



Integral Sliding Mode Control Based on Barrier Function for Servo Actuator with Friction

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ABSTRACT

This paper proposes the use of the integral sliding mode control (ISMC) based on the barrier function to control the servo actuator system with friction. Based on the barrier function, the main features of the ISMC design were preserved, additionally, the proposed control design is done without the need to know the bound on the system model uncertainty, accordingly, the overestimation of the control gain doesn't take place and the chattering is eliminated. Moreover, the steady-state error can be adjusted via selecting the barrier function parameter only.

The simulation results demonstrate the performance of the proposed ISMC based on the barrier function where the system angle successfully follows the desired angular position with a small pre-adjusted steady-state error. Additionally, the obtained results clarify superior features compared with a traditional ISMC designed to the same actuator.

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1. INTRODUCTION

One of the effective approaches to control nonlinear systems containing matched disturbances is sliding mode control (SMC), which has given efficiency in design has been widely used in industrial [1]. Despite the benefits of SMC, it has many disadvantages such as the chattering effectiveness, the reaching phase, and sensitiveness to matchless uncertainties [2, 3]. To solve these problems, various strategies of SMC have been proposed, including integral sliding mode control (ISMC), which is looking to remove the reaching phase by enforcing sliding mode during the full system response [4, 5]. In an integral sliding mode, the order of the motion equation is equal to the main system without reducing by using the dimension of the control input. Concepts of ISMC can expand for the purpose of building a new type that can estimate turbulence and solve the problem of chatter without losing

control accuracy and strength [6]. In the ISMC approaches; knowledge of parameters uncertainties bounds is required for the purpose of calculating the control gain [7].

The adaptive controller is defined as being able to regulate its behavior for the purpose of responding to make dynamic changes for the process and the characteristics of disturbances [8]. Recently, the Adaptive Sliding Mode controllers based on using barrier function have been established as is present in [9].

In this paper, an integral sliding mode control based on barrier function is proposed. Then it was applied for a DC servo actuator system containing friction, to deal with the above strategies.

The major advantages of the proposed algorithm in addition to the known features of ISMC are:

- The algorithm does not require knowledge of the upper bound on the model uncertainty and disturbance or its derivatives.
- Only one control parameter is needed, which also adjusts the steady-state error.
- As a by-product, the chattering is eliminated because the proposed ISMC is continuous.

The organization of this work is as follows. The second section introduces the problem statement. In the third section, the DC servo actuator system including the friction model is described. Then, Classical ISMC Design is discussed in section four. While in the fifth section the proposed controller is illustrated in brief. And, the sixth section explains the simulation results. Finally, in the seven sections conclusions are drawn.

2. PROBLEM STATEMENT

For the classical ISMC, the main difficulty is the determination of the discontinuous gain k , where it is required to know the upper bounds on the system parameters and on the friction and the external load components. This will lead to an excessive gain value which will cause undesirable chattering behavior.

In this paper, we propose the use of the barrier function instead of the discontinuous control term u_s . So, we will not need to determine the gain k in classical ISMC, and consequently, we do not need to know the upper bound on the perturbation as mentioned above. Additionally, because the barrier function is a continuous function, the chattering is eliminated.

3. DC SERVO ACTUATOR SYSTEM WITH FRICTION MODEL

The DC motor is a specific type of motor that is classified as one of the main machines that use electrical power to generate mechanical power. The servo actuator system model can be represented by a second-order dynamic system with friction [10],

$$J \ddot{x} = u - F - T_L \quad (1)$$

Where;

- x The actuator position
- J The moment of inertia
- F The friction torque
- T_L The external load torque
- u The control input

The friction torque is explained as static friction phenomena, which contain: Coulomb friction, viscous friction, and stiction friction [10]

$$F = \left\{ F_s e^{-\left(\frac{\dot{x}}{v_s}\right)^2} + F_c \left(1 - e^{-\left(\frac{\dot{x}}{v_s}\right)^2} \right) + \sigma |\dot{x}| \right\} \text{sign}(\dot{x}) \quad (2)$$

