25)$$

and for tray ( $i=2$  to  $N_F$ ) is:

$$L_i = L_{0B} + (M_i - M_{0i})/\tau + (V_{i-1} - V_0)\lambda \quad (26)$$

where  $L_0$  is the initial reflux flow,  $M_{0i}$  is the initial reboiler holdup (kmol) on tray  $i$ ,  $\tau$  is the time constant for liquid dynamics. In this model it is chosen to be 0.063 min, and  $\lambda$  represents the effect of vapor flow on liquid flow which it ignore by setting  $\lambda=0$ .  $L_{0B}$  is the initial liquid flow below feed, given by the formula:

$$L_{0B} = L_0 + q_0 F_0 \quad (20)$$

The initial feed rate  $F_0$  is taken 1 kmol/min and  $q_0 = 1$  is the initial liquid feed fraction.

#### **4. Dynamic simulation and discussion**

The dynamics behavior of the column is simulated by using MATLAB programme to study product composition response and test effect of disturbance which takes place in feed. It plays important rule in controlling the column. Firstly, a steady state is reached by simulating the column for 30000 minutes based on design data as shown in Table 2.

The tests were carried out to examine the dynamic responses behavior of compositions due to the different disturbances. After the column obtains steady state, the level controls of distillate and bottom drums are done that by manipulating the distillate ( $D$ ) and bottom ( $B$ ) flow rates. The constant level drums obtain by use of proportional control with gain equal to 10.

##### **4.1. The effect of feed flow**

Figure 3 shows the response in product composition to a 0.5% increase in feed rate  $F$  from 1 to 1.005 kmol/min. This work agrees with Skogestad<sup>[13]</sup>, when feed flow rate increased. It goes down to the bottom of the column, and then the action of the level controller (bottom) corresponds with an increase of the bottom flow rate. This action gives a large effect on the bottom product composition, and drifts quite far away (from 0.010 to about 0.021). On the other hand, there is a very small increase in the top product composition ranging (from 0.9998 to about 0.99981). Figure 3 illustrates the response in product composition to a 0.5% decrease in feed rate  $F$  from 1 to 0.995 kmol/min. The response behavior is an inverse response in  $\Delta x_D$  and  $\Delta x_B$ . The decrease of feed rate makes large effect on product compositions (distillate and bottom), where the distillate composition drops (from 0.9998 to 0.9995) and bottom composition drops (from 0.01 to 0.0009).

