Calculation of the Longitudinal Electron Scattering Form Factors for the 2s-1d Shell Nuclei

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Abstract

Inelastic longitudinal electron scattering form factors have been calculated for isoscaler transition $T=0$ of the $\left(0^+ \to 2^+\right)$ and $\left(0^+ \to 4^+\right)$ transitions for the ^{20}Ne ,^{24}Mg *and* ^{28}Si nuclei. Model space wave function defined by the orbits $1d_{5/2}$,$2s_{1/2}$ and $1d_{3/2}$ can not give reasonable result for the form factor. The core-polarization effects are evaluated by adopting the shape of the Tassie-Model, together with the calculated ground Charge Density Distribution CDD for the low mass 2s-1d shell nuclei using the occupation number of the states where the sub-shell 2s is included with an occupation number of protons (α).

حساب عوامل التشكل الطولية للأستطارة الألكترونين من نوى القشرة 25-1d

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الخلاصة

تم حساب عوامل التشكيل الطولية للأستطارة الألكترونية للأنتقالات العددية في فضاء البرم النظيري $T=0$ ، $\left(0^+ \to 2^+\right)$ ، $\left(0^+ \to 4^+\right)$ للنوى ^{20}Ne ,^{24}Mg *and* ^{28}Si . أن الأعتماد على فضاء النموذج فقط المعرف بالمدارات $1d_{5/2}$,$2s_{1/2}$, $1d_{3/2}$ لا يعطي نتائج مقبولة لعوامل التشكيل. تم أضافة أستقطاب القلب للحسابات بالأعتماد على شكل نموذج تاسي، مع الأخذ بنظر الأعتبار توزيعات الشحنة للحالة الأرضية بأستخدام عدد ملأ الحالات للفترة 2s .

Introduction

Electron scattering takes place by an electromagnetic interaction. There are many reasons why an electron is such a powerful tool for studying the nuclear structure [1,2]. The basic interaction between the electron and the target nucleus is known. Since the interaction is relatively weak, one can make measurements on the target nucleus without greatly disturbing its structure, while in the case of nuclear particles both of the target structure and interaction are unknown, so it is very complicated to distinguish between them in the analyzing of experimental results. The electron scattering one immediately relate the cross section to the transition matrix elements of the local charge and current density operator and thus directly to the structure of the target nucleus itself [3].In this work, longitudinal form factors has been calculated from ^{20}Ne ,^{24}Mg *and* ^{28}Si target nuclei. Calculation of form factors using the many particle shell model space alone were known to be inadequate in describing electron scattering data [3]. So effects out of the model space (core polarization) are necessary to be included in the calculations. This effect can be regarded as a polarization of core protons by the valance protons and neutrons. The shape of the transition density for the excitation considered in this work is given by the Tassie model [4], where this model is connected with the ground charge density. Introducing an additional parameter α that reflected the difference of the occupation numbers of states between the real states of the nuclei and simple shell model predictions, leads to a very good agreement between the calculated and experimental results of the longitudinal form factor for the transitions

$J_i^\pi T_i = 0^+ 0 \;\; to \;\; J_f^\pi T_f = 2^+ 0 \; and \; 4^+ \; 0$

for ^{20}Ne, ^{24}Mg and ^{28}Si nuclei.

Theory

1-The Nuclear Form Factors

The interaction of the electron with the charge distribution of the nucleus gives rise to the longitudinal or Coulomb scattering. The longitudinal form factor is related to the **CDD** through the matrix elements of multiple operators $\hat{T}_J^L(q)$ [3].

$$\left|F_J^L(q)\right|^2 = \frac{4\pi}{Z^2(2J_i+1)}\left|\left\langle f \left\| \hat{T}_J^L(q) \right\| i \right\rangle\right|^2 \left|F_{cm}(q)\right|^2 \left|F_{fs}(q)\right|^2 \quad (1)$$

where Z is the atomic number of the nucleus , $F_{cm}(q)$ is the center of mass correction , which removes the spurious state arising from the motion of the center of mass when shell model wave functions are used and given by [5]

$$F_{cm}(q) = e^{q^2 b^2 / 4A} \quad (2)$$

where b is the harmonic oscillator length parameter and A is the total number of the nucleons in the nucleus. The function $F_{fs}(q)$ is the free nucleon form factor and assumed to be the same for protons and neutrons and takes the form [6]

$$F_{fs}(q) = e^{-0.43q^2/4} \quad (3)$$

The longitudinal operator is defined as [1]

$$\hat{T}_{J t_z}^L(q) = \int dr \;\; j_J(qr) Y_J(\Omega)\, \rho(r,t_z) \quad (4)$$

where $j_J(qr)$ is the spherical Bessel function, $Y_J(\Omega)$ is the spherical harmonic wave function and $\rho(r,t_z)$ is the charge density operator.

