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Comparison of Classical and Bayesian methods to Estimate the shape parameter and Reliability function in Burr type X or two parameter of exponential Rayleigh distribution under different Loss function

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Abstract

The paper deals with estimation shape parameter of Burr distribution type X (which also called two parameter exponential Rayleigh).The methods of estimation are Maximum likelihood method and percentiles estimator (PER), as well as Bayesian estimator using Jeffery's prior distribution. The Bayes estimator of shape parameter (θ) is conducted using different loss function, like squared error loss function and also precautionary loss function .The comparison is done using statistical Measures Mean Square error (MSE) take different sample size (n=15,30,50,100) and different set of initial values for (θ,ʎ)

Keywords: Burr type X distribution, Maximum likelihood estimator، Percentiles Estimator، Bayes estimator ،Mean squared error.

مقارنة بين الطرائق الاعتيادية والبيزية لتقدير معلمة الشكل ودالة المعوليه لتوزيع بور نوع X او توزيع رآلي الأسي ذي المعلمتين تحت دوال خسارة مختلفة

أ.م.د. أنعام عبد الرحمن نعمان **م.م. هدى عامر عبد الامير**

المستخلص

في بحثنا هذا تناولنا فيه دراسة للتقدير معلمة الشكل لتوزيع بور نوع العاشر الذي يسمى ايضا (توزيع رالي الاسي ذو معلمتين) وكانت طرائق التقدير هي (طريقة الامكان الاعظم وطريقة المقدرات الجزئية) وكذلك المقدر البيزي باستخدام التوزيع الاولي الجفري والطريقة البيزية للتقدير استخدم فيها دوال خسارة مختلفة متماثلة وغير متماثلة وهما دالة الخسارة التربيعية ودالة الخسارة الوقائية وللمقارنه استخدمنا متوسط مربعات الخطأ بأخذ احجام عينة مختلفة الصغيرة والمتوسطة والكبيرة ومجموعة مختلفة من القيم الاولية للمعلمتين (θ,ʎ).

الكلمات المفتاحية : توزيع بور نوع العاشر ، مقدر الامكان الاعظم ، المقدرات الجزئية ، المقدر البيزي ، متوسط مربعات الخطأ .

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1. INTRODUCTION

Burr type X studied by some authors : Ahmad Sartawi and Abe-Salih(1991) , Jaheen (1995), Jaheen (1996), Ahmad et al.(1997), Raqab (1998) , Surles and Padgett (1998). Surles and Padgett (2001) proposed and observed that Eq (1) could be used quite effectively in modeling strength data as well as modeling general life time data. Kunda and Raqab (2006) studied the relationship of Burr type X with Weibull, Gamma, and Generalized Exponential Weibull distribution . Lio et al.(2014) studied the control charts for monitoring Burr type X and in same year (Athak and Chaturvedi (2014)) compare the performance of the uniformly minimum variance unbiased and maximum likelihood estimator of the reliability function R(t) = P(X>t) and P = P(X>Y)for the two parameter exponential Rayleigh distribution. Smith et al. (2015) studied the higher order inference for stress – strength reliability with independent Burr X.

2. Theoretical Aspects

Burr introduced twelve different forms of cumulative distribution function for modeling lifetime data or survival data. [2][5] , out of those twelve distribution , Burr type X and Burr type XII have received the maximum attention due to this application in the study of biological industrial , reliability and life testing and survival industrial and economic experiments [7] .

The Burr type x and named the two parameter exponential Rayleigh has the following distribution function for X>0 [3] [4] :

$$F(x;\theta,\lambda) = (1-e^{-\lambda^2 x^2})^{\theta} \quad x>0,\ \theta>0,\ \lambda>0 \qquad \text{....(1)}$$

There for the Burr type x has the density function for X>0:

$$f(x;\theta,\lambda) = 2\lambda^2\theta x e^{-\lambda^2 x^2}(1-e^{-\lambda^2 x^2})^{\theta-1} \quad x>0,\ \theta>0,\ \lambda>0 \qquad \text{....(2)}$$

Where θ and λ are the shape and scale parameters respectively distribution

The reliability function, R(t), and the hazard function ,h(t) are given as follows :

$$R(t) = 1-(1-e^{-\lambda^2 t^2})^{\theta} \quad t>0,\ \theta>0,\ \lambda>0 \qquad \text{.........(3)}$$

$$h(t) = \frac{f(t)}{R(t)} = \frac{2\lambda^2\theta t e^{-\lambda^2 t^2}(1-e^{-\lambda^2 t^2})^{\theta-1}}{1-(1-e^{-\lambda^2 t^2})^{\theta}} \qquad \text{.........(4)}$$

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The two parameter burr type x has several types of distribution like Rayleigh when (ɵ =1)

2.1 Different estimators of shape parameter θ and Reliability function R(t)

According to the estimations of ($\hat{\theta}$) we can obtain estimators of reliability function R(t), using different set of scale parameter (λ) and (n). Is necessary to know the probability of working system and life system mean time of failure since the estimators of reliability function R(t) is necessary to know the probability of working system and life system mean time of failure

2.1.1 Classical estimators

1- Maximum likelihood Estimator (MLE) of θ

Let $x_1, x_2, \ldots, x_n$ be a random sample of size n drown from the Burr type X distribution defined by (2). Then the likelihood function for the given random sample is given by :

$$L(\theta,\lambda|\underline{x}) = \prod_{i=1}^{n} f(xi/\theta,\lambda)$$

$$L(\theta,\lambda \setminus \underline{x}) = (2\theta\lambda^{\lambda})^{n} \prod_{i=1}^{n} x_i e^{-\lambda^2 \sum x_i^2} \prod_{i=1}^{n} (1-e^{-\lambda^2 x^2})^{\theta-1} \quad \ldots\ldots\ldots (5)$$

From which we calculate the log – likelihood function :

$$LnL(\theta,\lambda|\underline{x}) = C + n\ln\theta + n\ln\lambda^2 + \sum_{i=1}^{n} \ln x_i - \lambda^2 \sum_{i=1}^{n} \chi_{x_i}^{2} + (\theta-1)\prod_{i=1}^{n} \ln(1-e^{-\lambda^2 x^2}) \quad \ldots\ldots\ldots(6)$$

Finding the maximum with respect to θ by taking the derivative and setting it equal to zero.