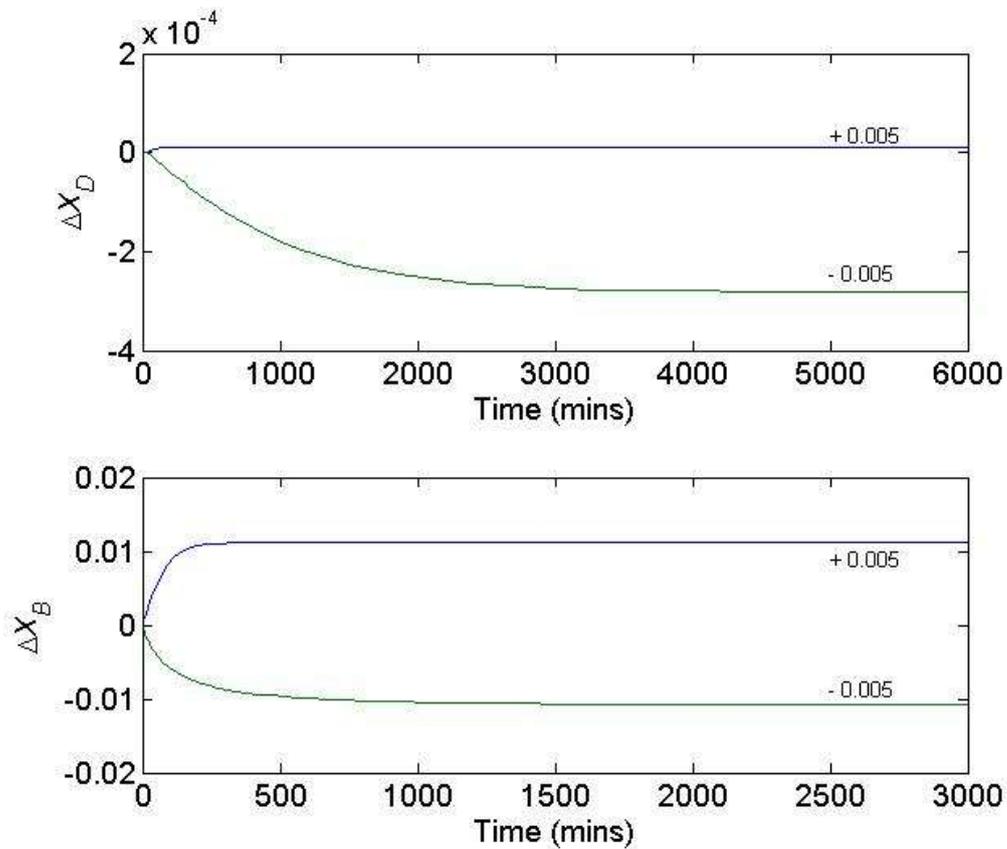


Figure 3. Dynamic composition responses to  $\pm 0.5\%$  step

change in feed flow rate.

Observably, from Figure 4a and Figure 4b, the different dynamic responses of change in distillate composition are accompanied by dead time at different feed flow rate perturbations. Figure 4c and Figure 4d are exhibited the different dynamic responses of change in bottom composition without dead time

