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Electronic Throttle Valve Control Design Based on Sliding Mode Perturbation Estimator †

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Abstract – The electronic throttling angle control system is the newly common requirement trend in automotive technology. It is used to regulate the amount of air flow into the engine. Due to the existence of multiple non-smooth nonlinearities, the controller design to the electronic throttle valve becomes a difficult task. These nonlinearities including stick–slip friction, backlash, and a discontinuous nonlinear spring involved in the system. The designed controller in the present work consists of the estimated perturbation term with a negative sign (used to cancel the perturbation term) and a stabilizing term used to stabilize the nominal system model. The perturbation term consists of the external unknown input and the uncertainty in throttle valve model including the nonlinear terms. The utilized estimator uses the sliding mode control theory and based on the equivalent control methodology. The simulation results show the effectiveness of the proposed controller in estimating the perturbation term. And then in forcing the angle of the throttle valve to follow the desired opening angle in presence of nonlinearities and disturbances in throttle system model and the variation in its parameters.

Keywords – Electronic Throttle Valve Control, Nonlinear Model, Sliding Mode Perturbation Observer, Equivalent Control.

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1. Introduction:

The electronic throttling angle control system is the newly common requirement trend in automotive technology. Controlling the throttling angle is the control of the plate opening angle which it controls the air amount that enters to the combustion engine. The air flow rate will directly control the output torque engine and consequently the speed will be raised or lowered according to the demand. This reveals the importance of controlling the air fuel ratio.

The air mass rate, traditionally, controlled according to the driver demand where the throttling angle is connecting directly to the accelerator by a wire. In this way, many internal and external conditions are ignored in determining the throttle angle such as fuel efficiency, road, or weather conditions which will negatively affect the engine overall efficiency [1]. To overcome the above deficiency an Electronic Control Module (ECM) is used to accurately determining the required opening angle. Figure 1 shows the details of the electronic control system [2]. System in the throttle plate is actuated electronically and, is therefore, mechanically decoupled from the gas pedal. Accordingly, the electronic control unit (ECU) determines the precise amount of fuel delivered to the engine. This amount of fuel is just enough to achieve an ideal air fuel ratio (A/F) (stoichiometry, about 14.7:1). The significance of controlling the air fuel ratio (A/F) is well clarified in Fig. (2) where the emissions lowered to a minimum amount (conversion efficiencies of 98% can be reached). The emission gases are like hydrocarbon (HC), carbon monoxide (CO) and nitrogen oxides (NO_x). For a deviations of ± 0.2 air fuel ratio (A/F) the conversion efficiencies of at least one of the emission components is drastically decreased [3] (Fig. (2)). This make known's the importance of controlling the air fuel ratio as a consequence of throttle angle control. Moreover a primary benefit of Electronic Throttle Control (ETC) is that it enables system designers to incorporate advanced functions into the control of throttle, such as traction control and vehicle stability control, as well as consolidate existing functions, such as cruise control and idle speed control, for lower the cost.

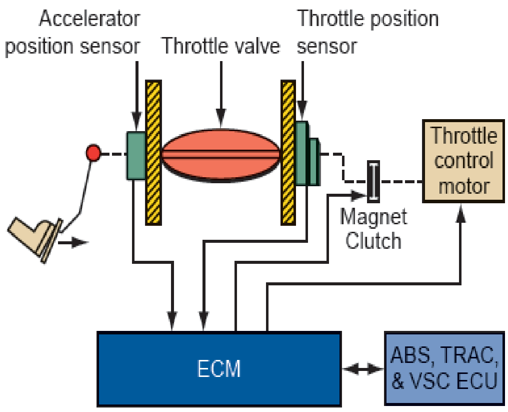

Figure 1: Electronic control module system diagram [2]

