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# Optimal Model Reference Control Scheme Design for Nonlinear Strict-Feedback Systems

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K E Y W O R D S

Model Reference, Most Valuable Player Algorithm, Optimal Control, Strict-Feedback, Variable Transformation Technique.

A B S T R A C T

*In this paper, a new design of the model reference control scheme is proposed in a class of nonlinear strict-feedback system. First, the system is analyzed using Lyapunov stability analysis. Next, a model reference is used to improve system performance. Then, the Integral Square Error (ISE) is considered as a cost function to drive the error between the reference model and the system to zero. After that, a powerful metaheuristic optimization method is used to optimize the parameters of the proposed controller. Finally, the results show that the proposed controller can effectively compensate for the strictly-feedback nonlinear system with more desirable performance.*

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## 1. Introduction

Most of the actual systems are inherently nonlinear in nature. However, some of the systems are analyzed to be acting as a linear system in the neighborhood of an operating point under some considerations. On the other hand, many physical processes are actually nonlinear models like coulomb friction, gravitational, etc. Recently, many research works were presented in terms of optimal model reference control which is considered an important topic that widely used in many fields of control systems. It has a key feature that it explicitly computes the error between the reference command signals and the corresponding plant outputs. This leads to a control structure with an integral term that renders itself or by using methods (adaptive/optimal) to find elegant and optimal parameters for the controller to the implemented problems. The strict-feedback system is a class of nonlinear systems that has a wide applicable area such as chaotic systems that need to be controlled using a single controller also applied for Duffing oscillator, Van Der Pol oscillator, Rossler system, and Chua's circuit [1]. Many contributions have been done in terms of optimal control design. An optimal control strategy was proposed by Sharma and Tewari [2] using Hamilton–Jacobian (H-J) equation for tracking maneuvers of rigid asymmetric spacecraft. Çimen and Banks [3] proposed optimal tracking controllers based on recursive approximation theory for a general class of nonlinear systems. The adaptive control method was applied by Luo et al. [4] for attitude tracking control problems of a rigid spacecraft with external disturbances and an uncertain inertia matrix. A tracking control concept for boom cranes was suggested by Neupert et al. [5] to make exact trajectory tracking behavior. Krishna et al [6] proposed a Model Reference Adaptive Control (MRAC) using MIT rule applied to a Single Input Single Output (SISO) nonlinear process spherical tank system. A Robust Model Reference Adaptive Control (MRAC) was designed by Babu [7] for the Single Input Single Output (SISO) spherical tank level control system with performance problem Coefficient Diagram Method (CDM). The Linear Quadratic Tracking (LQT) control of a UAV was suggested by Suicmez and Kutay [8] using the difference Riccati equation. Dias et al. designed [9] a sliding mode controller associated with the Variable Structure Model Reference Adaptive Controller (VS-MRAC) decouple the dynamic behavior of the joints. A nonlinear tracking controller based on Lyapunov stability was proposed by Tofigh et al. [10] for a novel aerial robot. Ali et al. [11] proposed an optimal nonlinear model reference controller for ball and plate systems. A transformation-based adaptive tracking control was proposed by Zhao et al [12] for uncertain strict-feedback nonlinear systems.

In this paper, a new nonlinear tracking control scheme is proposed to solve a class of nonlinear strict-feedback system problems. Generally, the strict-feedback problem is solved using the Backstepping technique. However, it is hard to use the Backstepping technique in designing when it comes to some kinds of nonlinear systems. Therefore, the strict-feedback problem will be solved using another technique called variable transformation technique. The system is analyzed using the Lyapunov quadratic function to derive the nonlinear controller that makes the system asymptotically stable. The proposed controller is used to drive the error between the reference command and the corresponding plant output to zero. After that, the Integral Square Error (ISE) performance index is used with the MVPA to find the optimal parameters of the proposed controller.

The rest of the paper is organized as follows. In section 2, the Most Valuable Player Algorithm (MVPA) is introduced as a metaheuristic optimization method. In section 3, the control design is given, and the necessary assumptions are required to analyze the research problem. In section 4, an illustrative example is given to demonstrate the effectiveness of the proposed control scheme. In section 5, the conclusion will be presented.

## 2. The Most Valuable Player Algorithm (MVPA)

The Most Valuable Player Algorithm (MVPA) is a sport-based optimization method. MVPA is inspired by the leagues and championships in sports where the players are competing within the same team and/or with other team players in order to be the best and win the MVP trophy. Also, there is an MVP trophy for the best team at the end of each league. The variables in this method represent the skills of the players. A team is a number of players are grouped together. Players and teams are represented in eqs. (1) and (2) respectively. An example of two players with the corresponding level of skills for each one as shown in Figure 1 [14]:

$$Player_k = [S_{k,1} \quad S_{k,2} \quad \cdot \quad \cdot \quad S_{k,ProblemSize}] \tag{1}$$

$$Team_i = \begin{bmatrix} S_{1.1} & S_{1.2} & \cdots & S_{1,ProblemSize} \\ S_{2.1} & S_{2.2} & \cdots & S_{1,ProblemSize} \\ \vdots & \vdots & \cdots & \vdots \\ S_{PlayerSize,1} & S_{PlayerSize,2} & \cdots & S_{PlayerSize,ProblemSize} \end{bmatrix} \tag{2}$$

Where $PlayersSize$ is the number of players in the league, $ProblemSize$ is the dimension of the problem and $S$ stands for skill. Each team has a franchise player (*i.e.* the best player for a sport team) and the league's MVP is the best player of the league.

