Journal of KerbalaUniversity , Vol. 11 No.2 Scientific . 2013

On The Factor GroupK($D_n \times C_3$When n =2^h)

حول الزمرة الكسريه($Dn \times C_3$) $cf(Dn \times C_3,Z)/\overline{R}$ عندما n عددزوجي

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Abstract

Let D_n be the dihedral group and C_3 be the cyclic group of order 3 . Let cf($Dn \times C_3$,Z) be the abelian group of Z-valued class function of the group $Dn \times C_3$. The intersection of cf($Dn \times C_3$,Z) with the group of generalized characters of $Dn \times C_3$ which is denoted by R($D_n \times C_3$) is a normal subgroup of the group cf($Dn \times C_3$,Z)denoted by $\overline{R}$ ($Dn \times C_3$) . The factor group cf($Dn \times C_3$,Z)/$\overline{R}$ ($Dn \times C_3$) is afinite abelian group denoted by K($Dn \times C_3$) .

In this paper, we prove that the rational valued characters table of the group $D_{2^h} \times C_3$ is equal to the tensor product of the rational valued characters table of D_{2^h} and the rational valued characters table of the cyclic group C_3 . Also, we find that $K(D_{2^h} \times C_3) = \overset{2}{\underset{i=1}{\oplus}} K(D_{2^h}) \overset{h+3}{\underset{i=1}{\oplus}} C_3$.

المستخلص

لتكن D_n زمرة ثنائي السطوح و C_3 الزمرة الدوارة ذات الرتبة 3. لتكن cf($Dn \times C_3$,Z) الزمرة الابدالية لدوال الصف ذات القيم الصحيحة للزمرة $Dn \times C_3$. ان تقاطع cf($Dn \times C_3$,Z) مع زمرة كل الشواخص العمومية للزمرة $Dn \times C_3$ والتي يرمز لها بالرمز R($Dn \times C_3$) تكون زمرة جزئية سوية من الزمرة(cf($Dn \times C_3$,Z ويرمز لها بالرمز $\overline{R}$ ($Dn \times C_3$).الزمرة الكسرية ($Dn \times C_3$) cf($Dn \times C_3$,Z)/$\overline{R}$تكون زمرة ابدالية منتهية ويرمز لها بالرمز K($Dn \times C_3$) .
برهنا في هذا البحث ان جدول الشواخص ذات القبم النسبية للزمرة $D_{2^h} \times C_3$ يساوي حاصل الضرب الممتد لجدول الشواخص ذات القيم النسبية للزمرة D_{2^h}وجدول الشواخص ذات القيم النسبية للزمرة C_3 وكذلك وجدنا ان :-

$$K(D_{2^h} \times C_3) = \overset{2}{\underset{i=1}{\oplus}} K(D_{2^h}) \overset{h+3}{\underset{i=1}{\oplus}} C_3$$

1.Introduction

Let G be a finite group ,two elements of G are said to be **Γ- conjugate** if the cyclic subgroups they generate are conjugate in G, this defines an equivalence relation on G .Its classes are called**Γ- classes** . The Z - valued class function on the group G , which is constant on the**Γ-classes** forms a finitely generated abelian group cf(G,Z) of a rank equal to the number of **Γ-classes** . The intersection of cf(G,Z) with the group of all generalized characters of G , R(G) is a normal subgroup of cf(G,Z) denoted by $\overline{R}(G)$,then cf(G,Z)/ $\overline{R}(G)$ is a finite abelian factor group which is denoted by K(G). Each element in $\overline{R}(G)$ can be written as $u_1\theta_1 + u_2\theta_2 + \ldots\ldots + u_l\theta_l$, where l is the number of **Γ - classes** , u_1 , u_2 , ... , $u_l \in Z$ and $\theta_i = \sum_{\sigma \in Gal(Q(\chi_i)/Q)} \sigma(\chi_i)$,where χ_i is an irreducible character of the group G and σ is any element in Galios group $Gal(Q(\chi_i)/Q)$. Let ≡*(G) denotes the $l \times l$ matrix which corresponds to the θ_i 's and columns correspond to the **Γ- classes** of G .This matrix expresses $\overline{R}(G)$ basis in terms of the cf(G,Z) basis .