Where;

- F_s The Coulomb friction
- F_c The stiction friction
- v_s The stribek velocity

σ The viscous friction coefficient

4. CLASSICAL INTEGRAL SMC DESIGN

For comparison purposes, the design of a classical ISM In order to design the classical integral sliding mode control (classical ISMC) system, the servo actuator system is rewritten in terms of the nominal and perturbation terms as follows:

$$\ddot{x} = \frac{1}{J_o} u + \delta(t) \quad (3)$$

Where $\delta(t)$ is the perturbation term which amount of the parameter variations and the external load, and it can be expressed as:

$$\delta(t) = \Delta \left(\frac{1}{J} \right) u - \frac{1}{J} (F + T_L) \quad (4)$$

Also, J_o is the nominal moment of inertia. In a state-space representation, Eq. (3) can be put by the following equations:

$$\left. \begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= \frac{1}{J_o} u + \delta(t) \end{aligned} \right\} \quad (5)$$

Define the error functions e_1 and e_2 which are the tracking error and its derivative, as

$$\left. \begin{aligned} e_1 &= x_1 - x_d \\ e_2 &= x_2 - \dot{x}_d \end{aligned} \right\} \quad (6)$$

where x_d is the reference signal which chosen to be differentiable function, accordingly, Eq. (5) in terms of the error functions is given by,

$$\left. \begin{aligned} \dot{e}_1 &= e_2 \\ \dot{e}_2 &= \frac{1}{J_o} u + \delta(t) - \ddot{x}_d \end{aligned} \right\} \quad (7)$$

Designing the classical ISMC is done here according to the fooling steps; let the control law be taken as

$$u = J_o(u_o + u_s) \quad (8)$$

Where J_o is the nominal value for the moment of inertia, u_o is the nominal control applied to stabilize the nominal system dynamics with the desired characteristics while u_s is the discontinuous control designed to reject the perturbation term.

Now, define the sliding manifolds(e) as

$$s(e) = s_o(e) + z \quad (9)$$

Which consists of two main parts: $s_o(e)$ is the conventional sliding manifold and z is the integral term, with $s(e), s_o(e), z \in R^1$. Let the conventional sliding manifold be chosen as

$$s_o(e) = e_2 \quad (10)$$

Then, the integral sliding manifold derivative is

$$\dot{s}(e) = \dot{e}_2 + \dot{z} \quad (11)$$

By substituting Eq. (7), (8) into Eq. (11), we can obtain

$$\dot{s}(e) = u_o + u_s + \delta - \ddot{x}_d + \dot{z} \quad (12)$$

Let the integral part derivative be defined as

$$\dot{z} = \ddot{x}_d - u_o \tag{13}$$

Accordingly, $\dot{s}(e)$ becomes

$$\dot{s}(e) = u_s + \delta \tag{14}$$

To rewrite the system dynamics described in Eqs. (7), we apply the equivalent control [10] to Eq. (14) yield;

$$[\dot{s}(e)]_{eq} = 0 = [u_s]_{eq} + \delta \\ \Rightarrow [u_s]_{eq} = -\delta$$

Substituting in Eq. (7), we obtain the following equivalent system dynamics;

$$\left. \begin{aligned} \dot{e}_1 &= e_2 \\ \dot{e}_2 &= u_o - \ddot{x}_d \end{aligned} \right\} \tag{15}$$

Therefore, u_o can be selected as in Eq. (16) which makes the origin of the error dynamics in (15) globally asymptotically stable

$$u_o = \ddot{x}_d - c_1 e_1 - c_2 e_2 \tag{16}$$

where c_1 and c_2 are positive constant values selected according to the desired characteristic.

