

Coflat Modules and Exact Sequence

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Abstract:

We know that a right R -module C is flat if and only if every short exact sequence $0 \rightarrow A \rightarrow B \rightarrow C \rightarrow 0$ of right R -modules is pure. Dually a right R -module A is coflat if ever short exact sequence $0 \rightarrow A \rightarrow B \rightarrow C \rightarrow 0$ of right R -modules is copure. Co flat modules are defined in several ways, (as shown in [2] and [4]), In this paper it has shown that these definitions are equivalents. And we shell prove that every direct sum of coflat modules is coflat.

الخلاصة:

من المعروف أن الموديول الأيمن C يكون مسطح إذا وفقط إذا كانت كل سلسلة مضبوطة من الموديولات على R تكون نقية. وبشكل مرادف فإن الموديول A يكون رديف مسطح إذا وفقط إذا كانت كل سلسلة مضبوطة من الموديولات تكون رديف نقية. لقد عرفت الموديولات رديف مسطحة بطرق متعددة كما هو مبين في [2] و [4] , في هذا البحث برهنا أن هذه التعاريف متكافئة . وسنبرهن أن المجموع المباشر لموديولات رديف مسطحة يكون رديف مسطح.

Introduction:

In 1979 Damiano [2] has defined coflat modules using matrices, and has given two other ways to define coflat modules, which he had show that they are equivalents. In the other hand we see that Hiremath [4] defined coflat modules that: an R -module A is said to be coflat if every short exact sequence $0 \rightarrow A \rightarrow B \rightarrow C \rightarrow 0$ of R -modules is copure. And he proved that every injective module is coflat, every direct union and direct sum of injective modules is coflat.

1-Coflat modules :

Definition1-1: A short exact sequence $0 \rightarrow A \rightarrow B \rightarrow C \rightarrow 0$ of R -modules is said to be copure if every related R -module is injective relative to this sequence.

Definition1-2[4]: An R -module A is said to be Coflat if and only if every short exact sequence $0 \rightarrow A \rightarrow B \rightarrow C \rightarrow 0$ of R -modules is copure.

It is an easy observation that an R -module A is coflat if and only if A is a copure submodule of every R -module in which it is embedded. So a coflat module can also be called as an absolutely copure module.

Definition1-3[2]: A module A satisfies the \mathfrak{N} -Baer criterion in case for every finitely generated right ideal I of R and every $R - homomorphism$:

$f: I \rightarrow A$ there exists an $a \in A$ with $f(x) = ax$ ($x \in I$).

Proposition1-4[2]: A module A is coflat if and only if it satisfies the \mathfrak{N} -Baer criterion .

Corollary1-5[2]: Every injective module is coflat.

Proof : Since every injective module satisfies the \mathfrak{N} -Baer criterion , so it is coflat .

Damiano has defined co flat as following :

Definition1-6[4]: For each $p \in \mathbb{N}$ and each $a \in R^p$ the cokernel $A^p / Im a$ is cogenerated as a set by $\{(C, A^p): C \in Mat_{p \times q}(R) s. t. aC = 0, p = 1, 2, \dots\}$.

But Singh defined co flat that is :

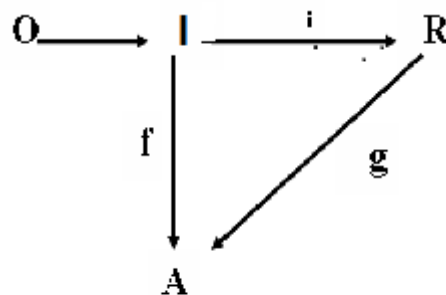
Definition1-7[3]: For each exact sequence $0 \rightarrow I \rightarrow R$ with I finitely generated the sequence $Hom_R(R, A) \rightarrow Hom_R(I, A) \rightarrow 0$ is exact.

The following proposition gathering between the definitions .

