On-MC-Functions

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Abstract
In this work we introduce a new concept namely MC - function. These are the function $f: X \to Y$ such that the inverse image of every compact subset in Y is closed in X.We proved several theorems about MC- functions and we study the relations of MC- function with other types of functions.

1-Introduction
Let $f: X \to Y$ be a continuous function from a topological space X in to a T_2 - space Y, if $K \subset Y$ is compact , then K is closed , but f is continuous, so $f^{-1}(K)$ is closed .This means that the inverse image of each compact set in Y is closed in X . This motivates the definition of MC- function**.**

2-Preliminaries
In this section we recall the basic definitions needed in this work .

(2-1) Definition [4]
If $f: X \to Y$ is a function , then the Graph of f (G (f)) is the subset $\{(x , y) : x \in X , y = f(x)\}$ of $X \times Y$.

(2-2) Definition [6]
A function f : $X \to Y$ has a closed graph (relative to $X \times Y$) if and only if G(f) is a closed subset of the product space $X \times Y$.

(2-3)Definition [8]
Let D be a non empty set and let $\geq$ be a binary relation on D we say that the relation $\geq$ directs D if and only if the following three conditions hold :
1-For every $a \in D$, $a \geq a$.
2- If $a \geq b$ and $b \geq c$ then $a \geq c$.
3-For each $a, b \in D$ $\exists$ $c \in D$ $\ni$ $c \geq a$ and $c \geq b$.
The pair $(D, \geq)$ is called a directed set.

(2- 4)Definition [8]
A net in the space X is a function $f: D \to X$ where $(D, \geq)$ is a directed set .

(2-5) Definition [7]
We will say that a function $f: X \to Y$ has a subclosed graph if for each $x \in X$ and net $\{x_\alpha\}$ in X- {x} with $x_\alpha \to x$ and net $\{y_\alpha\}$ in Y , with $y_\alpha = f\{x_\alpha\}$ for each α and $y_\alpha \to y$ in Y , we have $y = f(x)$.

(2-6) Definition [5]
A space (X , T) is said to be a T_2 - space if and only if given any two distinct points x and y of X , there are open sets U and V such that $x \in U$, $y \in V$, and $U \cap V = \varphi$. A T_2 - space is often called a Hausdorff space.

(2-7) Definition [5]
A space (X , T) is said to be compact if and only if given any open cover $\{U_i\}$, $i \in I$, of X , there is a finite subcover of $\{U_i\}$, $i \in I$. Suppose (X , T) is any space and $A \subset X$, A is said to be compact if and only if every open cover of A has a finite subcover.

(2-8) Theorem [3]
Let X and Y be topological spaces . A function $f: X \to Y$ is said to be continuous if and only if every closed set K of Y, the set $f^{-1}(K)$ is closed in X .

3 - MC - Functions
In this section we introduce a new concept, namely MC-function defined as follows:

(3-1) Definition
Let $f: X \to Y$ be a function we say that f is an MC-function if and only if for each compact $K \subset Y$ then $f^{-1}(K)$ is a closed subset in X . If $f: X \to Y$ also continuous then we say that f is an MC-mapping .
We will need the following fact (see[2]).

(3-2) Fact [2]
Let $f: X \to Y$ be a function from a space X into Y , then f has closed graph if and only if f has a subclosed graph and closed point image .

(3-3) Theorem
If $f: X \to Y$ is a function with subclosed graph then f is an MC-function .

Proof :
Let K be a compact subset of Y .We must prove that $f^{-1}(K)$ is a closed subset of X .Let a be a limit point of $f^{-1}(K)$.There are nets $\{x_\alpha\}$in X-{a} and $\{y_\alpha\}$in K with $x_\alpha \to a$ and $y_\alpha = f(x_\alpha)$ for each α . Some subnet $\{y_{\alpha n}\}$of $\{y_\alpha\}$ converges to some $y \in K$, this gives $y = f(a)$, since f has subclosed graph so $a \in f^{-1}(K)$.Thus , $f^{-1}(K)$ is closed in X .

