Nahrain University, College of Engineering Journal (NUCEJ) Vol.16 No.1, 2013 pp.9 - 19

Free Vibration Characteristics of Elastically Supported Pipe Conveying Fluid

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Abstract

In this study, the effect of support values on the natural frequency and critical flow velocity of a straight pipe conveying laminar flowing fluid is studied. The aim of this work is deriving a new analytical model to perform a general study to investigate the dynamic behavior of a pipe under general boundary conditions by considering the supports as compliant material with linear and rotational springs. This model describes both the classical (simply support, free, built, guide) and the restrained boundary condition and it is not required to derive a new frequency equation if the boundary conditions is changed ,also the result will be near to reality by knowing the physical parameters for the compliant material and the pipe.

Keywords: Pipe conveying fluid, Natural frequency, Critical velocity, and Elastic support

1.Introduction

The study of the dynamic behavior of a fluid conveying pipe, started in 1950, despite the great importance of this subject in pumps, heat exchanger, discharge lines, marines risers, etc.., the first observation of this phenomena was made by Ashley and Havilland [1**]**, when examining the above ground Trans-Arabian oil pipe line. They considered the problem as a simply supported pipe.

The free vibrations of pipes conveying fluid was studied by Huang [2], taking into account, the effects of rotary inertia on both the fluids and the pipes, the shear deformation of the pipes and the leteral inertia force due to the moving fluids. The theory of dynamics of pipes containing flowing fluid has been of a great interest for engineers in various fields. The results showed that the fluid forces and the pipe forced will interact with each other. ***Singh and*** Mallik [3] investigated the effect of harmonic fluid on stability using bolotin's concept, it was concluded that, the same regions of instability also exist in continuous pipe when it was parametrically exited and the phenomenon was like that of a beam subjected to an axial harmonic forces. The mass ratio did not have any significant effect on the regions of instability within the range of parametric values considered. With the increase in the pressure and the velocity of the fluid, the instability regions became wider and were shifted to lower frequencies, damping in the pipe reduced the extent of the instability regions and a finite value of the excitation parameters was required to start the instability.

Abraham [4] studied the vibration and stability of straight pipe systems conveying fluid, either steady or fluctuating flow. The supports considered are different in type and positions, this work perfferent a general study to investigate the dynamic behavior for a pipe with N-spans and general boundary conditions and that by considering the supports as compliant boundary material with linear and rotational springs and dampers. It was concluded that the support position and values had a significant on dynamic characteristics of the pipe.

Dian etal [5] analyzed the free lateral vibration of thin annular with variable thickness and circular plates. Study was adopted the finite element method to obtain the natural frequencies and mode shapes of the axisymmetric and non axisymmetric thin annular. The results showed that the finite element method was an efficient and convenient tool for analyzing the lateral vibration of annular and circular composite plates with variable thickness.

The effect of induced vibration of a simply supported pipe conveying fluid with a restriction, investigated theoretically and experimentally by ***Alaa*** [6], where transfer matrix approach was implemented to described the dynamic response of a pipe conveying fluid and a numerical technique for solving two-dimensional incompressible steady viscous flow for the rang of Reynolds number($5<Re<1000$). It was concluded that the fluid flow through a pipe with restriction affected the dynamic behavior of the pipe in addition to the flow field structure due to induced vibration.

Wang and Bloom [7] studied the static and dynamic instabilities of submerged and inclined concentric pipes conveying fluid mathematically.

The discrtized dynamical equations using spatial finite-difference schemes in the case of steady flow and pulsatile flow. The result was obtained, showed that the outer pipe length is a more important design factor than gravity and friction.

Shintaro and Masaki [8] studied experimentally the three-dimension dynamics of hanging tube conveying fluid with varying the length of the tube, Instabilities to static buckling, plane or rotating pendulum, sub-harmonic or to chaotic states, were found depending on boundary conditions.

In the present study, the free vibration of elastically supported pipe conveying fluid is analyzed under general boundary conditions by considering the supports as compliant material with linear and rotational springs. The convergence study is based on the numerical values. In the numerical examples, the first three eigenvalues of the Timoshenko beam are calculated for various values of stiffness of translational and rotational springs

2.Theoretical approach

Consider a straight uniform single-span pipe conveying fluid of length L where t_i and r_i are the translational and rotational spring constant, it will be considered that the pipe is supported at the two end points, where the parameters t_i and r_i are taken to have the same values at all the supports denoted by $t_i = t$ and $r_i = r$.

