

Some Applications Of Generalized Pre Regular Sets In Intuitionistic Fuzzy Topological Spaces

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Abstract

In this paper ,we introduce Intuitionistic Generalized Pre Regular Sets in an Intuitionistic topological spaces with some basic properties and some applications with counter examples .

1 –Introduction

One of the generalization of a generalized closed sets which was defined by [5],in this paper we defined the notion of an Intuitionistic Generalized Pre Regular Sets (IGPRS, for short).it follows easily from definitions that every IGC set is an IGPRCS . In this paper we define and study the properties of Intuitionistic Generalized Pre Regular Closed Set (IGPRCS ,for short) which is weaker from of the above mentioned generalizations .Moreover , in this paper we defined Intuitionistic Generalized Pre Regular D-Sets of some separation axioms and IGPR $T_{1/2}$ in an Intuitionistic topological Spaces.

2- Preliminaries :

We recall some definitions and results which are used in this paper

Definition:2-1[1]

Let X be anon empty fixed set . An Intuitionistic fuzzy set (IFS ,for short) A is an object having the form $A = \langle x, A_1, A_2 \rangle$, which A_1 and A_2 are subset of X and satisfying the following $A_1 \cap A_2 = \emptyset$.

Definition : 2-2 [2]

An Intuitionistic fuzzy topology (IFT ,for short)on a nonempty set X is a family T containing $\tilde{\phi} = \langle x, \phi, X \rangle$ and $\tilde{X} = \langle x, X, \phi \rangle$ under finite intersection and arbitrary union.

In this case the pair (X , T) is called an Intuitionistic fuzzy topological spaces (IFTS , for short) and each IFS in T is known as an intuitionistic fuzzy open set (IFOS , for short) in X . The complement \bar{A} of an IFOS A in an IFTS (X , T) is called an Intuitionistic fuzzy closed set (IFCS , for short) , in X .

Definition : 2-3[4]

let X be anon empty set and let the IFS's A and B be in the form $A = \langle x, A_1, A_2 \rangle$, $B = \langle x, B_1, B_2 \rangle$ and let $\{ A_i : i \in I \}$ be an arbitrary family of IFS's in X .Then

- $A \subseteq B \Leftrightarrow A_1 \subseteq B_1 \wedge A_2 \supseteq B_2$;
- $A = B \Leftrightarrow A \subseteq B \wedge B \subseteq A$;
- $\bar{A} = \langle x, A_2, A_1 \rangle$;
- $\bigcup A_i = \langle x, \bigcup A_1^i, \bigcap A_2^i \rangle$; $\bigcap A_i = \langle x, \bigcap A_1^i, \bigcup A_2^i \rangle$;
- $FA = \langle x, A_1, A_1^c \rangle$, $SA = \langle x, A_2^c, A_2 \rangle$

Definition : 2-4[1]

Let (X, T) be an IFS ,then

- $T_{0,1} = \{ FA : A = \langle x, A_1, A_2 \rangle \in T \}$;
- $T_{0,2} = \{ SA : A = \langle x, A_1, A_2 \rangle \in T \}$;
- $T^* = \{ A_1 : A = \langle x, A_1, A_2 \rangle \in T \}$;
- $T^{**} = \{ A_2 : A = \langle x, A_1, A_2 \rangle \in T \}$;

Are Intuitionistic topologies on X

Definition : 2-5[1]

Let A be an IFS , then the interior and closure of an IF'S A is defined by ;

Int

$$A = \bigcup \{ G : G \in T, G \subseteq A \} \quad \text{And} \quad \text{Pint} \quad A = A \cap \text{Int} \quad (\text{CLA}) \quad , \quad \text{PCL} \\ A = A \cap \text{CL}(\text{Int} A) \quad \text{if} \quad A \in T, A \subseteq k \}$$

Definition : 2-6 [5]

A subset A of an IFTS (X, T) is called :

- Intuitionistic Regular Open Set (IROS , for short) if $A = \text{Int}(\text{CLA})$ and an Intuitionistic Regular Closed Set (IRCS , for short) if $A = \text{CL}(\text{Int} A)$.
- Intuitionistic Pre Open Set(IPOS , for short) if $A \subseteq \text{Int}(\text{CLA})$ and Intuitionistic Pre Closed Set(IPCS ,for short) if $\text{CL}(\text{Int} A) \subseteq A$.
- Intuitionistic Generalized Closed Set (IGCS ,for short) if $\text{CLA} \subseteq U$ whenever $A \subseteq U$ and U is Open Set
- Intuitionistic Generalized Pre Closed Set (IGPCS ,for short) if $\text{PCLA} \subseteq U$ whenever $A \subseteq U$ and U is Open.

