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Optimal Quantitative H_2 Controller Design for Twin Rotor MIMO System

Mustafa K. Khreabet [a*], **Hazem I. Ali** [b]

[a] Control and Systems Engineering Department, University of Technology , Baghdad, Iraq, Mustafa.alnassar7@gmail.com

[b] Control and Systems Engineering Department, University of Technology , Baghdad, Iraq, 60143@uotechnology.edu.iq

*Corresponding author.

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K E Y W O R D S

Twin Rotor MIMO System (TRMS), Robust control, H_2 control, QFT, optimal control, PSO.

A B S T R A C T

In this paper, the H_2 control approach is used for achieving the desired performance and stability of the twin-rotor MIMO system. This system is considered one of the complex multiple inputs of multiple-output systems. The complexity because of the high nonlinearity, significant cross-coupling and parameter uncertainty makes the control of such systems is a very challenging task. The dynamic of the Twin Rotor MIMO System (TRMS) is the same as that in helicopters in many aspects. The Quantitative Feedback Theory (QFT) controller is added to the H_2 control to enhance the control algorithm and to satisfy a more desirable performance. QFT is one of the frequency domain techniques that is used to achieve a desirable robust control in presence of system parameters variation. Therefore, a combination between H_2 control and QFT is presented in this paper to give a new efficient control algorithm. On the other hand, to obtain the optimal values of the controller parameters, Particle Swarm Optimization (PSO) which is one of the powerful optimization methods is used. The results show that the proposed quantitative H_2 control can achieve more desirable performance in comparison to H_2 control especially in attenuating the cross-coupling and eliminating the steady-state error.

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1. Introduction

For controlling the system of twin-rotor there are many strategies considered. Tao et al. [1] in 2010 proposed a parallel distributed fuzzy LQR control for the twin rotor MIMO system. The design procedures of the fuzzy Takagi–Sugeno model of TRMS were presented. Ahmed et al. [2] in 2012 proposed to control the system of the Twin-Rotor System (TRS) by using H_2 and H_∞. The linearized model has been developed. Controlling on the lever bar angular position was the objective. Pradhn et al. [3] in 2013 proposed the decoupling compensation of the designing and implementation for the system of twin-rotor the multiple input and the multiple-output system. A 2 DOF controller (SISO) to achieve decoupling has been proposed. Van et al. [4] in 2014 proposed Multiple Sliding Surface (MSS) Control for a twin MIMO system. MSS controller was proposed for a nonlinear TRMS with mismatched uncertainties. Taimoor et al. [5] in 2015 proposed Sliding and Integral Sliding Mode Control for the twin-rotor system. The nonlinear model for the system has been developed. The integral sliding mode controller technique was better than the sliding mode controller. Pandey et al. [6] in 2015 proposed the use of LQR for optimal controlling of the system of the twin-rotor. The Riccati equation was used to find the optimal control gain matrix. The horizontal movement and also the vertical one of the system are controlled by the PID and the optimal controller. Chithra et al. [7] in 2016 proposed robust optimal sliding mode control (ROSMC) of the system of twin rotor MIMO. An integral action with the Linear Quadratic Regulator was designed and merged with the robust type of sliding mode controller.

One of the control techniques that considered important is H_2 control. It's used to construct a controller to stabilize the system if it is not stable. Also, it is used to achieve desirable performance in sense of signal disturbance, noise, dynamics of an unmodeled system and system perturbation [8]. Also, the Quantitative Feedback Theory (QFT) is considered one of the reliable approaches for handling the uncertainties in the motion control area. It is a frequency domain design approach. It is capable of rejecting input and output disturbances, decreasing steady-state error, and improving the transient response specifications [9].

In this paper, the design of the Quantitative H_2 controller is proposed. In section two the system mathematical model is adopted. The quantitative H_2 controller design is given in section three. In section four, the results and discussion are given.

2. System Mathematical Model

The main components of the Twin Rotor MIMO system are the tail rotor, the main rotor, the beam and also the counterbalance, the two rotors are situated. Figure 1 shows the main components of TRMS. In the horizontal plane, the main rotor is rotating, the rotation angle is named pitch, and in the vertical plane the tail rotor is rotating, the rotation angle is named yaw. These two movements considered as two degrees of freedom (2DOF) on yaw and pitch angles [10]. Control design of a multiple-input system such as the Twin-Rotor system is a very challenging task. This is because of the high non-linearity, significant cross-coupling. Further, the parameters of the system variation produce a system dynamics variation [11].

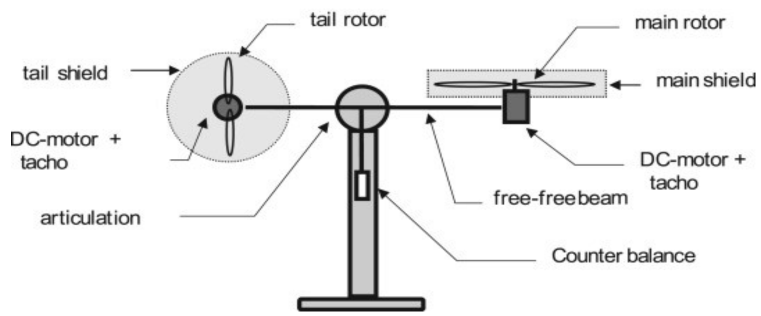


Figure 1: The twin-rotor MIMO system

The TRMS model equations are conducted in Figure 2 gives the model of the Phenomenological. In the natural state, the system is considered as nonlinear that which leads to the position of the rotor, or the current of the rotor is in an argument of a nonlinear function.