(3-4) Corollary
If $f: X \to Y$ is a function with a closed graph then f is MC-function. If $f: X \to Y$ is an MC-function, the G(f) is not necessarily closed.
The following theorem shows that if $f: X \to Y$ is an MC-function from a space X into a locally compact T_2 - space Y, then G(f) will be closed.

(3-5) Theorem
Let $f: X \to Y$ be an MC-function from a space X into a locally compact T_2 - space Y, then G(f) will be closed .

Proof :
Suppose $(x , y) \notin G(f)$, then $y \neq f(x)$ and so there exists disjoint open sets U_1, U_2 with $y \in U_1$ and $f(x) \in U_2$, further there is a compact neighborhood W of y such that $W \subset U_1$, then

$f^{-1}(W)$ is closed in X and $x \notin f^{-1}(W)$, thus there is an open set V such that $x \in V$ and V $\cap f^{-1}(W) = \varphi$. Hence V×W is neighborhood of (x , y) which misses G(f) and so G(f) is a closed.

(3-6) Proposition

If f :X→Y is a continuous function from a space X into a T_2 - space Y then f is an MC-mapping .

Proof :

Let K be a compact subset of Y, since Y is a T_2-space then K is Closed in Y{A compact subset of a Hausdorff space is a closed.[6]} since f is continuous , then $f^{-1}(K)$ is closed in X . Then f is an MC-mapping .

(3-7) Example

Let f :(R ,T_u)→(R ,T_u) be a function defined as follows :

$f(x) = x^2$, then f is an MC-mapping.

Before we state our next theorem , we recall the definition of a compact function.

(3-8) Definition [1]

Let f :X→Y be a function, we say that f is a compact function if and only if the inverse image of each compact set K in Y is compact in X.

(3-9) Proposition

If f : X→Y is a compact function from a T_2 -space X into a space Y, then f is an MC-function.

Proof :

Let K be a compact subset in Y, since f is a compact function. Then $f^{-1}(K)$ is compact of X .Since X is T_2-space then $f^{-1}(K)$ is closed of X. Then f is an MC-function.

(3-10) Proposition

If f : X→Y is an MC-function and W$\subset$ X , then $f|$ W : W→ Y is an MC-function.

Proof :

Let g = $f|$ W : W→ Y and let K be a compact subset of Y, Since f is an MC-function , then $f^{-1}(K)$ is a closed subset of X .Now $g^{-1}(K) = f^{-1}(K) \cap W$, then $g^{-1}(K)$ is a closed subset of W. Hence g = $f|$ W: W→ Y is an MC-function.

(3-11) Proposition

Let f :X→Y be a continuous function from a T_2-space X into Y, and let g : Y→ Z be an MC-function. Then $g \circ f$ is an MC-function.

Proof :

Let K be a compact subset of Z .Since g is MC-function then, $g^{-1}(K)$ is a closed subset in Y . Since f is continuous then , $f^{-1}(g^{-1}(K))$ is closed of X , i.e $(g \circ f)^{-1}(K)$ is a closed subset of X, then $g \circ f$ is an MC-function.

(3-12) Proposition

Let f : X→Y be an MC-function from a space X into a compact space Y and let g : Y→ Z be an MC-function, then $g \circ f$ is an MC-function.

Proof :

Let K be a compact subset of Z , since g is an MC-function then $g^{-1}(K)$ is a closed subset of Y. Since Y is a compact, then $g^{-1}(K)$ is a compact .Since f is an MC-function, then $f^{-1}(g^{-1}(K)) = (g \circ f)^{-1}(K)$ is a closed subset of X, then $g \circ f$ is an MC-function.

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دوال – MC

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الملخص

قدمنا في هذا البحث نوعاً جديداً من الدوال أسميناه الدالة – MC حيث f :X→Y بحيث أن الصورة العكسية $f^{-1}(K)$ تكون مغلقة في X لكل K حيث K متراصة في Y .

وقد برهنا عدة مبرهنات عن الدالة – MC وقمنا بدراسة العلاقة بين هذه الدالة وبعض أنواع الدوال الأخرى .