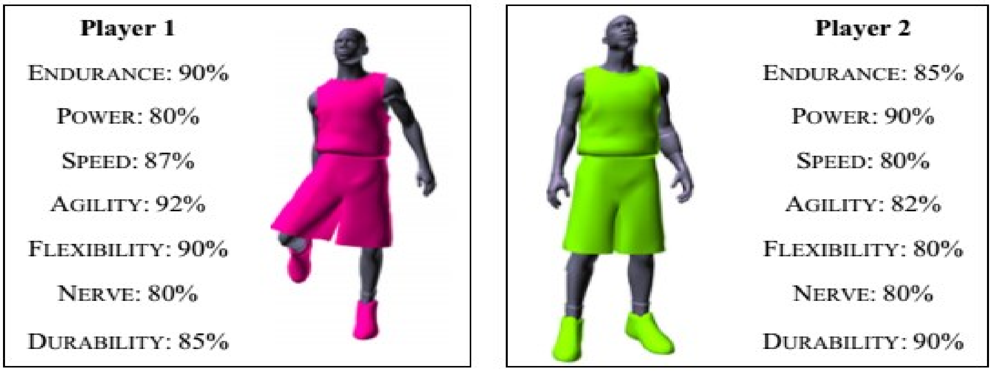


**Figure 1: Example of two players with their skills**

The following explains in detail the main phases of the MVPA:
1. Initialization; players and teams are randomly generated in the search space.
2. Team formation; the teams are named as '$nT_1$' and '$nT_2$' are first team and second-team respectively. Also, the players are named such as '$nP_1$' and '$nP_2$' are the players of the first and second team respectively. These variables are calculated as follows [14]:

$$nP_1 = ceil\left(\frac{PlayersSize}{TeamsSize}\right) \quad (3)$$

$$nP_2 = nP_1 + 1 \quad (4)$$

$$nT_1 = PlayersSize - nP_2 \times TeamsSize \quad (5)$$

$$nT_2 = TeamsSize - nT_1 \quad (6)$$

3. Team competition; players are debating each other individually to find the best player with the best skills. This competition is calculated by the following expressions [14]:

$$TEAM_i = TEAM_i + rand\left(FranchisePlayer_i - TEAM_i\right) + 2 \times rand\left(MVP - TEAM_i\right) \quad (7)$$

4. Application of greediness; a new solution is selected after the comparison of the population is done. Each selection is made based on better objective function value.
5. Application of elitism; the best (elite) players are selected and the other players are replaced with the better ones.
6. Remove duplicates; if the best players are selected twice. Then, one of them is dropped.
7. Termination criterion; in the MVPA, this criterion is an option implemented by the user himself or the number of the iterations will be the termination criterion [14].

The reason behind using MVPA is that the method is converging faster after compared with 13 well-known optimization methods, including Genetic Algorithm (GA), Particle Swarm Optimization (PSO), …, etc [14]. The aforementioned phases illustrate the MVPA calculations to find the optimal parameters. The following explains how this method is working on the system in this paper. First, assign the number of teams, players, and the problem dimensions which are the parameters of the controller that need to be optimized. These parameters represent the skills of a player in the MVPA. Next, assign the objective function which represents the cost function used in this paper (Integral Square Error). Then, the players gained skills (parameters) throughout the phases mentioned previously which are embodied in Matlab code (m-files). After that, the pre-optimized parameters are calculated and applied simultaneously in the controlled system to calculate the measured error. Consequently, the measured error used to find the cost function and compared to the previous cost at each iteration in order to obtain the best cost and then the optimal parameters. Finally, this process is repeated until the optimal parameters are obtained after a certain number of iterations. Figure 2 shows the process of how the MVPA calculates the optimal parameters and then applied it to the controller.