Yields the Maximum likelihood Estimation of θ parameter by $\hat{\theta}_{ML}$

$$\hat{\theta}_{ML} = \frac{-n}{\sum_{i=1}^{n} \ln(1-e^{-(\lambda x)^2})} \quad \ldots\ldots\ldots\ldots\ldots(7)$$

2- Percentiles Estimator (PERCE)

For burr type x distribution, We can use the percentiles point estimators for estimate [6] the parameter θ when λ is known, we use the C.D.F function defined by (1)

$$F(x;\theta,\lambda) = (1-e^{-\lambda^2 x^2})^{\theta}$$

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Take log for C.D.F

ln F(x;θ,λ)= $ln(1-e^{-\lambda^2x^2})^{\theta}$

let p_i is c.d.f F(x;θ, λ)

ln p_i= $ln(1-e^{-\lambda^2x^2})^{\theta}$ …….(8)

then

ln p_i- $\boldsymbol{ln(1-e^{-\lambda^2x^2})^{\theta}=0}$ **…….(9)**

take squared and sum for equation (8)

$\sum_{i=1}^{n}(\boldsymbol{lnp_i}-\theta\ \boldsymbol{ln(1-e^{-\lambda^2x^2}})^2=0$ …… (10)

Finding the percentiles to θ by taking the derivative

Yields the **Percentiles Estimator** of θ parameter by $\hat{\theta}_{PCE}$

$$\hat{\theta}_{PCE}=\frac{\sum_{i=1}^{n}(\boldsymbol{lnp_i\ ln\left(1-e^{-\lambda^2x_i^2}\right)})}{\sum_{i=1}^{n}(\boldsymbol{ln\left(1-e^{-\lambda^2x_i^2}\right)})^2} \quad ………(11)$$

Since the equation (8) depends on the estimator (P_i) which take formula of them such as :

$P_i=(i-1/2)/n$ .... (12)

2.1.2 Bayes estimators

In Bayesian estimation procedure we consider the parameter , or parameters are random variable have prior distribution determined from experience and past data so we use these priors g(θ) or g(θ_1) g(θ_2) …… to find the posterior distribution of parameters given the observations of samples i.e. $f(\theta/\underline{x})$ and then according to the obtained posterior , and different type of loss function , we can obtain the Bayesian estimator of θ which work on Minimizing the Expected loss function ,in our research we consider θ as r.v. having Jeffery prior, which the loss function used are squared error loss function , precautionary loss function. We shall Explain how to obtain Bayesian estimator for (θ) (shape parameter) under these different type of posterior distribution and find Bayesian Reliability Estimators and all the results are explained in tables.

1- prior and posterior Density function

For Bayesian estimation we need to specify a Jeffery prior distribution Then the posterior density function of θ , denoted by :

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$\Pi(\theta|\underline{x})$ for the given random sample X with prior information is given by combining the specified prior with likelihood (5) such that :

$$\prod_{i=1}^{n}(\theta|\underline{x}) = \frac{\prod_{i=1}^{n} f(x_i;\theta).g(\theta)}{\int \prod_{i=1}^{n} f(x_i;\theta).g(\theta)d(\theta)}$$

The consider prior and corresponding posterior density function are summarized in table (1)

Table (1)
the prior and posterior density functions:

prior distribution	posterior distribution
Jeffery prior[6] $g(\theta) \propto 1/\theta$	$\prod(\theta|\underline{x})_J = \frac{((\sum \ln(1-e^{-\lambda^2 x^2})^{-1})^n}{\Gamma(n)} \theta^{n-1} e^{-\theta \sum \ln(1-e^{-(\lambda x)^2})^{-1}}$ $\Rightarrow (\theta/\underline{x})_j \sim$ Gamma (n, $\sum \ln(1-e^{-(\lambda x)^2})^{-1}$)

The Jeffery's prior considered as [1]: $g(\theta) \propto \left[\sqrt{I(\theta)}\right]$

Where $I(\theta)$= -n E $\left[\frac{\partial^2 \ln f(t,\lambda,\theta)}{\partial \theta^2}\right]$ is the fisher's information matrix

2-Loss function

Here we have determined Bayes estimators of shape parameter θ based on different to symmetric loss function " squared error , an symmetric loss, entropy and precautionary loss function associates importance to the losses due to overestimation and under estimation of equal magnitude however in the real application .

Thus , the use of symmetric loss function is inappropriate .

In this case , an asymmetric loss function must be considered . the Bayes estimator corresponding to each loss function are given by the formulas which summarized in table (2) , the obtained Bayes estimator of θ for Burr type x distribution in table (2).

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Table (2)
Bayes Estimator under different Loss function.

