

Sliding Mode Control for Electro-Hydraulic Servo System

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Abstract – T his paper deals with the problem of designing a robust controller for the electro-hydraulic position servo system (EHPSS). The sliding mode control design methodology is utilized here to design a robust controller with respect to system parameters uncertainty. Because the relative degree of the mechanical sub-system with respect to hydraulic force F_L is two, the switching function is constructed in two stages in order to reduce the relative degree to one. As a result of the proposed controller, the switching function reaches the zero level in a finite time, after that the state tracks the reference value asymptotically. The simulations result using MATLAB/ Simulink tools reveal the effectiveness and the robustness of the proposed control in forcing the position to track the reference value in spite of the uncertainty in system parameters. The uncertainty in hydraulic system model depends mainly on the variation on load mass (20 kg to 250 kg). Namely, the robustness is tested for two load masses 75 kg and 200 kg.

Keywords -Electro hydraulic servo system, sliding mode control, variable load mass.

1. Introduction

In spite of the electro - hydraulic systems are widely used in many applications such as in robotics, factories of large objects, actuators in aircrafts, and many kinds of mechanizations, because of their high power-to-weight ratios, fast and smooth response characteristics, low cost in comparison to other types of drives; they have some problems in controlling of the position, force or pressure because of the nonlinearities of some parameters such as friction, fluid compressibility and leakages, in order to improve the performance of the hydraulic actuator, a suitable controller is required.

The sliding mode controller (SMC) is a discontinuous robust controller and it is used with systems in presence of the disturbances and the variation in their parameters. Different papers investigated the use of the sliding mode controller to the hydraulic systems. Here a literature review is presented. A nonlinear sliding made controller was used by M. Jerouane et al [1], for a nonlinear mathematical model of a hydraulic system interacting with the environment. The proposed controller shows its insensitivity to a large variation of parameters such as flow gain, pressure or environmental supply stiffness, and has an excellent tracking performance for various set point forces under uncertainties. The work of M. Mihajlov et al [2], used the Fuzzy PI controller to improve the performance of the variable structure control system for position tracking of an electro-hydraulic servo system. The results of the combined controller show much better results than using sliding mode controller only with regard to the position precision and disturbance rejection.

A time-varying sliding mode control approach combined with adaptive control is proposed based on Lyapunov analysis have been introduced by Ch. Guan et al [3], in order to overcome the influence to the uncertain parameters and disturbances of the electro-hydraulic servo system, the approach has nice control global robustness and improves position tracking accuracy considerably. Hong-Ming Chen et al [4] have introduced the varying boundary layer, as a new sliding mode control scheme which has effectively improved the tracking performance of the nonlinear electro-hydraulic position servo systems (EHPSS), as compared with the conventional continuous sliding control. In reference C. Guan [5] presented a nonlinear adaptive sliding controller with adaptation laws to compensate for the nonlinear uncertain parameters caused by the electro-hydraulic system; the proposed control method can compensate for all system uncertainties and satisfying the required performance. The method of adaptive fuzzy sliding mode control with friction torque dynamic compensation for the uncertain parameter and external disturbances of Direct Drive Electro-Hydraulic (DDEH) servo Rocket Rudder is designed in [6]. The simulation indicates that the proposed control theme has good dynamic performance and the steady state accuracy even though the parameter perturbation and the load disturbance exist. A fuzzy sliding mode controller is designed in [7] for an electrohydraulic system, which uses fuzzy logic to obtain equivalent control signal after system states reaching sliding manifold. Simulating result adequately satisfies control demand of an electro-hydraulic system with large moment of inertia of load.

Chao Jia1 et al. [8], have proved that the designed sliding mode controller can achieve the scheduled demand such as position tracking with high precision and better disturbance-resistant ability after introducing an exponent approaching law with variable parameters for the hydraulic nonlinear system. A performance study of SMC, P, PD and PID controllers for position tracking control to the Electrohydraulic Servo System has been introduced by R. Ghazali et al [9]. The numerical simulations show that the SMC provides better performance in tracking accuracy and response. The chattering problem has been introduced bv Indrawanto et al [10], where they proposed and applied a sliding mode controller in order to reduce of the singlehydraulically actuated manipulator. The simulation results have shown good performances of tracking with overcoming the chattering problem. J. Yu [11] proposed the robust et al stabilizability of the uncertainty for the equivalent linear parameter varying (LPV) model of the nonlinear hydraulic servo systems by using parameterdependent Lyapunov method.