(a)

……………………(b)¶

Figure (1) Considered Timoshenko beams with (a) the first type and (b) the second type of translational and rotational springs

The following assumptions are considered in the analysis of the system under consideration **[9]**:

1. Neglecting the effect of gravity.
2. The pipe considered to be horizontal.
3. Neglecting the material damping.
4. The pipe is inextensible.
5. Neglecting the shear deformation and rotary inertia.
6. All motion considered small.
7. Neglecting the velocity distribution through the cross- section of the pipe.

Derivation of the equation of motion for stright pipe with steady flow are available in the literature Ref.[10].For a single-span pipe conveying fluid,the equation based on beabeam theory is given by,

$$EI\frac{\partial^4 y}{\partial x^4} + \left(M_f U^2 + pA\right)\frac{\partial^2 y}{\partial x^2} + 2M_f U\frac{\partial^2 y}{\partial x \partial t} + \left(M_f + M_p\right)\frac{\partial^2 y}{\partial t^2} = 0 \quad 1$$

….Ref [11]……………..(1)

$EI\frac{\partial^4 y}{\partial x^4}$: Stiffness term

$\left(M_f U^2 + pA\right)\frac{\partial^2 y}{\partial x^2}$:Curvature term

$2M_f U\frac{\partial^2 y}{\partial x \partial t}$: Coriolis force term

$\left(M_f + M_p\right)\frac{\partial^2 y}{\partial t^2}$: Inertia force term

The Coriolis force is a result of the rotation of the system element due to the system lateral motion, since each point in the span rotates with angular velocity **[12]**. The equation of motion Eq.(1) can be written in the following non-dimensional form:

$$Y^{iv} + (u^2+\gamma)Y'' + 2\beta U\dot{Y}' + \ddot{Y} = 0 \qquad 2$$

where

$X = x/L$

$Y = y/L$

$Y^{iv} = \frac{\partial^4 y}{\partial x^4}$, $Y'' = \frac{\partial^2 y}{\partial x^2}$, $\dot{Y}' = \frac{\partial^2 y}{\partial x \partial t}$,

$\ddot{Y} = \frac{\partial^2 y}{\partial t^2}$, $\tau = \sqrt{EI/(M_f+M_p)}(\frac{t}{l^2})$.

$\beta = \sqrt{M_f/(M_f+M_p)}$, $u = UL\sqrt{(M_f/EI)}$

$\gamma = (\frac{pA_p}{EI})L^2$

β: Non-dimensional mass ratio.

γ: Non-dimensional fluid pressure.

The general solution to Eq. (2) is given by:

$$y(x,t) = \sum_{j=1}^{4} C_j e^{ix\lambda_j} . e^{i\Omega\tau} \quad \ldots \quad 3$$

where $\mathbf{C_j}$ is amplitudes of vibrations and $\boldsymbol{\lambda_j}$ is the wave numbers. Substituting Eq. (3) into Eq. (2), a relationship is obtained between the wave numbers $\boldsymbol{\lambda_j}$ and the eigenvalues $\mathbf{\Omega}$.

$$\lambda^4 - (u^2+\gamma)\lambda^2 - 2\beta U\lambda\Omega - \Omega^2 = 0 \quad 4$$

From these relationships four wave numbers $\boldsymbol{\lambda_j}$ can be determined as functions of $\mathbf{\Omega}$ and the pipe parameters.

To solve the problem of free vibration for a structure, first its boundary conditions must be known. The method of solution consists of formulating the support condition of a pipe in terms of the compliant boundary material. The parameter of this material will be represented by linear and rotational spring. To describe the classical boundary conditions impedance values are taken to be zero or infinity values.

2.1.Evaluating Natural Frequency

The flexible-flexible boundary condition may be written as:-

$$r_1 l y'(0,\tau) = EIy''(0,\tau)$$
$$t_1 y(0,\tau) = -EIy'''(0,\tau)$$
$$r_2 y'(l,\tau) = -EIy''(l,\tau) \qquad 5$$
$$t_2 y(l,\tau) = EIy'''(l,\tau)$$

Substituting Eq. (3) into the boundary conditions Eq. (5) gives:

1-at x=0, $r_1 l y'(0,\tau) = EIy''(0,\tau)$

$\lambda_1(\lambda_1 + R_1 i)\, C_1 + \lambda_2(\lambda_2 + R_1 i)\, C_2 +$
$\lambda_3(\lambda_3 + R_1 i)\, C_3 + \lambda_4(\lambda_4 + R_1 i)\, C_4 = 0$

where $R_1 = (r_1 l/EI)$: dimensionless rotational stiffness at x=0

2-at x=0, $t_1 y(0,\tau) = -EIy'''(0,\tau)$

$-i\lambda_1^3 + T_1)C_1 + (-i\lambda_2^3 + T_1)C_2 +$
$(-i\lambda_3^3 + T_1)C_3 + (-i\lambda_4^3 + T_1)C_4 =$