As a final step, we need to determine the gain k of the discontinuous control term u_s , where it is given by

$$u_s = -k \text{sign}(s) \tag{17}$$

The gain k is determined via the inequality

$$k > |\delta| \tag{18}$$

Then using Eq. (4), we obtain (Appendix A)

$$k = 420 + 1.174 |u_o| + 14.7|\dot{x}| \tag{19}$$

According to the above, the classical ISMC is given by

$$\left. \begin{aligned} s(e) &= e_2 + z \\ \dot{z} &= c_1 e_1 + c_2 e_2, \quad z(0) = -e_2(0) \\ u_o &= \ddot{x}_d - c_1 e_1 - c_2 e_2 \\ u_s &= -(420 + 1.174 |u_o| + 14.7|\dot{x}|)\text{sign}(s) \\ u &= J_o(u_o + u_s) \end{aligned} \right\} \tag{20}$$

5. THE PROPOSED INTEGRAL SMC

In this work, we propose to use the barrier function $g_\epsilon(s)$ instead of the discontinuous control term u_s . Firstly, let us define the barrier function as follows;

Definition [1]: Let's suggest that some $\epsilon > 0$ is given and fixed; the barrier function can be defined as even continuous function $f: z \in [-\epsilon, \epsilon] \rightarrow g(z) \in [b, \infty]$ strictly increasing on $[0, \epsilon]$.

- $\lim_{|z| \rightarrow \epsilon} g(z) = +\infty$
- $g(z)$ has a unique minimum at zero and $g(0) = b \geq 0$

Two different classes of BFs exist:

1. Positive definite BFs (PBFs): $g_p(z) = \frac{\epsilon F}{\epsilon - |z|}$, i.e. $g_p(0) = F > 0$.
2. Positive Semi-definite BFs (PSBFs): $g_{ps}(z) = \frac{|z|}{\epsilon - |z|}$, i.e. $g_{ps}(0) = 0$.

The PBFs $g_{ps}(z)$ was chosen and will be used when simulating the servo system in this paper. To this end, the following ISMC based on barrier function was proposed for the servo actuator system

$$\left. \begin{aligned} s(e) &= e_2 + z \\ \dot{z} &= c_1 e_1 + c_2 e_2, \quad z(0) = -e_2(0) \\ u_o &= \ddot{x}_d - c_1 e_1 - c_2 e_2 \\ u_s &= -g_\varepsilon = \frac{-s}{\varepsilon - |s|} \\ u &= J_o(u_o + u_s) \end{aligned} \right\} \tag{21}$$

where $g_\varepsilon = g_{ps}(s) * \text{sign}(s) = \frac{s}{\varepsilon - |s|}$ is a differentiable function of $s(e)$.

Remark 1: Since the proposed ISMC in Eq. (24) is continuous, the chattering will be eliminated (or attenuated if a smaller value of ε is selected), moreover; the steady-state error is function to ε and becomes smaller for smaller ε .

Remark 2: we do not need to determine the discontinuous gain k as in classical ISMC, which required knowing the bound on the perturbation term $\delta(t)$. Instead, we need only to select a suitable value for ε according to the wanted accuracy.

6. SIMULATION RESULTS

This section gives the simulation results of the DC actuator system with the proposed ISMC based on barrier function. The nominal and actual dynamic parameters of the model are presented in Table I and Table II respectively, which are selected based on the information provided in [10].

TABLE I: DC servo actuator and friction model nominal parameters.

Nominal Parameters	Value	Unit
J_o	0.2	$Kg.m^2$
T_{Lo}	2	N. m
F_{so}	2.19	N. m
F_{co}	16.69	N. m
ϑ_{so}	0.01	rad/sec
σ_o	0.65	N. m.sec/rad

TABLE II: DC servo actuator and friction model parameters used in the simulation.