3- Basic Properties Of Generalized Pre Regular Closed Set In Intuitionistic Topological Spaces .

We introduce the following definition

Definition : 3-1

A subset A of an IS (X, T) is called Intuitionistic Regular Closed Set (IGRCS ,for short) if $\text{PCL} A \subseteq U$,whenever $A \subseteq U$ and U is an Intuitionistic Regular Open in (X, T).

Remark :3-2

The complement of IGPCS in X is called Intuitionistic Generalized Pre Open Set (IGPOS ,for short) .

Proposition : 3-3

Let (X, T) be an ITS , A is IGPROS in X if and only if for each regular Closed Set F such that $F \subseteq A$,then $F \subseteq \text{Pint} A$

Proof : \rightarrow

Suppose that A is an IGPRO in X , then A^c is an IGPRCS, so for each IROS U in X , $A^c \subseteq U$, then $PCLA^c \subseteq U$, put $A^c = F$ and $U^c \subseteq PIntA$.

Suppose that A is an IGPRCS in X , then A is an IGPRO, so for each F is IRCS in X and $F \subseteq A^c$, $F \subseteq PIntA^c$, put $U = PCLF^c$, then $PCLA \subseteq U$, i.e for each $A \subseteq U$, $PCLA \subseteq U$.

Remark 3-4

i. Every Intuitionistic Closed Set (ICS, for short) is an IGPRC Set, but the converse is not true.

ii. Every Intuitionistic Open Set (IOS for short) is an IGPRO Set, but the converse is not true.

Proof : i

Suppose that A is an ICS, then for each IROS U , $A \subseteq U$, but $A = CLA$ so $CLA \subseteq U$

Since $CIA = CL(CLA) \subseteq PCL(CLA) = PCLA$, so $PCLA \subseteq U$ so A is an IGPRC Set.

ii is the dual of i.

Example :3-5

Let $X = \{a, b, c\}$, defined T by $T = \{ \tilde{\phi}, \tilde{X}, A \}$ where $A = \langle x, \{a\}, \{b, c\} \rangle$ so $B = \langle x, \{c\}, \{a, b\} \rangle$, is IGPRCS but not Closed Set, and $C = \langle x, \{a, b\}, \{c\} \rangle$, is IGPROS, but not Open Set.

Proposition :3-6

If A is an IOS and IGPRCS, then A is Pre Closed.

Proof :

Since A is an I open and IGPRCS, then for all U is IGRO in X , $A \subseteq U$, then $PCLA \subseteq U$ replacing U by A , we have $A \subseteq U$, then $PCLA \subseteq A$ ©

Since $PCLA = A \cup CL(IntA)$ so $A \subseteq PCLA$ ® From © and ®, we get $A = PCLA$ i.e A is Pre Closed Set.

Proposition : 3-7

If A is an IGPRCS in ITS (X, T) and $A \subseteq B \subseteq PCLA$, then B is IGPRCS.

Proof :

For each U is IROS and $B \subseteq U$, we have to prove that $PCLB \subseteq U$. Suppose A is an IGPRCS, then for each U is IROS in (X, T) , $A \subseteq U$, then $PCLA \subseteq U$, but $A \subseteq B \subseteq PCLA$. Thus $B \subseteq PCLA$, and $PCLB \subseteq PCL(PCLA) = PCLA$, then $PCLB \subseteq PCLA \subseteq U$ i.e B is IGPRCS.

Proposition : 3-8

i. Intersection of any family of IGPRCS is an IGPRCS.

ii. Any union of IGPROS is IGPROS.

