



A Hybrid Variable Step-Size MCMA Blind Equalizer Algorithm for QAM Signals

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Received: 27/9/2012

Accepted: 22/7/2013

Abstract: **B**lind equalization is a technique for adaptive equalization of a communication channel without the aid of the usual training sequence. The Modified Constant Modulus Algorithm (MCMA) is one of adaptive blind equalization algorithms. The step size in (MCMA) must be selected to get balancing between convergence rate and final accuracy (maladjustment). This paper proposes an enhanced technique based on an absolute difference error and iteration number to adjust a step size which is used in modified constant modulus algorithm (MCMA). The new proposed algorithm is called Combined Iteration and Absolute Error MCMA (CIAE-MCMA). It is observed from the simulation results that the proposed algorithm can speed up convergence rate and decreases steady state mean square error and performs significantly better than the conventional fixed-step size MCMA and other variable step size algorithms (VSS-MCMA).

Keyword: MCMA, step size, blind equalizer

1. Introduction

In practical application of the CMA and MCMA algorithms, a key parameter is the step size, which is used to adjust the update of equalizer weights. Most blind adaptive equalizer algorithms used a constant modulus algorithm (CMA) formula to calculate the weights [1], [2]:

$$W(k+1) = W(k) - \mu \cdot e(k) \cdot X^*(k) \quad (1)$$

Where: $W(k)$ is the equalizer tap weights vector, μ is step size, $e(k)$ is error and $X(k)$ is the input vector of the equalizer.

As it is well known, if the step size is large, the convergence rate of the CMA algorithm will be rapid, but the steady-state mean square error (MSE) will be increased. On the other hand, if the step size is small, the steady state MSE will be small, but the convergence rate will be slow. Thus, the step size provides a tradeoff between the convergence rate and the steady-state MSE of the CMA algorithm.

To improve the performance of the CMA algorithm, an excellent way is to make the step size variable rather than fixed, that is, choose large step size values during the initial convergence of the CMA algorithm, and use small step size values when the system is close to its steady state.

There are many methods available to improve the performance of the CMA and MCMA during the different stages of adaptation. One of these methods is proposed by Jones. D. L. [3] who controlled the step size by using the channel output signal vector energy $\|x(k)\|^2$. Chahed et al. [4] adjusted the step size by using a time varying step size parameter depending upon squared Euclidian norm of the channel output vector and on the equalizer output.

Xiong et al. [5] employed the lag(1) error autocorrelation function between $e(k)$ and $e(k-1)$. Here, $e(k)$ is the output error of the blind system. Zhao, B. [6], proposed that the variable step size of CMA algorithm can be controlled by difference between current and previous MSE. Kevin Banović., [7] proposed an adjustment process to step size based on the length of the equalizer output radius. An alternative scheme that considered a nonlinear function of instantaneous error for adjusting the step-size parameter was proposed by Liyi et. al. [8]. Meng Zhang [9], proposed an affine projection blind equalization CMA algorithm based on quantization estimation errors and variable step-size. Variable step size MCMA was proposed by Wei Xue, [10] in which the step size was adjusted according to the region where the received signal lied in the constellation plane.

In this paper, a new variable step-size method has been used for modified CMA algorithm (CIAEMCMA). In this proposal, the step size was controlled by two parameters; the first parameter was absolute difference between current and previous error, while the second one was the iteration number.

As it will be shown in the simulations, the proposed algorithm has better performance as compared with MCMA and other variable step size (VSSMCMA) algorithm which was proposed by Liyi et. al. [8].

This paper is organized as follows: the basic concept of the adaptive equalizer is shown in section 2 with basic idea about blind equalizer and its types CMA, MCMA and variable step size MCMA.

The proposed algorithm is described in section 3. In section 4, a simulation that confirms the analysis and the advantages of the proposed algorithm are presented. Then a comparison between the new proposal with the traditionally MCMA and VSS-MCMA algorithms is shown. Finally, section 5 provides the conclusion.