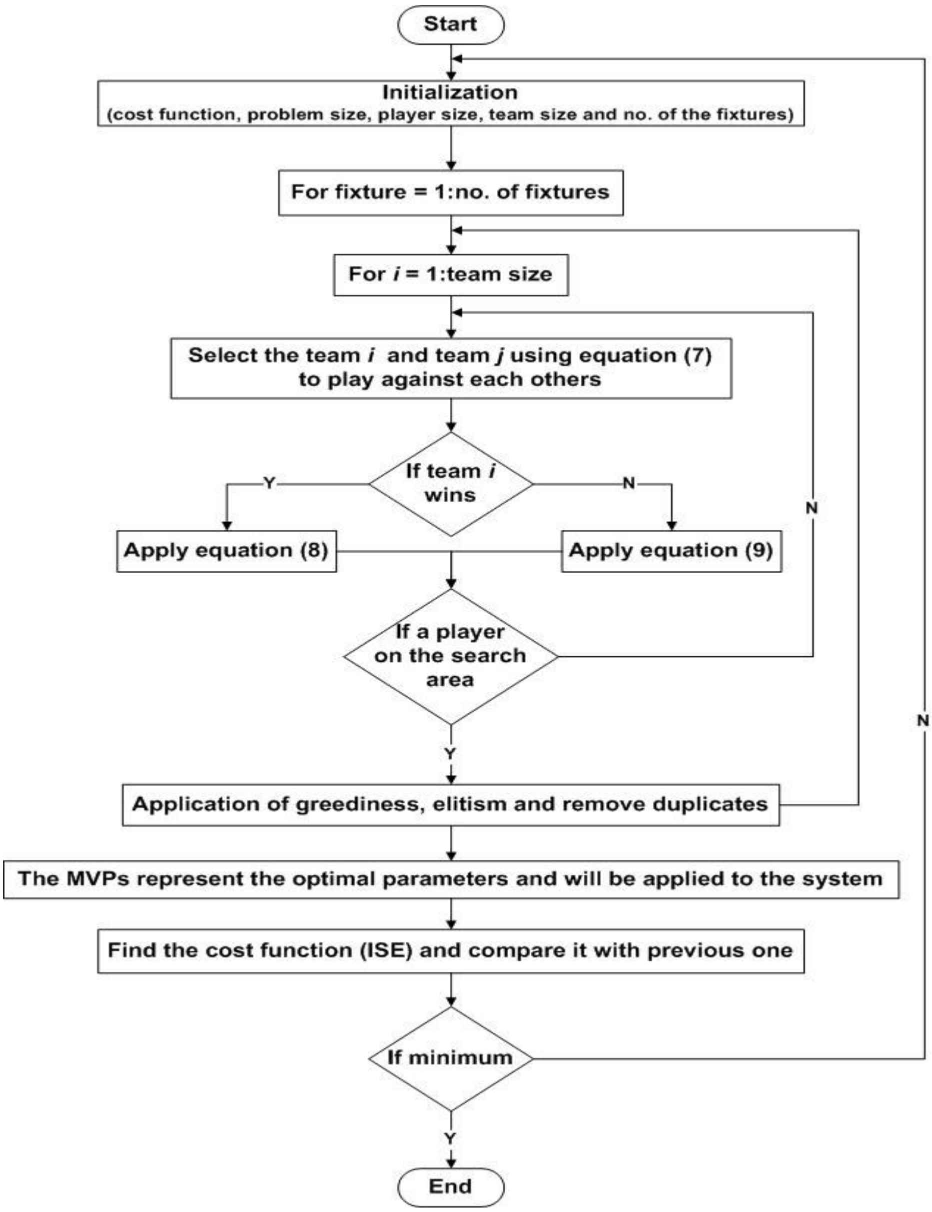


**Figure 2: The flowchart of MVPA with the system**

## 3. Controller Design

Consider the following strict-feedback nonlinear systems with time-varying [15]:

$$\dot{x}_i(t) = f_i(\bar{x}_i(t)) + g_i(\bar{x}_i(t))\bar{x}_{i+1}(t), \qquad i = 1,2,\dots,n-1$$
$$\dot{x}_n(t) = f_n(\bar{x}_n(t)) + g_n(\bar{x}_n(t))u(t) \quad y(t) = x_1(t) \qquad (8)$$

Where $\bar{x}_i(t) = (x_1(t),\cdots,x_i(t))^T \in \mathcal{R}^i$, $u(t) \in \mathcal{R}$ is the system states, control input and $y(t) \in \mathcal{R}$ is the system output, respectively. In addition, $f_i(\cdot): \mathcal{R}^i \rightarrow \mathcal{R}$ $(i = 1,\cdots,n)$ is satisfying the condition: $f_i(\bar{x}_i) = \theta^T \varphi_i(\bar{x}_i)$, where $\varphi_1,\cdots,\varphi_n$ are smooth functions and the vector $\theta$ satisfying $\theta \in \Omega_\theta = \{\theta \in \mathcal{R}^+\}$. Also, $g_i(\cdot): \mathcal{R}^i \rightarrow \mathcal{R}$ are known as smooth functions.

In this work, the proposed controller makes the system output tracks the reference model in order to converge to achieve the desired performance. To facilitate the discussion of the main results, the following assumptions and theorems are made [16, 17].

**Assumption 1**. The functions $g_i(\cdot): \mathcal{R}^i \rightarrow \mathcal{R}$ are known, and there exists a positive constant $g_0$ such that $0 < g_0 < |g_i(\cdot)|$; also, $g_i(\cdot)$ are all positive.

**Assumption 2.** There exist positive constants $Y_i$ $(i = 1,\cdots,n)$ such that the reference signal $y_r(t)$and its time derivatives satisfy $|y_r(t)| \leq Y_0 \leq k_{c1}$ & $\left|y_r^{(i)}(t)\right| \leq Y_i, i = 1,\cdots,n, \forall t \geq 0$ . In another words, the model reference trajectory $y_r(t)$ should be continuous and differentiable function up to the $n^{th}$ order.

**Theorem 1.** The second Lyapunov method is used to study the stability of a system $\dot{x} = f(x)$ without solving. It has a certain form with some conditions:

1. If there exists a function $V: \mathcal{R}^n \to \mathcal{R}$ that satisfies some conditions on $V$ and $\dot{V}$.
2. Then, the trajectories of the system satisfy some property.

**Theorem 2.** The system is globally and asymptotically stable (G.A.S.) in the sense of Lyapunov if there is a function $V$ called Lyapunov function such that:

3 . $V$ is positive definite (+ve def.).

4 . $\dot{V}(z) < 0\ \forall z \neq 0$ and $\dot{V}(0) = 0$.

Then, the system is globally asymptotically stable (G.A.S.).