The main problem that we aim to deal with it in this paper is to design a robust controller to the electro-hydraulic actuator system in spite of the variation in parameters values. A sliding mode controller is utilized here to direct the piston position to the desired value irrespective to parameters variation. The numerical simulations will demonstrate the effectiveness of the proposed controller with a broad range of the load mass variation.

The organization of the present work is as follows: an overview to the sliding mode control is first introduced, while the model of the electro hydraulic system is presented in the following section. After that the required steps and assumptions for designing a sliding mode control is introduced. The numerical investigations for the effectiveness of the proposed controller and the conclusions are presented in the latter sections.

2. Sliding Mode Control

Sliding modes as a phenomenon may appear in a dynamic system governed by ordinary differential equations with discontinuous right-hand sides. The term sliding mode first appeared in the context of relay systems [12].

The high-speed discontinues control action is used to drive the state trajectories into a specified surface in state space named as the sliding or switching surface. Then the state is maintained on this surface or within its neighborhood while moving toward the origin. Actually, such control scheme means a lot to the regulation matter since the only concern is to stay on the surface; hence, external excitations are no more allowed to destroy the desired performance during the sliding mode toward the equilibrium state.

Systems with sliding modes have been proven to be an efficient tool to control complex high-order nonlinear dynamic plants operating under uncertainty conditions, a common problem for many processes of modern technology. This explains the high level of research and publication activity in the area and unremitting interest of practicing engineers in sliding mode control during the past two decades [12]. As a procedure for designing the sliding mode control, we need first to construct a switching function and then we compute the required discontinuous control that will

force the state toward the switching surface and maintain it there for all future time. In this case it is required that the relative degree between the switching function and the control action is equal to one. This type of sliding mode controller is the conventional one and it is also named as the first order sliding mode control (FOSMC) [12]. Moreover it is required also that the reduced system dynamics when regulating the switching function to zero level with respect to the state is asymptotically stable.

In the subsequent section a FOSMC will be designed for electro-hydraulic actuator system after defining a switching function that has a relative degree equal to one with the control action.

3. Mathematical Model of the Electro-Hydraulic Actuator System

The Electro-Hydraulic Actuator system response will be studied in order to realize the dynamic problems, so a mathematical model should be written for the electrohydraulic servo system. The Basic components for electro-hydraulic actuator are shown in Figure 1 [11].



Figure 1 The Electro-Hydraulic Actuator system schematic diagram [11]

The dynamical model for the cylinder can be described, via Newton's Law, by the following equation [11]

$$m\ddot{x} = P_L \Omega - b\dot{x} - kx \tag{1}$$

where x represents the displacement of the actuator and m is the mass of the load, $P_L = P_1 - P_2$ is the load pressure of the cylinder, where the P_1 and P_2 are the pressure of the actuator of chamber 1 and chamber 2 respectively, Ω is the ram area of the cylinder, b represents the viscous damping coefficient, k is the spring constant. The load pressure of the cylinder can be represented by the following equation:

$$\frac{V_t}{4\beta_e}\dot{P}_L = -\Omega\dot{x} - C_{tm}P_L + Q_L \tag{2}$$

where V_t is the total volume of the cylinder and the hoses between the cylinder and the servo valve, β_e is the effective bulk modulus, C_{tm} is the coefficient of the total internal leakage of the cylinder due to pressure, and $Q_L = (Q_1 + Q_2)/2$ is the load flow. Q_L is related to the spool valve displacement of the servo valve, as in the equation below:

$$Q_L = C_d w x_v \sqrt{\frac{P_s - sgn(x_v)P_L}{\rho}}$$
(3)

where C_d is the discharge coefficient, w is the spool valve area gradient, P_s is the supply pressure of the fluid, ρ is the fluid density and x_v is the spool valve displacement of the servo valve. The spool valve dynamics is as described in equation (4):

$$\pi_v \dot{x}_v = -x_v + K_v u \tag{4}$$

where the spool valve displacement x_v is related to the current input u = i, τ_v and K_v are the time constant and gain of the servo-valve respectively. In this work the spool dynamics is omitted and hence we get:

$$x_v = K_v u \tag{5}$$

Now to write the mathematical model of the electro-hydraulic actuator in state

space form we define the following state variables;

$$x_1 = x - x_d, \quad x_2 = \dot{x} - \dot{x}_d,$$

 $x_3 = P_L \quad \& \quad u_o = \sqrt{P_s - sgn(u)x_3}u.$

Then we can write the mathematical model as:

$$\begin{array}{l} x_{1} = x_{2} \\ \dot{x}_{2} = \frac{a}{m} x_{3} - \frac{b}{m} x_{2} - \frac{k}{m} x_{1} - \frac{b}{m} \dot{x}_{d} - \frac{k}{m} x_{d} - \ddot{x}_{d} \\ \dot{x}_{3} = -\frac{4\beta_{e}C_{tm}}{V_{t}} x_{3} - \frac{4\beta_{e}\Omega}{V_{t}} x_{2} - \frac{4\beta_{e}\Omega}{V_{t}} \dot{x}_{d} + \frac{4\beta_{e}C_{d}wK_{v}}{V_{t}\sqrt{\rho}} u_{o} \end{array} \right\}$$

$$(6)$$

where u_o is a new control signal. By knowing that $sgn(u_o) = sgn(u)$, the actual control signal u (the current) can now be computed from the following relation:

$$u = \frac{u_o}{\sqrt{P_s - sgn(u_o)x_3}} \tag{7}$$

The state space model in (6) with a control signal u_o will be more appropriate for designing a controller as will be shown in the following section.

4. Sliding Mode Controller Design

The following steps are devoted to construct a switching function first and then design a control law for the spool valve displacement that will force the witching function to reach the zero level.

To construct the switching function, we define first the following function:

$$s_o = x_2 + \lambda_o x_1, \quad \lambda_o > 0$$

When differentiating s_o , the control u_o will not appear and this means that the relative degree between s_o and u is greater than one. Thus s_o cannot be taken as a switching function for the FOSMC. To overcome this problem, the switching function can be defined as:

$$s = \dot{s}_o + \lambda s_o$$

= $\dot{x}_2 + (\lambda + \lambda_o)x_2 + \lambda \lambda_o x_1$, for $\lambda > 0$ (8)

The switching function, as can be easily noted, has a relative degree equal to one. So when we differentiate it the control u_o will appear. Moreover, for $\lambda, \lambda_o > 0$, and when the switching function reaches its zero level the state will approach the origin asymptotically. The stability for s = 0 is granted when λ_o and λ are greater than zero.

The next step in designing the sliding come control is to determine a control law that ensures directing the state towards the sliding surface. The sliding condition when satisfied ensures both the reaching phase and the sliding phase is [12];

$$s\dot{s} < 0 \tag{9}$$

To derive the control low, such that the sliding mode condition in the inequality (9) is satisfied, we differentiate *s*

$$\dot{s} = \ddot{x}_2 + (\lambda + \lambda_o)\dot{x}_2 + \lambda\lambda_o\dot{x}_1$$

= F + Gu_o (10)

where

$$F = \frac{1}{m} \left\{ \left(\frac{bk}{m} - (\lambda + \lambda_o)k \right) x + \left(-\frac{4\beta_e \Omega^2}{V_t} + \frac{b^2}{m} - k - (\lambda + \lambda_o)b \right) \dot{x} + \left(-\frac{4\beta_e C_{tm} \Omega}{V_t} - \frac{b\Omega}{m} + (\lambda + \lambda_o)\Omega \right) x_3 - (\lambda + \lambda_o)m \ddot{x}_d - m \ddot{x}_d + \lambda_o m (\dot{x} - \dot{x}_d) \right\}$$

and

$$G = \frac{4\Omega\beta_e}{mV_t}$$

By substituting in condition (9), yields

$$s\{F + Gu_o\} < 0$$

Since G > 0, the above inequality can be put as

$$s\left\{\frac{F}{G}+u_o\right\} < 0 \tag{11}$$

Now let

$$\Gamma = \frac{F}{G} = \Gamma_{\rm n} + \Delta \Gamma \tag{12}$$

where Γ_n and $\Delta\Gamma$ are the nominal and the uncertain terms of Γ respectively:

$$\begin{split} \Gamma_{n} &= \frac{V_{t}}{4\beta_{e}\Omega} \left\{ \left(\frac{bk}{m_{n}} - (\lambda + \lambda_{o})k \right) x + \left(-\frac{4\beta_{e}\Omega^{2}}{V_{t}} + \frac{b^{2}}{m_{n}} - k - (\lambda + \lambda_{o})b \right) \dot{x} + \left(-\frac{4\beta_{e}C_{tm}\Omega}{V_{t}} - \frac{b\Omega}{m_{n}} + (\lambda + \lambda_{o})\Omega \right) x_{3} - (\lambda + \lambda_{o})m_{n}\ddot{x}_{d} - m_{n}\ddot{x}_{d} + \lambda_{o}m_{n}(\dot{x} - \dot{x}_{d}) \right\} \end{split}$$
(13)
$$\begin{aligned} |\Delta\Gamma| &\leq \frac{V_{t}|\Delta m|}{4\beta_{e}\Omega m_{n}m_{min}} |bkx + b^{2}\dot{x} - b\Omega x_{3}| + \frac{V_{t}|\Delta m|}{4\beta_{e}\Omega} |\lambda\lambda_{o}\dot{x} - (\lambda + \lambda_{o})\ddot{x}_{d} - \lambda\lambda_{o}\dot{x}_{d} - \ddot{x}_{d}| (14) \end{aligned}$$

Also m_n and m_{min} are the nominal and minimum mass values respectively and $|\Delta m|$ is the maximum mass variation with respect to nominal mass value.

The sliding mode control law, accordingly, can be taken as

$$u_o = -\Gamma_n - \mu * sgn(s) \tag{15}$$

where

$$\mu = \mu_o + \frac{V_t |\Delta m|}{4\beta_e \Omega m_n m_{min}} |bkx + b^2 \dot{x} - b\Omega x_3| + \frac{V_t |\Delta m|}{4\beta_e \Omega} |\lambda \lambda_o \dot{x} - (\lambda + \lambda_o) \ddot{x}_d - \lambda \lambda_o \dot{x}_d - \ddot{x}_d| > |\Delta \Gamma|, \quad \mu_o > 0$$
(16)

Furthermore u_o must also satisfy the sliding condition (9). This can be verified by substituting (15) into (9) as follows;

$$s\{\Gamma - \Gamma_n - \mu * sgn(s)\} = s\{\Delta\Gamma - \mu * sgn(s)\}$$
$$= s\Delta\Gamma - \mu|s| \le |s||\Delta\Gamma| - \mu|s|$$
$$= -|s|\{\mu - |\Delta\Gamma|\}$$

But from inequality (16) $\mu > |\Delta\Gamma|$, therefore

$$s\left\{\frac{F}{G}+u_o\right\} \le -|s|\{\mu-|\Delta\Gamma|\} < 0$$

This proves that the proposed SMC will direct the state to the sliding surface in a finite time and maintains it there for all future time (the sliding motion). Note that the signum function sgn(s) with gain μ is used here to enforce the state to reach and slide along the switching manifold (s = 0) in presence of the disturbances and the uncertainty in system parameters.

When the system is in sliding mode

the system dimension is reduced by one. This situation appears in the FOSMC, where the electro-hydraulic servo system dynamics becomes:

$$\ddot{x}_1 + (\lambda + \lambda_o)\dot{x}_1 + \lambda\lambda_o x_1 = 0$$
(17)

As mentioned above that for positive λ and λ_o , x_1 in (17) is exponentially asymptotically stable and the error between the piston position and the reference value x_1 will exponentially decay to zero with the required dynamic characteristics according to the values of λ and λ_o .

The above analysis proves the stability of the electro-hydraulic servo system that uses the proposed sliding mode controller.

5. Simulation Results and Discussions

The dominant parameter in the system model of the electro-hydraulic servo system that resembles the uncertainty is its load mass that ranging from (20 - 250) kg, as given in Table 1, which presents the value of the system parameters. The nominal term Γ_n is evaluated with the mean value of the mass (135 kg). In the simulations the designed SMC is applied to the hydraulic system for mass value equal to 200 kg and 75 kg.

The parameters of the switching function (8) are chosen as $\lambda_o = 30$ and $\lambda = 20$. The reduced dynamics, when *s* reachs zero value in a finite time, is exponentially asymptotically stable since λ and λ_o are positive. In deriving the sliding mode control law the nominal system parameters are considered first to get Γ_n (see (13)). The sliding mode controller will definitely overcome system uncertainties via the discontinuous term in (15) for suitable gain μ (see (16)) where

 $\begin{aligned} \max |\Delta m| &= m_{max} - m_n = 115 \ kg \\ m_{min} &= 20 \ kg. \end{aligned}$