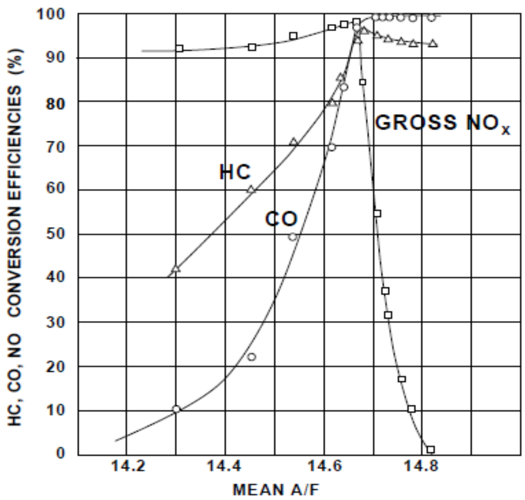

Fig. (2) Steady state conversion efficiency [3].

As shown in Fig. (3), the electronic throttling valve consists of a dc motor, a motor pinion gear, an intermediate gear, a sector gear, a valve plate, and a nonlinear spring.

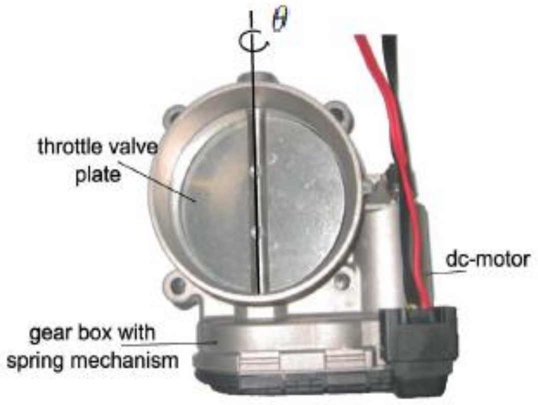

Figure 3: The outer frame of electronic throttling valve [4].

The sliding mode control theory had been used by many authors in designing a controller for the throttle control system considering the presence of the uncertainty in system model and the non-smooth terms coming from the friction model and the nonlinear spring. As a robust and active controller the sliding mode control is utilized to force the throttle valve angle to follow the desired value as in references [5, 6]. In [7] the author designed a continuous sliding mode control for the throttle system to eliminate system chattering by using saturation function and the integral of the saturation function in the sliding mode control law. The price of eliminating chattering is the steady state error which it is adjusted by selecting a suitable control parameter. In [4] the higher order sliding mode control theory is used to design a sliding mode controller to the throttling system with attenuated chattering effect.

In the following sections the mathematical model in terms of the error function (the difference between the throttle angle and the desired one), is presented first. Then the idea of the sliding mode perturbation observer is presented depending on equivalent control theory. In the last sections a nonlinear controller uses an estimate for the perturbation term that affects the throttle valve system is derived. Then the control system is simulated for many uncertainty levels in system parameters and with the presence of non-smooth terms via MATLAB software.

2. Mathematical Model and Problem Statement

As mentioned above the electronic throttling valve consists of a dc motor, a motor pinion gear, an intermediate gear, a sector gear, a valve plate, and a nonlinear spring. The mathematical model for the electronic throttle valve consists of the electrical system (the DC. motor mathematical model) [4]:

$$\frac{di}{dt} = -\frac{R}{L}i - \frac{k_m}{L}\omega_m + \frac{1}{L}u \quad (1)$$

And the mathematical model for mechanical system

$$\begin{aligned}\frac{d\theta}{dt} &= \omega \\ \frac{d\omega}{dt} &= \frac{k_m N}{J}i - \alpha(\theta-\theta_o) - \beta * sign(\theta-\theta_o) \\ &\quad -\gamma\omega - \delta * sign(\omega)\end{aligned} \quad (2)$$

where i is the armature current, ω_m is the motor angular velocity, u is the input voltage, k_m is the inductive voltage constant, L and R are the inductance and resistance in the armature circuit, respectively. Also, in Eq. (2), θ is the throttle valve angle, ω is the angular velocity of the throttle valve, N is the gear ratio and J is the equivalent moment around the throttle axis, and their values are defined by

$$N = \frac{\omega_m}{\omega} \ \& \ J = N^2 J_m + J_{th} \quad (3)$$

The model parameters α, β, γ and δ are taken as in reference [4] (see Table 1) and formally modeled the friction effect, includes both the viscous and Coulomb friction, and the nonlinear spring and which acts on the throttle valve.

