

On Artin Character Table of The Group $Q_{2m} \times C_2$ When m is a Prim Number

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Abstract

The main purpose of this thesis Result is to determine the of The Artin Characters Table of The Group $(Q_{2m} \times C_2)$ When m is a prim Number.

And we prove that the Artin Characters Table of The Group $(Q_{2m} \times C_2)$ When m is a prim Number to explain by using the examples.

الخلاصة

الغرض الرئيسي من هذا البحث هو تحديد جدول شواخص أرتن للزمرة $(Q_{2m} \times C_2)$ عندما m عدد أولي .
وبرهنا ايجاد جدول شواخص أرتن للزمرة $(Q_{2m} \times C_2)$ عندما m عدد أولي ووضحناه عن طريق الأمثلة .

Introduction

Let Q_{2m} be Quaternion Group Q_{2m} of order $4m$, and let $(Q_{2m} \times C_2)$ the group of order $8m$. In 1995, N. R. Mahamood [5] studied The Cyclic Decomposition of the Factor Group

$cf(Q_{2m}, Z)/\bar{R}(Q_{2m})$. In 2000, H.R. Yassien [2] studied the Artin Cokernel of Finite Groups . In 2008, A.H.Abdul-Munem [1] studied the Artin Cokernel Of The Quaternion group Q_{2m} When m is an Odd number. In 2009, J.R.Nime, [3] The Cyclic Decomposition of The Factor

Group $cf(Q_{2m} \times C_2, Z)/\bar{R}(Q_{2m} \times C_2)$ When m is an Odd Number.

In this Result we find the general Artin Characters table of the group $(Q_{2m} \times C_2)$ when m is a prim number.

1.The Quaternion Group Q_{2m}

The Generalized Quaternion Group $Q_{2m}(1.1):[5]$

For each positive integer m, the generalized quaternion group Q_{2m} of order $4m$ with two generators x and y satisfies

$$x^m y^2, yxy^{-1} = x^{-1} \text{ and } yx^m y^{-1} = x^{-m} \text{ which implies } y^4 = x^{2m} = 1$$

Any element $q \in Q_{2m}$ can be expressed uniquely in the form

$$q = x^h y^k, 0 \leq h \leq 2m-1, k = 0, 1$$

In general, it can be written as : $Q_{2m} = \{x^h y^k, 0 \leq h \leq 2m-1, k = 0, 1\}$

which has the following properties $\{x^{2m} = y^4 = 1, yx^m y^{-1} = x^{-m}\}$

Example(1.2)

$$Q_6 = \{1, x, x^2, x^3, x^4, x^5, y, xy, x^2y, x^3y, x^4y, x^5y\}$$

$$x^3 = y^2; yxy^{-1} = x^{-1}; x^6 = y^4 = 1$$

Theorem(1.3): [2]

Let H be a cyclic subgroup of G and $h_1, h_2, \dots, h_m$ are chosen representatives for Γ -conjugate classes of H contained in $CL(g)$, then:

$$\Phi(g) = \begin{cases} \frac{|C_G(g)|}{|C_H(g)|} \sum_{i=1}^m \phi(h_i) & \text{if } h_i \in H \cap CL(g) \\ 0 & \text{if } H \cap CL(g) = \emptyset \end{cases}$$

Definition(1.4): [6]

Let G be a finite group, every character of G induced from the principal character of a cyclic subgroup of G is called **Artin's character of G** .

Proposition(1.5): [4]

The number of all distinct Artin characters on a group G is equal to the number of Γ -classes on G . Furthermore, Artin characters are constant on each Γ -class.

Definition(1.6):[4]

Artin characters of finite group can be displayed in a table called Artin characters table of G which is denoted by $Ar(G)$, The first row is the Γ -conjugate classes, the second row is the number of elements in each conjugate class, the third row is the size of $|C_G(CL_\alpha)|$ the centralized and the other rows contains the values of Artin characters.