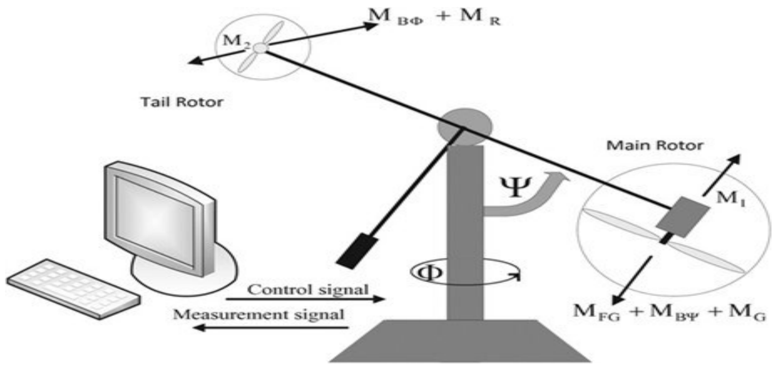


Figure 2: TRMS model

With respect to the model given in Figure 2, the nonlinear mathematical equations of TRMS can be formed in the following. The mathematical equation in a vertical plane is given by [12]:

$$I_1\ddot{\psi} = M_1 - M_{FG} - M_{B\psi} - M_G \tag{1}$$

where

$$M_1 = a_1{\tau_1}^2 + b_1\,\tau_1 \tag{2}$$

$$M_{FG} = M_g \sin(\psi) \tag{3}$$

$$M_{B\psi} = B_{1\psi}\ \dot{\psi} + B_{2\psi}\ \mathrm{Sign}\,(\dot{\psi}) \tag{4}$$

$$M_G = K_{gy}\, M_1\, \dot{\phi}\, \cos(\psi) \tag{5}$$

where M_1 the nonlinear static characteristics, M_{FG} is the gravity momentum, $M_{B\psi}$ the friction forces momentum, M_G is the gyroscopic momentum, ψ is the pitch angle, I_1 is the Pitch inertia moment, a_1 is the parameter of main rotor static characteristic, b_1 is the parameter of main rotor static characteristic, M_g is the gyroscopic forces parameter, $B_{1\psi}$ is the parameter of friction forces moment, $B_{2\psi}$ is the parameter of friction forces moment, K_{gy} is the parameter of gyroscopic forces.

The motor and the electrical control circuit are approximated as a first-order transfer function described by [6]:

$$\tau_1(s) = \left(\frac{K_1}{T_{11}\,S + T_{10}}\right) u_1(s) \tag{6}$$

where K_1 is the main rotor gain coefficient, T_{11} and T_{10} are the main rotor parameters.
The mathematical equation in a horizontal plane is given by [13]:

$$I_2\ddot{\phi} = M_2 - M_{B\phi} - M_R \tag{7}$$

where

$$M_2 = a_2{\tau_2}^2 + b_2\tau_2 \tag{8}$$

$$M_{B\phi} = B_{1\phi}\ \dot{\phi} + B_{2\phi}\ \mathrm{sign}\,(\dot{\phi}) \tag{9}$$

$$M_R = \left(\frac{K_C(T_0\,S+1)}{T_P S+1}\right) M_1 \tag{10}$$

where M_2 the nonlinear static characteristics, $M_{B\phi}$ the friction forces momentum, M_R is the cross-reaction momentum, ϕ is the yaw angle, I_2 is the Yaw inertia moment, a_2 is the parameter of main rotor static characteristic, b_2 is the main rotor static characteristic parameter, $B_{1\phi}$ is the friction forces moment parameter, $B_{2\phi}$ is the friction forces moment parameter, K_C is the cross-reaction gain coefficient, T_0 is the cross-reaction moment parameter andT_P is the cross-reaction moment parameter.

The tail rotor momentum transfer function is given by [14]:

$$\tau_2(s)=(\frac{K_2}{T_{21}\,S+T_{20}})\,u_2(s) \tag{11}$$

where K_2 is the tail rotor gain coefficient, T_{21} and T_{20} are the tail rotor parameters. The model parameters used in Eqs. (1) to (11) are obtained such that the TRMS nonlinear model will be a semi-phenomenological model. The bounds of the control signal are set to (-2.5 to +2.5 volt) [15]. By using the dynamical equations, the state-space model of the linearized plant can be expressed as:

$$\dot{x}=Ax+Bu \tag{12}$$

$$y=Cx+Du \tag{13}$$

$$x=[\psi\ \dot{\psi}\ \phi\ \dot{\phi}\ \tau_1\ \tau_2\,]^T \tag{14}$$

$$u=[u_1\ u_2]^T \tag{15}$$

$$y=[\psi\ \ \phi\,]^T \tag{16}$$

where x is the state vector, u is the input vector and y is the output vector.