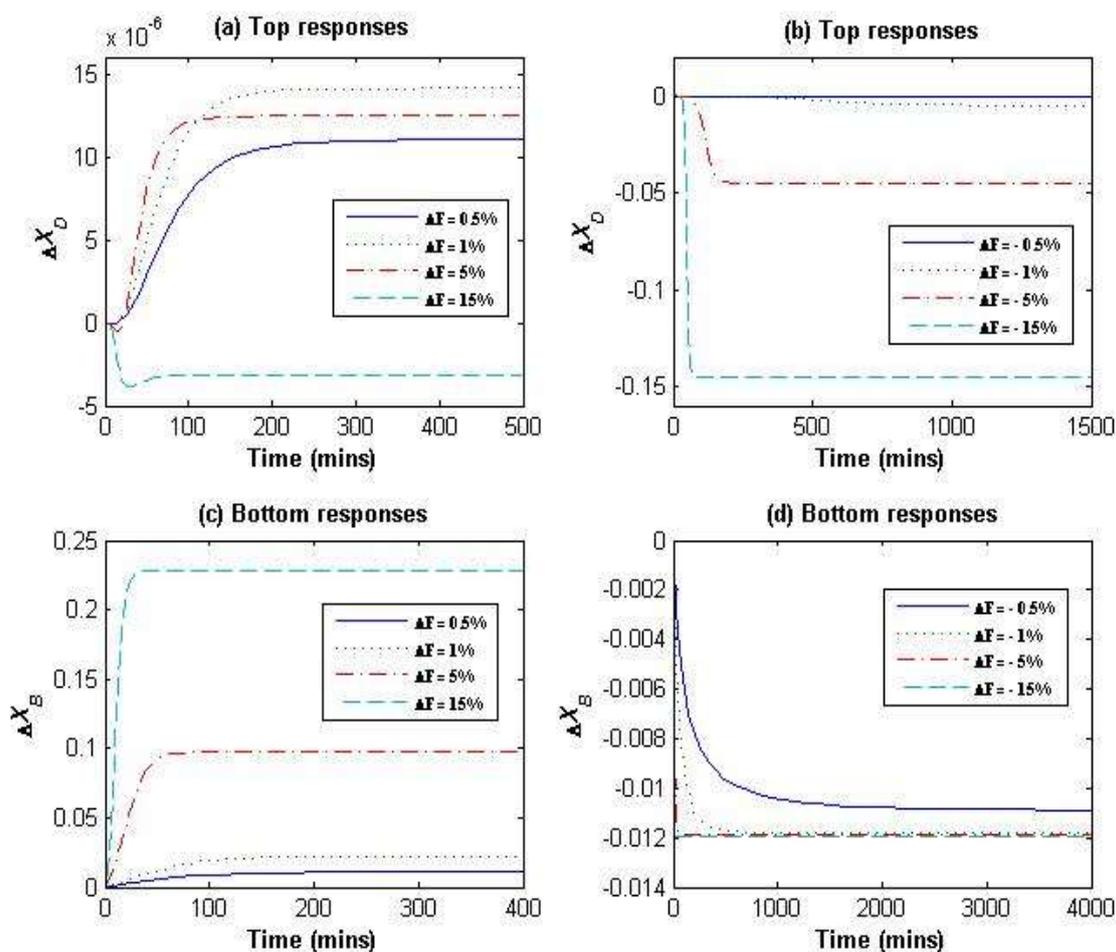


Figure 4. Dynamic composition responses to step change in feed flow rate.

For 0.5% increase in feed flow rate, the time constant of  $\Delta x_D$  response is about 75 min while dead time is about 14 min. The time constant of  $\Delta x_D$  response due to 0.5% decrease in feed flow rate is about 910 min with dead time about 23 min. These responses are slowest dynamic of the column where the time constants are large. The reason for that is the compositions inside the column may change significantly, while the change in product compositions is very small. The obtained result is consistent with work of Skogestad and Morari<sup>[14]</sup>.

Table 3. Time constants and dead times (in mins) of the dynamic composition responses due to different feed flow rate disturbances.

Step changes	- 15%	- 5%	- 1%	- 0.5%	0.5%	1%	5%	15%
$\Delta x_D$	39	110	567	910	75	66	26	-
Dead time of $\Delta x_D$	12	14	35	23	14	12	21	-
$\Delta x_B$	3	10	62	146	68	56	25	12

The time constants of  $\Delta x_D$  and  $\Delta x_B$  responses with the dead times are presented in Table 3. The responses in these cases are faster when an increment on the magnitude of the introduced step change is made (negative or positive change).

The behavior of composition responses resembles the response of a first order system with different time constants (the time it takes for the compositions to reach 63.2% of their steady-state change), which was corresponded to other works<sup>[8], [14], [15]</sup>. Exceptionally, the dynamic response behavior of  $\Delta x_D$  due to 15% increase in feed flow rate. It is an inverse response with overshoot like a second order system as shown in Figure 4a.

**4.2.The Effect of feed composition**

The responses of change in distillate composition are also accompanied by dead time for both positive and negative step changes in feed composition. These responses were illustrated in Figure 5a and Figure 5b. Figure 5c and Figure 5d display the bottom responses without dead time. All the responses are like the response of a first order system with different time constants.

The results of this simulation for the time constant and dead time were summarized in Table 4. The time constants vary as the magnitudes of the feed composition disturbances vary. Especially, Time constant is noted in negative step changes. These results were different when compared with work of Truong et al. [8]. The interpretation of these predicted dynamic behavior effects also takes place when small concentration changes from tray to tray, as purity increase due to highly nonlinear responses. With disturbances of these magnitudes, the response is completely different for a positive change than for a negative change. However, as purity increase, the dynamic responses (particularly to feed composition disturbances) begin to differ greatly for system. In other words, the disturbances in feed composition were felt very quickly in the bottom of the column in accordance with Fuentes and Luyben [15].