We can use the theory of invariant factors to obtain the direct sum of the cyclic Z-module of orders the distinct invariant factors of ≡*(G) to find the cyclic decomposition of K(G) .
In 1982, M.S.Kirdar [10] studied the factor group K(Cn) .In 1994, H.H.Abass [3] studied the factor cf(Q2m,Z) / $\overline{R}$(Q2m) .In 2005, N.S.Jasim [12] studied the factor group cf(G,Z)/ $\overline{R}(G)$ for the special linear group SL(2,p). In 2010,H.H.Abass and M.S.Mahdi [4] studied the Factor Group K(Dnh) when n is an Odd Number. In 2011, H.H.Abass and K.A.Layith [6] studied the Factor Group
K(Dnh ×C2) when n is an Odd Number
The aim of this paper is find the rational valued characters table of the group $D_{2^h} \times C_3$ and the cyclic decomposition of the group $K(D_{2^h} \times C_3)$.

2.Preliminaries

In this section, we review definitions and some results which will be used in later section

Definition(2.1):[8]

The set of all $n \times n$ non-singular matrices over the field F forms group under the operation of the matrix multiplication,this group is called the ***general lineargroup***of the dimension n over the field K, denoted by GL (n ,K).

Definition(2.2):[2]

A matrix representation of a group G is a homomorphism T from G into GL (n ,K) , n is called the degree of matrix representation T .

Definition(2.3):[8]

The trace of square matrix A is the sum of the elements on the main diagonal ; we denote the trace of A by tr(A) .

Definition (2.4): [2]

A matrix representation T: G→GL (n , K) is said to be***reducible*** if T is equivalent to the matrix representation of the form

$$\begin{bmatrix} T_1(g) & V(g) \\ 0 & T_2(g) \end{bmatrix}$$

Where T_1, T_2 are matrices ofrepresentations over Kof the dimension m×m, (n-m)×(n-m) respectively , V(g) is a matrix of the dimension (m)×(n-m) and 0 is zeros matrix of the dimension (n-m)×(m) such that 0<m<n .
Otherwise ,T is called an irreduciblematrix representation.
Moreover , if we could remove the off-diagonal block , i.e V(g) =0 for all g∈G ,then T is called completely reducible matrix representation

Definition (2.5): [7]

Let T be a matrix representation of a group G over the field K, ***the character***χ of a matrix representation T is the mapping χ: G→K defined by χ(g)=Tr(T(g)) for all g ∈G,where Tr(T(g)) refers to the trace of the matrix T(g) and χ(1) is the degree of χ .

Remark (2.6) :[8]

(I) A finite group G has a finite number of conjugacy classes and a finite number of distinct k-irreducible characters, the group character of a group representation is constant on a conjugacy class, the values of the characters can be written as a table known ***the characters table*** which is denoted by ≡(G) .

(II) If C_n=<*r*> is the cyclic group of order n generated by *r* and $\omega = e^{2\pi i/n}$ is the primitive n-th root of unity, then ≡(C_n) =

CL_α	[1]	[r]	[r^2]	…	[r^{n-1}]
$\lvert CL_\alpha \rvert$	1	1	1	…	1
$\lvert C_G(C_\alpha) \rvert$	n	n	n	…	n
χ_1	1	1	1	…	1
χ_2	1	ω	ω^2	…	ω^{n-1}
χ_3	1	ω^2	ω^4	…	ω^{n-2}
⋮	⋮	⋮	⋮	⋱	⋮
χ_n	1	ω^{n-1}	ω^{n-2}	…	ω

Table(2.1)

Theorem (2.7): [11]

1-Sum of characters is a character .
2- Product of characters is a character.

Theorem (2.8):[8]

Let T_1: G_1→GL(n ,K) and T_2: G_2→GL(m ,K) are two irreducible representations of the group G_1 and G_2 with characters χ_1 and χ_2 respectively. Then the tensor product $T_1 \otimes T_2$ is irreducible representation of the group $G_1 \times G_2$ with character $\chi_1\,\chi_2$.

Definition (2.9):[9]

A rational valued characterθ of G is a character whose valuesare in the set of integers Z, i.eθ(g) ∈ Z, for all g ∈G .

Proposition (2.10):[10]

The rational valued characters $\theta_i = \sum_{\sigma \in Gal(Q(\chi_i)/Q)} \sigma(\chi_i)$ form basis for $\overline{R}$(G), where χ_i are the irreducible characters of G and their numbers are equal to the number of all distinct Γ- classes of G.