In this work, the reference model is selected as a second-order system trajectory in order to improve the actual system performance by tracking this reference. In addition, the Lyapunov quadratic function is used to analyze the stability of the system as follows after assuming the reference model equation to be [18]:

$$\dot{z}_d = Az_d + Bv \tag{9}$$

$$\begin{bmatrix} \dot{z}_{1d} \\ \dot{z}_{2d} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\omega_n^2 & -2\zeta\omega_n \end{bmatrix} \begin{bmatrix} z_{1d} \\ z_{2d} \end{bmatrix} + \begin{bmatrix} 0 \\ \omega_n^2 \end{bmatrix} v \tag{10}$$

By defining the error vector and its derivative as:

$$e = z_d - z \tag{11}$$

$$\dot{e} = \dot{z}_d - \dot{z} \tag{12}$$

Hence

$$\dot{z} = f(z, u, t) \tag{13}$$

Then substituting eqs. (9) and (10) in eq. (12), gives

$$\dot{e} = Az_d + Bv - f(z,u,t)$$

$$\dot{e} = Az_d + Bv - f(z,u,t) + Az - Az$$

$$\dot{e} = A(z_d - z) + Az + Bv - f(z,u,t)$$

$$\dot{e} = Ae + Az + Bv - f(z,u,t) \tag{14}$$

By using Lyapunov quadratic function as:

$$V(e) = e^T P\, e \tag{15}$$

The time-derivative of Lyapunov quadratic function $\dot{V}(e)$ yields:

$$\dot{V}(e) = \dot{e}^T P\, e + e^T P\, \dot{e} \tag{16}$$

Substituting eq. (14) in Eq. (16) yields:

$$\dot{V}(e) = [Ae + Az + Bv - f(z,u,t)]^T\, P\, e + e^T P\, [Ae + Az + Bv - f(z,u,t)]$$

$$\dot{V}(e) = e^T(A^T P + PA)e + 2e^T P[Az - f(z,u,t) + Bv]$$

$$\dot{V}(e) = -e^T Q e + 2M \tag{17}$$

with

$$M = e^T P[Az + Bv - f(z,u,t)] \tag{18}$$

Where $Q$ and $P$ are Positive definite matrix.

A proper controller can be selected to compensate for the effect of the nonlinear terms in $f(z,u,t)$. Therefore, the bound of the term $M$ is decreasing, which makes the level of the surface of $\dot{V}(e)$ shrinking toward the origin. Therefore, the system becomes asymptotically stable based on theorems 2.

## 4. Illustrative Example

Consider the following strict-feedback system [19]:

$$\dot{x}_1 = x_2 + \theta_1 \cos(x_1) + \theta_2 x_1 + \theta_2 \cos(x_1)$$

$$\dot{x}_2 = u + \theta_1 \cos(x_1) + \theta_2 x_1 + \theta_2 \cos(x_2)$$

$$y = x_1 \tag{19}$$

Figure 3 shows the closed-loop response before applying the proposed controller for $v = e^{-0.1t} sin(t)$. There is a large error gap between the output and the input due to the nonlinear effects which mean the system has performance issues.

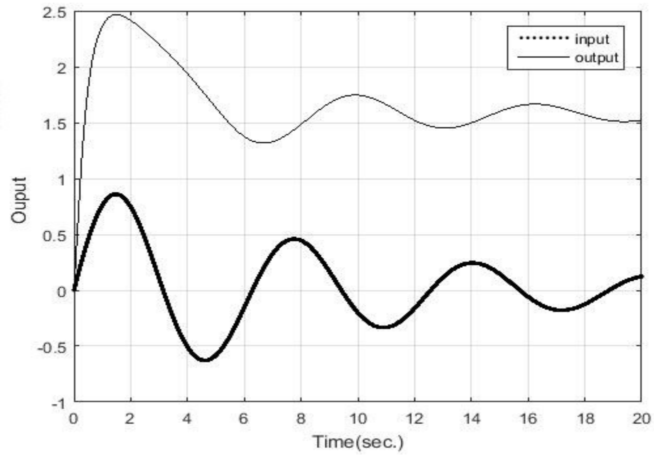


**Figure 3: System closed-loop response before applying the controller**

As long as the system is strict-feedback, the controller cannot compensate for the nonlinear terms in the first state. Consequently, Backstepping is limited to some kinds of nonlinear systems [13]. Therefore, the nonlinear controller can be designed based on change variables to make the control action reaches all the states as follows:

Let $$z_1 = x_1 \quad (20) \quad \dot{z}_1 = \dot{x}_1 = z_2$$

$$\dot{z}_1 = x_2 + \theta_1 \cos(x_1) + \theta_2 x_1 + \theta_2 \cos(x_1) \quad (21)$$

For $$x_2 = z_2 - \theta_1 \cos(z_1) - \theta_2 z_1 - \theta_2 \cos(z_1), z_1 = x_1, \dot{z}_1 = \dot{x}_1 \quad (22)$$

$$\dot{z}_2 = \ddot{x}_1 = \dot{x}_2 + \theta_1(\dot{x}_1)(-\sin(x_1)) + \theta_2 \dot{x}_1 + \theta_2(\dot{x}_1)(-\sin(x_1)) \quad (23)$$