Theorem(1.7) :[1]

The Artin's characters table of the Quaternion group Q_{2m} when m is a prime number is given as follows:

$Ar(Q_{2m})$

	$\Gamma - \text{Classes of } C_{2p}$				
$\Gamma - \text{Classes}$	$[1]$	$[x^p]$	$[x^2]$	$[x]$	$[y]$
$ CL_\alpha $	1	1	2	2	2p
$ C_{Q_{2m}}(CL_\alpha) $	4p	4p	2p	2p	2
Φ_1	4p	0	0	0	0
Φ_2	2p	2p	0	0	0
Φ_3	4	0	4	0	0
Φ_4	2	2	2	2	0
Φ_5	p	p	0	0	1

Example(1.8):

To construct $Ar(Q_{26})$ by using theorem(1.7):

	$\Gamma - \text{Classes of } C_{26}$				
$\Gamma - \text{Classes}$	$[I]$	$[x^{13}]$	$[x^2]$	$[x]$	$[y]$
$ CL_\alpha $	1	1	2	2	26
$ C_{Q_{26}}(CL_\alpha) $	52	52	26	26	2
Φ_1	52	0	0	0	0
Φ_2	26	26	0	0	0
Φ_3	4	0	4	0	0
Φ_4	2	2	2	2	0
Φ_5	13	13	0	0	1

Definition(1.9): [3]

The group $(Q_{2m} \times C_2)$ is the direct product group Q_{2m} and C_2 , where C_2 is a cyclic group of the order 2 consisting of elements $\{e, z\}$.

The order of the group $|Q_{2m} \times C_2| = |Q_{2m}| \cdot |C_2| = 4m \cdot 2 = 8m$.

Example(1.10):

$$Q_6 \times C_2 = \{(I, e), (x, e), (x^2, e), (x^3, e), (x^4, e), (x^5, e), (y, e), (xy, e), (x^2y, e), (x^3y, e), (x^4y, e), (x^5y, e), (I, z), (x, z), (x^2, z), (x^3, z), (x^4, z), (x^5, z), (y, z), (xy, z), (x^2y, z), (x^3y, z), (x^4y, z), (x^5y, z)\}$$

2.The Main Result

Theorem (2.1):

The Artin's characters table of the group $(Q_{2m} \times C_2)$ when m is a prime number is given as Follows:

$$\text{Ar}(Q_{2p} \times C_2) =$$

Γ -Classes	$[I, e]$	$[x^p, e]$	$[x^2, e]$	$[x, e]$	$[y, e]$	$[I, z]$	$[x^p, z]$	$[x^2, 1]$	$[x, z]$	$[y, z]$
$ CL_\alpha $	1	1	2	2	2p	1	1	2	2	2p
$ C_{Q_{2p} \times C_2}(CL_\alpha) $	8p	8p	4p	4p	4	8p	8p	4p	4p	4
$\Phi_{(1,1)}$	$2Ar(Q_{2p})$					O				
$\Phi_{(2,1)}$										
$\Phi_{(3,1)}$										
$\Phi_{(4,1)}$										
$\Phi_{(5,1)}$										
$\Phi_{(1,2)}$	$Ar(Q_{2p})$					$Ar(Q_{2p})$				
$\Phi_{(2,2)}$										
$\Phi_{(3,2)}$										
$\Phi_{(4,2)}$										
$\Phi_{(5,2)}$										

Proof :-

$$Q_{2m} = \{ \langle x, y \rangle : x^h y^k ; 0 \leq h \leq 2m-1, 0 \leq k \leq 1 \}, C_2 = \{e, z\}$$

$H_j, 1 \leq j \leq 10$ is cyclic subgroups of $(Q_{2p} \times C_2)$,to find $\text{Ar}(Q_{2p} \times C_2)$ by theorem (1.3):

$$\Phi(g) = \begin{cases} \frac{|C_{Q_{2m} \times C_2}(g)|}{|C_{H_j}(g)|} \sum_{i=1}^m \phi(h_i) & \text{if } h_i \in H \cap CL(g) \\ 0 & \text{if } H \cap CL(g) = \emptyset \end{cases}$$

There are ten cyclic subgroup of $(Q_{2p} \times C_2)$:

$$H_1 = \langle (I, e) \rangle, H_2 = \langle (x^p, e) \rangle, H_3 = \langle (x^2, e) \rangle, H_4 = \langle (x, e) \rangle, H_5 = \langle (y, e) \rangle,$$

$$H_6 = \langle (I, z) \rangle, H_7 = \langle (x^p, z) \rangle, H_8 = \langle (x^2, z) \rangle, H_9 = \langle (x, z) \rangle, H_{10} = \langle (y, z) \rangle$$

$g \in H_j ; g = (q, e); q \in Q_{2m}, e \in C_2, e$ is identity of $C_2, 1 \leq j \leq 5$

$$H_1 = \langle (I, e) \rangle = \{(I, e)\}$$

$$\Phi_{(1,1)}(g) = \frac{|C_{Q_{2p} \times C_2}(g)|}{|C_H(g)|} \phi(g) = \frac{8p}{|C_H(g)|} \cdot 1 = 2 \frac{4p}{|C_{(I)}(I)|} \cdot 1 = 2 \frac{|C_{Q_{2p}} \langle I \rangle|}{|C_{(I)}(I)|} \phi(I) = 2 \cdot 4p = 2 \cdot \Phi_1(I)$$

$$\Phi_{(1,1)}(g) = 0 = 2 \cdot \Phi_1(q); q \neq I \quad \text{for all } g \notin H_1 \quad (\text{since } H_1 \cap CL(g) = \emptyset)$$