To simplify the model, the motor inductance L is ignored and accordingly the throttle valve dynamical system in Eqs. (1) & (2) is reduced to a system with lower dimension as follows [8]; by ignoring the inductance effect on system model

$$L \approx 0 \tag{4}$$

Eq. (1) becomes;

$$I = \frac{1}{R}\, u - \frac{K_e\, N}{R}\, \omega \tag{5}$$

By substituting the value of I from Eq. (5) in Eq. (2), we get;

$$\begin{aligned}
\dot{\omega} &= \frac{K_e N}{J\, R} u - \frac{{K_e}^2 N^2}{J\, R}\, \omega - \frac{m_1}{J}\,(\theta - \theta_0) \\
&\quad - \frac{D}{J} sgn(\theta - \theta_0) \ - \frac{B}{J}\omega - \ \frac{K_d}{J} sgn(\omega) \\
&= bu - \frac{{K_e}^2 N^2}{J\, R}\, \omega - \frac{m_1}{J}\,(\theta - \theta_0) \\
&\quad - \beta sgn(\theta - \theta_0) \ - \gamma\omega - \ \delta sgn(\omega) \\
&= -a_1(\theta - \theta_0) - a_2\, \omega + bu \\
&\quad - \beta\, sgn(x_1 + \theta_r - \theta_0) - \delta\, sgn(\omega)
\end{aligned} \tag{6}$$

Now by defining the state variables $x_1$and x_2 as $x_1 = \theta - \theta_r$ and $x_2 = \dot{\theta} - \omega_r$ Eq. (6) in state space form becomes;

$$\left.\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x_2} &= -a_{1\,o}(x_1 + \theta_r - \theta_0) - a_{2\,o}(x_2 + \omega_r) \\
&\qquad + b_o\, u - \dot{\omega}_r + D\big(x_1, x_2\big)
\end{aligned}\right\} \tag{7}$$

where θ_r and ω_r are the reference throttle angle and angular velocity respectively, and $D(x_1, x_2)$ is the perturbation term collected the uncertainties in system parameters model and the non-smooth terms due to friction and the nonlinear spring. It is given by

$$\left.\begin{aligned}
&D(x_1, x_2) = \\
&-\beta\, sgn(x_1 + \theta_r - \theta_0) - \delta\, sgn(x_2 + \omega_r) \\
&-\Delta a_1(x_1 + \theta_r - \theta_0) - \Delta a_2(x_2 + \omega_r) + \Delta bu
\end{aligned}\right\} \tag{8}$$

Where a_{10}, a_{20}, and b_0 are the nominal parameters value and $\Delta a_1, \Delta a_2, \Delta b$ are the uncertainty in their values. In addition

$$a_1 = \alpha = \frac{m_1}{J},\, a_2 = \gamma + \frac{{k_e}^2 N^2}{J\, R},\ \ b = \frac{k_e N}{J\, R},$$
$$\gamma = \frac{B}{J},\ \beta = \frac{D}{J},\ \text{and}\ \delta = \frac{K_d}{J}$$

3. Sliding Mode Control

Control in the presence of uncertainty is one of the main topics of modern control theory. The idea of SMC is based on the introduction of a function, named the sliding variable. When it is designed properly the sliding variable becomes equal to zero, and it defined then as the sliding manifold (or the sliding surface). The proper design of the sliding variable yields suitable closed-loop system performance while the system trajectories belong to the sliding manifold. The control task is then to steer the trajectory of the system to the sliding manifold and maintain motion on the manifold thereafter. When the state on the sliding manifold, the control system is insensitive to external and internal disturbances matched by the control with ultimate accuracy, and sliding variables converge to zero value in finite-time [9].