2. Basic Concept of Adaptive Blind Equalizer

Quadrature Amplitude Modulation (QAM) schemes have been used widely in modern communication systems due to high bandwidth and power efficiency [1]. QAM signals are sensitive to Inter Symbol Interference (ISI) caused by multi-path propagation and the fading of the channel [2], so it is necessary to use equalization technique to reduce the effect of ISI. Conventional adaptive equalizers employ a training sequence which is known both at the transmitter and the receiver to update their filter coefficients. However, when sending a training sequence is impractical or impossible, it is desirable to equalize a channel without the training sequence. The first blind equalization algorithm was published by Sato in 1975 for equalization of pulse amplitude modulation (PAM) signals [11]. In blind equalization; the desired signal or input to the channel is unknown to the receiver, except for its probabilistic or statistical properties over some known alphabet A . As both the channel $h(k)$ and its input $a(k)$ are unknown, the objective of a blind equalization is to recover the unknown input sequence based solely on its probabilistic and statistical properties.[8].

The equivalent baseband model of blind equalization system is shown in Figure 1.

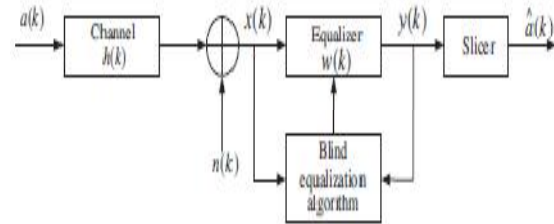


Figure. 1. Equivalent baseband model of blind equalization system.

The received signal $X(k)$ can be written as:

$$X(k) = \sum_{i=0}^{L-1} h(i)a(k-i) + n(k) \quad (2)$$

Where $h(k)$ is the overall complex baseband equivalent impulse response of the transmitter filter, and unknown channel and receiver filter. The length of $h(k)$ is L . The input data sequence $a(k)$ is assumed to be independent and identically distributed. $n(k)$ is the additive white Gaussian noise.

The output of the equalizer can be written as:

$$y(k) = X^T(k)W(k) \quad (3)$$

Where:

$$W(k) = [w_0(k), w_1(k), \dots, w_{N-1}(k)]^T \quad (4)$$

$$X(k) = [x(k), x(k-1), \dots, x(k-L+1)]^T \quad (5)$$

L is the length of the equalizer tap weights.

2.1. Constant Modulus Algorithm (CMA)

The constant modulus algorithm (CMA) [12] is one of widely used blind equalization algorithm for QAM signals. CMA was proposed by Godard in 1980. The cost function of CMA is given by:

$$J(k) = E([|y(k)|^2 - R_2]^2) \quad (6)$$

Where $E[\cdot]$ indicates the statistical expectation, $y(k)$ is the output of the equalizer and R_2 is a constant that depends on transmitted data statistics, which is given by [12] :

$$R_2 = \frac{E[|a(k)|^4]}{E[|a(k)|^2]} \quad (7)$$

The error signal is given by

$$e(k) = y(k)(|y(k)|^2 - R_2) \quad (8)$$

The update of the tap weights vector can be written as:

$$W(k+1) = W(k) - \mu \cdot e(k) \cdot X^*(k) \quad (9)$$

Where, μ is the fixed step size parameter.

CMA has the disadvantages of slow convergence rate, large steady state mean square error (MSE) and phase-blind nature. Therefore, several new algorithms have been introduced to overcome the disadvantages of CMA.

2.2. Modified Constant Modulus Algorithm (MCMA)

Modified constant modulus algorithm (MCMA) shows a performance improvement in the convergence behavior and can correct the phase error and frequency offset at the same time [13].

Modified CMA modifies the cost function of CMA to the form of real and imaginary parts, the modified cost function can be written as [14]:

$$J(k) = J_R(k) + J_I(k) \quad (10)$$

Where $J_R(k)$ and $J_I(k)$ are the cost function for real and imaginary parts of the equalizer output:

$$y(k) = y_R(k) + y_I(k)$$

respectively, they are defined as

$$J_R(k) = E \left(\left[(R_{2R} - |y_R(k)|^2)^2 \right] \right) \quad (11)$$

$$J_I(k) = E \left(\left[(R_{2I} - |y_I(k)|^2)^2 \right] \right) \quad (12)$$

Where R_{2R} and R_{2I} are the real constants determined by the real and imaginary parts of transmitted data sequence respectively, they are defined as:

$$R_{2R} = \frac{E[|a(k)|^4]}{E[|a(k)|^2]} \quad (13)$$

$$R_{2I} = \frac{E[|a(k)|^4]}{E[|a(k)|^2]} \quad (14)$$

Where the error signal is:

$$e(k) = e_R(k) + j \cdot e_I(k)$$

given by

$$e_R(k) = y_R(k)(R_{2R} - |y_R(k)|^2) \quad (15)$$

$$e_I(k) = y_I(k)(R_{2I} - |y_I(k)|^2) \quad (16)$$

The update of tap weights vector can be written as:

$$W(k+1) = W(k) + \mu(k+1) \cdot e(k) \cdot X^*(k) \quad (17)$$

In contrast, the cost function of MCMA separates the output of equalizer to real and imaginary parts and estimates the error signal for real and imaginary parts respectively [16]. The cost functions of MCMA use both modulus and phase of the equalizer output, so MCMA can correct the phase rotation and spinning phase error.