Then the final model is given by:

$$\dot{x}(t)=\begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -M_g/I_1 & -B_{1\psi}/I_1 & 0 & 0 & b_1/I_1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -B_{1\phi}/I_2 & -(\frac{K_C T_0 b_1}{T_P})/I_2 & b_2/I_2 \\ 0 & 0 & 0 & 0 & -T_{10}/T_{11} & 0 \\ 0 & 0 & 0 & 0 & 0 & -T_{20}/T_{21} \end{bmatrix}\begin{bmatrix}\psi\\ \dot{\psi}\\ \phi\\ \dot{\phi}\\ \tau_1\\ \tau_2\end{bmatrix}+ \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ K_1/T_{11} & 0 \\ 0 & K_2/T_{21} \end{bmatrix}\begin{bmatrix}u_1\\ u_2\end{bmatrix} \tag{17}$$

$$\begin{bmatrix}\psi\\ \phi\end{bmatrix}=\begin{bmatrix}1 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 1 & 0 & 0 & 0\end{bmatrix}\begin{bmatrix}\psi\\ \dot{\psi}\\ \phi\\ \dot{\phi}\\ \tau_1\\ \tau_2\end{bmatrix}+\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}\begin{bmatrix}u_1\\ u_2\end{bmatrix} \tag{18}$$

The parameters of the system are given in Table 1.

TABLE I: TRMS parameters [16]

Parameter	Value	Unit
I_1	6.12×10^{-2}	Kg.m^2
I_2	2×10^{-2}	Kg.m^2
M_g	0.32	N.m
b_1	9.24×10^{-2}	N/A
b_2	9×10^{-2}	N/A
$B_{1\psi}$	6×10^{-2}	N.m.s/rad
$B_{1\phi}$	6×10^{-3}	N.m.s/rad
K_1	1.1	N/A
K_2	0.8	N/A
T_{11}	1.1	N/A
T_{10}	1	N/A
T_{21}	1	N/A
T_{20}	1	N/A
K_C	-0.2	N/A
T_P	2	N/A
T_0	3.5	N/A

3. CONTROLLER DESIGN

In this section, the design of the quantitative H_2 controller is presented. The design procedure of each part of the proposed controller is given:

I. Controller of H_2 State Feedback

Control of the H_2 control is considered as optimal control. It is used to solve the quadratic cost function in many control problems. The objective of H_2 control is to design a robust and optimal control system. The designed H_2 controller will minimize the H_2 quadratic performance index of the controlled system. Moreover, the H_2 control offers a chance to combine the performance of quadratic with the design criteria of disturbance attenuation [17-19].

Consider Figure 3 and assume that

$$M=\begin{bmatrix} A & B_1 & B_2 \\ C_1 & 0 & D_{12} \\ I & 0 & 0 \end{bmatrix} \tag{19}$$

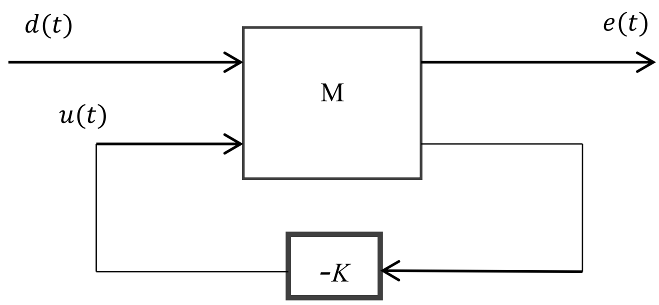


Figure 3: H_2 control structure

The following assumptions are made:

1) *(A, B_1) and (A, B_2) are stabilizable.*

2) *(C_1, A) is detectable.*

From Eq. (19),

$$\dot{x}=A\,x(t)+B_1\,d(t)+B_2\,u(t) \quad (20)$$

$$(t)=C_1x(t)+D_{12}\,u(t) \quad (21)$$

$$y(t)=x(t) \quad (22)$$

where e(t) is the signals to be minimize. Assuming that d(t) is the white noise vector with unit intensity.

The system error due to the input of the white noise for the H_2 norm is:

$$\|T_{ed}\|_{H_2}^2=E(e^T(t)e(t)) \quad (23)$$

where

$$e^T\mathrm{e}=x^TC_1^TC_1x+2x^TC_1^TD_{12}\mathrm{u}+u^TD_{12}^TD_{12}u \quad (24)$$

With Eqs. (20) and (23), the minimization of$\|T_{ed}\|_{H_2}^2$ equal to the solution of stochastic regulation problem settings Q_f=$C_1^TC_1$, N_f=$C_1^TD_{12}$ and R_f=$D_{12}^TD_{12}$

The optimal control action is:

$$u(t)=-K\,x(t) \quad (25)$$

where

$$K=R_f{}^{-1}(B_1{}^TP+N_f{}^T) \quad (26)$$

The cost function to be minimized is:

$$J=\int_{t_0}^{t_f}[x(t)^TQ_fx(t)+2x(t)^TN_fu(t)+u(t)^TR_f\,u(t)]dt \quad (27)$$

The Riccati equation:

$$\left(A-B_2R_f{}^{-1}N_f{}^T\right)^TP+P\left(A-B_2R_f{}^{-1}N_f{}^T\right)-PB_2\,R_f{}^{-1}B_1{}^TP+Q_f-N_f\,R_f{}^{-1}N_f{}^T=0 \quad (28)$$

It should be noted that the gain K is independent of the matrix B_1.