Actual Parameters	Value	Unit
J	0.23	$Kg.m^2$
T_L	2.25	N. m
F_s	2.5185	N. m
F_c	21.1935	N. m
ϑ_s	0.0115	rad/sec
σ	0.821	N. m.sec/rad

The simulation is performed using MATLAB with the initial condition $(x_1(0), x_2(0)) = (\frac{\pi}{360}, 0)$. The control objective is to satisfy the system stability as well as minimizing the tracking error so x_1 tracks the desired trajectory x_d . The position, velocity, and acceleration desired signals in the present work are chosen as [11].

$$\begin{aligned} x_d &= \frac{1}{16\pi} \sin(8\pi t) - \frac{1}{24\pi} \sin(12\pi t) \\ \dot{x}_d &= \sin(10\pi t) \sin(2\pi t) \tag{25} \\ \ddot{x}_d &= 10\pi \cos(10\pi t) \sin(2\pi t) - 2\pi \sin(10\pi t) \cos(2\pi t) \end{aligned}$$

The ISMC based on barrier function input u is given in Eq. (21), with $\varepsilon = 0.01$. While for the classical SMC input u is as given in Eq. (20), for the gain k with the nominal integral control parameters $c_1 = 6750$ and $c_2 = 195$.

To demonstrate the characteristics of the proposed control and highlight its robustness, two cases were used:

I. Constant load torque case

In this case, a constant load torque was used $T_L = 2.5 \text{ N.m}$. The results can be illustrated as follows: In Figure 1, the time required to reach the desired angle is less than 0.1 sec for both controllers classical ISMC and ISMC based on barrier function. This result is realized while plotting the error in Figure 2 where the maximum error of angle does not exceed 8.7×10^{-3} radian. The sliding manifolds as shown in Figure 3 does not exceed ε for the proposed controller from the first moment of operation, on the other hand, the classical ISMC also doesn't exceed ε but it represented by a very high amount of undesirable switching of the actuation torques. The control effort is clarified in Figure 4, where it can notice that the classical ISMC required more effort in addition to unwanted switching, unlike the continuous one for the ISMC based on barrier function.

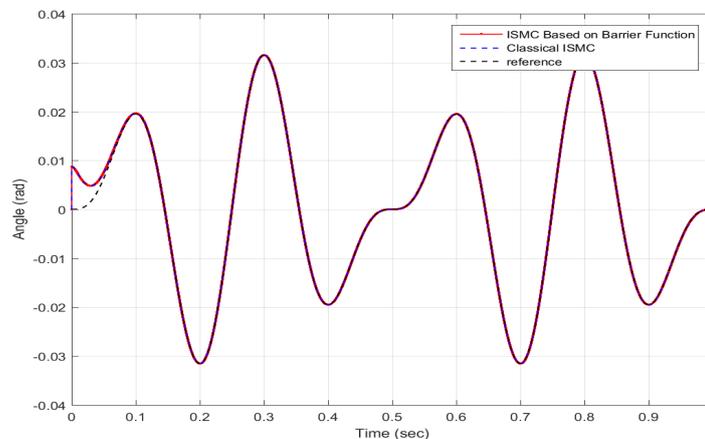


Figure 1: Angle x_1 vs. time, $T_L = \text{constant}$.

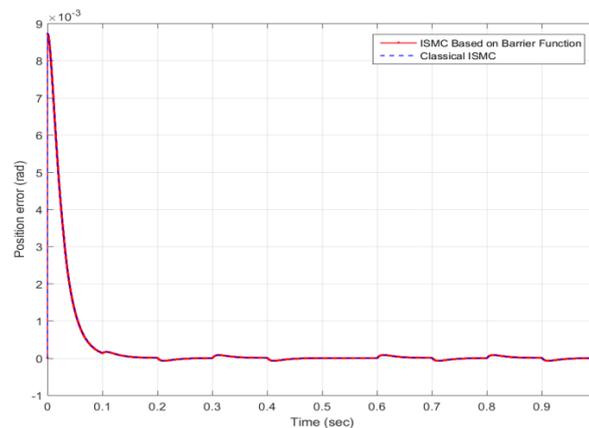


Figure 2: The position error e_1 vs. time, $T_L = \text{constant}$.