Substituting Eqs. (19)-(21) in Eq. (23) gives:

$$\dot{z}_1 = z_2$$

$$\dot{z}_2 = u + \theta_1 \cos(z_1) + \theta_2 z_1 + \theta_2 \cos\begin{pmatrix} z_2 - \theta_1 \cos(z_1) - \\ \theta_2 z_1 - \theta_2 \cos(z_1) \end{pmatrix} - \theta_1 z_2 \sin(z_1)$$

$$+\theta_2 z_2 - \theta_2 z_2 \sin(z_1) \quad (24)$$

Let the bad terms equal to $F$ after adding/subtracting the terms $\pm 2\theta_2 z_{1,2}$ to assure the stability of the system as follows:

$$\begin{bmatrix} \dot{z_1} \\ \dot{z_2} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\theta_2 & -\theta_2 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u + \begin{bmatrix} 0 \\ 1 \end{bmatrix} F$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} \quad (25)$$

where $F = 2\theta_2 z_1 + 2\theta_2 z_2 - \theta_1 z_2 \sin(z_1) - \theta_2 z_2 \sin(z_1) + \theta_2 \cos\begin{pmatrix} z_2 - \theta_1 \cos(z_1) - \\ \theta_2 z_1 - \theta_2 \cos(z_1) \end{pmatrix}$

By substituting Eqs. (9), (10) and (13) in Eq. (18) as follow:

$$M = \begin{bmatrix} e_1 & e_2 \end{bmatrix} \begin{bmatrix} p_{11} & p_{12} \\ p_{12} & p_{22} \end{bmatrix} \left\{ \begin{matrix} \begin{bmatrix} 0 & 1 \\ -\omega_n^2 & -2\zeta\omega_n \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \omega_n^2 \end{bmatrix} \nu \\ - \begin{bmatrix} 0 & 1 \\ -\theta_2 & -\theta_2 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} - \begin{bmatrix} 0 \\ 1 \end{bmatrix} u - \begin{bmatrix} 0 \\ 1 \end{bmatrix} F \end{matrix} \right\}$$

$$= \begin{bmatrix} e_1 & e_2 \end{bmatrix} \begin{bmatrix} p_{11} & p_{12} \\ p_{12} & p_{22} \end{bmatrix} \left\{ \begin{matrix} \begin{bmatrix} 0 & 0 \\ -\omega_n^2 + \theta_2 & -2\zeta\omega_n + \theta_2 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} \\ + \begin{bmatrix} 0 \\ \omega_n^2 \nu - u - F \end{bmatrix} \end{matrix} \right\}$$

$$= \begin{bmatrix} e_1 & e_2 \end{bmatrix} \begin{bmatrix} p_{11} & p_{12} \\ p_{12} & p_{22} \end{bmatrix} \left\{ \begin{bmatrix} 0 \\ (-\omega_n^2 + \theta_2) z_1 + (-2\zeta\omega_n + \theta_2) z_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \omega_n^2 \nu - u - F \end{bmatrix} \right\}$$

$$= \left[ \begin{pmatrix} e_1 p_{11} \\ +e_2 p_{12} \end{pmatrix} \begin{pmatrix} e_1 p_{12} \\ +e_2 p_{22} \end{pmatrix} \right] \begin{bmatrix} 0 \\ \begin{pmatrix} -\omega_n^2 \\ +\theta_2 \end{pmatrix} z_1 + \begin{pmatrix} -2\zeta\omega_n \\ +\theta_2 \end{pmatrix} z_2 + \omega_n^2 \nu - u - F \end{bmatrix}$$

$$= \begin{pmatrix} e_1 p_{12} \\ +e_2 p_{22} \end{pmatrix} [\begin{pmatrix} -\omega_n^2 \\ +\theta_2 \end{pmatrix} z_1 + \begin{pmatrix} -2\zeta\omega_n \\ +\theta_2 \end{pmatrix} z_2 + \omega_n^2 \nu - u - F]$$

Let $u = \begin{pmatrix} -\omega_n^2 \\ +\theta_2 \end{pmatrix} z_1 + \begin{pmatrix} -2\zeta\omega_n \\ +\theta_2 \end{pmatrix} z_2 + \omega_n^2 \nu - F - \theta_1 z_1^2 + \theta_2 z_1^2 sign \begin{pmatrix} e_1 p_{12} \\ +e_2 p_{22} \end{pmatrix} - \begin{pmatrix} k_d z_1^2 \\ +k_b z_2^2 \end{pmatrix}^2$ (26)

Then $M = \begin{pmatrix} e_1 p_{12} \\ +e_2 p_{22} \end{pmatrix} \left[ \left( \theta_2 - \theta_1 sign \begin{pmatrix} e_1 p_{12} \\ +e_2 p_{22} \end{pmatrix} \right) z_1^2 + \begin{pmatrix} k_d z_1^2 \\ +k_b z_2^2 \end{pmatrix}^2 \right]$ (27)

By substituting Eq. (27) in Eq. (17) as follow:

$$\dot{V}(e) = -\begin{pmatrix} q_{12} e_1^2 \\ +q_{22} e_2^2 \end{pmatrix} + 2 \begin{pmatrix} e_1 p_{12} \\ +e_2 p_{22} \end{pmatrix} \left[ \theta_2 - \theta_1 sign \begin{pmatrix} e_1 p_{12} \\ +e_2 p_{22} \end{pmatrix} \right] z_1^2 + \begin{pmatrix} k_d z_1^2 \\ +k_b z_2^2 \end{pmatrix}^2 \quad (28)$$