[since ϕ is the principal character of $Q_{2p} \times C_2$ and $|H| = |\langle I \rangle|$]

$$\text{if } H_2 = \langle (x^p, e) \rangle = \{(I, e), (x^p, e)\}, g \in H_2$$

$$\Phi_{(2,1)}(g) = \frac{|C_{Q2p \times C_2}(g)|}{|C_H(g)|} \phi(g) = \frac{8p}{|C_H(g)|} \cdot 1 = 2 \frac{4p}{|C_{\langle x^p \rangle}(x^p)|} \cdot 1 = 2 \frac{|C_{Q2p} \langle x^p \rangle|}{|C_{\langle x^p \rangle}(x^p)|} \phi(x^p)$$

$$= 2 \cdot 2p = 2 \cdot \Phi_2(x^p) = 2 \cdot \Phi_2(I)$$

$$\Phi_{(2,1)}(g) = 0 = 2 \cdot \Phi_2(q), q \notin \{I, x^p\} \quad \text{for all } g \notin H_2 \quad (\text{since } H_2 \cap CL(g) = \emptyset)$$

$$H_3 = \langle (x^2, e) \rangle = \{(I, e), (x^2, e), (x^4, e), \dots, (x^{2p-2}, e)\}$$

$$\text{If } g = (I, e)$$

$$\Phi_{(3,1)}(g) = \frac{|C_{Q2p \times C_2}(g)|}{|C_H(g)|} \phi(g) = \frac{8p}{|C_H(g)|} \cdot 1 = 2 \frac{4p}{|C_{\langle x^2 \rangle}(I)|} \cdot 1 = 2 \frac{|C_{Q2p} \langle I \rangle|}{|C_{\langle x^2 \rangle}(I)|} \phi(I) = 2 \cdot 4 = 2 \cdot \Phi_3(I)$$

$$\text{if } g \in H_3; H_3 \cap CL(g) = \{g, g^{-1}\}$$

$$\Phi_{(3,1)}(g) = \frac{|C_{Q2p \times C_2}(g)|}{|C_H(g)|} (\phi(g) + \phi(g^{-1})) = \frac{4p}{|C_H(g)|} \cdot (1+1) = 2 \frac{2p}{|C_{\langle x^2 \rangle}(x^2)|} \cdot (1+1)$$

$$= 2 \frac{|C_{Q2p} \langle x^2 \rangle|}{|C_{\langle x^2 \rangle}(x^2)|} (1+1) = 2 \cdot 4 = \Phi_3(x^2)$$

$$\Phi_{(3,1)}(g) = 0 = 2 \cdot \Phi_3(q), q \notin \{I, x^2, x^4, \dots, x^{2p-2}\} \quad \text{for all } g \notin H_3 \quad (\text{since } H_3 \cap CL(g) = \emptyset)$$

$$H_4 = \langle (x, e) \rangle = \{(I, e), (x, e), (x^2, e), \dots, (x^{2p-1}, e)\}$$

$$\text{if } g = (I, e) \quad \text{or } g = (x^p, e)$$

$$\Phi_{(4,1)}(g) = \frac{|C_{Q2p \times C_2}(g)|}{|C_H(g)|} \phi(g) = \frac{8p}{|C_H(g)|} \cdot 1 = 2 \frac{4p}{|C_{\langle x \rangle}(I)|} \cdot 1 = 2 \frac{|C_{Q2p} \langle I \rangle|}{|C_{\langle x \rangle}(I)|} \phi(I)$$

$$= 2 \cdot 2 = 2 \cdot \Phi_4(I) = 2 \cdot \Phi_4(x^p)$$

$$\text{if } g \in H_4; H_4 \cap CL(g) = \{g, g^{-1}\}$$

$$\Phi_{(4,1)}(g) = \frac{|C_{Q2p \times C_2}(g)|}{|C_H(g)|} (\phi(g) + \phi(g^{-1})) = \frac{4p}{|C_H(g)|} \cdot (1+1) = 2 \frac{2p}{|C_{\langle x \rangle}(x)|} \cdot (1+1)$$