where $T_1 = (t_1 l^3/EI)$: dimensionless longitudinal stiffness at x=0

3-at x=l, $r_2 y'(l,\tau) = -EIy''(l,\tau)$

$\lambda_1(-\lambda_1 + R_2 i)\; e^{i\lambda_1}\, C_1 +$
$\lambda_2(-\lambda_2 + R_2 i)\; e^{i\lambda_2}\, C_2 +$
$\lambda_3(-\lambda_3 + R_2 i)\; e^{i\lambda_3}\, C_3 +$
$\lambda_4(-\lambda_4 + R_2 i)\; e^{i\lambda_4}\, C_4 = 0$

where $R_2 = (r_2 l/EI)$: dimensionless rotational stiffness at x= l

4-at x=l, $t_2 y(l,\tau) = EIy'''(l,\tau)$

$i\lambda_1^3 + T_2)\, e^{i\lambda_1} C_1 + (i\lambda_2^3 +$
$T_2)\, e^{i\lambda_2} C_2 + (i\lambda_3^3 + T_2)\, e^{i\lambda_3} C_3 +$
$(i\lambda_4^3 + T_2)\, e^{i\lambda_4} C_4 = 0$

where $T_2 = (t_2 l^3/EI)$: dimensionless longitudinal stiffness at x= l

These four equations can be written in matrix form as follows:

$$\begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ b_1 & b_2 & b_3 & b_4 \\ c_1 & c_2 & c_3 & c_4 \\ d_1 & d_2 & d_3 & d_4 \end{bmatrix} * \begin{Bmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix}$$

This matrix can be written as follows:

$$[Hi,j]\{Cj\} = 0$$

$$\Delta = |\mathrm{Hi,j}|$$

where i,j=1,2,3,4

$$a_1 = \lambda_1(\lambda_1 + R_1 i), a_2 = \lambda_2(\lambda_2 + R_1 i)$$

$$a_3 = \lambda_3(\lambda_3 + R_1 i), a_4 = \lambda_{4(}\lambda_4 + R_1 i)$$

$$\mathrm{b}_1 = (-i\lambda_1{}^3 + T_1), \mathrm{b}_2 = (-i\lambda_2{}^3 + T_1)$$

$$\mathrm{b}_3 = (-i\lambda_3{}^3 + T_1), \mathrm{b}_4 = (-i\lambda_4{}^3 + T_1)$$

$$c_1 = \lambda_1(-\lambda_1 + R_2 i)\ e^{i\lambda_1},$$

$$c_2 = \lambda_2(-\lambda_2 + R_2 i)\ e^{i\lambda_2}$$

$$c_3 = \lambda_3(-\lambda_3 + R_2 i)\ e^{i\lambda_3}$$

$$c_4 = \lambda_4(-\lambda_4 + R_2 i)\ e^{i\lambda_4}$$

$$\mathrm{d}_1 = (i\lambda_1{}^3 + T_2)e^{i\lambda_1}$$

$$\mathrm{d}_2 = (i\lambda_2{}^3 + T_2)e^{i\lambda_2}$$

$$\mathrm{d}_3 = (i\lambda_3{}^3 + T_2)e^{i\lambda_3}$$

$$\mathrm{d}_4 = (i\lambda_4{}^3 + T_2)e^{i\lambda_4}$$

The non-dimension natural frequency is evaluated by setting term $[Hi,j]\{Cj\}$ equal to zero

$$[Hi,j]\{Cj\} = 0$$

For non trivial solution:

i.e. $\Delta = |\mathrm{Hi,j}| = 0$

This is determinant function to λ_i,but λ_i function to (x,β,γ,U) therefore the determinant can be written as,

$$\Delta(x,\beta,\gamma,U,R_1,T_1,R_2,T_2) = 0.$$

Therefore the frequency can be written as, $F(x,\beta,\gamma,U,\ R_1,T_1,R_2,T_2,) = 0.$

The solution is done by trial and error procedure, the value of **Ω** that makes the determinant vanish can be found which will represent the non dimensional natural frequency.

A MATLAB computer program was built for this purpose.

2.1. Evaluating Critical Velocity

If the natural frequencies of the pipe reach to zero, the flow velocity in this case is called critical flow velocity. When the flow velocity is equal to the critical velocity the pipe bows out and buckles, because the forces required to make the fluid deform to the pipe curvature are greater than the stiffness of the pipe.