Loss function	Shape parameter	Reliability function
Squared error loss [8] $L(\theta,\hat{\theta})=(\theta-\hat{\theta})^2$	$\hat{\theta}_s=E_{\Pi}(\theta,\underline{x})$	$\hat{R}(x)_s=E_{\Pi}(R(x)/\underline{x})$
Precautionary loss [7] $L(\theta,\hat{\theta})=\frac{(\theta-\hat{\theta})^2}{\theta}$	$\hat{\theta}_p=\sqrt{E_{\Pi}(\theta^2/\underline{x})}$	$\hat{R}(x)_P=\sqrt{E_{\Pi}(R(x))^2/\underline{x}}$

Table (3)
Bayes Estimator for Burr type x distribution

Prior dist	Bayes Estimator	
	Shape parameter	Reliability function
Square error		
Jeffery	$\hat{\theta}_{SJ}=\frac{n}{\sum\ln(1-e^{-(\lambda x)^2})^{-1}}$	$\hat{R}(x)_s=$ 1- $(1+\frac{\ln(1-e^{-(\lambda x)^2}}{\sum_{i=1}^{n}\ln(1-e^{-(\lambda x)^2}})^{-n}$
Precautionary error		
Jeffery	$\hat{\theta}_{PJ}=\frac{\sqrt{n(n-1)}}{\sum\ln(1-e^{-(\lambda x)^2})^{-1}}$	$\hat{R}(X)_{PJ=}\sqrt{1-2\left((\frac{\sum ln(1-e^{-\lambda^2x^2})^{-1}}{\sum ln(1-e^{-\lambda^2x^2})^{-1}+ln(1-e^{-\lambda^2x^2})}\right)^n+\left(\frac{\sum ln(1-e^{-\lambda^2x^2})^{-1}}{\sum ln(1-e^{-\lambda^2x^2})^{-1}+2ln(1-e^{-\lambda^2x^2})}\right)^n}$

3. Simulation and Results

in this section we introduce the simulation study which has been conducted assess the statistic performance of the shape parameter . The simulation program is written by using (Mat lab) program .

Simulation experience includes four basic and important stage to estimate the shape parameter and reliability function of Burr type x

First stage :

This is the most important stage the depends on it the rest of the stage . the first stage involves determine the default values (true values) for:

The parameter of Burr type x dist. (λ,θ): The default values are varied to observe the effect of parameter on the estimators when

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λ> θ , λ= θ, λ< θ we consider the vales (λ=0.5, 1) with (θ = 0.5, 1, 1.5,2) and different sample sizes (n=15,30,50,100)
to represent small ,median , and large at least .
Number of sample replicated size (L=1000) :

Second stage:

The stage involves data generating: A random data are generated as uniform distribution (U) for period (0,1) then data generated are converted from uniform distribution to data distribution as Burr type X distribution with shape parameter λ and θ:

$$x_i = F^{-1}(Ui) = \frac{\sqrt{(\ln(1-Ui)^{1/\theta})^{-1}}}{\lambda} \qquad i = 1,2,\dots\dots\dots \qquad (13)$$

Third stage:

At this stage of the research, a comparison between real and estimators values of the different methods by using the comparison scale (MSE) , according to the formula

$$MSE(\hat{\theta}) = \frac{\sum_{I=1}^{n}(\hat{\theta}_L - \theta)^2}{L} \quad , \quad MSE(\widehat{R}(t)) = \frac{\sum_{I=1}^{n}(\widehat{R}(t)_L - R(t))^2}{L} \qquad (14)$$

fourth stage:

In this stage, the simulation results are presented in order to find the best estimation of the parameter of the form and the function of the dependencies as shown in the tables below

Table (4)

Values of mean square error (MSE) for (***θ***) by different methods and due to different sample size with initial values (***θ*** =0.5,1,1.5,2), λ=0.5,R=1000).

θ=0.5, λ=0.5					
n	$\hat{\theta}$ MLE	$\hat{\theta}$ PEC	$\hat{\theta}$ SJ	$\hat{\theta}$ PJ	Best
15	0.3388	0.7283	0.1738	0.2765	$\hat{\theta}$ SJ
30	0.2078	8.3670	0.1271	0.1897	$\hat{\theta}$ SJ
50	0.1352	0.0058	0.0930	0.1332	$\hat{\theta}$ PEC
100	0.0779	0.0027	0.0569	0.0763	$\hat{\theta}$ PEC
θ=1, λ=0.5					
n	$\hat{\theta}$ MLE	$\hat{\theta}$ PEC	$\hat{\theta}$ SJ	$\hat{\theta}$ PJ	Best
15	0.2502	0.9730	0.1139	0.2089	$\hat{\theta}$ SJ
30	0.1594	11.8651	0.0930	0.1524	$\hat{\theta}$ SJ
50	0.1082	0.0159	0.0715	0.1103	$\hat{\theta}$ PEC

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100	0.0632	0.0076	0.0457	0.0647	$\hat{\theta}$ PEC
θ=1.5, λ=0.5					
n	$\hat{\theta}$ MLE	$\hat{\theta}$ PEC	$\hat{\theta}$ SJ	$\hat{\theta}$ PJ	Best
15	0.1953	1.1407	0.0774	0.1647	$\hat{\theta}$ SJ
30	0.1303	13.9638	0.0710	0.1274	$\hat{\theta}$ SJ
50	0.0910	0.0261	0.0573	0.0949	$\hat{\theta}$ PEC
100	0.0543	0.0126	0.0382	0.0568	$\hat{\theta}$ PEC
θ=2, λ=0.5					
n	$\hat{\theta}$ MLE	$\hat{\theta}$ PEC	$\hat{\theta}$ SJ	$\hat{\theta}$ PJ	Best
15	0.1571	1.2641	0.0537	0.1334	$\hat{\theta}$ SJ
30	0.1098	15.4274	0.0557	0.1092	$\hat{\theta}$ SJ
50	0.0786	0.0354	0.0471	0.0835	$\hat{\theta}$ PEC
100	0.0478	0.0172	0.0328	0.0510	$\hat{\theta}$ PEC

able (5)

Values of mean square error (MSE) for ($\boldsymbol{\theta}$) by different methods and due to different sample size with initial values ($\boldsymbol{\theta}$ =0.5,1,1.5,2), λ =1,R=1000).