Proof : i

Let A, B be two IGPRCS so for each U is an IROS in X , $A \subseteq U$, then $PCLA \subseteq U$, and for each V

is an IGPRCS in X , $B \subseteq V$, then $PCLB \subseteq V$, so $A \cap B \subseteq U \cap V$, and $PCL(A \cap B) \subseteq PCLA \cap PCLB \subseteq U \cap V$, so $A \cap B$ is IGPRCS in X .

ii is the dual of i.

Proposition 3-9

Let (X, T) be an ITS, then Every IGPCS is IGPRCS.

Proof :

Let $A \subseteq X$ be an IGPCS and $A \subseteq U$, where U is an IROS. Since every IROS is open and A is IGPCS, $PCLA \subseteq U$. Hence A is an IGPRCS.

Remark 3-10

The converse of the proposition 3-9 is not true in general, and the following example shows this case.

Example 3-11

Let $X = \{a, b\}$ and defined $T = \{ \tilde{\phi}, \tilde{X}, A, B \}$, where $A = \langle x, \{a\}, \emptyset \rangle$, $B = \langle x, \{a\}, \{b\} \rangle$. Then $IGPO(X) = T \cup \{C, D, E\}$, where $C = \langle x, \emptyset, \emptyset \rangle$, $D = \langle x, \{b\}, \emptyset \rangle$ and $E = \langle x, \emptyset, \{b\} \rangle$, so $IGPC(X) = \{ \tilde{\phi}, \tilde{X}, A, B, C, D, E \}$, and $IGPRC(X) = IGPC(X) \cup \{A, B\}$, so A and B are $IGPRC(X)$, but not $IGPC(X)$.

4- Ntutionistic Fuzzy Generalized Pre Regular –D Sets And Associated Separation Axioms

In this section we recall the definitions of GPR-Di spaces where

$i = 0, 1, 2$ which are some applications Of Intuitionistic fuzzy Generalized Pre Regular Sets.

Definition:4-1

A subset A of an IFTS X is called an IGPR-D set if there exists U and V are two IGPRO(X, T) such that $U \neq X$ and $A = U - V$.

Observe that every IGPROS U different from X is an IGPR-D set if $A = U$ and $V = \emptyset$

Definition : 4-2_ A sub set A of an IFTS (X, T) is called :

- IGPR-D₀ set (IGPR-T₀ set) if for any distinct pair of points x and y there exist an IGPR-D set (IGPRO set) of X containing x but not y or an IGPR-D set (IGPRO set) of X containing y but not x .
- IGPR-D₁ set (IGPR-T₁ set) if for any distinct pair of points x and y there exist an GPR-D set (IGPRO set) of X containing x but not y and an IGPR-D set (IGPRO set) of X containing y but not x .
- IGPR-D₂ set (IGPR-T₂ set) if for any distinct pair of points x and y of X , there exists disjoint IGPR-D set (IGPRO set) U and V of X containing x and y , respectively.

Remark : 4-3

i. If (X, T) is an IGPR-T_i, then its an IGPR-D_i, where $i = 1, 2$

ii. If (X, T) is an IGPR-D_i, then it is an IGPR-D_{i-1}, where $i = 1, 2$

Theorem : 4-4

For an ITS (X, T) , the following statements hold :

- i. An ITS (X, T) is an IGPR- T_0 if and only if it is an IGPR- D_0 .
- ii. An ITS (X, T) is an IGPR- D_2 if and only if it is an IGPR- D_1 .

Proof :

the sufficiency, let (X, T) be an IGPR- D_0 , then for each Distinct pair $x, y \in X$, at least one of x, y , say x belongs to an IGPR-D set $U = \langle x, U_1, U_2 \rangle$, where $y \in U$, let $U = G_1/G_2$, such that $G_1 \neq X$ and G_1, G_2 belongs to an IGPR-D (X, T) , then $x \in G_1$. For $y \notin U$, so either $y \in G_1$ or $y \notin G_1$, then if $y \notin G_1$ so $x \in G_1$ and $y \in G_2$, but $x \notin G_2$. Hence (X, T) is an IGPR- T_0 .

The sufficiency is stated in remark 4-3 i

ii by i

Remark : 4-5

If an ITS (X, T) is an IGPR- T_0 , then its not an IGPR- D_1 , so the following example show this case.