II. H_2 State Feedback Controller with Integral

The state feedback controller design using H_2 control gives one major disadvantage where a steady-state error will be introduced. Therefore, for this problem to be compensated, an integral part can be added to eliminate the system steady-state error [20]. Figure 4 shows the overall block diagram of the controlled system with an integral part.

From Figure 4, we obtain:

$$\dot{x}=Ax+Bu \quad (29)$$

$$y=Cx \quad (30)$$

$$u=-Kx+K_I\,x_h \quad (31)$$

$$\dot{x}_h=r-y=r-Cx \quad (32)$$

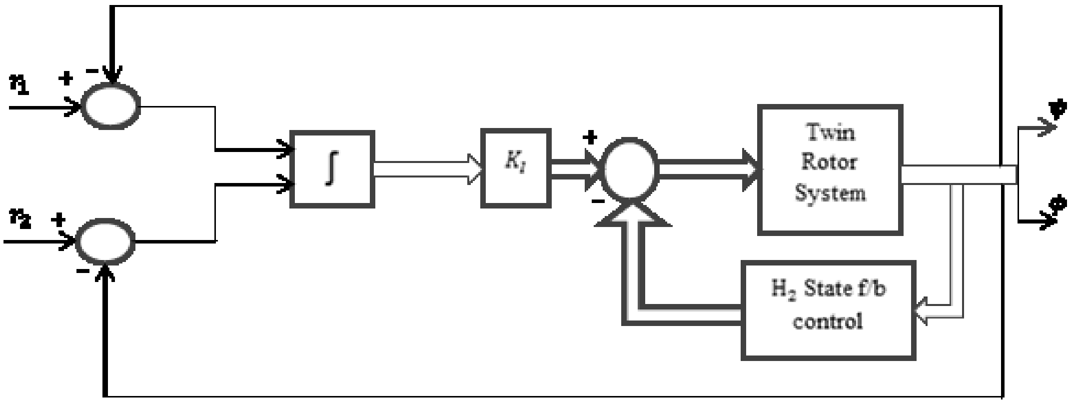


Figure 4: H_2 state feedback control block diagram with integral for twin-rotor system

where x is the vector state, u is the control signal, y is the output signal, x_h is the output of the integrator, K_I is the integral gain and r is the reference input.

The reference input can be assumed to be applied at t=0. For t >0, the system dynamics will be represented by combining Eqs. (29) and (32) to be [21]:

$$\begin{bmatrix}\dot{x}(t)\\ \dot{x}_h(t)\end{bmatrix}=\begin{bmatrix}A & 0\\ -C & 0\end{bmatrix}\begin{bmatrix}x(t)\\ x_h(t)\end{bmatrix}+\begin{bmatrix}B\\ 0\end{bmatrix}u(t)+\begin{bmatrix}0\\ 1\end{bmatrix}r(t) \quad (33)$$

For asymptotic stability of the system with $x(\infty)$, $x_h(\infty)$ and $u(\infty)$ approach constant values, the controlled system will be designed. Then, at steady state $\dot{x}_h(\infty)$=0, and we get $y(\infty) = r$.

Then

$$\begin{bmatrix}\dot{x}(\infty)\\ \dot{x}_h(\infty)\end{bmatrix}=\begin{bmatrix}A & 0\\ -C & 0\end{bmatrix}\begin{bmatrix}x(\infty)\\ x_h(\infty)\end{bmatrix}+\begin{bmatrix}B\\ 0\end{bmatrix}u(\infty)+\begin{bmatrix}0\\ 1\end{bmatrix}r(\infty) \quad (34)$$

Noting that $r(t)$ is a step input, we have $r(\infty) = r(t) = r(constant)$ for t $>$ 0. By subtracting Eq. (33) from Eq. (34), we obtain

$$\begin{bmatrix}\dot{x}(t)-\dot{x}(\infty)\\ \dot{x}_h(t)-\dot{x}_h(\infty)\end{bmatrix}=\begin{bmatrix}A & 0\\ -C & 0\end{bmatrix}\begin{bmatrix}x(t)-x(\infty)\\ x_h(t)-x_h(\infty)\end{bmatrix}+\begin{bmatrix}B\\ 0\end{bmatrix}[u(t)-u(\infty)] \quad (35)$$

Define

$x_e(t)= x(t) - x(\infty)$

$x_{he}(t)=x_h(t) - x_h(\infty)$

$u_e(t)= u(t) - u(\infty)$

Then Eq. (35) can be written as:

$$\begin{bmatrix}\dot{x}_e(t)\\ \dot{x}_{he}(t)\end{bmatrix}=\begin{bmatrix}A & 0\\ -C & 0\end{bmatrix}\begin{bmatrix}x_e(t)\\ x_{he}(t)\end{bmatrix}+\begin{bmatrix}B\\ 0\end{bmatrix}[u_e(t)] \quad (36)$$

where

$$u_e(t)=-Kx_e(t)+K_I x_{he}(t) \quad (37)$$

$e(t)=\begin{bmatrix}x_e(t)\\ x_{he}(t)\end{bmatrix}$

Then Eq. (36) becomes:

$$\dot{e}=\hat{A}e+\hat{B}e \tag{38}$$

where

$\hat{A}=\begin{bmatrix} A & 0 \\ -C & 0 \end{bmatrix}$, $\hat{B}=\begin{bmatrix} B \\ 0 \end{bmatrix}$

and Eq. (37) becomes

$$u_e(t)=-\hat{K}e \tag{39}$$

where

$\hat{K}=\{K \ \ |-K_I\}$

III. Quantitative Feedback Theory (QFT)

One of the frequency domain technique is (QFT) that's means Quantitative feedback theory. It's using the chart of Nichols to have the wanted robust design for the specified region of uncertainty plant, developed by Isaac Horowitz [22]. One of the engineering ways to devoted the practical design of a close loop system is the QFT, the idea is to put a set of linear, time-invariant (LTI) plant instead of the nonlinear plant, by using assumed response of input and output. Figure 5 shows the QFT feedback system, where $G_p(s)$ represents the plant, $G_c(s)$ and $F(s)$ are the controllers to be designed.

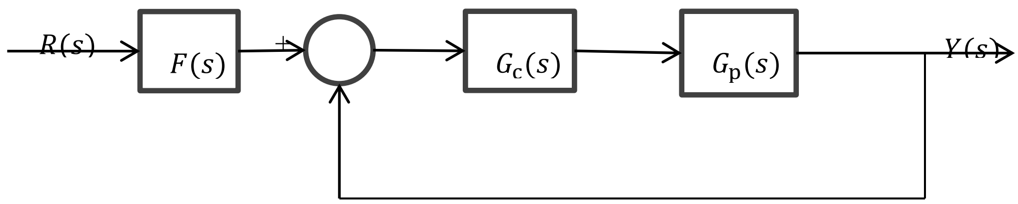


Figure 5: QFT feedback system

The open-loop function is:

$$L(s)=G_c(s)G_p(s) \tag{40}$$

The nominal open-loop function is:

$$L_o(s)=G_c(s)G_{po}(s) \tag{41}$$

The objective in QFT is to synthesize $G_c(s)$ and $F(s)$ that lead to achieving the following specifications [23]:

1) Robust stability margin

$$\left|\frac{G_p(jw)G_c(jw)}{1+G_p(jw)G_c(jw)}\right| \leq M_L \tag{42}$$

where M_L represents a constant, which is set according to the required gain margin and phase margin.

2) Lower gain margin

$$G.M=1+M_L^{-1} \tag{43}$$

3) Lower phase margin

$$P.M=180°-\theta \tag{44}$$

Where

$$\theta=\cos(0.5\mathrm{M}_{\mathrm{L}}^{-1}-1)>0 \tag{45}$$

4) Robust input disturbance rejection performance

$$\left|\frac{G_c(jw)}{1+G_p(jw)G_c(jw)}\right| \leq \mathrm{W}_{d0} \tag{46}$$

5) Robust output disturbance rejection performance

$$\left|\frac{1}{1+G_p(jw)G_c(jw)}\right| \leq \mathrm{W}_{d1} \tag{47}$$

Where
W_{d0} and W_{d1} are performance specifications.

IV. Tuning of Controller Parameters

Can use the PSO as an optimization method that tunes the controller parameters. In 1995 Eberhart and Kennedy developed a technique that uses population-based on the stochastic optimization and they get inspired by the bird flocking behavior or the behavior of fish schooling. According to the next equations of the motion can manipulate the particles [24, 25]:

$$V_i^{n+1}=wV_i^n+c_1rand_1\left(X_{pi}-X_i^n\right)+c_2rand_2(X_g-X_i^n) \tag{48}$$

$$X_i^{n+1}=X_i^n+V_i^{n+1} \tag{49}$$

where V_i^{n+1} is velocity of particle i at loop n + 1, X_i^{n+1} is position of particle i at loop n + 1, c_1, c_2 is cognitive and social parameters, $rand_1$, $rand_2$ is random numbers between 0 and 1 and w is the inertia weight. In this paper the use of PSO to have the controller weighting matrices (Q_f, R_f and N_f) optimal values following the minimization of cost function:

$$\mathrm{J}=\int_0^{t_f} e^2(t)dt \tag{50}$$

where t_f represents the final time, $e(t)$ represents the error of the system which is represented by $e_1(t)$ and $e_2(t)$ for the twin-rotor MIMO system.

V. Optimal Quantitative H_2 Controller

The algorithm of the PSO used to obtain the parameters that satisfy the desired specifications subject to H_2 and QFT constraints. The quantitative H_2 controller block diagram using PSO that proposed is shown in Figure 6.