In this work, the second-order model reference parameters are chosen such as $\omega_n = 5.477225575\ rad/sec$ and $\zeta = 0.8215838363$ which represent the undamped natural frequency and the damping ratio respectively. Based on the aforementioned analysis, the nonlinear controller $(u)$ can overcome the nonlinearity effects. In addition, the Integral Square of Error performance index (ISE) is used. It is expressed as [20]:

$$J = \int_0^t e^2(t)\, dt \quad (29)$$

Where $e(t)$ represents the difference between the model reference output and the system output.
The Most Value Player Algorithm settings are considered to give a fast convergence as follow: $LowerBound = 0.64234, UpperBound = 3.00882, ProblemSize\ 6, PlayerSize = 10,$ $TeamSize = 5$ and $Iterations = 50$. The parameters need to be optimized are: $P_{12}$, $P_{22}$, $\theta_1$, $\theta_2, k_b$ and $k_d$. The resulting optimal parameters are: $P_{12} = 3.0087, P_{22} = 3.0087$, $\theta_1 = 3.0087$, $\theta_2 = 1.0307$, $k_b = 0.6616$ and $k_d = 1.0929$. The block diagram of the nonlinear system with model reference control has shown in Figure 4. Figure 5 shows the nonlinear system state trajectories. It has shown that the proposed controller can stabilize the proposed can stabilize the system efficiently.

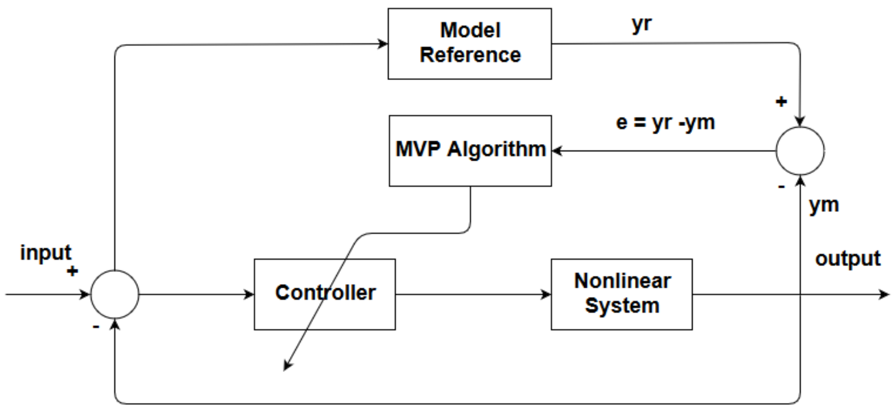


**Figure 4: The block diagram of the overall system with MVP algorithm**

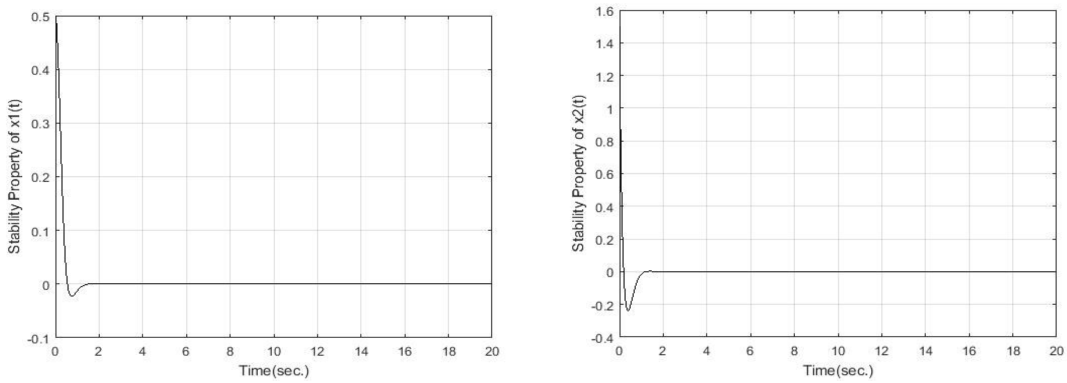


**Figure 5: Stabilization properties of the system states**

Figures 6 and 7 show the time response properties of the system states and output of the controlled system subjected to $\boldsymbol{v} = \boldsymbol{e^{-0.1t} sin(t)}$. Consequently, the proposed controller can achieve more desirable tracking which is clear by comparing Figure 3 and Figure 7.

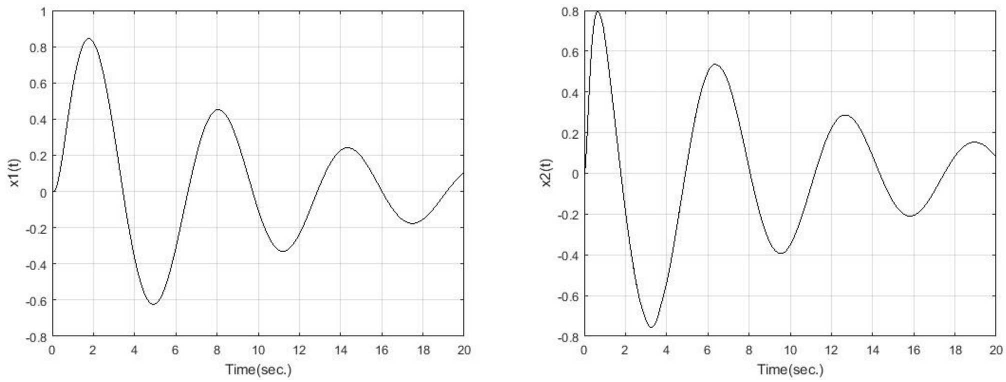


**Figure 6: System states trajectories**

Figure 8 shows the time-derivative of Lyapunov function $\left(\dot{V}(e)\right)$ which is converged to zero when time increases thereby the system is asymptotically stable in the sense of Lyapunov.