Generally speaking, automotive control problems are highly nonlinear and subject to high amount of disturbances and uncertainties. On the other hand, sliding mode control theory has been investigated in detail over the last there decades and it currently offers numerous systematic design methods applicable to industrial control problems. The use of sliding mode control ideas in automotive control applications has also been reported [10]. The sliding mode control theory can also be used to estimate the unknown but bounded disturbances and uncertainty terms in system model which it is known as Sliding Mode Perturbation Observer (SMPO).

In the following section, the theory of SMPO is presented, and then it will be utilized in designing a nonlinear controller for electronic throttle valve.

4. Perturbation Estimation and Compensation

One way of enhancing the robustness of a control system is to estimate the discrepancies between the model used for the control derivation purposes and the actual system by a perturbation estimator and to incorporate this information into the control law in a proper way. The disturbance estimation presented here is based on the equivalent control methodology and it is in the continuous-time domain [10]. First, consider a SISO nonlinear system

$$\dot{x} = f(x) + g(x)u + \delta(t,x,u) \tag{9}$$

Where $x \in R$ is the state, $u \in R$ is the control , $f(x), g(x)$ are smooth, known functions and $\delta(t,x,u)$ is the perturbation function which lumps all the disturbances and the uncertainties of the system. It is assumed that

$$|\delta(t,x,u)| \leq \rho(t) \tag{10}$$

Where $\rho(t)$ is a known bounding function. The objective is to estimate $\delta(t,x,u)$ from x . Consider now the perturbation estimator as;

$$\dot{\hat{x}} = f(x) + g(x)u + (\rho(t) + \eta)\, sgn\,(x - \hat{x}) \tag{11}$$

Which basically repeats what is known about the system with an additional discontinuous injection. The error dynamics follow from the subtraction of Eq. (11) from Eq. (9) as follows:

$$\dot{e}_x = \delta(t,x,u) - (\rho(t) + \eta)\, sgn(e_x) \tag{12}$$

Where $e_x = x - \hat{x}$ is the estimation error. Ideally $e_x = 0\ \forall t \geq t_0$ (sliding motion) with $\eta > 0$, and $x(t_0) = \hat{x}(t_0)$; hence

$$[(\rho(t) + \eta)\, sgn(e_x)]_{eq} = \delta(t,x,u) \tag{13}$$

The operator $[\blacksquare]_{eq}$ outputs the equivalent value of its discontinuous argument. In sliding mode control the equivalent control is the required control action to maintain the sliding motion [11]. The output of Eq. (13) is defined as the continuous injection which would satisfy the invariance conditions of the sliding motion ($e_x = 0$, $\dot{e}_x = 0$) that this discontinuous input induces. The equivalent value operator, $[\blacksquare]_{eq}$, can be approximately realized by an high bandwidth low-pass filter according to the equivalent control methodology [12] ; i.e.,

$$\tau\dot{v} + v = (\rho(t) + \eta) * sgne_x \tag{14}$$

Therefore the output of Eq. (14) approaches asymptotically the unknown perturbation. Namely

$$v \approx \delta(t,x,u) \tag{15}$$

5. Control Design Based on Perturbation Observer

The electronic throttle valve system model (Eq. (6)) is rewritten here for the case where $\dot{\omega}_r = 0$ (i.e., constant or piecewise constant reference throttle angle) as follows;

$$\left.\begin{aligned} \dot{x_1} &= x_2 \\ \dot{x_2} &= -a_{1\,0}(x_1 + \theta_r - \theta_0) - a_{2\,0}x_2 + b_0 u \\ &\quad + D(x_1, x_2) \end{aligned}\right\} \tag{16}$$

Our aim here is to propose a control law that consists of two parts; the first is an estimate to the disturbances and the uncertainty in throttle valve model and the second is a stabilizing controller for the certain or nominal throttle system model. The proposed control law in this work is as follows:

$$u = \frac{1}{b_0}\left(u_o - \widetilde{D}\right) \tag{17}$$

Where u_o is the stabilizing control term (designed for nominal system) and $\widetilde{D}$ is the estimate to the perturbation that acting the throttle valve system.