Table 1. The system parameters [11]

and

The parameter	Description	The value (SI units)
b	Viscous damping coefficient.	19.84*103 m/s
Ω	Ram area of the cylinder.	(5550/1000000) m2
Vt	Total volume of the cylinder and the hoses between the cylinder and the servo valve.	(1.75*106)/((103)3)m3
C _{tm}	Coefficient of the total internal leakage of the cylinder due to pressure.	(15/(103)5) m5/Ns
K	Effective bulk modulus of spring.	70*103 N/m
β	Effective bulk modulus.	(700*(103)^2) N/m2
$Cdw/\sqrt{\rho}$	Cd is the discharge coefficient, w is the spool valve area gradient and ρ is the fluid density.	(3.42*10^4/(103)3) m3√Ns
m	Mass of the load.	20~250 Kg
Ps	Supply pressure of the fluid.	10 MPa
kv	Gain of the servo-valve.	0.03

To avoid chattering that may appears as a result of applying a discontinuous control action, the signum function in the control law (15) is replaced with inverse tan function as a continuous and a differentiable function. Therefore the SMC law becomes:

 $u_o = -\Gamma_{\rm n} - \tfrac{\mu}{2\pi} * tan^{-1}(10 * s)$

This design for the case m = 200kg, where the desired reference position $x_d = 1 mm$. The proposed sliding mode controller will guide the states to the sliding surface in considerably proper time as shown in Figure 2. This is known

as the reaching time which it relates to the control parameters (15) and the switching function form as determined in (8). Figure (2) also reveals that the time required to regulate the switching function is not exceed 0.1 second. After reaching sliding manifold s = 0 the error state x_1 is regulated, according to (17), asymptotically to the origin. This is depicted in Fig. (3) which it needs 1.5 second to be equal to zero. As a result the displacement x reaches the desired position as shown in Fig. (4).



 $(m = 200 \, kg)$



Figure 4 Position of the system x(t) (response to a unit step input) (m = 200 kg).

The phase plot, load pressure P_L and the yielded hydraulic force FL(t) are plotted in Figures 5, 6 and 7 respectively.

It can be noted that for 1 mm desired position the load pressure P_L and the yielded hydraulic force FL(t) are within acceptable limits. The hydraulic force FL(t) represents the actual deriving force that controls the mechanical system.



Figure 5 The error phase plot $(x_1(t), x_2(t))$



Figure 6 load pressure P_L time response



 $(FL = PL * \Omega)$

The sliding mode control which is represented by current input u is plotted in Figure 8. The plot is smooth due to replacing the signum function by the arc tan function where the chattering effect is eliminated.



Figure 8 Sliding mode control action u

To demonstrate the robustness of the proposed controller the simulation is repeated for the case where m = 75kg. The simulation results are plotted in Figures 9 and 10 for the switching function and the position *x* respectively.



Figure 9 Switching function time response (m = 75 kg)



Figure 10 The system response x(t) (m = 75 kg)

It can be noted that the position response for m = 200 kg and m = 75 kg are nearly equal especially during the sliding phase where the system response influenced by the parameters variation and disturbances only in the reaching phase.

The ability of the proposed sliding mode controller is tested for the piecewise constant desired position with m =200 kg. The error state $x_1(t)$ with time is plotted in Figure 11. As seen from the figure the error is incredibly small and the position of the system x(t), as shown in Figure 12, reaches the desired level smoothly and firmly as expected.





Figure 12 Position of the system x(t) response $(m = 200 \ kg)$

Moreover the load pressure P_L is clarified in Figure 13 which it actuates the mechanical system to follow the desired position.



 $(m = 200 \, kg)$

6. Conclusion

The sliding mode control has been utilized in this work to design a robust control to the electro-hydraulic servo system. The switching function has been constructed such that the relative degree equals to one in order to apply the conventional sliding mode control which it also named as FOSMC. In sliding motion mode the response has been shown to be asymptotically stable for a positive selection of λ_o and λ with the desired system dynamics characteristics and irrespective to the variations in the system load mass. This proves the stability of the proposed control. The effectiveness and the robustness of the proposed control have been also proved via the numerical simulations for two different loads mass (200 kg and 75 kg). The results showed a fast response to track the reference step value in spite of the variation in load mass which is influenced also by the parameters selection of the sliding mode control law $(\lambda_{\rho}, \lambda \text{ and } \mu \text{ in } (8) \& (15) \text{ respectively}).$ Moreover the sliding mode controller has also been tested for a pricewise constant desired position which again proved its ability to force the position to follow the desired path.

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