The reduce matrix elements in spin and isospin space of the longitudinal operator between the final and initial many particles states of the system including the configuration mixing are given in terms of the One Body Density Matrix (OBDM) elements times the single particle matrix elements of the longitudinal operator [3].

$$\left\langle f \left\| \hat{T}_{JT}^L \right\| i \right\rangle = \sum_{a,b} OBDM^{JT}(i,f,J,a,b) \left\langle b \left\| \hat{T}_{JT}^L \right\| a \right\rangle \quad (5)$$

The OBDM is calculated in terms of the isospin –reduced matrix elements [3]

2- Tassie – Model for the Core-Polarization

The model space matrix elements is not adequate to the absolute strength of the observed gamma-ray transition probabilities, because of the polarization in nature of the core protons by the model space protons and neutrons. The many particle reduced matrix element of the longitudinal operator, consists of two parts one is for the model space and the other is for core polarization matrix element [7].

$$\left\langle f \left\| \hat{L}_J(\tau_Z,q) \right\| i \right\rangle = \left\langle f \left\| \overset{ms}{\hat{L}_J}(\tau_Z,q) \right\| i \right\rangle + \left\langle f \left\| \overset{cor}{\hat{L}_J}(\tau_Z,q) \right\| i \right\rangle \quad (6)$$

where the model space matrix element is given by[8]

$$\left\langle f \left\| \hat{L}_J^{ms} (\tau_Z , q) \right\| i \right\rangle = e_i \int_0^\infty dr . r^2 . j_J (qr) . \rho_{J,\tau_Z}^{ms} (i, f , r) \qquad (7)$$

where $\rho_J^{ms}(i, f , r)$ is the transition charge density of the model space and given by [3]

$$\rho_{J,\tau_Z}^{ms} (i, f , r) = \sum_{jj'(ms)}^{ms} OBDM(i, f, J, j, j', \tau_Z) \left\langle j \| Y_J \| j' \right\rangle R_{nl}(r)\, R_{n'l'}(r) \qquad (8)$$

The core- polarization matrix element in eq. (2.2.1) can be written as:

$$\left\langle f \left\| \hat{L}_J^{core} (\tau_Z , q) \right\| i \right\rangle = e_i \int_0^\infty dr . r^2 . j_J (qr)\, \rho_J^{core} (i, f , r) \qquad (9)$$

where ρ_J^{core} is the core polarization transition density which depends on the model used for core polarization. To take the core- polarization effects into consideration, the model space transition density is added to the core-polarization transition density that describes the collective modes of nuclei. The total transition density becomes

$$\rho_{J\tau_Z} (i, f , r) = \rho_{J\tau_Z}^{ms} (i, f , r) + \rho_{J\tau_Z}^{core} (i, f , r) \qquad (10)$$

where $\rho_J^{Cor}(i, f , r)$ is assumed to have the form of Tassie shape and given by [4] .

$$\rho_{Jt_Z}^{core} (i, f , r) = \frac{N}{2} (1 + \tau_Z) r^{J-1} \frac{d\rho_o(r)}{dr} \qquad (11)$$

where N is a proportionality constant. It is determined by adjusting the reduced transition probability $B(CJ)$ and given by [9]

$$N = \frac{\int_0^\infty dr\, r^{J+2} \rho_{Jt_Z}^{ms} - \sqrt{(2J_i + 1)\, B(CJ)}}{(2J + 1) \int_0^\infty dr\, r^{2J} \rho_o (i, f , r)}$$

$\rho_o(r)$ is the ground state charge density distribution. It is evaluated on the basis that the number of protons in the shells $1s_{1/2}, 1p_{3/2}, 1p_{1/2}, 1d_{5/2}$ *and* $2s_{1/2}$ are equal to $2,4,2,(Z-8-\alpha)$ *and* α respectively for $^{20}Ne, ^{24}Mg$ *and* ^{28}Si nuclei. Using this assumption, the ground charge density for low mass sd-shell nuclei is obtained as [9]

$$\rho_o(r) = \frac{e^{-r^2/b^2}}{\pi^{3/2} . b^3} \left[2 + \frac{3}{2}\alpha + (4 - 2\alpha)\left(\frac{r}{b}\right)^2 + \left\{ \frac{4}{15}(Z-8) + \frac{2}{5}\alpha \right\} \left(\frac{r}{b}\right)^4 \right] \qquad (13)$$

The parameter α is determined from the central **CDD** $\rho(\mathrm{r} = 0)$ of eq. (13), i.e

$$\rho(0) = \frac{1}{\pi^{3/2} b^3} \left[2 + \frac{3}{2}\alpha \right] \qquad (14)$$

where the value of $\rho(0)$ can be taken from the experiments. The mean square radii (MSR) of the considered nuclei are obtained by

$$\left\langle r^2 \right\rangle = \frac{4\pi}{Z} \int \rho_o (r) . r^4 . dr \qquad (15)$$

Putting eq. (13) when α= 0, in eq. (15) we obtain:

$$\left\langle r^2 \right\rangle = \frac{b^2}{Z}\left[-10+\frac{7}{2}Z\right] \qquad (16)$$

The harmonic oscillator size parameter b is obtained by introducing the experimental MSR of considered nuclei into eq. (16).