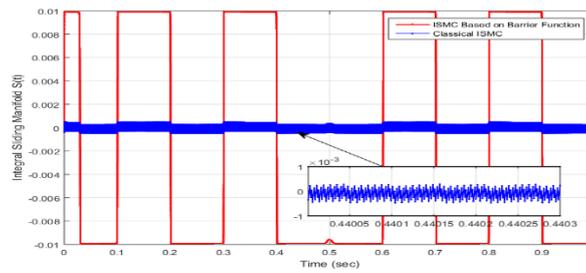


Figure 3: The integral sliding manifold $s(t)$ vs. time, $T_L = \text{constant}$.

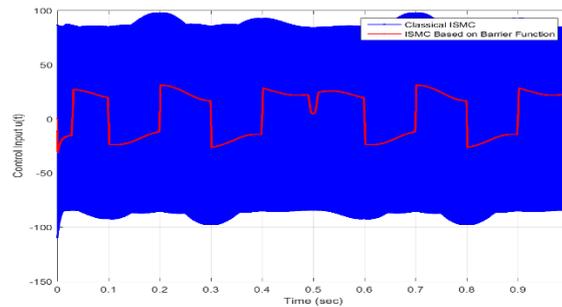


Figure 4: The control input u vs. time using saturation function for Classical ISMC, $T_L = \text{constant}$.

To solve the problem of chattering in the classical ISMC for the above case, the saturation function is used as an approximation for the $sign(s(t))$ function. With this change, the controller becomes as follow

$$u_s = -k(t, s(t)) \text{sat}(s(t), \alpha) \tag{22}$$

In this case, the results can be clarified as follows: In Figure 5 the time required to reach the desired angle is still less than 0.1 sec for both controllers classical ISMC and ISMC based on barrier function. This can be checked while plotting the error in Figure 6 where the maximum error of angle does not exceed 8.7×10^{-3} radian. The sliding manifold as shown in Figure 7 does not exceed ϵ for the proposed controller from the first moment of operation, on the other hand, the classical ISMC also doesn't exceed ϵ and the chattering problem has been solved using a saturation function as an approximation for $sign(s(t))$ function with α is taken equal to 0.009. This function caused the two controllers to work similarly. The control effort is clarified in Figure 8, where it can be noted that the classical ISMC and the barrier ISMC required the same effort.

The similarity between the control performance for the classical ISMC and the ISMC based barrier function is the result of using the same sliding mode control method (the ISMC) and both controllers provide the control input with the efficient gain through u_s which enables the control system from maintaining the state in the vicinity of the sliding manifold from the first instant. Additionally, both controllers use the same nominal controller which led to the same state response.

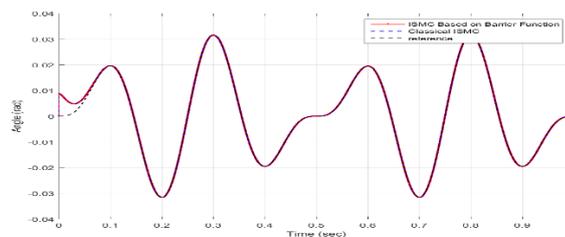


Figure 5: Angle x_1 vs. time using saturation function for Classical ISMC, $T_L = \text{constant}$

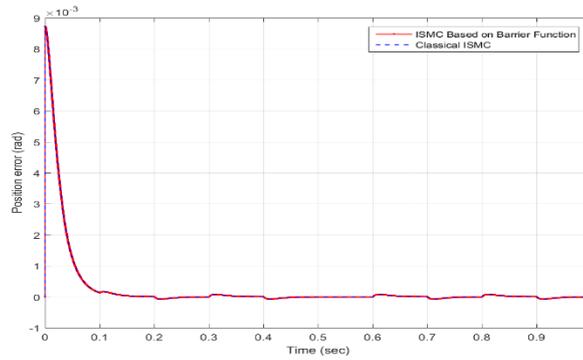


Figure 6: The position error e_1 vs. time using saturation function for Classical ISMC, $T_L = \text{constant}$.