θ=0.5, λ=1					Best
n	$\hat{\theta}$ MLE	$\hat{\theta}$ PEC	$\hat{\theta}$ SJ	$\hat{\theta}$ PJ	
15	0.1621	0.1147	0.1401	0.1375	$\hat{\theta}$ PEC
30	0.0913	0.6953	0.0885	0.0948	$\hat{\theta}$ SJ
50	0.0586	0.0014	0.0607	0.0690	$\hat{\theta}$ PEC
100	0.0447	6.3889	0.0347	0.0429	$\hat{\theta}$ SJ
θ=1, λ=1					
n	$\hat{\theta}$ MLE	$\hat{\theta}$ PEC	$\hat{\theta}$ SJ	$\hat{\theta}$ PJ	Best
15	0.1223	0.1619	0.1093	0.1069	$\hat{\theta}$ PJ
30	0.0741	0.8762	0.0722	0.0784	$\hat{\theta}$ SJ
50	0.0480	0.0042	0.0507	0.0588	$\hat{\theta}$ PEC
100	0.0364	0.0020	0.0296	0.0377	$\hat{\theta}$ PEC
θ=1.5, λ=1					
n	$\hat{\theta}$ MLE	$\hat{\theta}$ PEC	$\hat{\theta}$ SJ	$\hat{\theta}$ PJ	Best
15	0.0966	0.2019	0.0870	0.0850	$\hat{\theta}$ PJ
30	0.0615	0.9920	0.0603	0.0663	$\hat{\theta}$ MLE
50	0.0405	0. 078	0.0433	0.0512	$\hat{\theta}$ MLE
100	0.0314	0. 038	0.0258	0.0338	$\hat{\theta}$ PEC
θ=2, λ=1					
n	$\hat{\theta}$ MLE	$\hat{\theta}$ PEC	$\hat{\theta}$ SJ	$\hat{\theta}$ PJ	Best
15	0.0782	0.2367	0.0706	0.0687	$\hat{\theta}$ PJ
30	0.0520	1.0787	0.0113	0.0571	$\hat{\theta}$ SJ
50	0.0348	0.0114	0.0376	0.0454	$\hat{\theta}$ PEC
100	0.0278	0. 056	0.0229	0.0308	$\hat{\theta}$ MLE

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Table (6)
Real values and MSE for Reliability function when
($\boldsymbol{\theta}$=0.5, λ=0.5,R=1000)

n	t_i	Real R(t)	$\hat{R}$ MLE	$\hat{R}$PEC	$\hat{R}$ SJ	$\hat{R}$ PJ	Best
15	1.5	0.7539	0.96897	0.21589	0.36624	0.503	$\hat{R}$PEC
	2	0.25337	0.0182	0.05847	0.23764	0.46305	$\hat{R}$ MLE
	2.5	0.3441	0.22426	0.015523	0.10399	0.31862	$\hat{R}$ SJ
	3	0.2049	0.025268	0.0038804	0.030801	0.17414	$\hat{R}$PEC
	3.5	0.1110	0.0020373	0.00090767	0.006239	0.081697	$\hat{R}$PEC
	4	0.0542	0.0001141	0.00020459	0.00084934	0.034848	$\hat{R}$ MLE
30	1.5	0.7539	0.239533	0.8617	0.247533	0.258862	$\hat{R}$ MLE
	2	0.5297	0.427164	0.176998	0.223639	0.282675	$\hat{R}$PEC
	2.5	0.3441	0.553561	0.0389825	0.136949	0.263657	$\hat{R}$PEC
	3	0.2049	0.101923	0.00836045	0.0526497	0.195081	$\hat{R}$PEC
	3.5	0.1110	0.00885895	0.00166268	0.0128262	0.115305	$\hat{R}$PEC
	4	0.0542	0.000577936	0.000305261	0.00205782	0.0575189	$\hat{R}$PEC
50	1.5	0.7539	0.13679	0.0032232	0.15487	0.15537	$\hat{R}$PEC
	2	0.5297	0.11734	0.0011148	0.16356	0.17279	$\hat{R}$PEC
	2.5	0.3441	0.37312	0.0003915	0.13142	0.17924	$\hat{R}$PEC
	3	0.2049	0.22194	0.00013845	0.066002	0.1626	$\hat{R}$PEC
	3.5	0.1110	0.024459	4.9216e-05	0.019422	0.1194	$\hat{R}$PEC
	4	0.0542	0.0015738	1.7606e-05	0.0035252	0.070841	$\hat{R}$PEC
100	1.5	0.7539	0.077188	0.0015069	0.077687	0.077687	$\hat{R}$PEC
	2	0.5297	0.084504	0.00052396	0.086345	0.086466	$\hat{R}$PEC
	2.5	0.3441	0.068045	0.00018558	0.087672	0.091767	$\hat{R}$PEC
	3	0.2049	0.20406	6.6355e-05	0.066346	0.093992	$\hat{R}$PEC
	3.5	0.1110	0.08069	2.3871e-05	0.028264	0.088412	$\hat{R}$PEC
	4	0.0542	0.0059657	8.6324e-06	0.0064807	0.069813	$\hat{R}$PEC

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Table (7)