Therefore the mechanism underlying instability may be illustrated by static method **[13]**, so deleting time dependent term from equation (2) yields:

$$Y^{iv} + (u^2 + \gamma)Y'' = 0 \ldots \qquad 6$$

whose its solution takes the form

$$Y(X) = C_1 + C_2 X + C_3 \sin(\alpha X) + \ C_4 \cos(\alpha X)$$

where $\alpha = \sqrt{\gamma + u_{cr}{}^2}$ and $C_1, C_2, C_3, C_4,$ are constant and can be evaluated by using the boundary conditions as following:

1- at x=0, $r_1 l y'(0,\tau) = EIy''(0,\tau)$

$$R_1 C_2 + R_1 \alpha C_3 + C_4 \alpha^2 = 0$$

where $R_1 = (r_1 l/EI)$: dimensionless rotational stiffness at x=0.

2-at x=0, $t_1 y(0,\tau) = -EIy'''(0,\tau)$

$$T_1 C_1 - \alpha^3 C_3 + T_1 C_4 = 0.$$

where $T_1 = (t_1 l^3/EI)$:dimensionless longitudinal stiffness at x=0

3- at x=l, $r_2 y'(l,\tau) = -EIy''(l,\tau)$

$$R_2 C_2 + (\alpha R_2 \cos(\alpha) - \alpha^2 \sin(\alpha))C_3 + (-\alpha R_2 \sin(\alpha) - \alpha^2 \cos(\alpha))C_4 = 0$$

where $R_2 = (r_2 l/EI)$:dimensionless rotational stiffness at x= l

4- at x=l, $t_2 l^3 y(l,\tau) = EIy'''(l,\tau)$

$$T_2 C_1 + T_2 C_2 + (T_2 \sin(\alpha) + \alpha^3 \cos(\alpha))C_3 + (T_2 \cos(\alpha) - \alpha^3 \sin(\alpha))C_4 = 0$$

where $T_2 = (t_2 l^3/EI)$:dimensionless longitudinal stiffness at x= l

These four equations can be written in matrix form as follows:

$$\begin{bmatrix} 0 & R_1 & R_1\alpha & \alpha^2 \\ T_1 & 0 & -\alpha^3 & T_1 \\ 0 & R_2 & c_3 & c_4 \\ T_2 & T_2 & d_3 & d_4 \end{bmatrix} * \begin{Bmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix}$$

This matrix can be written as follows:

$$[Wi,j]\{Cj\} = 0$$

$$\Delta = |\mathrm{Wi,j}|$$

where i,j=1,2,3,4

$$c_3 = \alpha R_2 \cos(\alpha) - \alpha^2 \sin(\alpha)$$

$$c_4 = -\alpha R_2 \sin(\alpha) - \alpha^2 \cos(\alpha)$$

$$d_3 = T_2 \sin(\alpha) + \alpha^3 \cos(\alpha)$$

$$d_4 = T_2 \cos(\alpha) - \alpha^3 \sin(\alpha)$$

Trial and error procedure is used to find the value of (α) that makes the determinant

$|Wi, j|$ Vanish. From $\left(\alpha = \sqrt{\gamma + {u_{cr}}^2}\right)$ the non-dimensional critical velocity $(u_{cr})$is obtained.

A MATLAB computer program was built for this purpose.

3. Results and Discussions.

3.1. Effect of the Numerical Value of Support Stiffness.

In order to investigate the influence of stiffness of the supports on the free vibration characteristics of a pipe conveying fluid, the first three eigen values of the pipe at $(u = 2, \beta = 0.1, \gamma = 0)$ with the two types of translational and rotational springs as shown in Figure (1) are calculated and three dimensional plots in Figures. (2), (3), (4), (5), (6) and (7) respectively to illustrate how the frequency parameters change with the spring constants. The stiffness parameters T_i and R_i are taken as having the values at all the supports denoted by $T_1 - T_2 - T$, $R_1 - R_2 - R$. for the pipe with the first type of the springs, and by $T_1 = 10^8$, $T_2 = T$, $R_1 = R$, $R_2 = 0$ for the beam with the second type of the springs.

It is possible to simulate infinite support stiffness by setting the translational or rotational stiffness coefficient equal to 10^8 at all the supports for comparing the obtained results with the existing results of the classically supported pipe. Therefore, comparative study of the pinned-pinned $(T_1 = T_2 = 10^8, \quad R_1 = R_2 = 0)$ and clamped-clamped $(T_1 = T_2 = 10^8,$

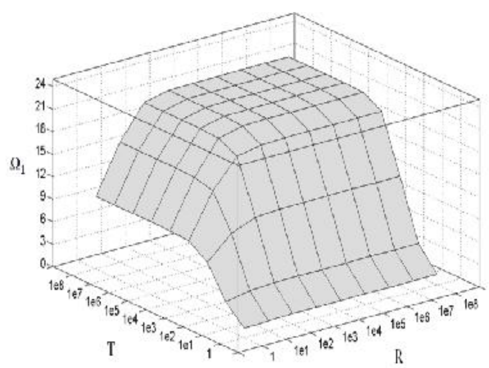
$R_1 = R_2 = 10^8)$ pipe with the classical solutions given in the Ref. **[14]** is carried out, and the results are given in tables 1 and 2. Also, by setting the translational and rotational stiffness coefficients equal to zero at all the supports, a completely free pipe situation can be obtained.