The structure for the observer used to estimate D is proposed here as follows;

$$\left.\begin{aligned}\dot{\hat{x}}_2 = -a_{1\,0}(x_1+\theta_r-\theta_0) - a_{2\,0}x_2 + b_0 u \\ +k * sgn(x_2-\hat{x}_2)\end{aligned}\right\} \quad (18)$$

The subtraction of Eq. (18) from Eq. (16), yields

$$\begin{aligned}\dot{x}_2 - \dot{\hat{x}}_2 &= -a_{1\,0}(x_1+\theta_r-\theta_0) - a_{2\,0}x_2 + b_0 u \\ &\quad +D(x_1,x_2) - \{-a_{1\,0}(x_1+\theta_r-\theta_0) \\ &\quad -a_{2\,0}x_2 + b_0 u + k * sgn(x_2-\hat{x}_2)\} \\ &= D(x_1,x_2) - k * sgn(x_2-\hat{x}_2) \qquad (19)\end{aligned}$$

Now define $e = x_2 - \hat{x}_2$, and let

$$k = k_o + \rho$$
$$k_o = max_{\Omega}|D(x_1,x_2)|, \rho > 0$$

where Ω is a compact set as given in appendix A. Then Eq. (19) becomes:

$$\dot{e} = D - (k_o+\rho) * sgn(e) \qquad (20)$$

Since $e(0) = 0$, where $x_2(0) = \hat{x}_2(0)$ and $(k_o+\rho) > |D(x_1,x_2)|$, the error will be in sliding mode $\forall t \geq 0$. This leads to $e = \dot{e} = 0 \;\; \forall t \geq 0$. To this end by considering Eq. (20) for $\dot{e} = 0$ and utilizing the equivalent control concept we have:

$$[(k_o+\rho) * sgn(e)]_{eq} = D(x_1,x_2) \qquad (21)$$

Where the equivalent operator is a low pass filter given by:

$$\tau\dot{v} + v = (k_o+\rho) * sgn(e) \qquad (22)$$

Therefore the output of Eq. (22), and after a small period of time T, is close to the unknown perturbation. Namely

$$v = \widetilde{D} \rightarrow D \;\; \forall t > T$$

By using the output of the low pass filter as un estimate for D , the control law becomes;

$$\left.\begin{aligned}\dot{v} &= \tfrac{1}{\tau}\{-v + (k_o+\rho) * sgn(e)\} \\ u &= \tfrac{1}{b_0}(u_o - v)\end{aligned}\right\} \qquad (23)$$

Eventually the nominal controller u_o is designed as follows;

$$u_0 = a_{1\,0}(x_1+\theta_r-\theta_0) + a_{2\,0}x_2 - k_1x_1 - k_2x_2 \qquad (24)$$

Where k_1 & k_2 the state proportional gains selected according to the required angle response characteristics. By considering Eqs. (17) & (24), Eq. (16) becomes;

$$\left.\begin{aligned}\dot{x_1} &= x_2 \\ \dot{x_2} &= -k_1x_1 - k_2x_2\end{aligned}\right\} \qquad (25)$$

The error state x_1 will be regulated exponentially to the origin with a dynamic characteristic according to the values of k_1 & k_2 .

6. Simulation Results and Discussion

The full electronic throttle valve mode that is used in the simulations is given in Eqs. (1) & (2). The nominal parameters values are given in Table (1) below [4].