$$= 2 \frac{|C_{Q2p} \langle x \rangle|}{|C_{\langle x \rangle}(x)|} (1+1) = 2 \cdot 2 = 2 \cdot \Phi_4(x)$$

$$\Phi_{(4,1)}(g) = 0 = 2 \cdot \Phi_4(q), q \notin \{I, x, x^2, \dots, x^{2p-1}\} \quad \text{for all } g \notin H_4 \quad (\text{since } H_4 \cap CL(g) = \emptyset)$$

$$\text{If } H_5 = \langle (y, e) \rangle = \{(I, e), (y, e), (y^2, e), (y^3, e)\}$$

$$\Phi_{(5,1)}(g) = \frac{|C_{Q_{2p} \times C_2}(g)|}{|C_H(g)|} \phi(g) = \frac{8p}{|C_H(g)|} \cdot 1 = 2 \frac{4p}{|C_{\langle y \rangle}(I)|} \cdot 1 = 2 \frac{|C_{Q_{2p}} \langle I \rangle|}{|C_{\langle y \rangle}(I)|} \phi(I) = 2 \cdot p = 2 \cdot \Phi_5(I) \text{ if } g=(I,e)$$

$$\Phi_{(5,1)}(g) = \frac{|C_{Q_{2p} \times C_2}(g)|}{|C_H(g)|} \phi(g) = \frac{8p}{|C_H(g)|} \cdot 1 = 2 \frac{|C_{Q_{2p}} \langle y^2 \rangle|}{|C_{\langle y \rangle}(y^2)|} \phi(y^2) = 2 \cdot P = 2 \cdot \Phi_5(x^p) \text{ if } g=(y^2,e) = (x^p,e)$$

$$\Phi_{(5,1)}(g) = 0 = 2 \cdot \Phi_5(q), q \notin \{I, y, y^2, y^3\} \quad \text{for all } g \notin H_5 \quad (\text{since } H_5 \cap CL(g) = \emptyset)$$

$$\text{If } g=(y,e) \text{ or } g=(y^3,e) \quad (\text{since } H_5 \cap CL(g) = \{g, g^{-1}\})$$

$$\Phi_{(5,1)}(g) = \frac{|C_{Q_{2p} \times C_2}(g)|}{|C_H(g)|} (\phi(g) + \phi(g^{-1})) = \frac{4}{|C_H(g)|} \cdot (1+1) = 2 \frac{2}{4} \cdot (1+1) = 2 = 2 \cdot \Phi_5(y) \quad \phi(g) = \phi(g^{-1}) = 1$$

$$g \in H_j : g=(q,z), q \in Q_{2m}, z \in C_2, 6 \leq j \leq 10$$

$$H_6 = \langle (I, z) \rangle = \{(I, e), (I, z)\}$$

$$\Phi_{(1,2)}(I, e) = \Phi_{(1,2)}(I, z) = \frac{|C_{Q_{2p} \times C_2}(g)|}{|C_H(g)|} \phi(g) = \frac{8p}{|C_H(g)|} \cdot 1 = \frac{8p}{|C_{\langle x \rangle}(I)|} \cdot 1 = \frac{2|C_{Q_{2p}} \langle I \rangle|}{2|C_{\langle x \rangle}(I)|} \phi(I) = \Phi_1(I)$$

$$\Phi_{(1,2)}(g) = 0 = 2 \cdot \Phi_1(q); q \neq I \quad \text{for all } g \notin H_6 \quad (\text{since } H_6 \cap CL(g) = \emptyset)$$

[since ϕ is the principal character of $Q_{2p} \times C_2$]

$$H_7 = \langle (x^p, e) \rangle = \{(I, e), (x^p, e), (I, z), (x^p, z)\}$$

$$\text{If } g \in H_7$$

$$\Phi_{(2,2)}(g) = \frac{|C_{Q_{2p} \times C_2}(g)|}{|C_H(g)|} \phi(g) = \frac{8p}{|C_{\langle x \rangle}(x^p)|} \cdot 1 = \frac{2|C_{Q_{2p}} \langle x^p \rangle|}{2|C_{\langle x \rangle}(x^p)|} \phi(x^p) = 2p = \Phi_2(x^p) = \Phi_2(I)$$

$$\Phi_{(2,2)}(g) = 0 = \Phi_2(q), q \notin \{I, x^p\} \quad \text{for all } g \notin H_7 \quad (\text{since } H_7 \cap CL(g) = \emptyset)$$

$$H_8 = \langle (x^2, z) \rangle = \{(I, e), (x^2, e), (x^4, e) \cdots (x^{2p-2}, e), (I, z), (x^2, z), (x^4, z), \cdots (x^{2p-2}, z)\}$$

