Approximate Solution of Coupled Burger's Equation and Advection Diffusion Equation

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Abstract

This paper is devoted to find the approximate solution of the coupled Burgers equation and advection diffusion equation by the Variational Iteration Method (VIM). This method provides a sequence of functions which converges to the exact solution of the system. It has been shown that the (VIM) is quite efficient and suitable for finding the approximate solution of this couple of PDE.

Key words: Variational iteration method, Coupled Burger's equation.

1. Introduction

The understanding and the design of energy efficiency in buildings, (the adjustment and optimization of ventilation, heating and air conditioning), has a considerable attention in the scientific community [2]. However, the coupling of a temperature field and a velocity field is a major problem of understanding because the complex dynamics of thermal flow in buildings. This coupling naturally involves Burger's equation and heat equation. Burger's equation has a wide variety of application in physics and engineering and is defined as,

$$u_t(t,x) + u(t,x)u_x(t,x) = \mu\, u_{xx}(t,x),$$

where $u(t,x)$ is a function in time and space and represent a velocity field and μ is the viscosity coefficient. This equation is a model that captures the interaction of convection and diffusion, so it's used to study the fluid flow. As well as this equation can be coupled with another convection diffusion equation (heat equation) to study the interaction between the temperature field and the velocity field. This couple of equations describes the incompressible fluid flow coupled to the thermal dynamics, which used to model the thermal fluid dynamics of air in building. Our system is defined by the coupled of partial differential equations

$$u_t(t,x) + u(t,x)u_x(t,x) = \mu\, u_{xx}(t,x) - \kappa\, T(t,x) + f_1(t,x) \quad \ldots (1.1a)$$

$$T_t(t,x) + u(t,x)T_x(t,x) = \nu\, T_{xx}(t,x) + f_1(t,x) \quad \ldots (1.1b)$$

The function T(t,x) can be viewed as a temperature field where ν is the thermal conductivity and κ is the coefficient of the thermal expansion, f_1 and f_2 are the forces on the system. Herein, the temperature drives the velocity field and the velocity field provides the convective term. In Section 2 we describe the VIM. The

Convergence of the method and its application to our coupled is describe in section 3. A test problem and numerical result are given in section 4.

2. VIM Analysis.

The variational iteration method (VIM) was proposed In 1999 by the Chinese mathematician He [4-6] as a modification of a general Lagrange multiplier method. This method is used to solve linear and nonlinear partial differential equation and gives rapidly convergent successive approximations of the exact solution if such a solution exists; otherwise a few approximations can be used for numerical purposes [10]. Unlike the traditional numerical methods, VIM needs no discretization, linearization, transformation or perturbation. For our purpose we write our system in an operator form

$$L_t u + R_1(u,T) + N_1(u,T) = f_1,$$
$$L_t T + R_2(u,T) + N_2(u,T) = f_2,$$

with the initials

$u(0,x) = g_1(x),$

$T(0,x) = g_2(x),$

where $L_t u, R_1$ and R_2 are linear operators and $N_1,\ N_2$ are nonlinear operators and f_1, f_2 are source terms. According to variational iteration method the correction functional for our system can be written as

$$u_{n+1} = u_n + \int_0^t \lambda_1(\tau)\big(L\, u_n(\tau) + R_1\big(\tilde{u}_n, \tilde{T}_n\big) + N_1(\tilde{u}_n, \tilde{T}_n) - f_1(\tau)\big)\, d\tau$$

$$T_{n+1} = T_n + \int_0^t \lambda_2(\tau)\big(L\, T_n(\tau) + R_2\big(\tilde{u}_n, \tilde{T}_n\big) + N_2(\tilde{u}_n, \tilde{T}_n) - f_2(\tau)\big)\, d\tau$$

(2.1)

where λ_1 and λ_2 are general Lagrange multipliers, which can be identified optimally via the variational theory and using the integration by parts. The subscript n denotes the nth approximation and $\tilde{u}_n$ and $\tilde{T}_n$ are restricted variations, that is, $\delta\tilde{u}_n = 0$ and $\delta\tilde{T}_n = 0$. The successive approximations u_{n+1} and T_{n+1}; $n \geq 1$,

of the solution $u(t,x)$ and $T(t,x)$ will be readily obtained upon using the obtained Lagrange multiplier and by using any selective function $u_0(t,x)$ and $T_0(t,x)$. Consequently, the exact solution may be obtained by using

$$u(t,x) = \lim_{n\to\infty} u_n(t,x) \; and \; T(t,x) = \lim_{n\to\infty} T_n(t,x)$$