Example : 4-6

Let $X = \{1, 2\}$ be an IS, defined T by $T = \{\tilde{\phi}, \tilde{X}, A\}$, where $A = \langle x, \{2\}, \{1\} \rangle$, then $IGPRO(X) = \{\tilde{\phi}, \tilde{X}, A, B, C, D, E\}$, where $B = \langle x, \emptyset, \emptyset \rangle$, $C = \langle x, \{1\}, \emptyset \rangle$, $D = \langle x, \emptyset, \{1\} \rangle$, $E = \langle x, \{2\}, \emptyset \rangle$, so $IGPR-D(x) = \{\tilde{\phi}, \tilde{X}, A, B, D, G, H\}$, where $G = \langle x, \emptyset, \{2\} \rangle$, $H = \langle x, \{3\}, \{2\} \rangle$, clearly the space (X, T) is an IGPR- T_0 , but not an IGPR- D_1 .

Theorem : 4-7

An ITS (X, T) is an IGPR- T_0 if and only for each pair of distinct points x and y of X , $PCL(\{x\}) \neq PCL(\{y\})$.

Proof : \rightarrow

Let (X, T) be an IGPR- T_0 , so there exists $x, y \in X$, $x \neq y$ and $x \in G = \langle x, G_1, G_2 \rangle$, and $y \notin G$, where G is an IGPRO set in (X, T) , then $G^c = \langle x, G_2, G_1 \rangle$, be an IGPRC set which $x \notin G^c$ and $y \in G^c$. since $PCL(\{y\})$ is the smallest IGPRCS containing y , so $PCL(\{y\}) \subseteq G^c$ and there for $x \notin PCL(\{y\})$. Hence $PCL(\{y\}) \neq PCL(\{x\}) \leftarrow$

suppose that $x, y \in X$ and $PCL(\{y\}) \neq PCL(\{x\})$ and let $z \in X$, such that $z \in PCL(\{x\})$ so $x \notin PCL(\{y\})$. for if $x \in PCL(\{y\})$, then $PCL(\{x\}) \subseteq PCL(\{y\})$, which is contradiction, that is z

$\notin PCL(\{y\})$, so $x \in IGPROS$, so $z \in (PCL(\{y\}))^c$ to which y does not belong.

Theorem : 4-8

An ITS (X, T) is an IGPR- D_1 if and only if the singletons are IGPRCS.

Proof :

Let (X, T) be an IGPR- D_1 , and x be any point of X , and suppose that $y \in \{x\}^c$. since (X, T) is an IGPR- D_1 , so there exist two IGPRO (X, T) , $U = \langle x, U_1, U_2 \rangle$ and $V = \langle y, V_1, V_2 \rangle$, such that $y \in V$, but $x \notin V$. Since $y \in V \subseteq \{x\}^c$, i.e. $\{x\}^c = \bigcup \{V | y \in V\}$ Which is an IGPRO sets so $\{x\}$ is an IGPRC set.

Conversely, suppose $\{p\}, \{z\}$ are an IGPRCS, for every $p = \langle x, \{p\}, \{p\}^c \rangle$, $z = \langle x, \{z\}, \{z\}^c \rangle \in X$, so $\{p\}^c$ and $\{z\}^c$ be an IGPRO in (X, T) , such that $p \notin \{p\}^c$, $p \in \{z\}^c$ and $z \notin \{z\}^c$, $z \in \{p\}^c$, so (X, T) is an IGPR- D_1 spaces.

Theorem : 4-9

If (X, T) be an ITS, then the following are equivalent :

- (X, T) is IGPR- D_1 ;
- (X, T^*) is IGPR- D_1 ;
- $(X, T_{0,1})$ is IGPR- D_1

Proof : i \rightarrow ii

Let $x, y \in X$ s.t. $x \neq y$, then there exist two IGPRO sets say $U_x = \langle x, U_1, U_2 \rangle$ and $V_y = \langle y, V_1, V_2 \rangle$ in (X, T) s.t. $x \in U_x$ and $y \in V_y$, and $x \notin V_y$, $y \notin U_x$ thus x in U_1 and not in V_1 and y in V_1 not in U_1 , therefore (X, T^*) is IGPR- D_1 .
ii \rightarrow iii