3.The Factor Group K(G)

In this section, we study the factor K(G) and discuss the cyclic decomposition of the factor groups K(C_n) and K(D_n) .

Definition (3.1):[8]

Let M be a matrix with entries in a principal ideal domain R. A k-minor of M is the determinant of k×ksubmatrix preserving rows and columns order.

Definition (3.2):[9]

A k-th determinant divisor of the matrix M is the greatest common divisor (g.c.d) of all the k-minors of M . This is denoted by D_k (M).

Theorem (3.3):[8]

Let M be an k×k matrix with entries in a principal domain R. Then there exists matrices P and W such that:

1- P and W are invertible.
2- P M W = D.
3- D is diagonal matrix.
4- If we denote D_{ii} by d_i then there exists a natural number m ; $0 \le m \le k$ such that $j > m$ implies $d_j \ne 0$ and $1 \le j \le m$ implies $d_j \mid d_{j-1}$.

Definition (3.4):[9]

Let M be a matrix with entries in a principal domain R, be equivalent to a matrix D = diag $\{d_1, d_2, \dots, d_m, 0, 0, \dots, 0\}$ such that $d_j \mid d_{j-1}$ for $1 \le j < m$ .We call D ***the invariant factors matrix of*** M and $d_1, d_2, \dots, d_m$ the invariant factors of M.

Proposition(3.5):[10]

Let A and B be two non-singular matrices of the rank n and m respectively, over a principal ideal domain R . Then the invariant factor matrices of $A \otimes B$ equals D(A) $\otimes$ D(B), where D(A) and D(B) are the invariant factor matrices of A and B respectively.

Proposition(3.6):[10]

Let H and L be p_1-group and p_2-group respectively ,where P_1 and P_2 are distinct primes.Then$\equiv^*(H \times L) = \equiv^*(H) \otimes \equiv^*(L)$.

Remark (3.7):[10]

Suppose cf(G,Z) is of the rank *r*, the matrix expressing the $\bar{R}$ (G)basis in terms of the cf(G,Z) = Z^r basis is $\equiv^*$(G).Hence by theorem (2.4),we can find two matrices P and Q with a determinant ± 1 such that $P.\equiv^*(G).Q = D(\equiv^*(G)) = diag\{d_1, d_2, \dots\dots\dots, d_r\}, d_i = \pm D_i(\equiv^*(G)) / \pm D_{i-1}(\equiv^*(G))$.

Theorem (3.8):[10]

Let P be a prime number , then

$K(G) = \oplus \sum C_{d_i}$ Such that $d_i = \pm D_i(\equiv^*(G)) / \pm D_{i-1}(\equiv^*(G))$.

Theorem (3.9):[10]

$|K(G)| = \det(\equiv^*(G))$

Proposition (3.10): [10]

The rational valued characters table of the cyclic group $C_{p^s} = \langle r \rangle$ of the rank s+1 where p is a prime number which is denoted by $(\equiv^*(C_{p^s}))$, is given as follows:

Γ-classes	[1]	$[r^{p^{S-1}}]$	$[r^{p^{S-2}}]$	$[r^{p^{S-3}}]$	…	$[r^{p^2}]$	$[r^p]$	$[r]$
θ_1	p^{s-1}(p-1)	- p^{s-1}	0	0	…	0	0	0
θ_2	p^{s-2} (p-1)	p^{s-2} (p-1)	- p^{s-2}	0	…	0	0	0
θ_3	p^{s-3}(p-1)	p^{s-3}(p-1)	p^{s-3}(p-1)	- p^{s-3}	…	0	0	0
⋮	⋮	⋮	⋮	⋮	⋱	⋮	⋮	⋮
θ_{s-1}	p(p-1)	p(p-1)	p(p-1)	p(p-1)	…	p(p-1)	-p	0
θ_s	p-1	p-1	p-1	p-1	…	p-1	p-1	-1
θ_{s+1}	1	1	1	1	…	1	1	1

Table (3.1)

Where its rank s+1 represents the number of all distinct Γ-classes

Proposition (3.11): [10]

If p is a prime number , thenD($\equiv^*(C_p{}^s)$) = diag{p^s,p^{s-1} ,……,p,1}.