Results, Discussion and Conclusions

An extensive investigation of the ground state and some excited states of $^{20}Ne\ ,^{24}Mg\ \ and\ ^{28}Si$ is performed, through elastic and inelastic electron scattering. The longitudinal form factors for these nuclei are calculated for the positive parity states. The core-polarization effects are considered, in this work, by using the Tassie model, which depends on the ground CDD. The CDD of the $^{20}Ne\ ,^{24}Mg\ \ and\ ^{28}Si$ nuclei are formulated by means of the wave functions of the harmonic oscillator on the assumption that the occupation numbers of the states in a real nucleus differ from the predictions of the simple shell model, as given in eq.(13). These occupation numbers of the states can determined from the comparison between the calculated and fitted to the experimental CDD. For all considered nuclei that are under investigation, the experimental values of the root means square charge radii and central CDD at $r=0$, the calculated values of the size parameter b and the parameter α are presented in table (1). The occupation numbers of the 1d and 2s orbits of the lower half of sd-shell nuclei are presented in table (2) and compared with those of theoretical [3] and experimental [10] results.

The C2 and C4 form factors for the transitions $J_i^{\pi}\,T_i\ =0^+\,0\ \ to\ J_f^{\pi}\,T_f\ =2^+\,0\ and\ 4^+\,0$ of the $^{20}Ne\ ,^{24}Mg\ \ and\ ^{28}Si$ nuclei are displayed in Fig.1 and Fig.2, respectively. In these figures, the cross symbols represent the contribution of the model space where the configuration mixing is taken into account, the dashed curves represent the core polarization contribution where the collective modes are considered and the solid curves represent the total contribution which is obtained by taking the model space together with the core polarization effects. Core polarization effects enhance the C2 form factors as the first and second maximum and bring the calculated values very close to the experimental data. For higher q values, the core-polarization results are shifted towards lower values of q, bringing the theoretical results very close to the experimental data[11]. So core-polarization effects give a strong q dependence modification to the form factors. The modifications of the form factors due to core-polarization effects are also reflected in C4 form factors. There is a significant improvement in the form factors over the low mass sd-shell nuclei, as shown in Fig.(2). For higher q there is an effective decrease in the form factors, when core polarization are included and the data are reproduced very well. The available experimental data [12] are very well reproduced in all momentum transfer values. The experimental values for the B(C2) and B(C4) are given in Refs. [3] and [13], while the OBDM values are given by [3].

It is concluded that the core polarization effects, which represent the collective modes, are essential in obtaining a remarkable agreement between the calculated longitudinal $C2$ and $C4$ form factors and those of experimental data.

Table (1)
Parameters of the 2s-1d Shell Nuclei

Nucleus	***Z***	$\rho(0)$ **e.fm**$^{-3}$ **exp[14]**	$\langle r^2\rangle^{1/2}$**(fm)** **exp[14]**	**b (fm)**	α
^{20}Ne	10	0.08	2.9	1.77	0.34
^{24}Mg	12	0.0817	3.03	1.85	0.6
^{28}Si	14	0.0847	3.07	1.644	0.638

Table (2)
Proton Occupation Numbers of States

Nucleus	**Present Work**		**Theo.[3]**		**Exp.[10]**	
	1d	**2s**	**1d**	**2s**	**1d**	**2s**
^{20}Ne	1.66	0.34	1.94	0.51	1.03	0.97
^{24}Mg	3.4	0.6	3.55	0.45	3.81	0.19
^{28}Si	5.362	0.638	5.3	0.7	5.21	0.79