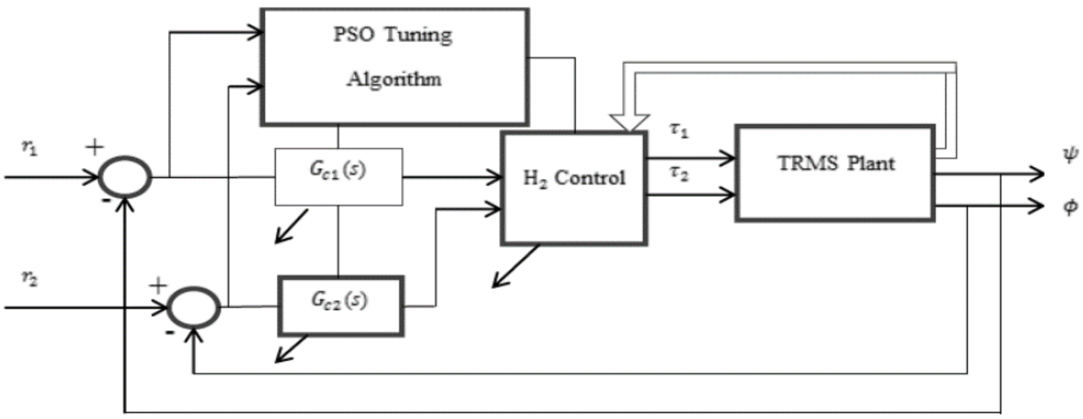


Figure 6: PSO based quantitative H_2 controller block diagram

The objectives of the proposed controller are:

1) To satisfy the desired robustness in stability and performance.

2) To attenuate the coupling effect as small as possible.

To achieve the above mentioned specifications, a suitable cost function is proposed which is expressed by:

$$J=\int_0^{t_f} e^2(t)dt + \left|\frac{G_p(jw)G_c(jw)}{1+G_p(jw)G_c(jw)}\right| + \left|\frac{G_c(jw)}{1+G_p(jw)G_c(jw)}\right| \quad (51)$$

where t_f represents the final time.

The algorithm of the PSO that use to find the optimal elements of weighting functions required for H_2 controller design. Eq. (51) shows the obtaining minimized suggested cost function of the parameters of the controller. The next settings of the PSO used to carry out the suggested controller design:

1. *The members of each individual in PSO algorithm are* q_{f11}, q_{f22}, q_{f33}, q_{f44}, q_{f55}, q_{f66}, n_{f11}, n_{f22}, n_{f32}, n_{f42}, n_{f52}, n_{f62}, r_{f11} *and* r_{f22}.
2. *Population size = 100*
3. *Inertia weight factor = 1.5*
4. c_1=c_2=*2*
5. *Maximum iteration is set to 30*
6. *The number of function evaluations = 3000*

4. Results and Discussions

Figure 7 shows the open-loop time response for the system. It is shown that the system is unstable.

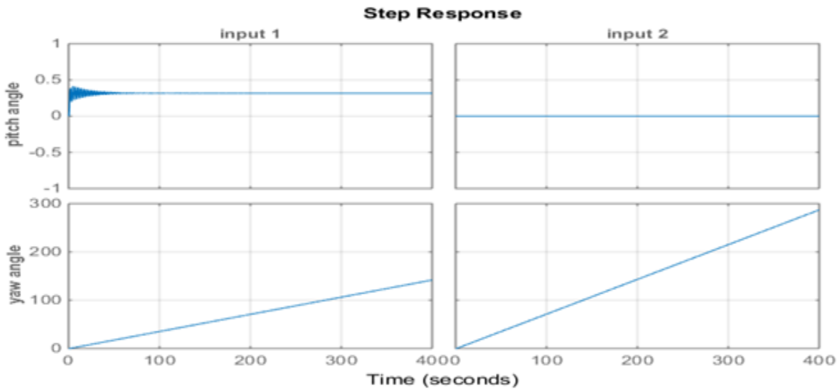


Figure 7: open-loop system time response

The H_2 state feedback controller is used to achieve system performance and stability. The properties of the time response of the system using the H_2 controller is showing in Figure 8. It is shown the system stability has been satisfied and the performance in terms of all rise time, settling time and overshoot was improved with an effect of the coupling and the steady-state error.

The resulting controller gain *K* is:

$$K=10^3 \times \begin{bmatrix} 2.7062 & 0.3890 & -0.0001 & 0.0027 & 0.0317 & -0.0074 \\ 0.0123 & -0.0039 & 0.0336 & 0.0081 & -0.0001 & 0.0083 \end{bmatrix}$$