Table (1) Comparison study of the first two dimensionless frequencies parameter Ω_i of the pinned-pinned pipe conveying fluid for various values of u at $\beta = \sqrt{0.1}$.

u	Ω_1		Ω_2		Ω_3
	present work	Ref.[14]	present work	Ref.[14]	present work
0.1	9.865	9.864	39.474	39.478	88.822
1	9.347	9.3460	39.000	39.013	88.340
1.5	8.652	8.652	38.400	38.424	87.728
2	7.579	7.579	37.569	37.583	86.865
3	2.899	2.898	34.946	35.075	84.354

Table (2) Comparison study of the first two dimensionless frequencies parameter Ω_i of the Clamped-Clamped pipe conveying fluid for various values of u at $\beta = \sqrt{0.1}$.

u	Ω_1		Ω_2		Ω_3
	present work	Ref.[14]	present work	Ref.[14]	present work
0.1	22.371	22.370	61.670	61.669	120.900
1	22.082	22.081	61.328	61.340	120.510
1.5	21.711	21.712	60.900	60.921	120.010
2	21.183	21.185	60.301	60.330	119.300
3	19.598	19.613	58.564	58.604	117.280



Figure(2)The first frequency parameter of the pipe with $T_1 = T_2 = T$, $R_1 = R_2 = R$.

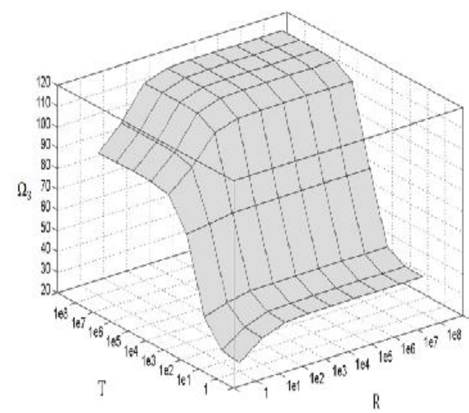


Figure (4) The third frequency parameter of the pipe with $T_1 = T_2 = T$, $R_1 = R_2 = R$.

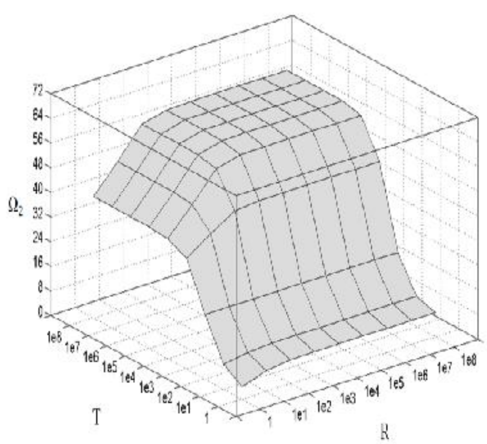


Figure (3) The second frequency parameter of the pipe with $T_1 = T_2 = T$, $R_1 = R_2 = R$.

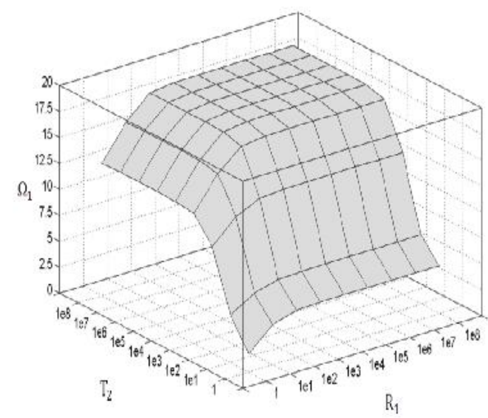


Figure (5) The first frequency parameter of the pipe with $T_1 = \infty$, , $R_2 = 10$.