Theorem (3.12) :[10]

Let p be a prime number. Then K($C_p{}^s$) = $\oplus\sum_{i=1}^{s} C_{P^i}$

Definition (3.13):[8]

For a fixed positive integer n≥3 , ***the dihedral group D_n*** is a certain non-abelian group of the order 2n. In general can write it as: D_n={ $S^j\ r^k$:0 ≤ k ≤ n-1 , 0 ≤ j ≤ 1}

which has the following properties r^n =1 , $Sr^kS = r^{-k}$, $(Sr^k)^2$ =1

Remark (3.14):

The group $D_n \times C_3$ is the direct product group of the dihedral group D_n and the cyclic group C_3 of order 3

Proposition (3.15): [3]

The rational valued character table of the dihedral D_n when n= 2^h can be given by $\equiv^*(D_{2^h})$=

Γ-classes	[1]	$[r^{2^{h-1}}]$	$[r^{2^{h-2}}]$	$[r^{2^{h-3}}]$	…	$[r^2]$	$[r]$	$[s]$	$[sr]$
θ_1	2^{h-1}	-2^{h-1}	0	0	…	0	0	0	0
θ_2	2^{h-2}	2^{h-2}	-2^{h-2}	0	…	0	0	0	0
θ_3	2^{h-3}	2^{h-3}	2^{h-3}	-2^{h-3}	…	0	0	0	0
⋮	⋮	⋮	⋮	⋮	⋱	⋮	⋮	⋮	⋮
θ_{h-1}	2	2	2	2	…	-2	0	0	0
θ_h	1	1	1	1	…	1	-1	-1	1
θ_{h+1}	1	1	1	1	…	1	1	1	1
θ_{h+2}	1	1	1	1	…	1	1	-1	-1
θ_{h+3}	1	1	1	1	…	1	-1	1	-1

Table (3.2)

Theorem (3.16):[3]

The invariant factors matrix of$\equiv^*(D_n)$ when n is an even number and $n=2^h$ can be given by
$D(\equiv^*(D_{2^h})) = diag\{2^{h+1}, -2^{h-1}, -2^{h-2}, \ldots, -2,-2,-2,-1\}$.

Theorem (3.17):[3]

The cyclic decomposition of the group $K(D_n)$ when $n=2^h$ can be given by :
$K(D_{2^h}) = K(C_{2^{h-1}}) \oplus C_{2^{h+1}} \oplus C_2 \oplus C_2$

Example(3.18) :

To find the cyclic decomposition of the group $K(D_{32})$ by theorem(3.17)
$K(D_{32}) = K(D_{2^5}) = K(C_{2^{5-1}}) \oplus C_{2^{5+1}} \oplus C_2 \oplus C_2 = K(C_{2^4}) \oplus C_{2^6} \oplus C_2 \oplus C_2$
By theorem(3.12) we can write $K(C_{2^4})$ as follows :-
$K(C_{2^4}) = C_{2^4} \oplus C_{2^3} \oplus C_{2^2} \oplus C_2 = C_{16} \oplus C_8 \oplus C_4 \oplus C_2$

Then $K(D_{2^4}) = C_{16} \oplus C_8 \oplus C_4 \oplus C_2 \oplus C_{64} \oplus C_2 \oplus C_2 = C_{64} \oplus C_{16} \oplus C_8 \oplus C_4 \bigoplus_{i=1}^{3} C_2$

4.The Main Results

In this section, we find the general form of the rational valued characters table of the group $D_n \times C_3$, when $n=2^h$

Theorem(4.1) :

The rational valued character table of the group $D_{2^h} \times C_3$ is equal to the tensor product of the rational valued characters table of the group D_{2^h} and the rational valued characters table of the group C_3; that is
$\equiv^*(D_{2^h} \times C_3) = \equiv^*(D_{2^h}) \otimes \equiv^*(C_3)$.

Proof:

Since the group D_{2^h} is a 2-group and the group C_3 is a 3-group and g.c.d(2,3)= 1 , then by proposition (3.8) , we have
$\equiv^*(D_{2^h} \times C_3) = \equiv^*(D_{2^h}) \otimes \equiv^*(C_3)$.■

Example (4.2):

To findthe rational valued characters table of the group $D_{2^5} \times C_3$ .By proposition (3.17) the rational valued characters table of D_{32} is equal to $\equiv^*(D_{32})$=