$$\text{if } g=(I,e) \text{ or } g=(I,z)$$

$$\Phi_{(3,2)}(g) = \frac{|C_{Q_{2p} \times C_2}(g)|}{|C_H(g)|} \phi(g) = \frac{8p}{|C_H(g)|} \cdot 1 = \frac{2|C_{Q_{2p}} \langle I \rangle|}{2|C_{\langle x^2 \rangle}(I)|} \phi(I) = 4 = \Phi_3(I)$$

$$\text{if } g \in H_8; H_8 \cap CL(g) = \{g, g^{-1}\}$$

$$\Phi_{(3,2)}(g) = \frac{|C_{Q_{2p} \times C_2}(g)|}{|C_H(g)|} (\phi(g) + \phi(g^{-1})) = \frac{4p}{|C_H(g)|} \cdot (1+1) = \frac{2|C_{Q_{2p}} \langle x^2 \rangle|}{2|C_{\langle x^2 \rangle}(x^2)|} (1+1) = 4 = \Phi_3(x^2)$$

$$\Phi_{(3,2)}(g) = 0 = \Phi_3(q), q \notin \{I, x^2, x^4 \cdots x^{2p-2}\} \quad \text{for all } g \notin H_8 \quad (\text{since } H_8 \cap CL(g) = \emptyset)$$

$$H_9 = \langle (x, z) \rangle = \{(I, e), (x, e), (x^2, e) \cdots (x^{2p-1}, e), (I, z), (x, z), (x^2, z) \cdots (x^{2p-1}, z)\}$$

$$\text{if } g=(I,e) \text{ or } g=(I,z)$$

$$\Phi_{(4,2)}(g) = \frac{|C_{Q_{2p} \times C_2}(g)|}{|C_H(g)|} \phi(g) = \frac{8p}{|C_H(g)|} \cdot 1 = \frac{2|C_{Q_{2p}} \langle I \rangle|}{2|C_{\langle x \rangle}(I)|} \phi(I) = 2 = \Phi_4(I)$$

if $g=(x^p, e)$ or $g=(x^p, z)$

$$\Phi_{(4,2)}(g) = \frac{|C_{Q_{2p} \times C_2}(g)|}{|C_H(g)|} \phi(g) = \frac{8p}{|C_H(g)|} \cdot 1 = \frac{2|C_{Q_{2p}} \langle x^p \rangle|}{2|C_{\langle x^p \rangle}(x^p)|} \phi(x^p) = 2 = \Phi_4(x^p)$$

if $g \in H_4; H_4 \cap CL(g) = \{g, g^{-1}\}$

$$\Phi_{(4,2)}(g) = \frac{|C_{Q_{2p} \times C_2}(g)|}{|C_H(g)|} (\phi(g) + \phi(g^{-1})) = \frac{4p}{|C_H(g)|} \cdot (1+1) = \frac{2|C_{Q_{2m}} \langle x \rangle|}{2|C_{\langle x \rangle}(x)|} \phi(x) = 2 = \Phi_4(x)$$

$$\Phi_{(4,2)}(g) = 0 = \Phi_4(q), q \notin \{I, x, \dots, x^{2p-1}\} \quad \text{for all } g \notin H_9 \quad (\text{since } H_9 \cap CL(g) = \emptyset)$$

$$H_{10} = \langle (y, z) \rangle = \{(I, e), (y, e), (y^2, e), (y^3, e), (I, z), (y, z), (y^2, z), (y^3, z)\}$$

$$\Phi_{(5,2)}(g) = \frac{|C_{Q_{2p} \times C_2}(g)|}{|C_H(g)|} \phi(g) = \frac{8p}{|C_H(g)|} \cdot 1 = \frac{2|C_{Q_{2p}} \langle I \rangle|}{2|C_{\langle y \rangle}(I)|} \phi(I) = p = \Phi_5(I) \quad \text{if } g=(I, e) \text{ or } g=(I, z)$$

$$\Phi_{(5,2)}(g) = \frac{|C_{Q_{2p} \times C_2}(g)|}{|C_H(g)|} \phi(g) = \frac{8p}{|C_H(g)|} \cdot 1 = \frac{2|C_{Q_{2p}} \langle y^2 \rangle|}{2|C_{\langle y \rangle}(y^2)|} \phi(y^2) = p = \Phi_5(x^p) \quad \text{if } g=(y^2, e) = (x^p, e) \text{ or } g=(y^2, z)$$