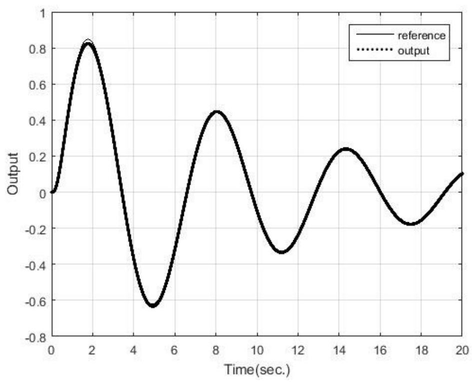


**Figure 7: The time response properties of the controlled system**

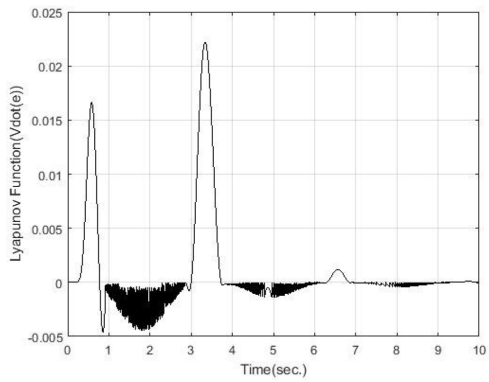


**Figure 8: The time-derivative of Lyapunov function**

In addition, it can be seen in Figure 9 that the resulting control action is low which is preferable for many practical issues. Finally, Figure 10 shows the cost function convergence properties for the MVP algorithm. It has shown that the number of iterations equals to 15 is fair enough to satisfy the required minimization.

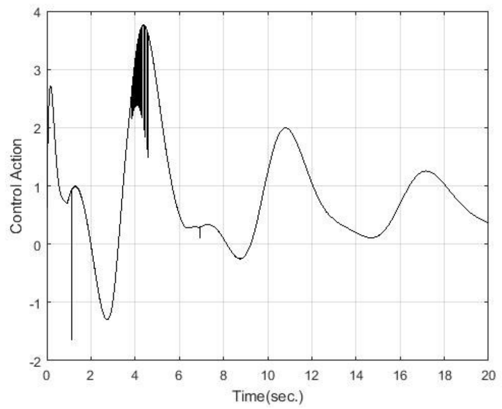


**Figure 9: The resulting control action**

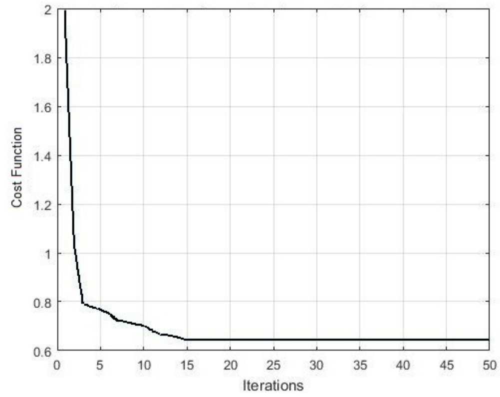


**Figure 10: Convergence properties**

Figure 11 shows the disturbance rejection properties of the controller with an excessive disturbance with different percentages (d=0, 10%, 20%, and 30%) of the actual input. It has shown that the proposed controller can effectively reject the disturbance and achieve the disturbance rejection properties.

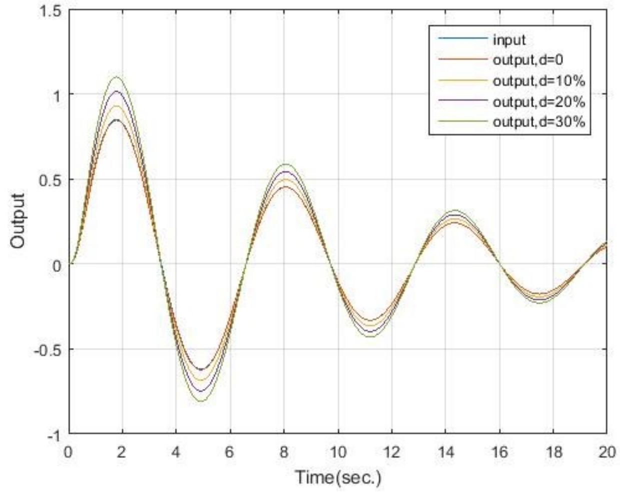


**Figure 11: The time response properties of the controlled system under an excessive disturbance effect**

## 5. Conclusion

In this paper, the optimal model reference control scheme design for a nonlinear system was proposed to compensate for the strictly-feedback system performance. The nonlinear system has been analyzed based on the Lyapunov quadratic function. The analysis has been done after changing the variables to make the controller be able to control all states. The nonlinear control scheme was subjected to a powerful metaheuristic optimization method (MVPA) which is inspired by sport to find optimum values for the parameters of the nonlinear control. Therefore, a desirable performance with optimum parameters of the controller has been achieved. Finally, the proposed controller has been successively improved the stability and the performance of the system. In addition, the optimal parameters have been obtained by using the MVPA. Also, the disturbance rejection of the controlled system has been achieved after an excessive disturbance is applied to the system. Moreover, the output results are preferable and applicable to many physical nonlinear systems.