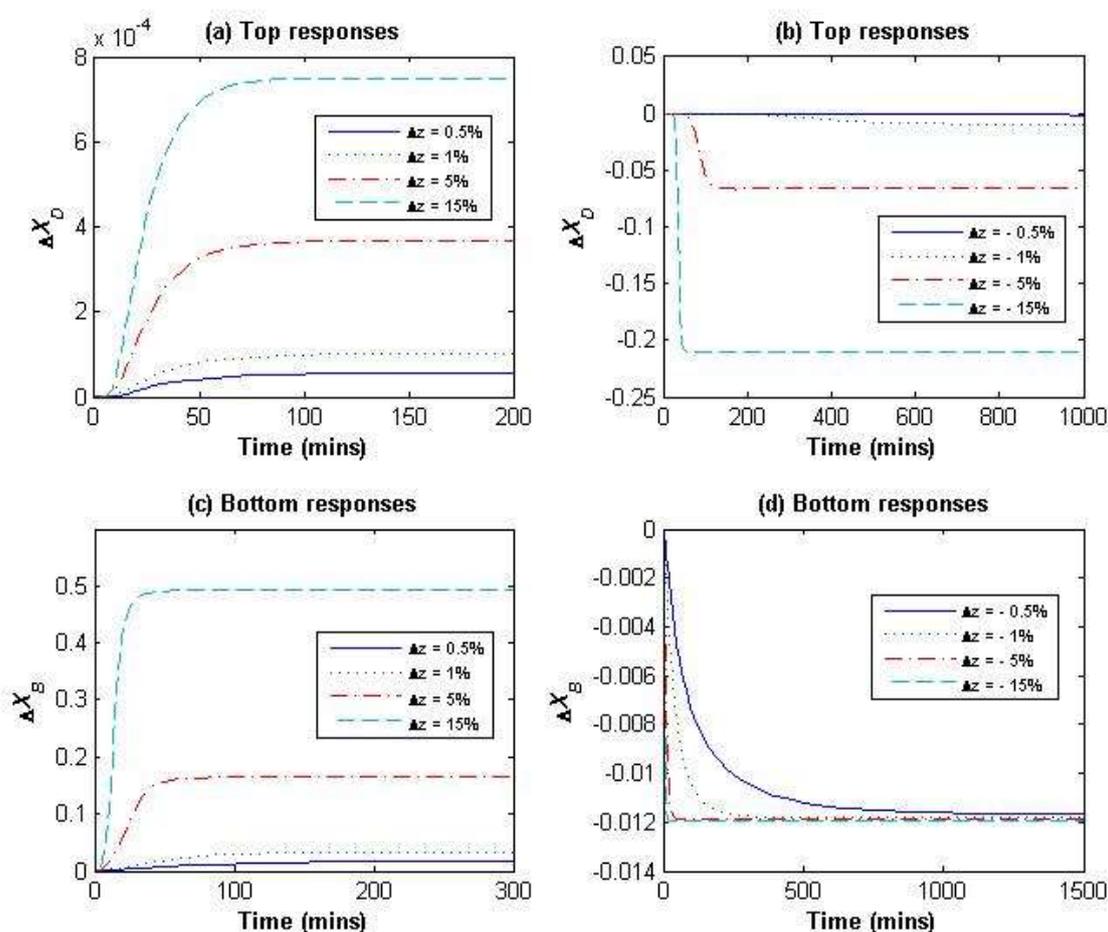


Figure 5. Dynamic composition responses to step change in feed composition.

Table 4. Time constants and dead times (in mins) of the dynamic composition responses due to different feed composition disturbances.

Step changes	- 15%	- 5%	- 1%	- 0.5%	0.5%	1%	5%	15%
$\Delta x_D$	35	89	237	381	34	31	25	24
Dead time of $\Delta x_D$	3	3	175	550	4	7	5	3
$\Delta x_B$	5	11	47	99	74	55	27	16

Note, the effect of feed composition disturbances is greater than the effect of feed flow rate disturbances, when comparing the steady-state gains between the dynamic response due to change in feed flow rate and the dynamic response due to change in feed composition.

#### 4.3. The effect of external flows

The two dynamic simulations of the column were done, as the reflux rate  $L$  is increase by 0.1% while keeping the boilup rate  $V$  constant (i.e. decrease  $D$  from 0.6971 to 0.6966). At steady-state,  $x_D$  increases from 0.9998 to about 0.99983 and  $x_B$  increases from 0.01 to about 0.0132. These dynamic responses resemble the response of a first order system with time constants about 31 min of  $\Delta x_D$  response and 79 min of  $\Delta x_B$  response.