3. Convergence of the variational iteration method.

Definition[10]. A variable quantity v is a functional dependents on a function $u(x)$ if to each function $u(x)$ of a certain class of functions $u(x)$ there corresponds a value v, $v[u(x)]$.

Theorem[10]. *(Banach's fixed point theorem). Assume that X be a Banach space and $H{:}X \to X$ is a nonlinear mapping, and suppose that*

$$||H[u] - H[\bar{u}]|| \leq \gamma \, ||u - \bar{u}||, \quad u, \bar{u} \in X,$$

for some $\gamma \leq 1$. Then H has a unique fixed point. Furthermore, the sequence $u_{n+1} = H[u_n]$ with an arbitrary choice of $u_0 \in X$, converges to the fixed point of H.

According to this theorem, a sufficient condition for convergence of the variational iteration method is strictly contraction of H, i.e.,

$$||H[u_n] - H[\bar{u}_n]|| \leq \gamma \, ||u_n - \bar{u}_n||, \quad u_n, \bar{u}_n \in X.$$

3.1. Convergence of the method for the coupled equations

The proof of convergence of VIM for our system comes from theorem 1, as the following way.

Consider the following nonlinear mapping

$$H[u] = u - \int_0^t \left[\frac{\partial u}{\partial \xi} + u \frac{\partial u}{\partial x} - \frac{\partial^2 u}{\partial x^2} + T - f_1 \right] d\xi = u - \int_0^t \left[\frac{\partial u}{\partial \xi} + \frac{1}{2} \frac{\partial u^2}{\partial x} - \frac{\partial^2 u}{\partial x^2} + T - f_1 \right] d\xi$$

$$H[\bar{u}] = \bar{u} - \int_0^t \left[\frac{\partial \bar{u}}{\partial \xi} + \bar{u} \frac{\partial \bar{u}}{\partial x} - \frac{\partial^2 \bar{u}}{\partial x^2} + T - f_1 \right] d\xi = \bar{u} - \int_0^t \left[\frac{\partial \bar{u}}{\partial \xi} + \frac{1}{2} \frac{\partial \bar{u}^2}{\partial x} - \frac{\partial^2 \bar{u}}{\partial x^2} + T - f_1 \right] d\xi$$

where, $u = u(x,t)$. According to the theorem 1, we get

$$H[u]-H[\bar{u}]=(u-\bar{u})-\int_0^t[\frac{\partial(u-\bar{u})}{\partial\xi}+\frac{1}{2}\frac{\partial(u^2-\bar{u}^2)}{\partial x}-\frac{\partial^2(u-\bar{u})}{\partial x^2}]d\xi$$

By using norm we get

$$\left\|H[u]-H[\bar{u}]\right\|=\left\|(u-\bar{u})-\int_0^t[\frac{\partial(u-\bar{u})}{\partial\xi}+\frac{1}{2}\frac{\partial(u^2-\bar{u}^2)}{\partial x}-\frac{\partial^2(u-\bar{u})}{\partial x^2}]d\xi\right\|$$

$$\geq\left\|(u-\bar{u})\right\|-\int_0^t\left\|[\frac{\partial(u-\bar{u})}{\partial\xi}+\frac{1}{2}\frac{\partial(u^2-\bar{u}^2)}{\partial x}-\frac{\partial^2(u-\bar{u})}{\partial x^2}]\right\|d\xi$$

since

$\left\|\frac{\partial(u-\bar{u})}{\partial\xi}\right\|\leq\delta_1\left\|u-\bar{u}\right\|$, $\left\|\frac{\partial(u^2-\bar{u}^2)}{\partial x}\right\|\leq\delta_2\left\|(u^2-\bar{u}^2)\right\|=\delta_2\left\|(u-\bar{u})(u+\bar{u})\right\|\leq 2\delta_2 N\left\|u-\bar{u}\right\|$ and $\left\|\frac{\partial^2(u-\bar{u})}{\partial x^2}\right\|\leq\delta_3\left\|u-\bar{u}\right\|$