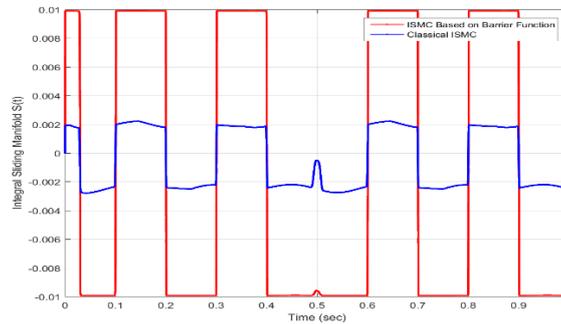


Figure 7: The integral sliding manifold $s(t)$ vs. time using saturation function for Classical ISMC, $T_L = \text{constant}$.

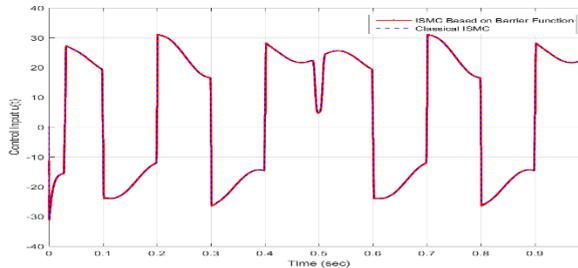


Figure 8: The control input u vs. time using saturation function for Classical ISMC, $T_L = \text{constant}$.

II. Variable load torque case

In this case, an increasing variable load torque was used which expressed in Eq. (23),

$$T_L = \begin{cases} 2.5 \sin(35 \pi t), & t \leq 0.5 \\ 5 \sin(35 \pi t), & 0.5 < t \leq 1 \\ 10 \sin(35 \pi t), & t > 1 \end{cases} \quad (23)$$

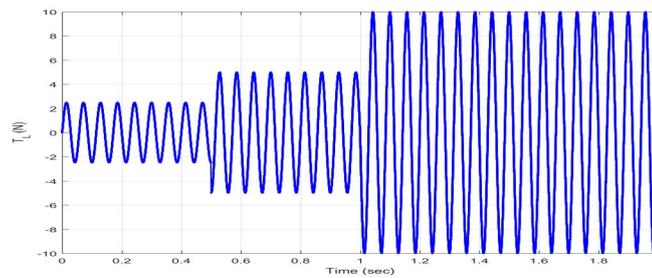


Figure 9: The increasing variable torque load

Figure 10 represents the DC actuator angle, where the time required to reach the desired signal is less than 0.1 sec for both controllers' classical ISMC and ISMC based on barrier function. This result is realized while plotting the error in Figure 11 where the maximum error of angle does not exceed 8.725×10^{-3} radian. The sliding manifold as shown in Figure 12 does not exceed ε for the proposed controller while the classical ISMC also doesn't exceed. The control input is seen in Figure 13, where for the barrier ISMC the controller is still continuous in spite of increasing the external load by four times.

The above results reveal the ability of the proposed continuous controller in enforcing the position to follow the desired reference with an error not exceed ε in spite of system uncertainty and variable external disturbances and without chattering. The results of the classical ISMC also show the ability to make the position to follow the desired reference but with discontinuity and inducing chattering. Moreover, the only assigned parameter for the barrier ISMC is ε , which represents the tracking accuracy also. For the classical ISMC, the situation is different where it required calculating discontinues gain k according to our knowledge about the bound of the uncertain parameters and on the external disturbance, which also leads to a high amplitude of the control input.

Finally, the benefits of using the ISMC based on barrier function can be summarized as follows

1. The design of the proposed controller is done without the need to know the bound on the system model uncertainty.
2. Calculating the value of the control gain is not required for the proposed controller; on the other hand, it is essential for the classical ISMC.
3. The ultimate bound on the steady-state error can be adjusted when using the proposed controller and because the bound on the sliding variable is preselected by ε , which is not the case for the classical ISMC, where the bound on the sliding variable is a function for many parameters including the uncertainty of the system model besides the parameters of the approximating function.