Table 1: the nominal parameters values [4]

parameters	*minimal value*	*maximal value*	*unite*
α	69	95	$1/s^2$
β	143.5	157	rad/s^2
γ	32	54	$1/s$
δ	57	76	rad/s^2
θ_0	0.095	0.095	rad
k_e	0.02	0.02	$v\,s\,/rad$
R	1.3	1.7	(Ω)
L	0.8×10^{-3}	1×10^{-3}	H
J	1.6×10^{-3}	1.65×10^{-3}	$kg\,m^2$
N	20	20	

The nominal system parameters values in control law are:

$$a_{1\,0} = 82 \;, a_{2\,0} = 110 \,, b_0 = 167.5$$

Accordingly the maximum uncertainties in system parameters are:

$$\delta_{a_1} = 13, \qquad \delta_{a_2} = 21 \,, \qquad \delta_b = 24.9$$

Also the extreme values for the non-smooth terms are:

$$\beta_{max} = 15\,,\ \delta_{max} = 76$$

For simulations the throttle valve system parameters value are taken as the nominal value plus percent of the uncertainty and the results are classified below according to the reference angle value. In addition, the controller parameters in our work are: $\tau = \frac{1}{1000}, \rho = 3,$ and

$$\left.\begin{aligned} k_o &= 233 + 13 * |x_1 + \theta_r - \theta_0| \\ &\quad +21 * |x_2| + 0.1486 * (|u_o| + v) \end{aligned}\right\} \quad (26)$$

where $|v| \leq 3022$ (see appendix A), and $|u_0| \leq (a_{1\,0} + k_1)|x_1| + (a_{2\,0} + k_2)|x_2| + a_{1\,0}|\theta_r|$.

Finally the control law is:

$$\left.\begin{aligned} \dot{v} &= \frac{1}{\tau}\{-v + (k_o + \rho) * sgn(e)\} \\ u &= \frac{1}{b_0}(u_o - v) \end{aligned}\right\} \quad (27)$$

where $= x_2 - \hat{x}_2$, $\tau = \frac{1}{1000}, \rho = 3,$ and

$$\dot{\hat{x}}_2 = -a_{1\,0}(x_1 + \theta_r - \theta_0) - a_{2\,0}x_2 + b_0 u + k * sgn(x_2 - \hat{x}_2)$$

$$k_o = 233 + 13 * |x_1 + \theta_r - \theta_0| + 21 * |x_2| + 0.1486 * (1307 * |x_1| + 180 * |x_2| + 82 * |\theta_r| + v)$$

***6.1.** $\theta_r = 60^o$ (uncertainty=0% from max uncertainty)*

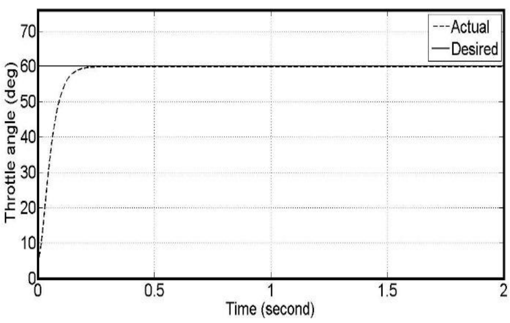

Figure 4: Throttle angle verses time.

In Fig. (4), initially the throttle angle is $5.44^o = 0.095\,rad = \theta_0$ while the reference angle is $\theta_r = 60^o$. In this figure the time required to reach the desired angle is less than $0.25\,sec.$. Figures (5) and (6) plot the phase plane and the control input (the voltage) respectively. Although the control design is carried out after reducing the model for a small inductance value but the simulation is done for complete model. The result proves that ignoring the dynamics of the electrical system does not affect the system response performance. Moreover the present of the electrical subsystem attenuate the chattering occurs in the estimating quantity. When the disturbance is in switching mode, the estimation takes the average value. The average value does not only relate to disturbance magnitude but also to the time duration for which the disturbance stay on either side of the switching line. This can be deduced in Fig. (7) where the discontinuous disturbance is estimated by a smoother quantity.

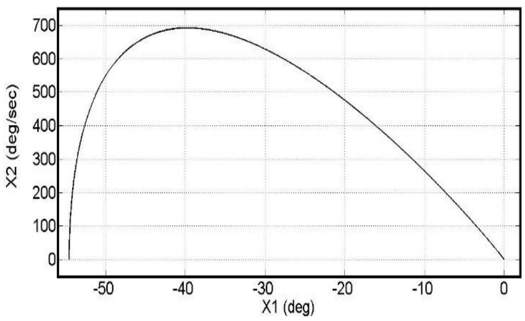

Figure (5) the phase plane plot.