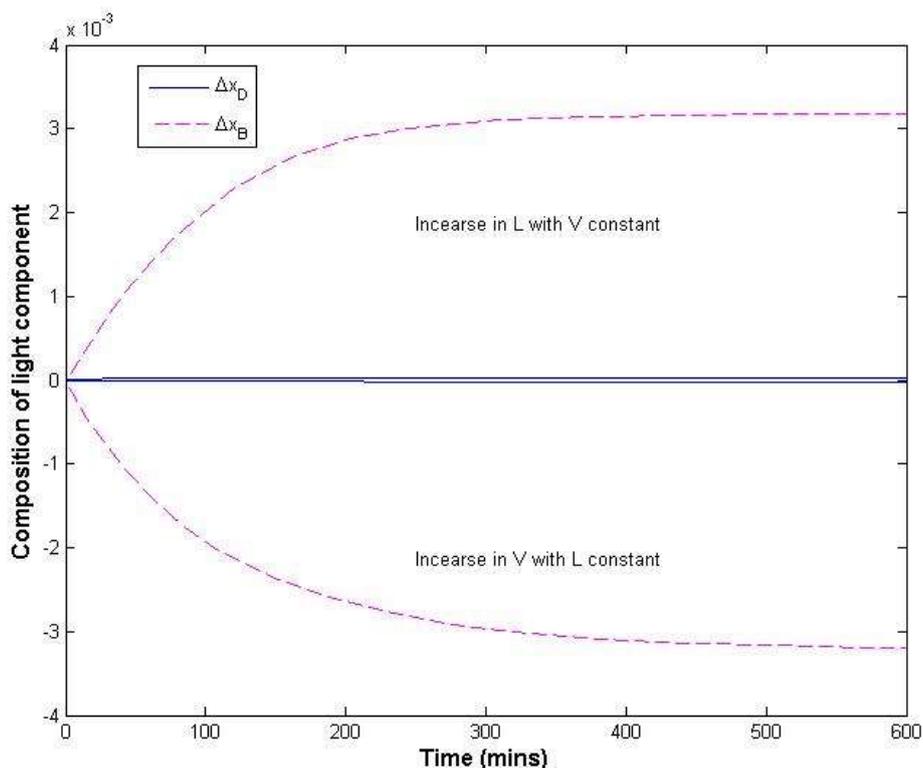


Figure 6. Dynamic composition responses to external flows changes: 0.1% individual increase in  $L$  and  $V$ .

Similarly, the boilup rate  $V$  was increased by the same amount while the reflux rate  $L$  was kept constant (i.e. increase  $D$  from 0.6971 to 0.6976). In this case, the dynamic responses got opposite direction with time constants about 64 min and 106 min respectively. These results were illustrated in Figure 6.

From the simulation results, the small changes in the external flows had large effects on the product compositions of the distillation column, which has been acknowledged by theoretical work of Truong et al. [8] and Skogestad [13].

#### 4.4. The effect of internal flows

Another important dynamic of the column was the effect of internal flow on the compositions of the products. Figure 7 illustrates the effect of changes in the internal flow. It was simulated by applying a  $\pm 10\%$  simultaneous step changes in reflux rate  $L$  and boilup rate  $V$  while kept distillate product  $D$  and bottom product  $B$  were kept constants.