$$\Phi_{(5,2)}(g) = 0 = \Phi_5(q), q \notin \{I, y, y^2, y^3\} \quad \text{for all } g \notin H_{10} \quad (\text{since } H_{10} \cap CL(g) = \emptyset)$$

$$\text{If } g=(y, e), (y, z) \text{ or } g=(y^3, e), (y^3, z) \quad (\text{since } H_{10} \cap CL(g) = \{g, g^{-1}\})$$

$$\Phi_{(5,2)}(g) = \frac{|C_{Q_{2p} \times C_2}(g)|}{|C_H(g)|} (\phi(g) + \phi(g^{-1})) = \frac{4}{8} (1+1) = \Phi_5(y) \quad \phi(g) = \phi(g^{-1}) = 1$$

Example(2.2):

To construct $\text{Ar}(Q_{26} \times C_2)$ by theorem(3), There are ten cyclic subgroup of $(Q_{26} \times C_2)$:

$$H_1 = \langle (I, e) \rangle, H_2 = \langle (x^{13}, e) \rangle, H_3 = \langle (x^2, e) \rangle, H_4 = \langle (x, e) \rangle, H_5 = \langle (y, e) \rangle,$$

$$H_6 = \langle (I, z) \rangle, H_7 = \langle (x^{13}, z) \rangle, H_8 = \langle (x^2, z) \rangle, H_9 = \langle (x, z) \rangle, H_{10} = \langle (y, z) \rangle$$

And by using theorem(1.3)

$$\Phi(g) = \begin{cases} \frac{|C_{Q_{2m} \times C_2}(g)|}{|C_{H_j}(g)|} \sum_{i=1}^m \phi(h_i) & \text{if } h_i \in H \cap CL(g) \\ 0 & \text{if } H \cap CL(g) = \emptyset \end{cases}$$

$$1 \leq j \leq 10$$

$$g \in H, g = (q, e); q \in Q_{2m}, e \text{ is identity of } C_2$$

$$H_1 = \langle (I, e) \rangle = \{(I, e)\}$$

$$\Phi_{(1,1)}(I, e) = \frac{8 \cdot 13}{1} \cdot 1 = 104 = 8 \cdot p = 2 \cdot \Phi_1(I) \quad (\text{since } H_1 \cap CL(I, e) = \{(I, e)\})$$

$$\Phi_{(1,1)}(g) = 0 = 2 \cdot \Phi_1(q), q \neq I \quad \text{for all } g \notin H_1 \quad (\text{since } H_1 \cap CL(g) = \emptyset)$$

$$H_2 = \langle (x^{13}, e) \rangle = \{(I, e), (x^{13}, e)\}$$

$$\Phi_{(2,1)}(g) = \frac{8 \cdot 13}{2} \cdot 1 = 52 = 4 \cdot p = 2 \cdot \Phi_2(q) \quad \text{if } g \in H_2$$

$$\Phi_{(2,1)}(g) = 0 = 2 \cdot \Phi_2(q), q \notin \{I, x^{13}\} \quad \text{for all } g \notin H_2 \quad (\text{since } H_2 \cap CL(g) = \emptyset)$$

$$H_3 = \langle (x^2, e) \rangle = \{(I, e), (x^2, e), (x^4, e), \dots, (x^{24}, e)\}$$

$$\Phi_{(3,1)}(I, e) = \frac{8 \cdot 13}{13} \cdot 1 = 8 = 2 \cdot \Phi_3(I)$$

$$\Phi_{(3,1)}(g) = \frac{4 \cdot 13}{13} \cdot (1+1) = 8 = 2 \cdot \Phi_3(x^2) \quad \text{if } H_3 \cap CL(g) = \{g, g^{-1}\}$$

$$\Phi_{(3,1)}(g) = 0 = 2 \cdot \Phi_3(q), q \notin \{I, x^2, \dots, x^{24}\} \quad \text{for all } g \notin H_3 \quad (\text{since } H_3 \cap CL(g) = \emptyset)$$

$$H_4 = \langle (x, e) \rangle = \{(I, e), (x, e), (x^2, e), \dots, (x^{25}, e)\}$$

$$\Phi_{(4,1)}(g) = \frac{8 \cdot 13}{26} \cdot 1 = 4 = 2 \cdot \Phi_4(I) = 2 \cdot \Phi_4(x^{13}) \quad \text{if } g = (I, e) \text{ or } g = (x^{13}, e)$$

$$\Phi_{(4,1)}(g) = \frac{4 \cdot 13}{26} \cdot (1+1) = 4 = 2 \cdot \Phi_4(x) \quad \text{if } H_4 \cap CL(g) = \{g, g^{-1}\}$$