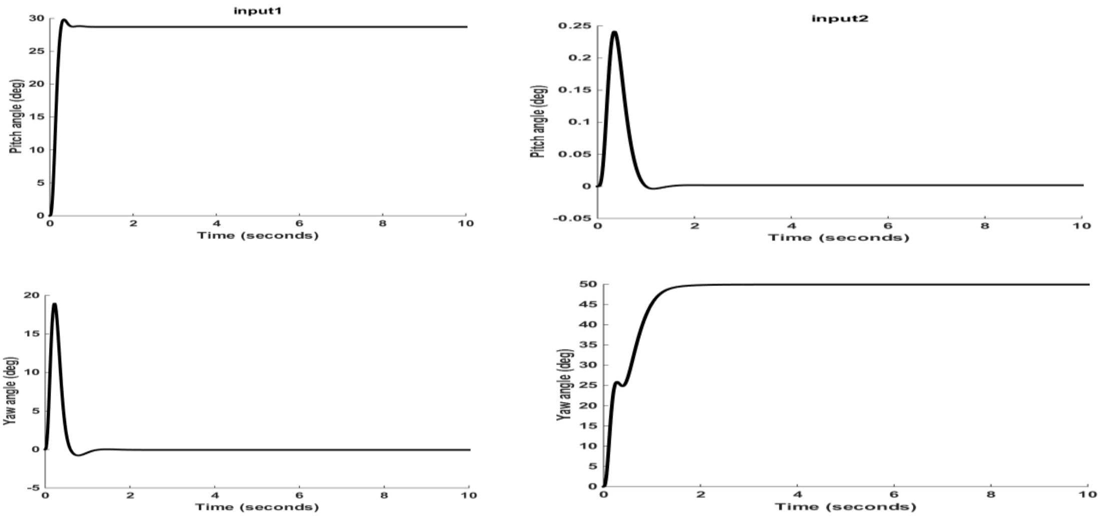


Figure 8: Time properties of the twin-rotor system with H_2 controller

For further improvement in performance, the method of PSO is used to achieve the optimal parameters of the controller of H_2. The time response properties of the system used with optimal H_2 controller is showing in Figure 9. It is shown that a more desirable performance can be obtained using the optimal H_2 controller. Moreover, more attenuation in the coupling effect has been achieved. Also, it is shown that the steady-state error was eliminated.

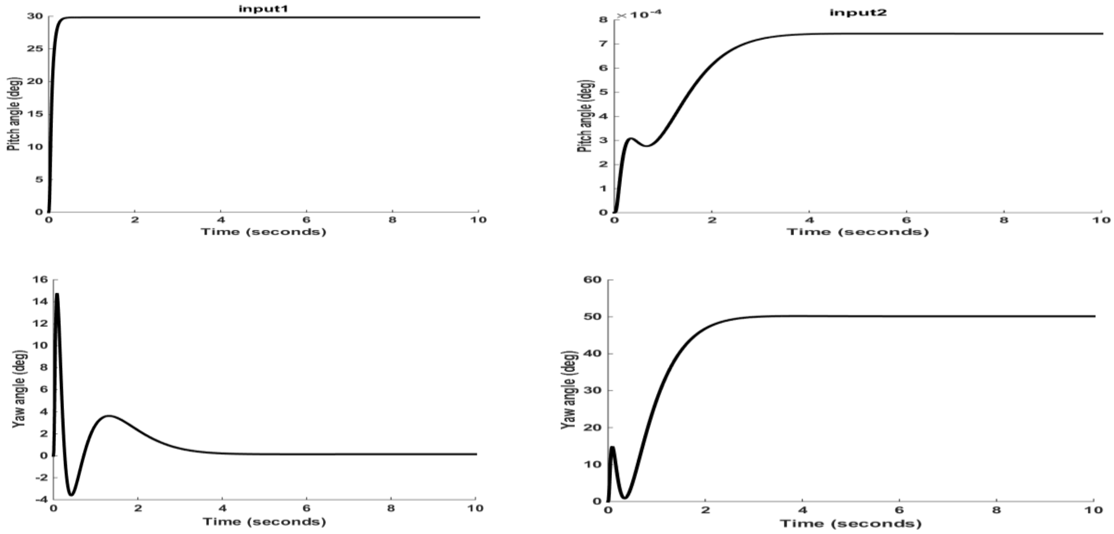


Figure 9: Time properties of the TRMS with optimal H_2 controller

The resulting controller gain K is:

$$K=\begin{bmatrix} 66866 & 5450 & -1 & 1 & 120 & -1 \\ -1.4 & 0.1 & 7 & -0.1 & 3 & 6 \end{bmatrix}$$

Designing the state feedback controller by using only the H_2 controller gave a steady-state error and a large control effort. Therefore, in order to attenuate this problem, integral control is added to the H_2 controller where it eliminates the steady-state error. The time response of H_2 with the integral controller is shown in Figure 10. It is shown that a more attractive performance and a large attenuation in coupling effect were obtained using quantitative H_2 with an integral controller. The output control signal with the minimum control effort is showing in Figure 11.

The resulting H_2 state feedback controller gains K is:

$$K=\begin{bmatrix} 6000 & 5450 & -1 & 1 & 290 & -1 \\ -1.4 & 0.1 & 7 & -0.1 & 3 & 6 \end{bmatrix}$$