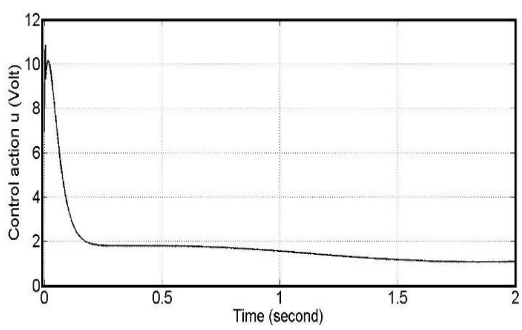

Figure (6) the control input versus time

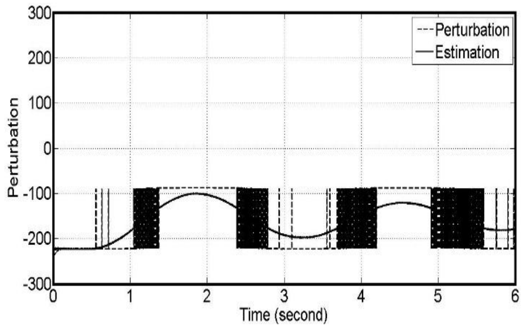

Figure (7) the perturbation and its estimation versus time

6.2. $\theta_r = 60^o$ (uncertainty =75% from max uncertainty)

The figures below (Figs. (8) & (9)) plot the throttle angle response and the phase plane plot for the case where the uncertainty in system parameters is 75% from the max uncertainty in their values. The figures show that the angle response is similar to the first case above. This is because the controller cancels the perturbation whatever the uncertainty in system parameters and leaves the throttle system with nominal terms only and for this reason it behaves in a similar manner.

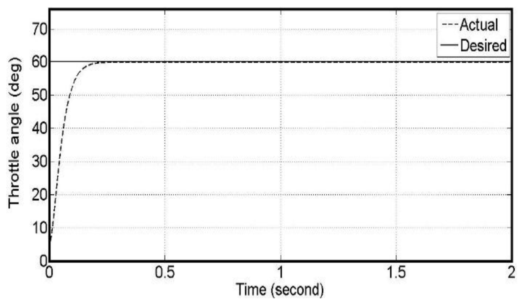

Figure (8): Throttle angle verses time.

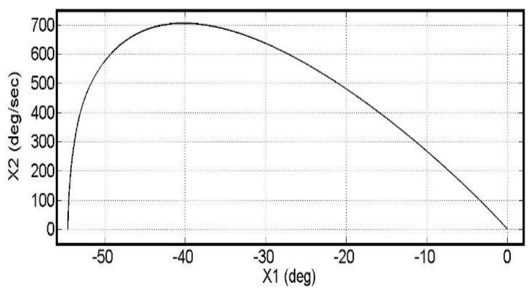

Figure (9) the phase plane plot.

6.3. Variable reference angle θ_r

In this simulation test the initial throttle angle value is $5.44^o = 0.095\, rad = \theta_0$ with a stepwise changed throttle angle reference. The throttle angle for variable reference angle is plotted versus time in Fig. (10). Since the throttle angle started from its initial value ($5.44^o = 0.095\, rad = \theta_0$), so there is no transient response from $(0 \rightarrow 2.5\, second)$ as shown in this figure. Also the phase plot and the control action are clarified in Figs. (11) & (12) respectively.