Γ-classes	[1]	$[r^{16}]$	$[r^8]$	$[r^4]$	$[r^2]$	$[r]$	$[S]$	$[Sr]$
θ_1	16	-16	0	0	0	0	0	0
θ_2	8	8	-8	0	0	0	0	0
θ_3	4	4	4	-4	0	0	0	0
θ_4	2	2	2	2	-2	0	0	0
θ_5	1	1	1	1	1	-1	-1	1
θ_6	1	1	1	1	1	1	1	1
θ_7	1	1	1	1	1	1	-1	-1
θ_8	1	1	1	1	1	-1	1	-1

Table(4.1)

And by proposition (3.12) the rational valued characters table of C_3 is equale to

$\equiv^*(C_3)$ =

	$[1']$	$[r']$
θ'_1	2	-1
θ'_2	1	1

Table (4.2)

Then $\equiv^*(D_{32}\times C_3)$ =

Γ-Clases	$[1,1']$	$[1,r']$	$[r^{16},1']$	$[r^{16},r']$	$[r^8,1']$	$[r^8,r']$	$[r^4,1']$	$[r^4,r']$	$[r^2,1']$	$[r^2,r']$	$[r,1']$	$[r,r']$	$[S,1']$	$[S,r']$	$[Sr,1']$	$[Sr,r']$
$\theta_{(1.1)}$	32	-16	-32	16	0	0	0	0	0	0	0	0	0	0	0	0
$\theta_{(1.2)}$	16	16	-16	-16	0	0	0	0	0	0	0	0	0	0	0	0
$\theta_{(2.1)}$	16	-8	16	-8	-16	8	0	0	0	0	0	0	0	0	0	0
$\theta_{(2.2)}$	8	8	8	8	-8	-8	0	0	0	0	0	0	0	0	0	0
$\theta_{(3.1)}$	8	-4	8	-4	8	-4	-8	4	0	0	0	0	0	0	0	0
$\theta_{(3.2)}$	4	4	4	4	4	4	-4	-4	0	0	0	0	0	0	0	0
$\theta_{(4.1)}$	4	-2	4	-2	4	-2	4	-2	-4	2	0	0	0	0	0	0
$\theta_{(4.2)}$	2	2	2	2	2	2	2	2	-2	-2	0	0	0	0	0	0
$\theta_{(5.1)}$	2	-1	2	-1	2	-1	2	-1	2	-1	-2	1	-2	1	2	-1
$\theta_{(5.2)}$	1	1	1	1	1	1	1	1	1	1	-1	-1	-1	-1	1	1
$\theta_{(6.1)}$	2	-1	2	-1	2	-1	2	-1	2	-1	2	-1	2	-1	2	-1
$\theta_{(6.2)}$	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
$\theta_{(7.1)}$	2	-1	2	-1	2	-1	2	-1	2	-1	2	-1	-2	1	-2	1
$\theta_{(7.2)}$	1	1	1	1	1	1	1	1	1	1	1	1	-1	-1	-1	-1
$\theta_{(8.1)}$	2	-1	2	-1	2	-1	2	-1	2	-1	2	-1	-2	1	-2	1
$\theta_{(8.2)}$	1	1	1	1	1	1	1	1	1	1	1	1	-1	-1	-1	-1

Table(4.3)

Theorem(4.3):

The cyclic decomposition of K($D_n \times C_3$), when n=2^h is equal to

$$K(D_n \times C_3) = \overset{2}{\underset{i=1}{\oplus}} K(D_n) \overset{h+3}{\underset{i=1}{\oplus}} C_3$$

Proof:

By theorem (3.18)

$D(\equiv^*(D_{2^h})) = diag\{2^{h+1}, -2^{h-1}, -2^{h-2}, \ldots\ldots, -2, -2, -2, -1\}$

And by proposition (3.13)

D(≡∗ (C_3))=diag {3,1}

From theorem (4.1) and proposition (3.7) we have

D(≡*(D_{2^h} ×C_3))= D(≡*(D_{2^h})⊗≡*(C_3))= D(≡*(D_{2^h}))⊗D(≡*(C_3))

$= diag\{2^{h+1}, -2^{h-1}, -2^{h-2}, \ldots\ldots, -2, -2, -2, -1\} \otimes diag\ \{3,1\}$

$=\{\, 3.2^{h+1}, 3(-2^{h-1}), 3(-2^{h-2}), \ldots\ldots, 3(-2), 3(-2), 3(-2), 3(-1),\ 2^{h+1}, -2^{h-1}, -2^{h-2}, \ldots\ldots, -2, -2, -2, -1 \}$