## References

[1].S. Ge, T. Lee and C. Wang, "Adaptive backstepping control of a class of chaotic systems," *Proceedings of the 38th IEEE Conference on Decision and Control*, 2000. Available: 10.1109/cdc.1999.832872.

[2].R. Sharma and A. Tewari, "Optimal Nonlinear Tracking of Spacecraft Attitude Maneuvers," *IEEE Transactions on Control Systems Technology*, vol. 12, no. 5, pp. 677-682, 2004. Available: 10.1109/tcst.2004.825060.

[3].T. Çimen and S. Banks, "Nonlinear optimal tracking control with application to super-tankers for autopilot design," *Automatica*, vol. 40, no. 11, pp. 1845-1863, 2004. Available: 10.1016/j.automatica.2004.05.015.

[4].W. Luo, Y. Chu and K. Ling, "Inverse optimal adaptive control for attitude tracking of spacecraft," *IEEE Transactions on Automatic Control*, vol. 50, no. 11, pp. 1639-1654, 2005. Available: 10.1109/tac.2005.858694.

[5].J. Neupert, E. Arnold, K. Schneider and O. Sawodny, "Tracking and anti-sway control for boom cranes," *Control Engineering Practice*, vol. 18, no. 1, pp. 31-44, 2010. Available: 10.1016/j.conengprac.2009.08.003.

[6].M. Bharathi, C. Selvakumar and A. Kalpana, "Model based Controller Design for a Spherical Tank", *IOSR Journal of Electrical and Electronics Engineering*, vol. 9, no. 2, pp. 74-79, 2014. Available: 10.9790/1676-09267479.

[7].R. Babu, "Design and Implementation of Model Reference Adaptive Controller using Coefficient Diagram Method for a nonlinear process," *IOSR Journal of Electrical and Electronics Engineering*, vol. 6, no. 4, pp. 70-76, 2013. Available: 10.9790/1676-0647076.

[8].E. Suicmez and A. Kutay, "Optimal path tracking control of a quadrotor UAV," *2014 International Conference on Unmanned Aircraft Systems (ICUAS)*, 2014. Available: 10.1109/icuas.2014.6842246.

[9].S. Dias, K. Queiroz, A. Araujo and A. Dias, "Robust control of robotic manipulators based on left inverse system and variable structure model reference adaptive control," *International Journal of Adaptive Control and Signal Processing*, vol. 30, no. 8-10, pp. 1389-1407, 2016. Available: 10.1002/acs.2663.

[10].M. Tofigh, M. Mahjoob and M. Ayati, "Dynamic modeling and nonlinear tracking control of a novel modified quadrotor," *International Journal of Robust and Nonlinear Control*, vol. 28, no. 2, pp. 552-567, 2017. Available: 10.1002/rnc.3885.

[11].H. Ali, H. Jassim and A. Hasan, "Optimal Nonlinear Model Reference Controller Design for Ball and Plate System," *Arabian Journal for Science and Engineering*, vol. 44, no. 8, pp. 6757-6768, 2018. Available: 10.1007/s13369-018-3616-1.

[12].K. Zhao, Y. Song and L. Chen, "Tracking control of nonlinear systems with improved performance via transformational approach," *International Journal of Robust and Nonlinear Control*, vol. 29, no. 6, pp. 1789-1806, 2019. Available: 10.1002/rnc.4460.

[13].G. Xiaojun, H. Yanli, H. Yunan and Z. Youan, "CMAC NN-based adaptive control of non-affine nonlinear systems," *Proceedings of the 3rd World Congress on Intelligent Control and Automation*, 2000. Available: 10.1109/wcica.2000.863137.

[14].H. Bouchekara, "Most Valuable Player Algorithm: a novel optimization algorithm inspired from sport," *Operational Research*, vol. 20, no. 1, pp. 139-195, 2017. Available: 10.1007/s12351-017-0320-y.

[15].C. Wang and Y. Wu, "Finite-time tracking control for strict-feedback nonlinear systems with full state constraints," *International Journal of Control*, vol. 92, no. 6, pp. 1426-1433, 2017. Available: 10.1080/00207179.2017.1397290.

[16].H. Khalil, "*Nonlinear systems,"* Upper Saddle River, N.J.: Prentice Hall, 2000.

[17].C. Wang, Y. Wu and Z. Zhang, "Tracking control for strict-feedback nonlinear systems with time-varying full state constraints," *Transactions of the Institute of Measurement and Control*, vol. 40, no. 14, pp. 3964-3977, 2018. Available: 10.1177/0142331217737596.

[18].M. Rao, "New Design of Model Reference Adaptive Control Systems," *Journal of Applied Mechanical Engineering*, vol. 03, no. 01, 2013. Available: 10.4172/2168-9873.1000134.

[19].W. Chen, J. Sun, C. Chen and J. Chen, "Adaptive control of a class of nonlinear systems using multiple models with smooth controller", *International Journal of Robust and Nonlinear Control*, vol. 25, no. 6, pp. 865-877, 2013. Available: 10.1002/rnc.3113.

[20].M. Rahimian and M. Tavazoei, "Improving integral square error performance with implementable fractional-order PI controllers," *Optimal Control Applications and Methods*, vol. 35, no. 3, pp. 303-323, 2013. Available: 10.1002/oca.2069.