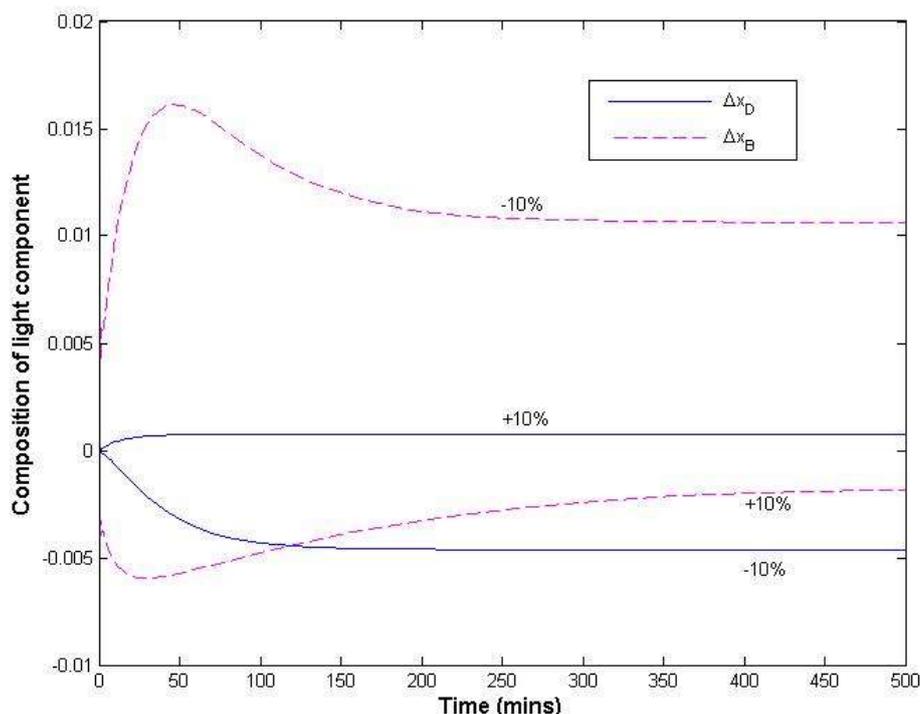


Figure 7. Dynamic composition responses to internal flow change:  $\pm 10\%$  simultaneous step change in both  $V$  and  $L$  with  $D$  and  $B$  constants.

From the plot, the time constant of  $\Delta x_D$  response at  $+10\%$  step change is close to 13 min with positive direction. Another  $\Delta x_D$  response at  $-10\%$  step change is close to 43 min with negative direction. The both composition responses were imitated the response of a first order system. On the other hand, the dynamic behaviors of  $\Delta x_B$  responses are different from the dynamic behaviors of previous  $\Delta x_D$  responses in directions. It has overshoot like a second order system as shown in Figure 7.

The simulation results also compare between the effects of internal flow and external flow. A small change in external flow gives large effect on the purity of product composition with same direction. A large change in internal flow makes the purity of product composition change with much faster dynamics in the different direction. This study agrees with the results introduced by works of Truong et al. [8] and Skogestad [13].

## 5. Conclusion

This research presents a methodology for design and dynamics of binary continuous distillation column. The result of increasing feed flow rate affects the product bottom composition largely while it has little effect on the product top composition. The inverse response behavior of decrease in feed flow rate gives large effect on both product compositions (distillate and bottom). The time constants vary so much when varying the magnitudes of the feed composition disturbances in negative step changes.

The effect of feed composition disturbances was greater than the effect of feed flow rate disturbances. The different dynamic responses of change in distillate composition were accompanied by dead time at different feed perturbations (flow rate and composition). The behaviors of composition responses resemble the response of a first order system with different time constants. Exceptionally, the response behavior of change in distillate composition may be like the response of a second order system, when the feed flow rate increases by 15%.

A small change in external flow gives large effects on the purity of product composition with same directions. A large change in internal flow makes the purity of product composition change with much faster dynamics in different directions and different behaviors.

**Nomenclature**

$B$	bottom product flow rate (kmol/min)
$D$	distillate (top) product flow rate (kmol/min)
$F$	feed flow rate (kmol/min)
$h$	heavy component and less volatile component (water)
$\Delta H^{vap}$	heat of vaporization (kJ/mol)
$\Delta \bar{H}^{vap}$	geometric average vaporization heat (kJ/mol)
$L$	reflux flow rate (kmol/min)
$l$	light component and more volatile component (methanol)
$M$	liquid holdup (kmol) ( $M_B$ reboiler holdup, $M_D$ condenser holdup)
$N$	number of theoretical trays including reboiler (N=21)
$N_T$	number of stages in the top section of distillation column
$N_B$	number of stages in the bottom section of distillation column
$N_{tot} = N + 1$	total number of stages (including total condenser)
$q$	fraction of liquid in feed ( $q=1$ )
$R$	gas law constant ( $R=8.314 \times 10^3 \text{ kg m}^2 \text{ s}^{-2} \text{ kmol}^{-1} \text{ K}^{-1}$ )
$S$	separation factor
$T_b$	boiling point temperature ( $^{\circ}\text{K}$ )
$\bar{T}_b$	geometric average boiling temperature ( $^{\circ}\text{K}$ )
$V = V_B$	boilup flow rate (kmol/min)
$x$	liquid composition of the light component (mol fraction)
$y$	vapor composition of the light component (mol fraction)
$z$	feed composition (mole fraction)
<i>Greek letter</i>	
$\alpha = \alpha_{lh}$	relative volatility between light and heavy component
$\lambda$	parameter for the effect of vapor flow on liquid
$\tau$	time constant for liquid flow dynamics (min)

*Subscripts*

$B$	bottom product
$D$	distillate product
$F$	feed position
$h$	heavy component
$i$	tray number
$l$	light component
$L$	liquid
$V$	vapor

**References**

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