Proposition1-8: For an R -module A the following conditions are equivalent :

- (i) – For each exact sequence $0 \rightarrow I \rightarrow R$ with I finitely generated the sequence $Hom_R (R,A) \rightarrow Hom_R (I,A) \rightarrow 0$ is exact .
- (ii) – For each $p \in N$ and each $a \in R^p$ the cokernel $A^p/Im a$ is cogenerated as a set by $\{(C, A^p): C \in Mat_{p \times q}(R) \text{ s.t. } aC = 0, p = 1, 2, \dots\}$.
- (iii) – For each $r \in R^p$, if $a \in A^p/Ar$, then $aC \neq 0$ for some $p \times q$ matrix C such that $rC = 0$.
- (iv) – For each $n \in N$, each $r \in R^n$, and each $a \in A^{(n)}$, if $a \notin Ar$, then there is $rc \in R^n$ with $rc^t = 0$ and $ac^t \neq 0$.
- (v) – A satisfies the \aleph – Baer criterion .

Proof : For (ii) \Leftrightarrow (iii) \Leftrightarrow (iv) \Leftrightarrow (v) see [2] , now we shall prove (v) \Rightarrow (i) : define $f: I \rightarrow A$, $\forall r \in I$, $f(r) = ar$ for some $a \in A$



We claim that $g: R \rightarrow A$, s.t. $g(r) = ar$

It is clear that : $(g \circ i)(s) = g(i(s)) = g(s) = as = f(s)$.

And for (i) \Rightarrow (v) :

Let $i: I \rightarrow R$, $g: R \rightarrow A$, $g(r) = ra$, for some

$a \in A$, we claim that $f = g \circ i$, it is clear that

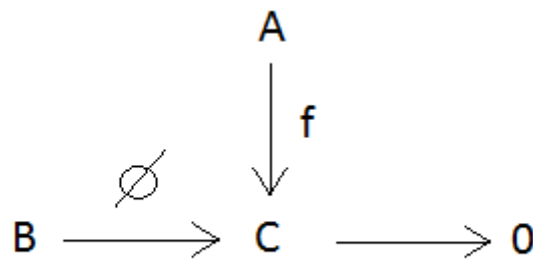
$\exists a_1 \in A$, with $a = a_1 r$ which ends the proof.

EXAMPLE

1.9. Let V_F be an infinite dimensional F -vector space and let $I = \{f \in End (V_F) \mid dim Im f < \infty\}$. If R is the subring of $End (V_F)$ generated by I , then it is easy to see that R is von Neumann regular, and ${}_R R/I$ is a simple noninjective R -module. Thus ${}_R R/I$ is coflat [2].. In the other hand if I is infinitely generated, then ${}_R R/I$ satisfies \aleph -Baer criterion, which mean it is coflat.

2-Exact sequence:

Definition2-1: An R-module A is called B-projective if given a diagram



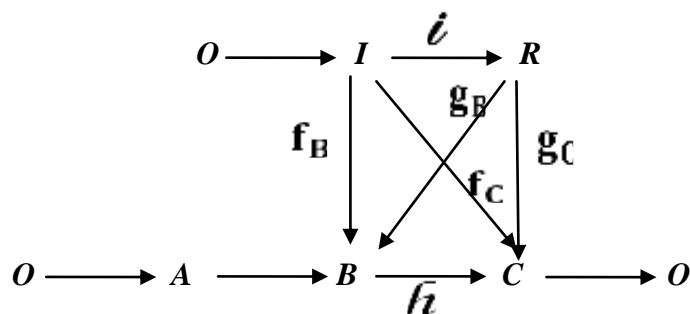
of maps of R-modules with horizontal sequence exact, \exists a map $h: A \rightarrow B$ such that $\emptyset \circ h = f$.

Proposition2-2: For an exact sequence :

$$0 \rightarrow A \rightarrow B \rightarrow C \rightarrow 0$$

If B is coflat I is A – projective , then C is coflat .

Proof : suppose $0 \rightarrow I \rightarrow R, f_B: I \rightarrow B, g_B: R \rightarrow B, f_B = g_B \circ i$



then if $g_C: R \rightarrow C, h: B \rightarrow C$, define $f_C: I \rightarrow C$, such that $f_C = h \circ f_B$ (I is A – projective)

$$\begin{aligned}
 \text{then } (g_C \circ i)(x) &= g_C(x) = g \circ g_B(x) \\
 &= g \circ g_B \circ i(x)
 \end{aligned}$$

So that $g_B = g \circ g_B$.

Proposition2-3[2].: Every direct union and direct sum of injective modules is cflat .

Proposition2-4: Every direct sum of coflat modules is coflat .

Proof : Since every injective module is coflat (Corollary5), so every direct sum of coflat modules is direct sum of injective modules, and hence every direct sum of coflat modules is coflat .

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