Since (X, T^*) is IGPR- D_1 so there exist $x, y \in X$ s.t. $x \neq y$ and two IGPROS say $\{U_1 : U_x = \langle x, U_1, U_2 \rangle\}$ and $\{V_1 : V_y = \langle y, V_1, V_2 \rangle\}$ and x in U_1 not in V_1 and y in V_1 not in U_1 , so x in U_1 and $x \notin U_1^c$ and y in V_1 , $y \notin V_1^c$ thus $x \in FU_x$, $y \in FV_y$, $x \notin FV_y$ and $y \notin FU_x$, so $(X, T_{0,1})$ is IGPR-
iii \rightarrow i

let $x, y \in X$ such that $x \neq y$, then there exist two IGPRO sets in $(X, T_{0,1})$ say $FU_x = \langle x, A_1, A_1^c \rangle$ and $FV_y = \langle y, V_1, V_1^c \rangle$ s.t. $x \in FU_x$, $y \in FV_y$, $x \notin FV_y$

and $y \notin FU_x$. Thus $x \in A_1$ and $x \notin A_1^c$ and $y \in v_1$ and $y \notin v_1^c$, also $x \notin v_1$, $y \in A_1$. Since $U_x = \langle x, A_1, A_2 \rangle$ and $V_y = \langle y, v_1, v_2 \rangle$ Then $x \in A_1$ and $x \notin A_2$, so $x \in U_x$, $x \notin FV_y$ implies that $x \notin v_1$ or $x \in v_1^c$.

If $x \notin v_1$, then $x \in v_2$, therefore $x \notin V_y$ and if $x \in v_1^c$, then $x \notin v_1$.

Since $v_1 \cap v_2 = \emptyset$, so $x \in v_2$. Thus $x \notin V_y$.

Similarly $y \in V_y$ and $x \notin V_y$, therefore (X, T) is IGPR-D₁.

Proposition : 4-10

Let (X, T) be an IGPR-D₁, then :

i. $(X, T_{0,2})$ is an IGPR-D₁ ;

ii. (X, T^{**}) is an IGPR-D₁

Proof :

Let (X, T) be an IGPR-D₁ and let $x, y \in X$ and $x \neq y$, then there exists $U_x = \langle x, u_1, u_2 \rangle$ and $V_y = \langle y, v_1, v_2 \rangle$ are two IGPRO(X, T) s.t $x \in U_x$, $y \in V_y$, $x \notin V_y$ and $y \notin U_x$. Thus $x \in u_1$ and $x \notin u_2$ and $y \in v_1$ and $y \notin v_2$, also $x \notin v_1$, $y \notin u_1$. Since $Su_x = \langle x, u_2^c, u_2 \rangle$ and $Sv_y = \langle y, v_2^c, v_2 \rangle$, so $x \in u_2^c$ and $y \in v_2^c$, thus $x \in Su_x$ and $y \in Sv_y$. Similarly $y \notin Su_x$ and $x \notin Sv_y$. Therefore $(X, T_{0,2})$ is IGPR-D₁.

In similar way we can prove ii .

Remark : 4-11

The converse of proposition 4-10 is not true in general .The following example show the case .

Example : 4-12

Let $X = \{a, b\}$ and defined $T = \{\tilde{\phi}, \tilde{X}, A, B\}$, where $B = \langle x, \emptyset, \{b\} \rangle$, $A = \langle x, \{a\}, \{b\} \rangle$ and the IGPRO(X) = $T \cup \{C, D, D^-\}$, where $C = \langle x, \emptyset, \emptyset \rangle$, $D = \langle x, \emptyset, \{a\} \rangle$ and $D^- = \langle x, \{a\}, \emptyset \rangle$, then $T^{**} = \{\emptyset, X, \{a\}, \{b\}\}$, so (X, T^{**}) is IGPR-D₁, but (X, T) is not . Also $T_{0,2} = \{\tilde{\phi}, \tilde{X}, A, A^-\}$, is IGPR-D₁.