By theorem (3.10), we get

$$K(D_{2^h} \times C_3) = C_{3.2^{h+1}} \oplus C_{3.2^{h-1}} \oplus C_{3.2^{h-2}} \oplus \ldots \oplus C_{3.2} \oplus C_{3.2} \oplus C_{3.2} \oplus C_3$$

$$\oplus C_{2^{h+1}} \oplus C_{2^{h-1}} \oplus C_{2^{h-2}} \oplus \ldots \oplus C_2 \oplus C_2 \oplus C_2 \ .$$

$$= C_{2^{h+1}} \oplus C_{2^{h-1}} \oplus C_{2^{h-2}} \oplus \ldots \oplus C_2 \oplus C_2 \oplus C_2$$

$$\oplus C_{2^{h+1}} \oplus C_{2^{h-1}} \oplus C_{2^{h-2}} \oplus \ldots \oplus C_2 \oplus C_2 \oplus C_2 \overset{h+3}{\underset{i=1}{\oplus}} C_3$$

By theorem (3.14) , we have

$$K(C_{2^{h-1}}) = C_{2^{h-1}} \oplus C_{2^{h-2}} \oplus \ldots \oplus C_2$$

From theorem (3.19) , we have

$$K(D_{2^h}) = K(C_{2^{h-1}}) \oplus C_{2^{h+1}} \oplus C_2 \oplus C_2$$

This implies

$$K(D_{2^h} \times C_3) = \overset{2}{\underset{i=1}{\oplus}} K(D_{2^h}) \overset{h+3}{\underset{i=1}{\oplus}} C_3 \ \square$$

Example(4.4):

To find the cyclic decomposition of K(D_{16}×C_3) and K(D_{32} ×C_3) :

K(D_{16}×C_3)= K(D_{2^4}×C_3)

From theorem (3.4)

$$K(D_{2^4} \times C_3) = \overset{2}{\underset{i=1}{\oplus}} K(D_{2^4}) \overset{h+3}{\underset{i=1}{\oplus}} C_3$$

By theorem (3.19)

K(D_{2^4}) $= K(C_{2^3}) \oplus C_{2^5} \oplus C_2 \oplus C_2$

Then, $K(D_{2^4} \times C_3) = \overset{2}{\underset{i=1}{\oplus}} \left[K(C_{2^3}) \oplus C_{2^5} \oplus C_2 \oplus C_2 \right] \overset{7}{\underset{i=1}{\oplus}} C_3$

$$= \overset{2}{\underset{i=1}{\oplus}} \left[C_{2^3} \oplus C_{2^2} \oplus C_2 \oplus C_{2^5} \oplus C_2 \oplus C_2 \right] \overset{7}{\underset{i=1}{\oplus}} C_3$$

$$= \bigoplus_{i=1}^{2} C_{2^5} \bigoplus_{i=1}^{2} C_{2^3} \bigoplus_{i=1}^{2} C_{2^2} \bigoplus_{i=1}^{6} C_2 \bigoplus_{i=1}^{7} C_3$$

To find $K(D_{32} \times C_3)$ by theorem (4.3)

$$K(D_{32} \times C_3) = K(D_{2^5} \times C_3) = \bigoplus_{i=1}^{2} K(D_{2^5}) \bigoplus_{i=1}^{8} C_3$$

By theorem (3.19)

$$K(D_{32}) = K(D_{2^5}) = K(C_{2^{5-1}}) \oplus C_{2^{5+1}} \oplus C_2 \oplus C_2$$

$$= K(C_{2^4}) \oplus C_{2^6} \oplus C_2 \oplus C_2$$

$$K(D_{32} \times C_3) = \bigoplus_{i=1}^{2} \left[K(C_{2^4}) \oplus C_{2^6} \oplus C_2 \oplus C_2 \right] \bigoplus_{i=1}^{8} C_3$$

Then ,

$$= \bigoplus_{i=1}^{2} \left[C_{2^6} \oplus C_{2^4} \oplus C_{2^3} \oplus C_{2^2} \oplus C_2 \oplus C_2 \oplus C_2 \right] \bigoplus_{i=1}^{8} C_3$$

$$= \bigoplus_{i=1}^{2} C_{32} \bigoplus_{i=1}^{2} C_{16} \bigoplus_{i=1}^{2} C_8 \bigoplus_{i=1}^{2} C_4 \bigoplus_{i=1}^{6} C_2 \bigoplus_{i=1}^{8} C_3 .$$

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