Real values and MSE for Reliability function when (θ=1, λ=0.5,R=1000)

n	t_i	Real R(t)	$\hat{R}$ MLE	$\hat{R}$PEC	$\hat{R}$ SJ	$\hat{R}$ PJ	Best
15	1.5	0.9394	0.73438	0.31867	0.27022	0.38922	$\hat{R}$ SJ
	2	0.7788	0.78033	0.090929	0.18856	0.39342	$\hat{R}$PEC
	2.5	0.5698	0.19411	0.025714	0.083919	0.28267	$\hat{R}$PEC
	3	0.3679	0.024156	0.0070021	0.024161	0.15779	$\hat{R}$PEC
	3.5	0.2096	0.0019164	0.0018413	0.0044561	0.074853	$\hat{R}$PEC
	4	0.1054	6.5093e-05	0.00048202	0.00047092	0.032125	$\hat{R}$ MLE
30	1.5	0.9394	0.176086	1.07571	0.191122	0.201092	$\hat{R}$PEC
	2	0.7788	0.370365	0.225419	0.189448	0.244059	$\hat{R}$ SJ
	2.5	0.5698	0.445331	0.0515182	0.120324	0.240381	$\hat{R}$PEC
	3	0.3679	0.0943217	0.0117044	0.0463396	0.182749	$\hat{R}$PEC
	3.5	0.2096	0.00938059	0.00254432	0.0109417	0.109505	$\hat{R}$PEC
	4	0.1054	0.000627527	0.000534141	0.00160804	0.0550222	$\hat{R}$PEC
50	1.5	0.9394	0.11466	0.010741	0.12026	0.12071	$\hat{R}$PEC
	2	0.7788	0.1124	0.0039204	0.14082	0.1494	$\hat{R}$PEC
	2.5	0.5698	0.31104	0.0014273	0.11872	0.16436	$\hat{R}$PEC
	3	0.3679	0.18997	0.00051857	0.060476	0.15386	$\hat{R}$PEC
	3.5	0.2096	0.024769	0.00018831	0.017599	0.017599	$\hat{R}$PEC
	4	0.1054	0.0018229	6.8476e-05	0.0030577	0.068687	$\hat{R}$PEC
100	1.5	0.9394	0.060301	0.0051717	0.060352	0.060353	$\hat{R}$PEC
	2	0.7788	0.0728	0.0018951	0.074652	0.074764	$\hat{R}$PEC
	2.5	0.5698	0.066803	0.00069353	0.080312	0.084233	$\hat{R}$PEC
	3	0.3679	0.17156	0.00025353	0.062403	0.089287	$\hat{R}$PEC
	3.5	0.2096	0.072468	9.2661e-05	0.026694	0.085617	$\hat{R}$PEC
	4	0.1054	0.0065166	3.389e-05	0.0060249	0.068298	$\hat{R}$PEC

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Table (8)
Real values and MSE for Reliability function when (θ=1.5, λ=0.5 ,R=1000)

n	t_i	Real R(t)	$\hat{R}$ MLE	$\hat{R}$PEC	$\hat{R}$ SJ	$\hat{R}$ PJ	Best
15	1.5	0.9851	0.58581	0.41747	0.19767	0.30101	$\hat{R}$ SJ
	2	0.8960	0.64442	0.12524	0.14802	0.33376	$\hat{R}$PEC
	2.5	0.5698	.16558	0.037274	0.066703	0.25025	$\hat{R}$PEC
	3	0.3679	0.020172	0.010776	0.018462	0.14264	$\hat{R}$PEC
	3.5	0.2096	0.001309	0.0030465	0.0029928	0.068406	$\hat{R}$ MLE
	4	0.1054	7.463e-06	0.0008655	0.0002055	0.029536	$\hat{R}$ MLE
30	1.5	0.9851	0.1339449	1.237248	0.1474415	0.1562134	$\hat{R}$ MLE
	2	0.8960	0.32511	0.2673568	0.1601985	0.2106948	$\hat{R}$ SJ
	2.5	0.5698	0.387287	0.06348831	0.1054045	0.2190898	$\hat{R}$PEC
	3	0.3679	0.08565869	0.01516124	0.0405762	0.1711148	$\hat{R}$PEC
	3.5	0.2096	0.008593801	0.00352826	0.0092322	0.1039392	$\hat{R}$PEC
	4	0.1054	0.0005033071	0.00081161	0.0012170	0.05260315	$\hat{R}$ MLE
50	1.5	0.9851	0.09084	0.020993	0.093381	0.093772	$\hat{R}$PEC
	2	0.8960	0.10022	0.0080564	0.12121	0.12917	$\hat{R}$PEC
	2.5	0.5698	0.27768	0.0030253	0.10716	0.1507	$\hat{R}$PEC
	3	0.3679	0.1713	0.0011217	0.055321	0.14557	$\hat{R}$PEC
	3.5	0.2096	0.023483	0.00041306	0.015892	0.11039	$\hat{R}$PEC
	4	0.1054	0.0017237	0.00015166	0.0026273	0.066587	$\hat{R}$PEC
100	1.5	0.9851	0.046881	0.010215	0.046886	0.046886	$\hat{R}$PEC
	2	0.8960	0.063217	0.003933	0.064541	0.064646	$\hat{R}$PEC
	2.5	0.5698	0.062859	0.001483	0.073563	0.077318	$\hat{R}$PEC
	3	0.3679	0.15612	0.00055228	0.058675	0.084817	$\hat{R}$PEC
	3.5	0.2096	0.067282	0.0002042	0.025191	0.082908	$\hat{R}$PEC
	4	0.1054	0.0064296	7.5307e-05	0.0055897	0.066813	$\hat{R}$PEC