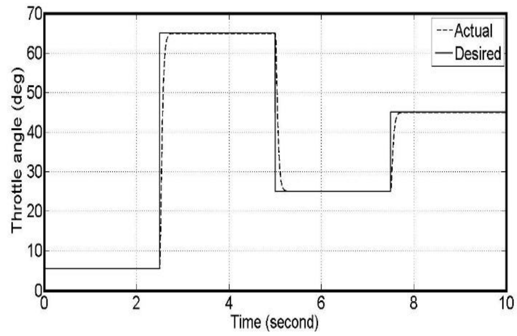

Figure 10: Throttle angle verses time for variable reference throttle angle.

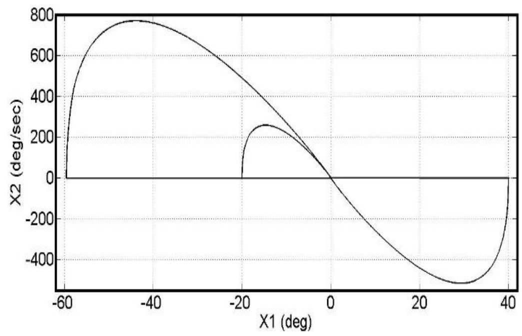

Figure 11: Phase plot for the throttle angle for variable throttle angle reference.

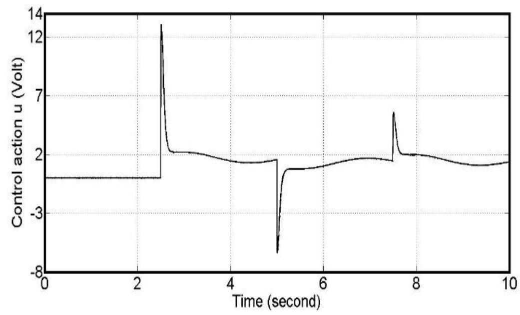

Figure 12: Control action (u) verses time for variable reference throttle angle.

7. Conclusion

The sliding mode perturbation estimator (SMPO) is utilized in the present work to estimate a collected term consists of the disturbances, nonlinearity and the uncertainty in the electronic throttle valve model. Cancelling the effect of disturbance and uncertainty from any dynamic system, and then regulating the error between the state and the reference value to origin with improved system response characteristics is the idea behind the present work. Consequently the designed electronic throttle control is a robust control.

The estimated perturbation term is found very close after a small period of time to the actual value and then it is used to cancel the effects of the perturbation on system dynamics. In addition a stabilizing controller designed for the nominal system is designed to regulate the error state to the origin. The simulation results for constant and variable reference throttle angle and with uncertainty in system parameter values validate the effectiveness of our proposed controller. In addition the throttle angle response is identical for different system parameters values as can be seen from Figs. (4) and (8) which represent the most important feature for the proposed controller.

8. References

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Appendix A: Maximum estimation of v

To estimate the maximum value for v we start from Eq. (22)

$$\tau\dot{v} + v = (k_o + \rho) * sgn(e) \quad \text{(A-1)}$$

Equation (A-1) can be solved as follows;

$$v(t) = e^{(-t/\tau)} * \left\{v(0) + \int_0^t \frac{(k_o+\rho)}{\tau} e^{(s/\tau)} * sgn(e)ds\right\}$$

$$\Rightarrow |v(t)| \leq e^{(-t/\tau)} * \left\{v(0) + |k_o + \rho|_{max} \int_0^t \frac{e^{(s/\tau)}}{\tau} ds\right\}$$

$$\leq e^{(-t/\tau)} * \left\{v(0) + |k_o + \rho|_{max}\left(e^{(t/\tau)} - 1\right)\right\}$$

Let $v(0) = 0$, the above inequality becomes;

$$|v(t)| \leq |k_o + \rho|_{max}, \forall t \geq 0$$

Or

$$|v(t)| \leq |k_o|_{max} + \rho, \quad \forall t \geq 0 \quad \text{(A-2)}$$

By substituting Eq. (26) in the inequality (A-2) and for $(x_1, x_2) \in \Omega = \left\{|x_1| < 70 * \frac{\pi}{180}, |x_2| < 2500 * \frac{\pi}{180}\right\}$, the bound on $v(t)$ is;

$$|v| \leq 3022$$