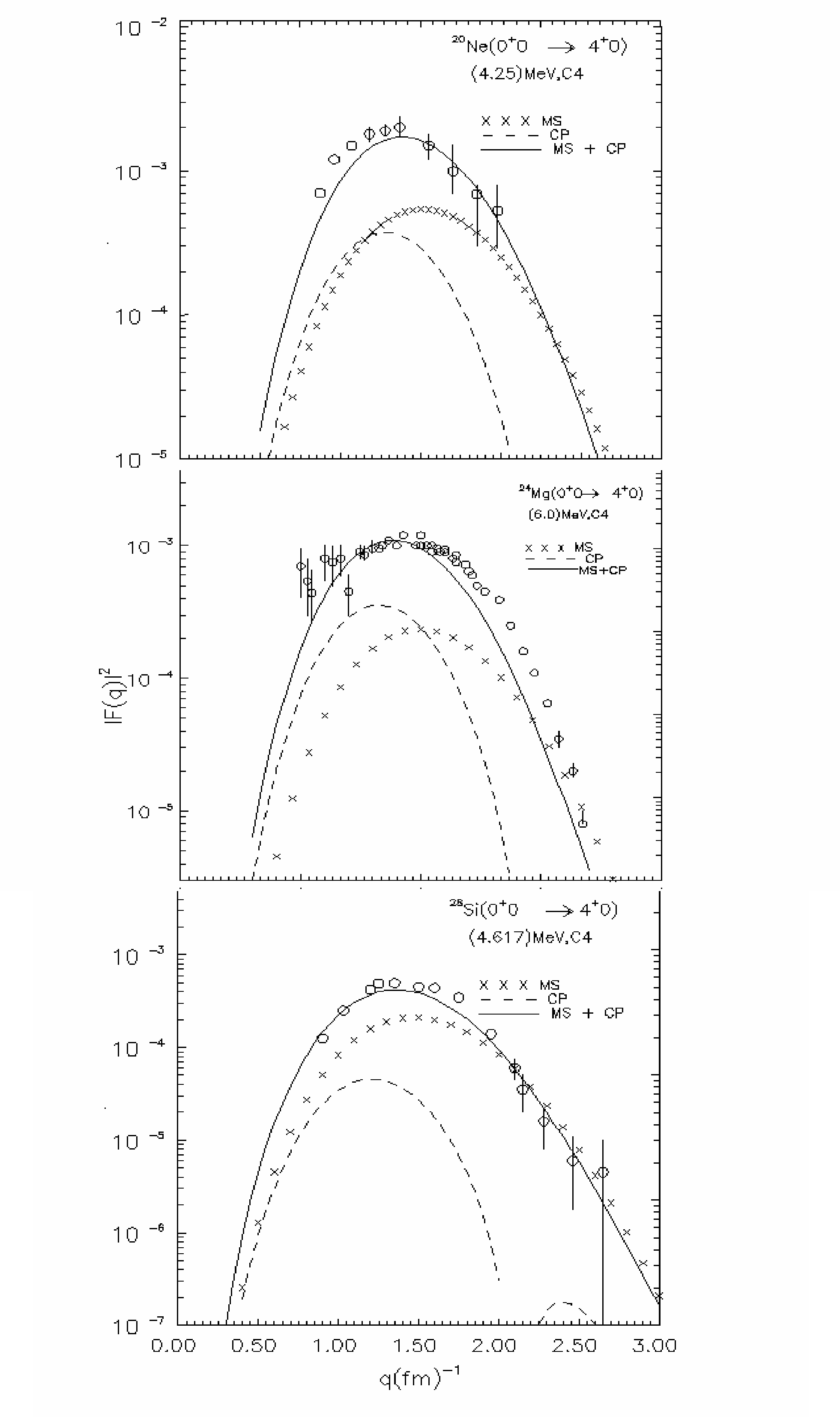

Fig.2. Inelastic longitudinal C4 form factors. The cross symbol is the model space (MS) and the dashed curve is the core-polarization effect (CP) calculations. The solid curve is the total form factor (MS and CP).The experimental values are shown as circles and taken from Ref.[12]

References

[1] T. de Forest and J.D.Walecka, Adv.Phys. 15 (1966) 1.
[2] J.D.Walecka, Nucl.Phys. A 574 (1994) 271.
[3] B. A. Brown, R.A.Radhi and B.H.Wildenthal; Phys.Rep. 101(1983) 313.
[4] L.J.Tassie; Austr.J.Phys. 9 (1956) 407.
[5] L.J.Tassie and F. C. Baker; Phys.Rev. 111 (1958) 940.
[6] R. S. Willy, Nucl. Phys. 40 (1963) 529.
[7] R. A. Radhi, A. Al. Rahmani and A. K. Hamoudi, Iraq J. Sci, C34, (2002) 2.
[8] R.A.Radhi and A.A.Aouda; Proceeding of the first Scientific. Conference, College of Science, Al-Nahreen University, 1(1997)297.
[9] K. N. Flaih, M.Sc Thesis, University of Baghdad (2003).
[10] B. Castal, I. P. Johnstone and B. P. Sinch, Nucl.Phys. A157 (1970) 137.
[11] G.C.Li and I.Sick, Phys.Rev.C9 (1974)1861.
[12] Y.Horikawa, Y.Tovizuka and A.Nakada, Phys.Lett. 39B (1971)9.
[13] B. A. Brown, W. Chung and B. H. Wildenthal, Phys. Rev. C21 (1980) 2600.
[14] Atomic Data and Nuclear Data Tables, vol.36, no. 3 (1987).