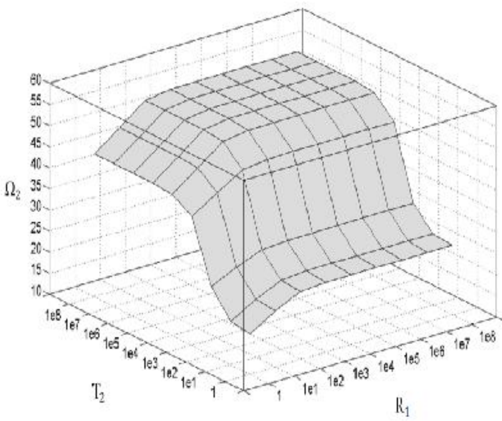


Figure (6) The second frequency parameter of the pipe with $T_1 = \infty$, , $R_2 = 10$

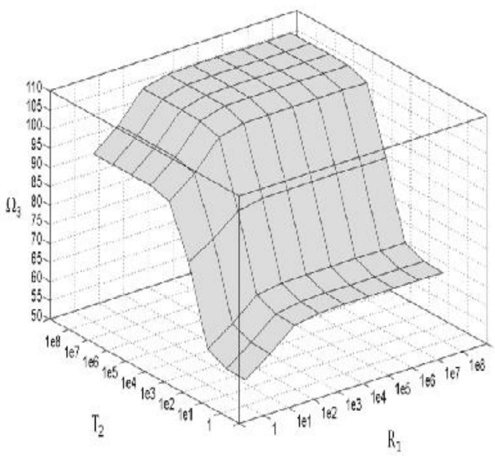


Figure (7) The third frequency parameter of the pipe with $T_1 - \infty$, , $R_2 - 10$.

It is seen from the figures that translational springs are much more effective on the frequency parameters than rotational springs. The reason of this behavior, the value of transverse displacement decreasing when the value of linear stiffness is increasing, this tends to increasing in natural frequency. But the value of slope is small decreasing when the value of rotational stiffness increasing, this tends to small increasing in natural frequency.For example, in Figure (3), when the spring parameter T is taken constant, value of 10^0 and the parameter R is changed from 10^0 to 10^8 , the frequency parameter Ω_2 changes from 4.096 to 7.87 but, while the parameter R is taken as 10^0 and the parameter T is changed from 10^0 to 10^8, the frequency parameter Ω_2 changes from 4.096 to 39.351 .

Increment in the values of parameters T and R are more effective on the first frequency parameter of the pipe than the second and third frequency parameters. For instance, for the pipe with the first type of the springs, when the parameters T and R are both changed from 10^0 to 10^0 , the first frequency parameter Ω_1 changes from 1.226 to 21.243, namely, Ω_1 becomes 17 times greater in this change. On the other hand, Ω_2 and $\Omega_3$increase approximately 14 and 5 times, Because the value of natural frequency in the first mode is small and any increasing in T and R strong effect in the value of natural frequency but the natural frequency in the second and third modes is high and the increasing in T and R small effect in the value of natural frequency.

respectively in the considered change. When the values of T and R are greater than $T = 10^5$and $R = 10^5$, there is no remarkable change in the frequency parameters. This situation can be observed from the flat area of Figures. (2) to (7). Also, it is evident from the obtained values of frequency parameters that, when the parameters T_i and R_i are taken as $T_i = R_i = 10^8$, then, the pipe can be considered as a pipe fixed at the both ends.

3.2. Effect of Fluid Velocity and Mass Ratio.

The fluid parameters (velocity, pressure, and mass ratio) have direct effects on the dynamic characteristics of the system under consideration. The effect of the fluid flow velocity and mass ratio will be discussed.

In general the natural frequencies for steady flow decrease with increasing the fluid flow velocity. If the velocity of the flow in the pipe equal zero, then the case will be a normal beam system and when the flow velocity equal the critical velocity the pipe bows out and buckles, because the forces required to make the fluid deform to the pipe curvature is greater than the stiffness of the pipe.

Mathematically the buckling instability arises from the mixed derivative term(Coriolis) $[2\beta U\dot{y}']$ in equation (4) which represents a forces imposed on the pipe by the flowing fluid that is always 90^o of phase with the displacement of the pipe, and is always in phase with the velocity of the pipe. This force is essentially a negative damping mechanism which extracts energy from the fluid flow and inputs energy into the bending pipe to encourage initially, vibration, and ultimately buckling **[15]**.

Many studies indicate that the natural frequencies of a pipe with both ends stationary, such as the clamped-clamped or clamped-pinned ends, are nearly independent of the mass ratio while the natural frequencies of pipes with one end free to move, such as cantilever pipe, are strongly affected by the mass ratio **[15]**.

Figures (8) and (9) shows the variation of the frequency parameter Ω_1 with flow velocity parameter u for different values of the mass ratio β for the clamped –clamped and pinned-pinned boundary condition respectively.

There is very good agreement in the values of Ω_1 with those obtained by Païdoussis and Issid **[16]**, for the pinned-pinned and clamped-clamped cases.