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Table (9)
Real values and MSE for Reliability function when
(θ=2,λ=0.5 ,R=1000)

n	t_i	Real R(t)	$\hat{R}$ MLE	$\hat{R}$PEC	$\hat{R}$ SJ	$\hat{R}$ PJ	Best
15	1.5	0.9963	0.47542	0.51304	0.14311	0.23265	$\hat{R}$ SJ
	2	0.9511	0.54481	0.1613	0.11471	0.11471	$\hat{R}$ SJ
	2.5	0.8149	0.14048	0.050147	0.05206	0.22104	$\hat{R}$PEC
	3	0.6004	0.01596	0.015182	0.013645	0.1286	$\hat{R}$ SJ
	3.5	0.3753	0.00072294	0.0045159	0.0018355	0.062341	$\hat{R}$ MLE
	4	0.1997	1.1402e-05	0.0013527	4.9279e-05	0.027078	$\hat{R}$PEC
30	1.5	0.9963	0.1021877	0.375023	0.1136334	0.1213498	$\hat{R}$PEC
	2	0.9511	0.2866359	0.3069305	0.1351996	0.1818697	$\hat{R}$ SJ
	2.5	0.8149	0.3450904	0.07558117	0.09203664	0.1996169	$\hat{R}$PEC
	3	0.6004	0.07749229	0.01884551	0.0353263	0.1601424	$\hat{R}$PEC
	3.5	0.3753	0.007524686	0.004630874	0.007690339	0.09860028	$\hat{R}$PEC
	4	0.1997	0.0003526774	0.001139216	0.0008832473	0.05025995	$\hat{R}$PEC
50	1.5	0.9963	0.071221	0.032847	0.072504	0.072849	$\hat{R}$PEC
	2	0.9511	0.087748	0.013222	0.10428	0.11168	$\hat{R}$PEC
	2.5	0.8149	0.25266	0.005112	0.096635	0.13817	$\hat{R}$PEC
	3	0.6004	0.1572	0.0019307	0.050517	0.13771	$\hat{R}$PEC
	3.5	0.3753	0.021888	0.00071952	0.014296	0.10611	$\hat{R}$PEC
	4	0.1997	0.001533	0.00026627	0.0022331	0.064537	$\hat{R}$PEC
100	1.5	0.9963	0.036428	0.016068	0.036424	0.036424	$\hat{R}$PEC
	2	0.9511	0.05482	0.0064888	0.0558	0.055897	$\hat{R}$PEC
	2.5	0.8149	0.058432	0.002517	0.067375	0.07097	$\hat{R}$PEC
	3	0.6004	0.14525	0.00095398	0.055152	0.080571	$\hat{R}$PEC
	3.5	0.3753	0.063246	0.00035678	0.023753	0.080283	$\hat{R}$PEC
	4	0.1997	0.0061779	0.00013243	0.0051746	0.065358	$\hat{R}$PEC

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Table (10)
Real values and MSE for Reliability function when (θ=0.5,λ=1 ,R=1000)

n	t_i	Real R(t)	$\hat{R}$ MLE	$\hat{R}$PEC	$\hat{R}$ SJ	$\hat{R}$ PJ	Best
15	1.5	0.9963	0.025268	0.0038804	0.17632	0.17414	$\hat{R}$PEC
	2	0.9511	0.0001141	0.00020459	0.0086678	0.034848	$\hat{R}$ MLE
	2.5	0.8149	1.2176e-06	1.1992e-05	7.1155e-05	0.0054481	$\hat{R}$PEC
	3	0.6004	8.26e-07	1.1188e-06	8.5781e-08	0.00077749	$\hat{R}$PEC
	3.5	0.3753	1.3568e-07	1.3996e-07	1.1864e-07	0.00010729	$\hat{R}$ MLE
	4	0.1997	1.8733e-08	1.877e-08	1.858e-08	1.4625e-05	$\hat{R}$PEC
30	1.5	0.9963	0.10192	0.0083605	0.19392	0.19508	$\hat{R}$PEC
	2	0.9511	0.00057794	0.00030526	0.016438	0.057519	$\hat{R}$PEC
	2.5	0.8149	1.8469e-07	1.0296e-05	0.00018959	0.010589	$\hat{R}$PEC
	3	0.6004	3.2391e-07	6.3428e-07	9.0811e-08	0.0016089	$\hat{R}$ MLE
	3.5	0.3753	6.6366e-08	7.0977e-08	5.0095e-08	0.00022728	$\hat{R}$ SJ
	4	0.1997	9.3536e-09	9.3937e-09	9.2005e-09	3.1249e-05	$\hat{R}$ PJ
50	1.5	0.9963	0.22194	0.00013845	0.16178	0.1626	$\hat{R}$PEC
	2	0.9511	0.0015738	1.7606e-05	0.023902	0.070841	$\hat{R}$PEC
	2.5	0.8149	2.6659e-06	2.3035e-06	0.00034587	0.01593	$\hat{R}$PEC
	3	0.6004	1.3301e-07	3.082e-07	5.7458e-07	0.0026248	$\hat{R}$MLE
	3.5	0.3753	3.8657e-08	4.1604e-08	2.3404e-08	0.00038238	$\hat{R}$ SJ
	4	0.1997	5.6019e-09	5.6278e-09	5.4493e-09	5.3181e-05	$\hat{R}$ PJ
100	1.5	0.9963	0.20406	6.6355e-05	0.093916	0.093992	$\hat{R}$PEC
	2	0.9511	0.0059657	8.6324e-06	0.033022	0.069813	$\hat{R}$PEC
	2.5	0.8149	1.316e-05	1.1453e-06	0.00071443	0.023812	$\hat{R}$ MLE
	3	0.6004	1.5021e-08	1.5395e-07	2.1358e-06	0.0047492	$\hat{R}$ SJ
	3.5	0.3753	1.7913e-08	2.08e-08	5.2044e-09	0.00074606	$\hat{R}$ MLE
	4	0.1997	2.788e-09	2.8139e-09	2.6369e-09	0.00010675	$\hat{R}$ SJ