The resulting gain of integral is: $K_{I1}=10^4$ and $K_{I2}=4$

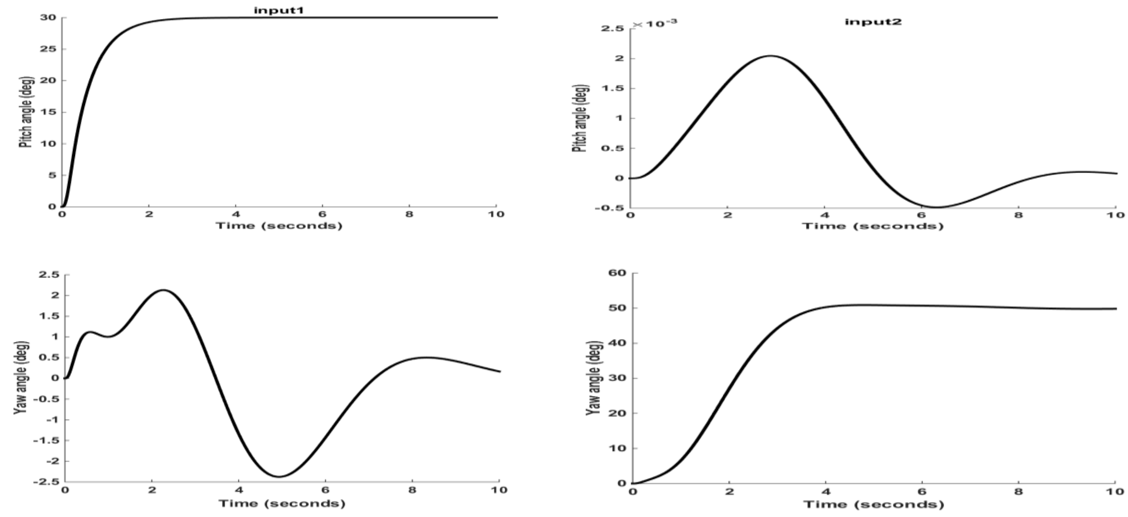


Figure 10: Time response properties of Quantitative H_2 with integral controller

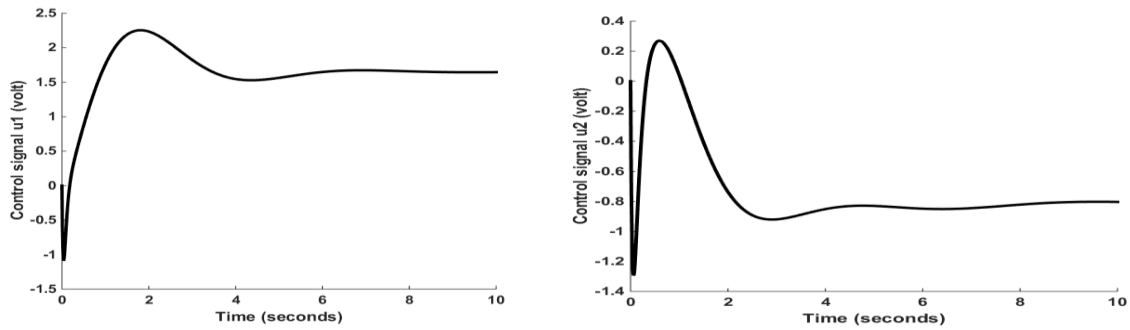


Figure 11: The resulting control signals

To check the robustness of the quantitative H_2 controller, a disturbance of 3 *N* is applied to the system. These disturbances signals are subjected at time (2 – 5) *sec*. it is clear that the proposed controller can effectively reject the disturbance as shown in Figure 12.

On the other hand, the 50% perturbation is used for checking the robustness of the suggested controller. The time response properties of TRMS with uncertain parameters are shown in Figure 13. Figure 13 can check the suggested controller is effective to compensate parameters of the system, change and exhibits performance and robust stability.

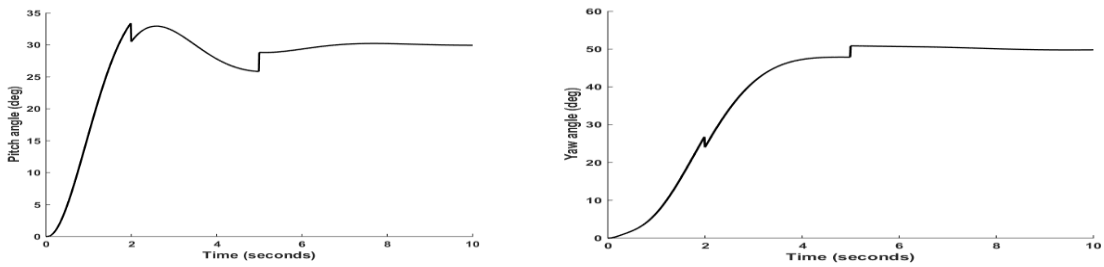
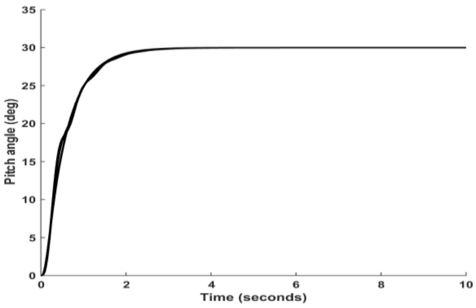


Figure 12: The properties of disturbance rejection for TRMS using quantitative H_2 with integral controller



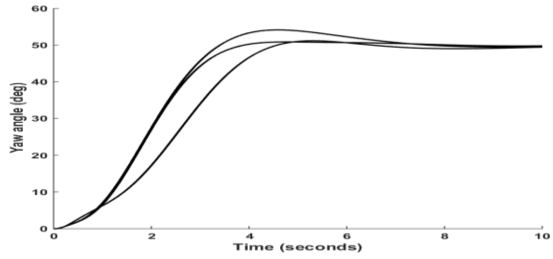


Figure 13: Time properties of the uncertain TRMS with Quantitative H_2 control

5. CONCLUSIONS

In this work, the design of the quantitative H_2 controller for the TRMS was considered. The optimal controller of H_2 designed to make the horizontal and vertical movements stable. The QFT has been added to the H_2 control to enhance the design and obtain an effective controller in eliminating the resulting steady-state error and attenuating the coupling. The results showed that the quantitative H_2 controller can give a desirable Performance in the sense of the steady-state and also transient. In the suggested controller the PSO used to achieve optimal parameters.

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