then the inequality becomes

$$\left\|H[u]-H[\bar{u}]\right\|\geq\left\|(u-\bar{u})\right\|-\int_0^t\alpha\left\|u-\bar{u}\right\|d\xi,$$

$$\geq\left\|(u-\bar{u})\right\|(1-\int_0^t\alpha\,d\xi)$$

$$\leq\left\|(u-\bar{u})\right\|(\int_0^t\alpha\,d\xi-1)$$

$$\leq\gamma_1\left\|(u-\bar{u})\right\|.$$

where, $\gamma_1=(\int_0^t\alpha\,d\xi-1)$ and $\alpha=\delta_1+\delta_2 N-\delta_3$.

Using the same way to proof the convergence of the component $T(x,t)$,

$$H[T]=T-\int_0^t[\frac{\partial T}{\partial \xi}+u\frac{\partial T}{\partial x}-\frac{\partial^2 T}{\partial x^2}-e^{-\xi}(x-\frac{x^2}{2})(1-2x)-2]d\xi$$

$$H[\bar{T}]=\bar{T}-\int_0^t[\frac{\partial \bar{T}}{\partial \xi}+u\frac{\partial \bar{T}}{\partial x}-\frac{\partial^2 \bar{T}}{\partial x^2}-e^{-\xi}(x-\frac{x^2}{2})(1-2x)-2]d\xi$$

Where, $T=T(x,t)$. According to the above theorem, we get

$$H[T]-H[\bar{T}]=(T-\bar{T})-\int_0^t[\frac{\partial (T-\bar{T})}{\partial \xi}+u\frac{\partial (T-\bar{T})}{\partial x}-\frac{\partial^2 (T-\bar{T})}{\partial x^2}]d\xi \,.$$

By using L^2norm, we have

$$\left\|H[T]-H[\bar{T}]\right\|=\left\|(T-\bar{T})-\int_0^t[\frac{\partial (T-\bar{T})}{\partial \xi}+u\frac{\partial (T-\bar{T})}{\partial x}-\frac{\partial^2 (T-\bar{T})}{\partial x^2}]d\xi\right\|$$

$$\geq\left\|(T-\bar{T})\right\|-\int_0^t\left\|[\frac{\partial (T-\bar{T})}{\partial \xi}+u\frac{\partial (T-\bar{T})}{\partial x}-\frac{\partial^2 (T-\bar{T})}{\partial x^2}]\right\|d\xi.$$

Since $\left\|\frac{\partial (T-\bar{T})}{\partial \xi}\right\|\leq\delta_4\left\|T-\bar{T}\right\|$, $\left\|\frac{\partial (T-\bar{T})}{\partial x}\right\|\leq\delta_5\left\|T-\bar{T}\right\|$, $\left\|\frac{\partial^2 (T-\bar{T})}{\partial x^2}\right\|\leq\delta_6\left\|T-\bar{T}\right\|$ and $\left\|u\right\|\leq N$

So, the inequality becomes

$$\left\|H[T]-H[\bar{T}]\right\| \geq\left\|(T-\bar{T})\right\|-\int_0^t\alpha_1\left\|T-\bar{T}\right\|d\xi,$$

$$\geq(1-\int_0^t\alpha_1\, d\xi)\,\left\|(T-\bar{T})\right\|$$

$$\leq(\int_0^t\alpha_1\, d\xi-1)\,\left\|(T-\bar{T})\right\|$$

$$\leq\gamma_2\,\left\|(T-\bar{T})\right\|.$$

where, $\alpha_1=\delta_4+N\delta_5-\delta_6$ and $\gamma_2=(\int_0^t\alpha_1\, d\xi\; -1)$

(where δ's are the absolute value of differential operators which appear in partial differential equations)