$$\Phi_{(4,1)}(g) = 0 = 2 \cdot \Phi_4(q), q \notin \{I, x, x^2, \dots, x^{25}\} \quad \text{for all } g \notin H_4 \quad (\text{since } H_4 \cap CL(g) = \emptyset)$$

$$H_5 = \langle (y, e) \rangle = \{(I, e), (y, e), (y^2, e), (y^3, e)\}$$

$$\Phi_{(5,1)}(I, e) = \frac{8 \cdot 13}{4} = 2 \cdot 13 = 26 = 2 \cdot p = 2 \cdot \Phi_5(I)$$

$$\Phi_{(5,1)}(y^2, e) = \frac{8 \cdot 13}{4} = 2 \cdot 13 = 26 = 2 \cdot p = 2 \cdot \Phi_5(x^{13}) \quad ; (y^2, e) = (x^{13}, e)$$

$$\Phi_{(5,1)}(g) = 0 = 2 \cdot \Phi_5(q), q \notin \{I, y, y^2, y^3\} \quad \text{for all } g \notin H_5 \quad (\text{since } H_5 \cap CL(g) = \emptyset)$$

$$\text{If } g = (y, e) \text{ or } g = (y^3, e)$$

$$\Phi_{(5,1)}(g) = \frac{4}{4} (1+1) = 2 = 2 \cdot \Phi_5(y) \quad (\text{since } H_5 \cap CL(g) = \{g, g^{-1}\})$$

$$H_6 = \langle (I, z) \rangle = \{(I, e), (I, z)\}$$

$$\Phi_{(1,2)}(I, e) = \Phi_{(1,2)}(I, z) = \frac{8 \cdot 13}{2} = 52 = \Phi_1(I) \quad (\text{since } H_6 \cap CL(I, e) = \{(I, e)\})$$

$$(H_6 \cap CL(I, z) = \{(I, z)\})$$

$$\Phi_{(1,2)}(g) = 0 = \Phi_1(q) ; q \neq I \quad \text{for all } g \notin H_6 \quad (\text{since } H_6 \cap CL(g) = \emptyset)$$

$$H_7 = \langle (x^{13}, z) \rangle = \{(I, e), (x^{13}, e), (I, z), (x^{13}, z)\}$$

$$\Phi_{(2,2)}(I, e) = \Phi_{(2,2)}(I, z) = \frac{8 \cdot 13}{4} = 26 = \Phi_2(I)$$

$$\Phi_{(2,2)}(x^{13}, e) = \Phi_{(2,2)}(x^{13}, z) = \frac{8 \cdot 13}{4} = 26 = \Phi_2(x^{13})$$

$$\Phi_{(2,2)}(g) = 0 = \Phi_2(q); q \notin \{I, x^{13}\} \quad \text{for all } g \notin H_7 \quad (\text{since } H_7 \cap \text{CL}(g) = \emptyset)$$

$$H_8 = \langle (x^2, z) \rangle = \{(I, e), (x^2, e) \cdots (x^{24}, e), (I, z), (x^2, z) \cdots (x^{24}, z)\}$$

$$\Phi_{(3,2)}(I, e) = \Phi_{(3,2)}(I, z) = \frac{8 \cdot 13}{26} = 4 = \Phi_3(I)$$

$$\Phi_{(3,2)}(x^2, e) = \Phi_{(3,2)}(x^2, z) = \frac{4 \cdot 13}{26}(1+1) = 4 = \Phi_3(x^2) \quad (\text{since } H_8 \cap \text{CL}(g) = \{g, g^{-1}\})$$

$$\Phi_{(3,2)}(g) = 0 = \Phi_3(q); q \notin \{I, x^2, \dots, x^{24}\} \quad \text{for all } g \notin H_8 \quad (\text{since } H_8 \cap \text{CL}(g) = \emptyset)$$

$$H_9 = \langle (x, z) \rangle = \{(I, e), (x, e), (x^2, e) \cdots (x^{25}, e), (I, z), (x, z), (x^2, z) \cdots (x^{25}, z)\}$$

$$\Phi_{(4,2)}(I, e) = \Phi_{(4,2)}(I, z) = \frac{8 \cdot 13}{52} = 2 = \Phi_4(I)$$

$$\Phi_{(4,2)}(x^{13}, e) = \Phi_{(4,2)}(x^{13}, z) = \frac{8 \cdot 13}{52} = 2 = \Phi_4(x^{13})$$