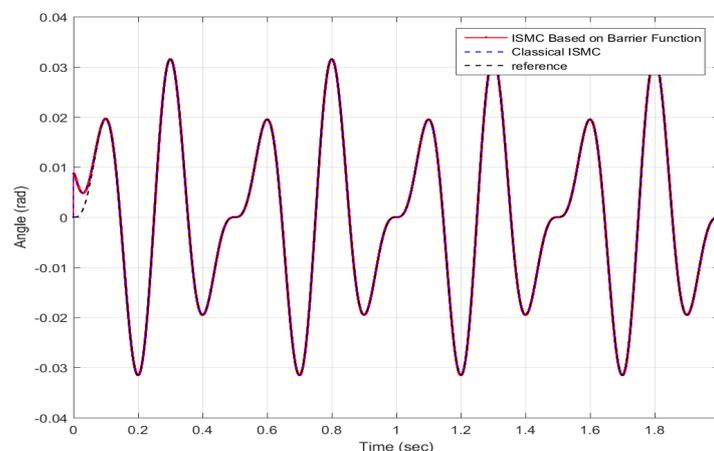
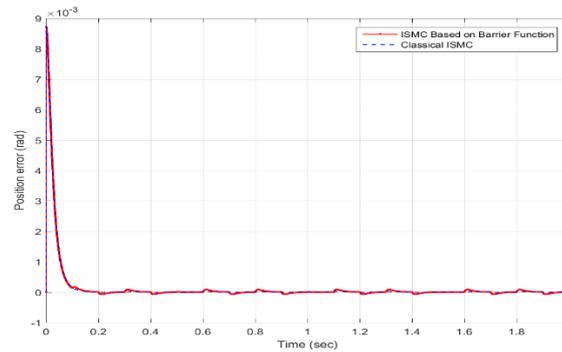
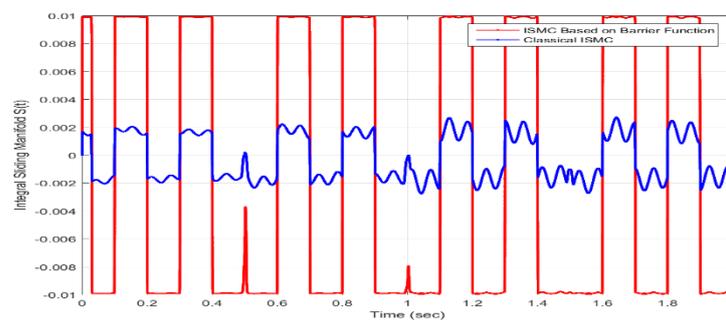
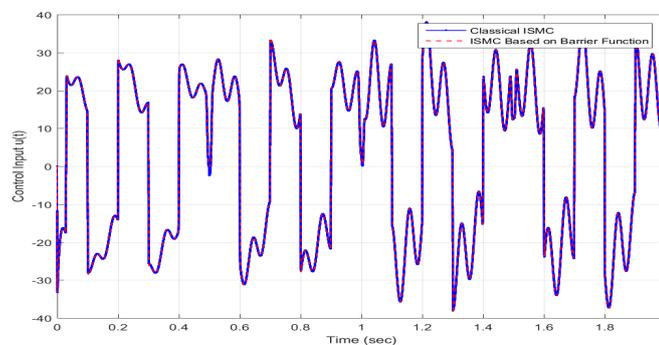


Figure 10: Angle x_1 vs. time using saturation function for Classical ISMC, $T_L = \text{variable}$.**Figure 11: The position error e_1 vs. time using saturation function for Classical ISMC, $T_L = \text{variable}$.****Figure 12: The integral sliding manifold $s(t)$ vs. time using saturation function for Classical ISMC, $T_L = \text{variable}$.****Figure 13: The control input u vs. time using saturation function for Classical ISMC, $T_L = \text{variable}$.**