Proposition : 4-13

Let (X, T) be IGPR-D₂, then :

i. $(X, T_{0,1})$ is an IGPR-D₂ ;

ii. (X, T^*) is an IGPR-D₂ ;

iii. $(X, T_{0,2})$ is an IGPR-D₂ ;

iv. (X, T^{**}) is an IGPR-D₂

Proof : direct

Remark : 4-14

The converse of proposition 4-13 is not true in general ,the following examples show the cases .

Example : 4-15

i. Let $X = \{a, b\}$ and defined

$T = \{\tilde{\phi}, \tilde{X}, A, B, C, D\}$, where $A = \langle x, \{a\}, \emptyset \rangle$, $B = \langle x, \{b\}, \emptyset \rangle$, $C = \langle x, \{a\}, \{b\} \rangle$, and $D = \langle x, \emptyset, \emptyset \rangle$. Hence an IGPRO(X) = T. Then the ITS(X, T_{0,1}), where $T_{0,1} = \{\emptyset, X^-, C, C^-\}$ and $T_1 = \{\emptyset, X, \{a\}, \{b\}\}$ are IGPR-D₂, but (X, T) is not .

ii. Let $X = \{a, b\}$ and defined

$T = \{\tilde{\phi}, \tilde{X}, A, B\}$, where $A = \langle x, \{a\}, \{b\} \rangle$, $B = \langle x, \emptyset, \{b\} \rangle$. Then the IGPRO(X) = $T \cup \{C, D, D^-\}$, where $C = \langle x, \emptyset, \emptyset \rangle$ and $D = \langle x, \emptyset, \{a\} \rangle$ so

$T^{**} = \{\emptyset, X, \{a\}, \{b\}\}$ and $T_{0,2} = \{\tilde{\phi}, \tilde{X}, A, A^-\}$, are an IGPR-D₂, but (X, T) is not .

Proposition : 4-16

Let (X, T) be an ITS, such that $(X, T_{0,1})((X, T_{0,2}))$ is an IGPR-D₂, then (X, T) is An IGPR-D₂ .

Proof :

Let $(X, T_{0,1})$ be an IGPR-D₂, then for each x, y in X and $x \neq y$, there exist $A = \langle x, A_1, A_2 \rangle$ and $B = \langle y, B_1, B_2 \rangle$ in IGPR-D₂, such that $x \in FA$ and $y \in FB$ and $FA \cap FB = \emptyset$, so $x \in A$ and $y \in B$ and $A \cap B = \emptyset$ i.e (X, T) is IGPR-D₂ .

Similarly we can prove (X, T) is IGPR-D₂, when $(X, T_{0,2})$ is an IGPR-D₂ .

Proposition : 4-17

Let (X, T) be an IGPR-D₂ such that $(X, T^*), ((X, T^{**}))$ is an IGPR-D₂, then (X, T) is an IGPR-D₂ .

Proof :

Let (X, T^*) be an IGPR-D₂, then for each $x, y \in X$, there exist $U_x = \langle x, u_1, u_2 \rangle$ and $V_y = \langle y, v_1, v_2 \rangle$ are two IGPRO(X, T) s.t $x \in u_1, y \in v_1$ and $u_1 \cap v_1 = \emptyset$, so $x \in U$ and $y \in V$ and $U \cap V = \emptyset$ i.e (X, T) is an IGPR-D₂ .

To prove (X, T) is an IGPR-D₂, when (X, T^{**}) is an IGPR-D₂, let $x, y \in X$ and $x \neq y$, so There exist $U_x = \langle x, u_1, u_2 \rangle$ and $V_y = \langle y, v_1, v_2 \rangle \in \text{IGPRO}(X)$ s.t $x \in u_2^c, y \in v_2^c$ and $u_2^c \cap v_2^c = \emptyset$, then $u_2 \cup v_2 = X$ and we have $u_1 \cap u_2 = \emptyset$ and $v_1 \cap v_2 = \emptyset$, then $((u_1 \cap u_2) \cap v_1) \cup ((u_1 \cap u_2) \cap v_2) = \emptyset$, from this we can use set theory operation to get $u_1 \cap v_1 = \emptyset$ so $U \cap V = \emptyset$ i.e (X, T) is an IGPR-D₂ .