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Table (11)
Real values and MSE for Reliability function when
(θ=1,λ=1 ,R=1000)

n	t_i	Real R(t)	$\hat{R}$ MLE	$\hat{R}$PEC	$\hat{R}$ SJ	$\hat{R}$ PJ	Best
15	1.5	0.9963	0.024156	0.0070021	0.15988	0.15779	$\hat{R}$PEC
	2	0.9511	6.5093e-05	0.00048202	0.0073374	0.032125	$\hat{R}$MLE
	2.5	0.8149	1.0953e-05	3.8821e-05	3.2367e-05	0.0050503	$\hat{R}$ SJ
	3	0.6004	3.5549e-06	4.2868e-06	1.7009e-06	0.00072211	$\hat{R}$ SJ
	3.5	0.3753	5.4639e-07	5.5704e-07	5.1356e-07	9.9722e-05	$\hat{R}$ PJ
	4	0.3925	7.4954e-08	7.5047e-08	7.4664e-08	1.3597e-05	$\hat{R}$ PJ
30	1.5	0.9963	0.094322	0.011704	0.18162	0.18275	$\hat{R}$PEC
	2	0.9511	0.00062753	0.00053414	0.015116	0.055022	$\hat{R}$PEC
	2.5	0.8149	5.6858e-07	2.6926e-05	0.00013989	0.010194	$\hat{R}$PEC
	3	0.6004	1.5256e-06	2.2922e-06	1.712e-07	0.001552	$\hat{R}$ SJ
	3.5	0.3753	2.6925e-07	2.8057e-07	2.3717e-07	0.00021941	$\hat{R}$ PJ
	4	0.1997	3.7443e-08	3.7541e-08	3.7153e-08	3.0176e-05	$\hat{R}$ PJ
50	1.5	0.9963	0.18997	0.00051857	0.15305	0.15386	$\hat{R}$PEC
	2	0.9511	0.0018229	6.8476e-05	0.022658	0.068687	$\hat{R}$PEC
	2.5	0.8149	1.2042e-06	9.133e-06	0.00029218	0.015553	$\hat{R}$MLE
	3	0.6004	7.3152e-07	1.2301e-06	4.1651e-08	0.0025683	$\hat{R}$PEC
	3.5	0.3753	1.5841e-07	1.6632e-07	1.2733e-07	0.00037445	$\hat{R}$ SJ
	4	0.1997	2.2438e-08	2.2507e-08	2.2149e-08	5.2092e-05	$\hat{R}$ PJ
100	1.5	0.9963	0.17156	0.00025353	0.089213	0.089287	$\hat{R}$PEC
	2	0.9511	0.0065166	3.389e-05	0.031983	0.068298	$\hat{R}$PEC
	2.5	0.8149	1.52e-05	4.5532e-06	0.0006587	0.023485	$\hat{R}$ MLE
	3	0.6004	1.8046e-07	6.1473e-07	1.1444e-06	0.0046953	$\hat{R}$PEC
	3.5	0.3753	7.5355e-08	8.3157e-08	4.6781e-08	0.00073821	$\hat{R}$PEC
	4	0.1997	1.1185e-08	1.1254e-08	1.0897e-08	0.00010566	$\hat{R}$PEC

Table (12)
Real values and MSE for Reliability function when
(θ=1.5,λ=1 ,R=1000)

n	t_i	Real R(t)	$\hat{R}$ MLE	$\hat{R}$PEC	$\hat{R}$ SJ	$\hat{R}$ PJ	Best
15	1.5	0.9963	0.020172	0.010776	0.14462	0.14264	$\hat{R}$PEC
	2	0.9511	7.463e-06	0.0008655	0.0061284	0.029536	$\hat{R}$MLE
	2.5	0.8149	3.3813e-05	8.0584e-05	8.7166e-06	0.0046688	$\hat{R}$ MLE
	3	0.6004	8.2978e-06	9.4932e-06	5.3558e-06	0.00066885	$\hat{R}$ SJ
	3.5	0.3753	1.2335e-06	1.2508e-06	1.1852e-06	9.2431e-05	$\hat{R}$ PJ
	4	0.1997	1.6866e-07	1.6881e-07	1.6824e-07	1.2606e-05	$\hat{R}$ PJ
30	1.5	0.9963	0.085659	0.015161	0.17002	0.17111	$\hat{R}$PEC
	2	0.9511	0.00050331	0.00081161	0.013861	0.052603	$\hat{R}$MLE
	2.5	0.8149	5.5926e-06	5.0992e-05	9.7857e-05	0.0098065	$\hat{R}$PEC
	3	0.6004	3.7183e-06	4.9696e-06	1.2724e-06	3.7183e-06	$\hat{R}$ SJ
	3.5	0.3753	6.1016e-07	6.2851e-07	5.6258e-07	0.00021169	$\hat{R}$ SJ
	4	0.1997	8.4272e-08	8.4432e-08	8.3849e-08	2.9121e-05	$\hat{R}$ PJ
50	1.5	0.9963	0.1713	0.0011217	0.14479	0.14557	$\hat{R}$PEC
	2	0.9511	0.0017237	0.00015166	0.021458	0.0017237	$\hat{R}$PEC
	2.5	0.8149	1.4195e-10	2.0441e-05	0.00024318	0.015181	$\hat{R}$PEC
	3	0.6004	1.9081e-06	2.7634e-06	1.2191e-07	0.0025126	$\hat{R}$PEC
	3.5	0.3753	3.6086e-07	3.7404e-07	3.1428e-07	0.00036661	$\hat{R}$ SJ
	4	0.1997	5.0518e-08	5.0633e-08	5.0094e-08	5.1016e-05	$\hat{R}$ PJ
100	1.5	0.9963	0.15612	0.00055228	0.084745	0.084817	$\hat{R}$PEC
	2	0.9511	0.0064296	7.5307e-05	0.030969	0.066813	$\hat{R}$PEC
	2.5	0.8149	1.1594e-05	1.0201e-05	0.00060542	0.023161	$\hat{R}$PEC
	3	0.6004	6.047e-07	1.3812e-06	4.6043e-07	0.0046419	$\hat{R}$PEC
	3.5	0.3753	1.7397e-07	1.8702e-07	1.2987e-07	0.0007304	$\hat{R}$SJ
	4	0.1997	2.5201e-08	.5316e-08	2.478e-08	0.00010457	$\hat{R}$PEC