$$\Phi_{(4,2)}(x^2, e) = \Phi_{(4,2)}(x^2, z) = \frac{4 \cdot 13}{52}(1+1) = 2 = \Phi_4(x^2) \quad (\text{since } H_9 \cap \text{CL}(g) = \{g, g^{-1}\})$$

$$\Phi_{(4,2)}(x, e) = \Phi_{(4,2)}(x, z) = \frac{4 \cdot 13}{52}(1+1) = 2 = \Phi_4(x) \quad (\text{since } H_9 \cap \text{CL}(g) = \{g, g^{-1}\})$$

$$\Phi_{(4,2)}(g) = 0 = \Phi_4(q); q \notin \{I, x, \dots, x^{25}\} \quad \text{for all } g \notin H_9 \quad (\text{since } H_9 \cap \text{CL}(g) = \emptyset)$$

$$H_{10} = \langle (y, z) \rangle = \{(I, e), (y, e), (y^2, e), (y^3, e), (I, z), (y, z), (y^2, z), (y^3, z)\}$$

$$\Phi_{(5,2)}(I, e) = \Phi_{(5,2)}(I, z) = \frac{8 \cdot 13}{8} = 13 = p = \Phi_5(I)$$

$$\Phi_{(5,2)}(y^2, e) = \Phi_{(5,2)}(y^2, z) = \frac{8 \cdot 13}{8} = 13 = p = \Phi_5(y^2); (y^2, e) = (x^{13}, e)$$

$$\Phi_{(5,2)}(g) = 0 = 2 \cdot \Phi_5(q); q \notin \{I, y, y^2, y^3\} \quad \text{for all } g \notin H_{10} \quad (\text{since } H_{10} \cap \text{CL}(g) = \emptyset)$$

$$\text{If } g = (y, e), (y^3, e) \quad \text{or} \quad g = (y, z), (y^3, z)$$

$$\Phi_{(5,2)}(g) = \frac{4}{8}(1+1) = 1 = \Phi_5(y) \quad (\text{since } H_{10} \cap \text{CL}(g) = \{g, g^{-1}\})$$

	Γ – Classes of $(Q_{26} \times C_2)$									
Γ – Classes	$[I, e]$	$[x^{13}, e]$	$[x^2, e]$	$[x, e]$	$[y, e]$	$[I, z]$	$[x^{13}, z]$	$[x^2, z]$	$[x, z]$	$[y, z]$
$ CL_\alpha $	1	1	2	2	26	1	1	2	2	26
$ C_{Q_{26} \times C_2}(CL_\alpha) $	104	104	52	52	4	104	104	52	52	4
$\Phi_{(1,1)}$	104	0	0	0	0	0	0	0	0	0
$\Phi_{(2,1)}$	52	52	0	0	0	0	0	0	0	0
$\Phi_{(3,1)}$	8	0	8	0	0	0	0	0	0	0
$\Phi_{(4,1)}$	4	4	4	4	0	0	0	0	0	0
$\Phi_{(5,1)}$	26	26	0	0	2	0	0	0	0	0
$\Phi_{(1,2)}$	52	0	0	0	0	52	0	0	0	0
$\Phi_{(2,2)}$	26	26	0	0	0	26	26	0	0	0
$\Phi_{(3,2)}$	4	0	4	0	0	4	0	4	0	0
$\Phi_{(4,2)}$	2	2	2	2	0	2	2	2	2	0
$\Phi_{(5,2)}$	13	13	0	0	1	13	13	0	0	1

Reference

- [1] A.H.Abdul-Mun'em,"On Artin Cokernel of Quaternion Group Q_{2m} when m is a prime Number",M.Sc.thesis,University of Technology,2008.
- [2] H.R.Yassien,"On Artin Cokernel of Finite Group" ,M.Sc. Thesis, University of Babylon,2000
- [3] J.R.Nime," The Cyclic Decomposition of The Factor Group $cf(Q_{2m} \times C_2, Z) / \overline{R}(Q_{2m} \times C_2)$ When m is an Odd Number", ,M.Sc. , thesis , University of Kufa , 2009 .
- [4] Lam.T.Y," Artin Exponent of Finite Group",Columbia University ,New York1967.
- [5] N.R.Mahamood"the Cyclic Decomposition of the Factor Group $cf(Q_{2m}, Z) / \overline{R}(Q_{2m})$ ",M.Sc.thesis,University of Technology,1995.
- [6] R.N.Mirza,"On Artin Cokernel of Dihedral Group D_n when n is An Odd Number" ,M.Sc.Thesis, University of Kufa,2007.