4. Test Problem.

For a particular case, we consider the following coupled of equation

$$u_t(t,x) + u(t,x)u_x(t,x) = \mu\, u_{xx}(t,x) - \kappa\, T(t,x) + f_1(t,x)$$

$$T_t(t,x) + u(t,x)T_x(t,x) = \nu\, T_{xx}(t,x) + f_1(t,x)$$

(3)

subject to the initial condition

$u(0,x) = sinx,$

$T(0,x) = \frac{1}{2} sin2x$

the exact solution is given by

$u(0,x) = e^{-t} sinx,$

$T(0,x) = \frac{1}{2} e^{-2t} sin2x$

where $\mu = \nu = \kappa = 1$ and

$f_1 = e^{-2t} sin2x\,, \qquad f_2 = e^{-3t} sinx\, cos2x + e^{-2t} sin2x$

To solve the problem (4) by using VIM, we consider the correction functionals (2.1) as

$$u_{n+1}(x,t) = u_n(x,t) + \int_0^t \lambda_1(\xi)\left[\frac{\partial u_n(x,\xi)}{\partial \xi} + \tilde{u}_n(x,\xi)\frac{\partial \tilde{u}_n(x,\xi)}{\partial x} - \frac{\partial^2 \tilde{u}_n(x,\xi)}{\partial x^2} + \tilde{T}_n(x,\xi) - f_1(x,\xi)\right] d\xi,$$

$$T_{n+1}(x,t) = T_n(x,t) + \int_0^t \lambda_2(\xi)\left[\frac{\partial T_n(x,\xi)}{\partial \xi} + \tilde{u}_n(x,\xi)\frac{\partial \tilde{T}_n(x,\xi)}{\partial x} - \frac{\partial^2 \tilde{T}_n(x,\xi)}{\partial x^2} - e^{-\xi}(x - \frac{x^2}{2})(1-2x) - 2\right] d\xi$$

(4)

where, λ_1, λ_2 are the general Lagrange multiplier. The value of λ_1, λ_2 can be found by considering $\tilde{u}_n(x,\xi), \frac{\partial \tilde{u}_n(x,\xi)}{\partial x}, \frac{\partial^2 \tilde{u}_n(x,\xi)}{\partial x^2}, \tilde{T}_n(x,\xi), \frac{\partial \tilde{T}_n(x,\xi)}{\partial x}, \frac{\partial^2 \tilde{T}_n(x,\xi)}{\partial x^2}$ and $f_2(x,\xi)$ as restricted variations in equations (5), then by integration by part we obtain $\lambda_1 = \lambda_2 = -1$. Hence, the correction functionals (4) become

$$u_{n+1}(x,t) = u_n(x,t) - \int_0^t \left[\frac{\partial u_n(x,\xi)}{\partial \xi} + \tilde{u}_n(x,\xi)\frac{\partial \tilde{u}_n(x,\xi)}{\partial x} - \frac{\partial^2 \tilde{u}_n(x,\xi)}{\partial x^2} + \tilde{T}_n(x,\xi) - f_1(x,\xi) \right] d\xi,$$

$$T_{n+1}(x,t) = T_n(x,t) - \int_0^t \left[\frac{\partial T_n(x,\xi)}{\partial \xi} + \tilde{u}_n(x,\xi)\frac{\partial \tilde{T}_n(x,\xi)}{\partial x} - \frac{\partial^2 \tilde{T}_n(x,\xi)}{\partial x^2} - e^{-\xi}(x - \frac{x^2}{2})(1-2x) - 2 \right] d\xi$$

(5)

Using the above iteration formulas (5) and the initial approximation, we can obtain the following approximations

$$u_1 = (1-t)sinx - \left(\frac{e^{-2t}}{2} + t - \frac{1}{2}\right)sin2x,$$

$$T_1 = \left(1 + 2t - \frac{e^{-2t}}{2}\right)sin2x - \left(\frac{e^{-3t}}{3} + t - 1\right)sinx\,cos2x$$