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Table (13)
Real values and MSE for Reliability function when
(θ=2,λ=1 ,R=1000)

n	t_i	Real R(t)	$\hat{R}$ MLE	$\hat{R}$PEC	$\hat{R}$ SJ	$\hat{R}$ PJ	Best
15	1.5	0.9963	0.01596	0.01518	0.13048	0.1286	$\hat{R}$ PJ
	2	0.9511	1.1402e-05	0.0013527	0.0050378	0.027078	$\hat{R}$PEC
	2.5	0.8149	7.0648e-05	0.00013712	5.0532e-08	0.0043035	$\hat{R}$ MLE
	3	0.6004	1.5059e-05	1.673e-05	1.1043e-05	0.00061769	$\hat{R}$ SJ
	3.5	0.3753	2.1967e-06	2.2209e-06	2.1331e-06	8.5419e-05	$\hat{R}$ PJ
	4	0.1997	2.9983e-07	3.0004e-07	2.9927e-07	1.1652e-05	$\hat{R}$ PJ
30	1.5	0.9963	0.077492	0.018846	0.15909	0.16014	$\hat{R}$PEC
	2	0.9511	0.00035268	0.0011392	0.012671	0.05026	$\hat{R}$MLE
	2.5	0.8149	1.6822e-05	8.2438e-05	6.3418e-05	0.0094282	$\hat{R}$MLE
	3	0.6004	6.9123e-06	8.6627e-06	3.3905e-06	0.0014416	$\hat{R}$ SJ
	3.5	0.3753	1.0891e-06	1.1146e-06	1.0262e-06	0.00020411	$\hat{R}$ SJ
	4	0.1997	1.4983e-07	1.5006e-07	1.4928e-07	2.8086e-05	$\hat{R}$ PJ
50	1.5	0.9963	0.1572	0.0019307	0.13695	0.1572	$\hat{R}$PEC
	2	0.9511	0.001533	0.00026627	0.020302	0.064537	$\hat{R}$PEC
	2.5	0.8149	1.5313e-06	3.6183e-05	0.00019882	0.014815	$\hat{R}$PEC
	3	0.6004	3.6778e-06	4.9057e-06	8.1306e-07	0.0024575	$\hat{R}$MLE
	3.5	0.3753	6.4603e-07	6.6465e-07	5.8411e-07	0.00035885	$\hat{R}$ SJ
	4	0.1997	8.9836e-08	8.9998e-08	8.928e-08	4.995e-05	$\hat{R}$ PJ
100	1.5	0.9963	0.14525	0.00095398	0.0805	0.080571	$\hat{R}$PEC
	2	0.9511	0.0061779	0.00013243	0.029981	0.065358	$\hat{R}$PEC
	2.5	0.8149	7.1956e-06	1.8065e-05	0.00055455	0.02284	$\hat{R}$PEC
	3	0.6004	1.3123e-06	2.4522e-06	8.2851e-08	0.0045888	$\hat{R}$MLE
	3.5	0.3753	3.1386e-07	3.3232e-07	2.5441e-07	0.00072264	$\hat{R}$ SJ
	4	0.1997	4.4837e-08	4.4999e-08	4.4283e-08	0.00010349	$\hat{R}$ SJ

4. Conclusions and recommendations

4.1. Conclusions

1- In table (4) :The best estimator for the parameter **θ** is the method Bayes estimator (**$\hat{\theta}$ *PEC***) ,Which has less MSE **.**

2. In table (5) :The best estimator for the parameter **θ** is the method Bayes estimator (**$\hat{\theta}$ SJ**) ,Which has less MSE **.**

2- In Table (6): when (**θ** =0.5, λ=0.5) the reliability function is for the PES is best because have the less MSE compare with other method when The larger the sample size .

3- In Table (6): when (**θ** =0.5, λ=0.5) the reliability function is for the MLE is best because have the less MSE compare with other method when The larger the sample size.

4- In Table (6-7-8-9-10-11-12) The MSE values were different and the larger the sample size was the preference for the methods (MLE and PEC) .

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4.2. Recommendations

1. Use of classical and Bayes methods for another continuous distributions,
2. Use other loss functions to get more estimators for shape parameter (**θ**) and reliability function R(t) and test it.
3. The use of another scale to measure the best estimators and reliability function R(t).

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