Figures (10, 11, 12and 13) show the variation of the frequency parameter Ω_1 with flow velocity parameter u for different values of the mass ratio β for four types of flexible support. For $u = 0$ and for $u = u_{cr}$, there is no difference in the frequency parameter for any value of β. For intermediate values of u, there is a slight decrease in the frequency parameter for increasing values of β in classical boundary condition **[16]**. The effect of mass ratio on the natural frequency depended on the value of rotational and translational impedance of support. The more effect happen when the value of stiffness is small and this effect decreasing when the value of stiffness is increasing, so less effect happens at the value of stiffness is large.

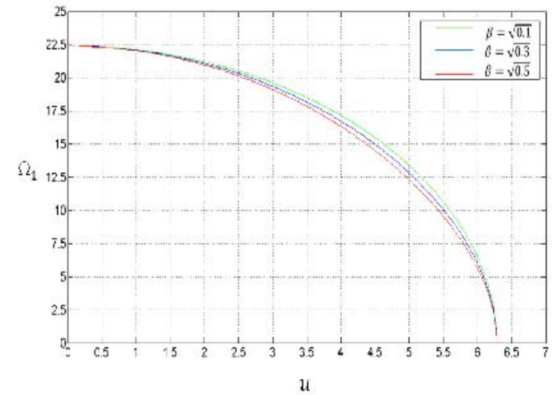


Figure (8) Variation of Ω_1 with u for various values of β for Clamped- Clamped pipe

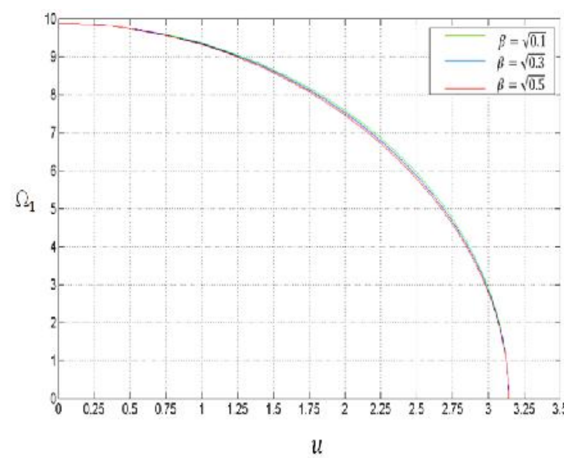


Figure (9) Variation of Ω_1 with u for various values of β for pinned- pinned pipe.

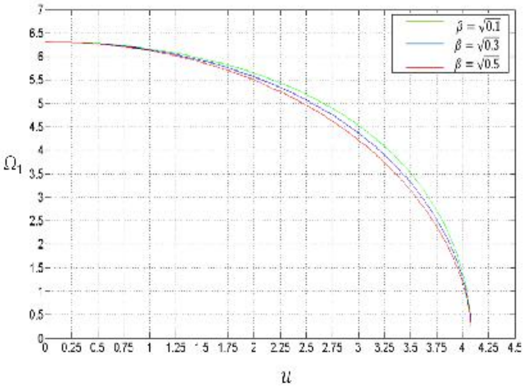


Figure (10) Variation of Ω_1 with u for various values of β at T_1=100, R_1=10 , T_2=10, R_2=100

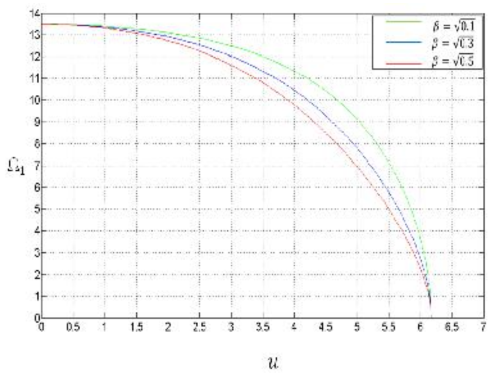


Figure (11) Variation of Ω_1 with u for various values of β
at T_1=110, R_1=130 , T_2=170, R_2=90

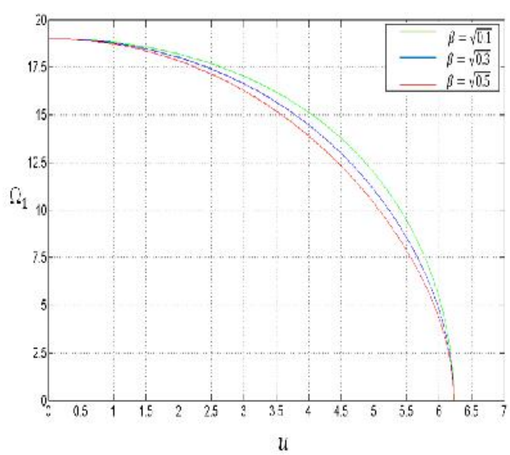


Figure (12) Variation of Ω_1 with u for various values of β
at T_1=750, R_1=180 , T_2=450, R_2=250