Remark : 4-18

An IGPR-D₂ property is independent between (X,T*) and (X,T**), is showing by the examples .

Example : 4-19

i. Take the example 4-12 ,we see that (X,T**) is an IGPR-D₂,but

(X,T*) is not .

ii. let $X=\{a,b\}$ and defined

$T=\{\tilde{\phi}, \tilde{X}, A, B, C, D\}$ where $A=\langle x, \{a\}, \emptyset \rangle$,
 $B=\langle x, \{b\}, \emptyset \rangle$, $C=\langle x, \{a\}, \{b\} \rangle$, and
 $D=\langle x, \emptyset, \emptyset \rangle$. Hence an IGPR(X)
 $=T$, then $T^*=\{\emptyset, X, \{a\}, \{b\}\}$ is an IGPR-D₂, but
 $T^{**}=\{\emptyset, X, \{a\}\}$ is not .

Remark : 4-20

Every IGPR-D₂ spaces is an IGPR-D₁ spaces ,but the converse is not true in general , and the following example in 4-12 show this case .

5- Pre Regular T_{1/2} Spaces

In this section we introduce another application of IGPRO sets namely Intuitionist Pre Regular T_{1/2} Spaces .

Definition : 5-1

Let (X,T) be an ITS ,so its called Intuitionistic Pre Regular T_{1/2} Space (IPR-T_{1/2} S for Short) , if every Intuitionistic Generalized Pre Regular Closed Set is Intuitionistic pre Closed set .

Remark : 5-2

If A is an IGPRCS in (X,T) ,then PCLA-A does not contain any non empty Regular Closed set .

Theorem : 5-3

For any ITS (X,T) ,X is Pre Regular T_{1/2} if and only if every singleton of X is either Regular closed or pre open .

Proof :→

Let $x \in X$ and suppose that $\{x\}=\langle x, \{x\}, \{x\}^c \rangle$ is not regular closed ,then

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$X-\{x\}$ is not regular open and trivially generalized

pre regular closed ,but $\tilde{X}=\langle x, X, \phi \rangle$ is pre regular T_{1/2} so it is pre closed and thus $\{x\}$ is pre open .

←

Let $A=\langle x, A_1, A_2 \rangle$,be generalized pre Regular Closed and let $x \in PCLA$,we will Show that $x \in A$ i.e $x \in A_1$ and $x \in A_2$,so either the set $\{x\}$ is pre open or regular Closed,since $x \in PCLA$ so $\{x\} \in PCLA$ thus $x \in A$.

Remark :5-4

In any ITS (X,T) , $IPO(X) \subseteq IGPRO(X)$,by an IGPRO(X) ,we mean the family of all IGPRO subset of the IS(X,T) .

Theorem : 5-5

An IS (X,T) is an Intuitionistic Pre Regular T_{1/2} if and only if $IPO(X)=IGPRO(X)$.

Proof :→

Let $A=\langle x, A_1, A_2 \rangle$ be an Intuitionistic pre open set ,then $A^c=\langle x, A_2, A_1 \rangle$ is an I Pre closed Set so an IGPRC ,this implies A is an IGPRO . Hence an $IPO(X) \subseteq IGPRO(X)$.

← let $IPO(X)=IGPRO(X)$

Suppose $A=\langle x, A_1, A_2 \rangle$ be an IGPRCS ,then $\bar{A}=\langle x, A_2, A_1 \rangle$ is an IGPROS .Hence $\bar{A} \in IPO(X)$.Thus A is an IPCS ,so (X,T) is an I pre Regular T_{1/2} space .

In the next paper we try to study the properties and relations between an I pre Regular T_{1/2} and Intuitionistic Generalized Pre Regular with other kind of separation axioms .

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بعض التطبيقات للمجموعات المعممة المنتظمة أوليا في الفضاءات التوبولوجية الحدسية

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الملخص

إن الهدف من هذا البحث هو دراسة المجموعات المعممة المنتظمة أوليا ودراسة بعض التطبيقات على هذه المجموعات وبعض الخواص والعلاقات بين هذه التطبيقات مع بعض الأمثلة التوضيحية.