7. CONCLUSION

In this paper, a barrier strategy to adjust the gain of ISMC for the DC servo actuator system with a friction model was proposed. The main advantage of this strategy is unlike the classical ISM where the computation of the discontinuous gain needs information on the upper bound of the system parameters and disturbances, where here the only required design parameter is the ϵ value which quantifies the tracking accuracy. The obtained results for different torque loads showed that the proposed ISMC based barrier function has a similar control system performance to the case of employing ISMC, but with a smaller control input effort. In addition, in order to eliminate chattering in classical ISMC, the discontinuous term is approximated using the saturation function, which

required selecting a suitable design parameter α . Although in this case, the results seemed very close to the results of the proposed controller, it still needs many tries times to select the suitable value for α which is not the case for the proposed ISMC which based on the barrier function, where due to the differentiability nature of the barrier function which it prevents chattering in the system response.

Appendix A

To determine the discontinuous control gain $k(t)$, the first step starts from inequality (22) and using Eq. (4)

$$\begin{aligned}
 k &> \left| \Delta \left(\frac{1}{J_o} \right) u - \frac{1}{J} (F + T_L) \right| \\
 &> \left| \Delta \left(\frac{1}{J} \right) J_o |u_o + u_s| + \left| \frac{F}{J} \right| + \left| \frac{T_L}{J} \right| \right| \\
 &> \left| \Delta \left(\frac{J_o}{J} \right) |u_o| + \left| \Delta \left(\frac{J_o}{J} \right) |u_s| + \left| \frac{F}{J} \right| + \left| \frac{T_L}{J} \right| \right| \\
 \left(1 - \left| \Delta \left(\frac{J_o}{J} \right) \right| \right) k &> \left| \Delta \left(\frac{J_o}{J} \right) |u_o| + \left| \frac{F}{J_{min}} \right| + \left| \frac{T_L}{J_{min}} \right| \right| \\
 k &> \frac{\left| \Delta \left(\frac{J_o}{J} \right) |u_o| + \left| \frac{F}{J_{min}} \right| + \left| \frac{T_L}{J_{min}} \right| \right|}{\left(1 - \left| \Delta \left(\frac{J_o}{J} \right) \right| \right)} \\
 k &= k_o + \frac{\left| \Delta \left(\frac{J_o}{J} \right) |u_o| + \left| \frac{F}{J_{min}} \right| + \left| \frac{T_L}{J_{min}} \right| \right|}{\left(1 - \left| \Delta \left(\frac{J_o}{J} \right) \right| \right)}
 \end{aligned}$$

Where $k_o > 0$, taking the value of $k_o = 0.5$ and using the system parameters with maximum uncertainty (35%) as presented in **Table A. I**, the following terms in the above formula for k are calculated;

$$\begin{aligned}
 \left| \Delta \left(\frac{J_o}{J} \right) \right| &\leq \left| \frac{J_o}{J_{min}} - \frac{J_o}{J_o} \right| \leq 0.54 \\
 \left| \frac{T_L}{J_{min}} \right| &\leq 20 \\
 \left| \frac{F}{J_{min}} \right| &\leq 173.5 + 6.75|\dot{x}|
 \end{aligned}$$

Table A. I: System parameters with 35% uncertainty used for Classical ISMC.

Parameters	Value	Unit
J_{min}	0.13	$Kg.m^2$
T_{Lmax}	2.5	N. m
F_{smax}	2.9565	N. m
F_{cmax}	22.5315	N. m
σ_{max}	0.8775	N. m. sec/rad

Then, for $k_o = 0.5, k$ is equal to

$$k = 420 + 1.174 |u_o| + 14.7|\dot{x}| \tag{A.1}$$

For the variable load torque case we have $|T_L| \leq 10$, accordingly k becomes

$$k = 544 + 1.174 |u_o| + 14.7|\dot{x}| \tag{A.2}$$

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