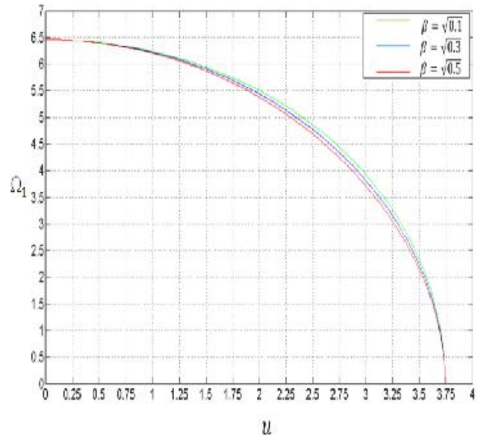


Figure (13) Variation of Ω_1 with u for various values of β at T_1=5, R_1=175 , T_2=520, R_2=380

3. Conclusions

Following the main summarized conclusions raised by this research:

1-The technique used for modeling the compliant boundary material in terms of linear and rotational impedance allows the designer to describe both the classical and restrained boundary conditions.

2-The natural frequency of a pipe increases with the increasing of the linear and rotational impedance.

3-The linear impedance is much more effective on the frequency parameters than rotational impedance. The reason of this behavior, the value of transverse displacement decreasing when the value of linear stiffness is increasing, this tends to increasing in natural frequency. But the value of slope is small decreasing when the value of rotational stiffness increasing, this tends to small increasing in natural frequency.

4-The values of linear and rotational impedance are more effective on the first frequency parameter of the pipe than the second and third frequency parameters.

5-When the values of linear and rotational impedance are greater than 10^5, there is no remarkable change in the frequency parameters.

6-The natural frequency of a pipe increases with the increasing of the linear and rotational intermediate impedance.

7- When flow velocity equal zero or critical value, there is no difference in the frequency parameter for any value of mass ratio, for intermediate values of flow velocity, there is decrease in the natural frequency parameter with increasing values of mass ratio.

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Appendix (A): parameters of Figure (3)

Table (A1)
Variation of the second frequency parameter of the pipe with $T_1 = T_2 = T$, $R_1 = R_2 = R$.

Ω_2	R								
T	10^0	10^1	10^2	10^3	10^4	10^5	10^6	10^7	10^8
10^0	٤.٠٩٦	٦.٧٦٧	٧.٧٣١	٧.٨٥٦	٧.٨٦٩	٧.٨٧	٧.٨٧	٧.٨٧	٧.٨٧
10^1	٧.٩٣٧	٩.٢٣١	٩.٧٩٨	٩.٨٧٥	٩.٨٨٣	٩.٨٨٤	٩.٨٨٤	٩.٨٨٤	٩.٨٨٤
10^2	٢٠.٨٣٨	٢٠.٨٢٢	٢٠.٨١٤	٢٠.٨١٣	٢٠.٨١٢	٢٠.٨١٢	٢٠.٨١٢	٢٠.٨١٢	٢٠.٨١٢
10^3	٣٥.٨٧٨	٤١.٢٨٤	٤٦.٠٩١	٤٦.٩٨٤	٤٧.٠٨٢	٤٧.٠٩١	٤٧.٠٩٢	٤٧.٠٩٢	٤٧.٠٩٢
10^4	٣٨.٩٨٨	٤٧.٥٤٤	٥٦.٦١٩	٥٨.٤٨٧	٥٨.٦٩٣	٥٨.٧١٤	٥٨.٧١٦	٥٨.٧١٦	٥٨.٧١٦
10^5	٣٩.٣١٥	٤٨.٢٣٢	٥٧.٧٩٩	٥٩.٧٧٣	٥٩.٩٩٢	٦٠.٠١٤	٦٠.٠١٦	٦٠.٠١٦	٦٠.٠١٦
10^6	٣٩.٣٤٧	٤٨.٣٠١	٥٧.٩١٧	٥٩.٩٠٢	٦٠.١٢١	٦٠.١٤٣	٦٠.١٤٥	٦٠.١٤٦	٦٠.١٤٦
10^7	٣٩.٣٥١	٤٨.٣٠٨	٥٧.٩٢٩	٥٩.٩١٤	٦٠.١٣٤	٦٠.١٥٦	٦٠.١٥٨	٦٠.١٥٩	٦٠.١٥٩
10^8	٣٩.٣٥١	٤٨.٣٠٩	٥٧.٩٣	٥٩.٩١٦	٦٠.١٣٥	٦٠.١٥٧	٦٠.١٦	٦٠.١٦	٦٠.١٦