$$\begin{aligned} u_2 = {} & sinx\left(1 - t + \frac{t^2}{2}\right) - sin2x\left(\frac{t}{2} - \frac{7e^{-2t}}{4} + \frac{7t^2}{4} + \frac{5}{4}\right) - 2t\,cos2x \\ & - sin4x\left(\frac{t}{4} + \frac{e^{-2t}}{8} - \frac{e^{-4t}}{16} - \frac{e^{-2t}}{2} - \frac{3t^2}{4} + \frac{t^3}{3} - \frac{1}{16}\right) \\ & + sinx\,cos2x\left(\frac{t\,e^{-2t}}{2} - \frac{e^{-2t}}{4} - \frac{e^{-3t}}{9} - \frac{4t}{3} + 2t^2 - \frac{2t^3}{3} + \frac{13}{36}\right) \\ & - cosx\,sin2x\left(\frac{t}{2} + \frac{e^{-2t}}{8} - \frac{t\,e^{-2t}}{4} - \frac{3t^2}{4} + \frac{t^3}{3} - \frac{1}{8}\right) \end{aligned}$$

$$T_2 = sin2x\left(4t^2 - \frac{3\,e^{-2t}}{2} - 4t + 2\right)$$
$$+ sin4x\left(\frac{e^{-3t}}{27} - \frac{7t}{12} + \frac{e^{-4t}}{16} + \frac{3te^{-2t}}{4} + \frac{te^{-3t}}{36} + \frac{4t^2}{3} - \frac{3t^3}{4} - \frac{43}{432}\right)$$
$$- sinx\,cos2x\left(\frac{t}{3} + \frac{7e^{-2t}}{16} - \frac{8e^{-3t}}{9} - \frac{te^{-2t}}{8} - \frac{t^2}{2} + \frac{t^3}{3} + \frac{65}{144}\right)$$
$$+ cosx\,sin2x\left(2t^2 - \frac{4e^{-3t}}{9} + \frac{4}{9} - \frac{4t}{3}\right)$$
$$- \sin^2 x\ sin2x\left(\frac{8e^{-3t}}{27} - \frac{2t}{3} + \frac{2te^{-3t}}{9} + \frac{4t^2}{3} - \frac{2t^3}{3} - \frac{8}{27}\right)$$
$$+ sinx\sin^2 2x\left(\frac{t}{3} - \frac{e^{-2t}}{12} + \frac{e^{-3t}}{27} - \frac{e^{-5t}}{15} - \frac{te^{-2t}}{2} - \frac{2te^{-3t}}{9} - \frac{5t^2}{6} + \frac{2t^3}{3} + \frac{61}{540}\right)$$
$$- cosx\,sin4x\left(\frac{t}{12} - \frac{e^{-2t}}{48} + \frac{e^{-3t}}{108} - \frac{e^{-5t}}{60} - \frac{te^{-2t}}{8} - \frac{te^{-3t}}{18} - \frac{5t^2}{24} + \frac{t^3}{6} + \frac{61}{2160}\right)$$

and we can continue to find the other iterations. The values of VIM for u and T are depicted in Tables 1, 2, 3, and 4.

Table 1: The absolute error of the approximate velocity at t=0.5, (where u_e is the exact solution at t=0.5).

x	$\lvert u_e - u_2\rvert$	$\lvert u_e - u_3\rvert$
0	*0*	*0*
0.314159265358979	*5.379899999865633 e-005*	*3.86498837799179 e-006*
0.628318530717959	*4.343799875878512 e-005*	*3.44003648004498 e-007*
0.942477796076938	*7.249699876889492 e-005*	*6.61009865904166 e-006*
1.256637061435917	*2.722998764778933 e-006*	*1.63499999994432 e-006*
1.570796326794897	*1.394284376589445 e-005*	*2.60056436004583 e-007*
1.884955592153876	*1.642999865389401 e-006*	*2.25199999998238 e-007*
2.199114857512855	*2.499408765438004 e-005*	*5.96063880001936 e-006*
2.513274122871835	*8.288999464785661 e-005*	*2.44500097501957 e-006*
2.827433388230814	*1.976008656601246 e-005*	*1.73547684770766 e-007*
3.1415926535897	*0*	*0*

Table 2: The absolute error of the approximate velocity at t=1, (where u_e is the exact solution at t=1).

x	$\lvert u_e - u_2 \rvert$	$\lvert u_e - u_3 \rvert$
0	0	0
0.314159265358979	2.312289974958399 e-004	1.444000973600320e-006
0.628318530717959	4.543900853000887 e-005	5.988598853998120e-006
0.942477796076938	1.638520000000263 e-005	2.114199754999740e-005
1.256637061435917	1.232810000000200 e-004	1.079700050875113e-005
1.570796326794897	5.427199999848737 e-005	9.097009948001350e-006
1.884955592153876	7.762610009875000 e-004	1.133090476539978e-005
2.199114857512855	2.707180085470035 e-004	2.907993864399891e-005
2.513274122871835	4.400069999999217 e-005	1.708076320888049e-005
2.827433388230814	3.290780008430010 e-004	1.860700865400022e-005
3.1415926535897	0	0

Table 3: The absolute error of the approximate temperature at t=0.5, (where u_e is the exact solution at t=0.5).

x	$\lvert T_e - T_2 \rvert$	$\lvert T_e - T_3 \rvert$
0	0	0
0.314159265358979	8.88293657899918 e-005	1.65804545500260 e-006
0.628318530717959	2.83099967400232 e-005	4.62046587400662 e-006
0.942477796076938	1.79657464740904 e-005	1.66275750001338 e-007
1.256637061435917	5.19700957769007 e-006	3.84000565900867 e-006
1.570796326794897	2.69369075494000 e-005	2.24345567554000 e-007
1.884955592153876	7.17899349998983 e-006	1.40006798000262 e-005
2.199114857512855	1.94079967892985 e-006	7.54996484699718 e-006
2.513274122871835	1.38491638949926 e-005	6.85069697900227 e-007
2.827433388230814	3.14699457990640 e-005	1.09903005700996 e-006
3.1415926535897	0	0

Table 4: The absolute error of the approximate temperature at t=1, (where u_e is the exact solution at t=1).

x	$\lvert T_e - T_2 \rvert$	$\lvert T_e - T_3 \rvert$
0	*0*	*0*
0.314159265358979	*2.20199000325504 e-004*	*6.43710035690006 e-005*
0.628318530717959	*3.77225996543994 e-003*	*4.83835789879996 e-005*
0.942477796076938	*1.91654488530001 e-004*	*3.60810764380072 e-005*
1.256637061435917	*2.29150006433405 e-005*	*2.84870006864007 e-005*
1.570796326794897	*3.02334000007740 e-004*	*1.59229003650000 e-005*
1.884955592153876	*7.56250999642899 e-004*	*1.06710008750038 e-005*
2.199114857512855	*2.98446006374008 e-003*	*1.06470802530025 e-005*
2.513274122871835	*1.77713964903995 e-004*	*5.66000000340074 e-006*
2.827433388230814	*2.93936068600002 e-004*	*6.08240075400010 e-005*
3.1415926535897	*0*	*0*

4.1. Discussion

In this paper the approximate solution of the coupled Burger's equation and advection diffusion equation is obtained by using the variational iteration method in the domain $[0,\pi]$, where the initials are functions of the variable x, and the external Forces f_1 and f_2 are given. We calculate the absolute error of the second and third iteration with the exact solution at times t=0.5 and t=1 as shown in the tables 1, 2, 3, and 4. The results shows that the errors are decreasing with increasing the iteration and increasing with time. Moreover, the results proved that this method is converge rapidly to the exact solution with less iteration.

Conclusion

The aim of this work is employing the powerful variational iteration method to investigate coupled of nonlinear equations, which is a coupled Burger's equation and advection diffusion equation (heat equation). It is obvious that the method gives rapidly convergent successive approximations through determining the Lagrange multipliers. Moreover, this method is quite efficient and suitable for finding the approximate solution of this couple of PDE.

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الحل التقريبي لثنائي معادلة بركر و معادلة الانتقال والتشتت

المستخلص

يهدف هذا البحث لايجاد حل تقريبي لثنائي معادلة برجر ومعادلة الانتقال والتشتت باستخدام طريقة التغاير التكرارية. حيث تولد هذه الطريقة متتابعة من الدوال تتقارب الى الحل المضبوط لثنائي المعادلات قيد الدراسة. وقد اثبت ان هذه الطريقة ملائمة تماما لايجاد حل تقريبي لهذا النوع من المعادلات.