3. A NEW PROPOSED ALGORITHM (CIAEMCMA)

In the proposed algorithm, a larger step size is set initially to speed the convergence up, and then the step size is gradually decreased to a smaller value to obtain smaller MSE after convergence. An

exponential function is carefully designed in this paper in order to adjust the value of the step size. Two parameters are used together for adjusting the step size: the first is the absolute difference error; the second is the iteration number. The proposed formulas that used to adjust the step size μ is the exponentially relation to mince combined between absolute difference of current and previous error and exponential value to symbol number (k) (i.e. iteration),

3.1. The Proposed Algorithm

The proposed algorithm is given by:

$$\mu(k+1) = \mu(k) * \rho \quad (18)$$

Where (ρ) is:

$$\rho = e^{-(f+g)} = e^{-(\exp(-|e(k)-e(k-1)|)) + (k/\gamma)} \quad (19)$$

Where β is the proportionality factor. It is used to control the value scope of $\mu(n)$ and γ which is a constant used to control the convergence rate. It is set according to the convergence rate of the conventional MCMA and depends on the number of symbols (iterations) used in convergence.

$\mu(k+1)$ is bounded between two values (μ_{max}) and (μ_{min}) as:

$$\mu(k+1) = \mu_{max} \text{ if } \mu(k) > \mu_{max} \quad (20)$$

$$\begin{aligned} \mu(k+1) &= \mu_{min} \text{ if } \mu(k) < \mu_{min} \\ \text{otherwise } \mu(k+1) &= \mu(k+1) \end{aligned} \quad (21)$$

3.2. Analysis of Proposed Variable Step-size

To analysis equation (19), we start with:

- First part $f = \exp(-|e(k) - e(k-1)|)$ at starting of the adaptation process, the difference between current and previous error is high so the value of this part $\rho = e^{-(f)}$ starts with large value and as iteration number increases it becomes a small value.
- The second part ($g = (k/\gamma)$) depends on the number of the iterations: at starting, the iteration number is small, so the value of ($\rho = e^{-(k/\gamma)}$) is equal to initial value of step size (μ_i) and gives the adjustment formula for the original MCMA algorithm.
- As the number of iteration will increase, then the value of (ρ) decreases. The relation between (ρ) and the iteration number is illustrated in figure 2.

By combining both parts as in (19) i.e. ($\rho = e^{-(f+g)}$), we observe that they cause large step size at a low number of iterations (dedicate a faster convergence rate) and a small value at the steady state that reduce low MSE and low misadjustment at steady state.

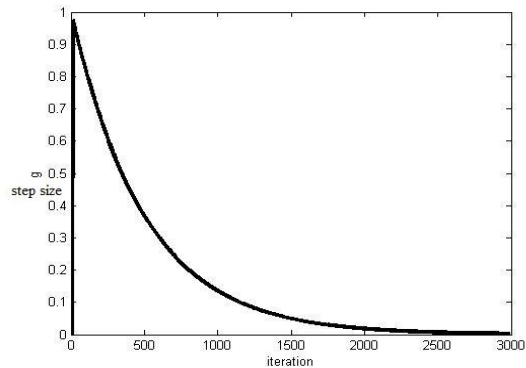


Figure. 2. Updating the step size using iteration number part

4. Simulation Results

The performance of proposed, traditional MCMA, and another variable step size VSSCMA [8] algorithms are validated by simulations of adaptive blind equalizer as shown in Figure 1. The variable step size updating equations for VSSCMA are as follows:

$$\begin{aligned}\mu(k) &= \beta * [1 - e^{-\sigma|e(k)|}] \\ \mu(k) &= \mu_{max} \text{ if } \mu(k) > \mu_{max} \\ \mu(k) &= \mu_{min} \text{ if } \mu(k) < \mu_{min}\end{aligned}\quad (23)$$

Figure 1 was simulated using two schemes; 16-QAM and 64-QAM sources respectively. The parameters used in MCMA, VSSMCMA and CIAE-MCMA algorithms were chosen to achieve better performance in terms of fast convergence time and low level misadjustment in order to make fairly comparison between these algorithms. They are chosen as following:

- 1- The number of symbols equal to 3000.
- 2- The signal to noise ratio at the primary input for all simulation is 30 dB. The source noise used for all simulation was white Gaussian noise.
- 3- The length of the equalizer was taken as (11). The middle tapped of the equalizer was initialized to the unity value, and the rests of the equalizer weights were initialized to zero.
- 4- The impulse response of the channel is multipath fading channel with four paths [17], $h = [-0.4 \ 0.84 \ 0.336 \ 0.134]$.
- 5- The step size (μ) used in MCMA algorithm is equal to (0.001).
- 6- The optimum value for the VSSMCMA of μ_{max} and μ_{min} was chosen to be (0.08) and (0.00008) respectively and the

constants (α, β) to be (0.6, 0.005) respectively.

- 7- The optimum value for the CIAE-MCMA of μ_{max} and μ_{min} was chosen to be (0.08) and (0.00002) respectively and initial step size (μ_i) is equal to (0.04).

A. The Mean Square Error (MSE) Curve

Figure 3 shows the comparison of convergence curve for three algorithms for 16-QAM and 64-QAM systems.

As can be seen from these figures, the proposed algorithm has faster convergence rate and low level of steady-state error compared with the other two algorithms.

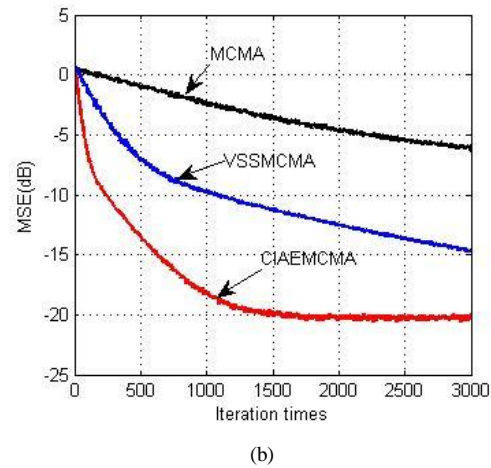
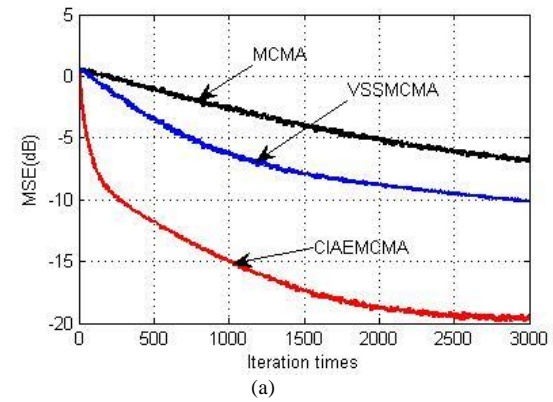
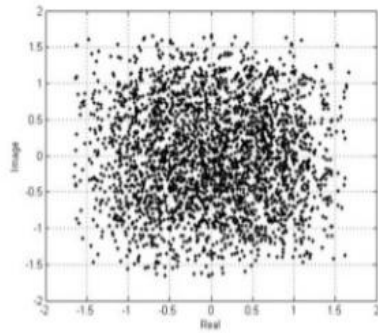


Figure. 3. Comparison of MSE between MCMA, VSS- MCMA and CIAE-MCMA for a) 16-QAM scheme b) 64-QAM scheme

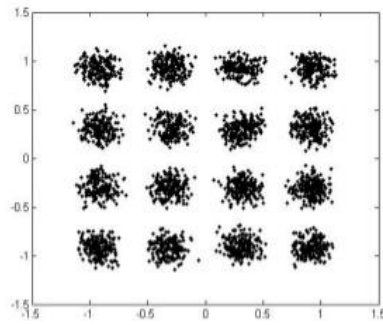
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B. Equalizer Output

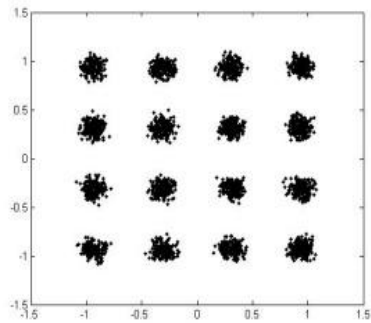
The comparison between equalizer input and output using three algorithms with 16-QAM is shown in figure 4.



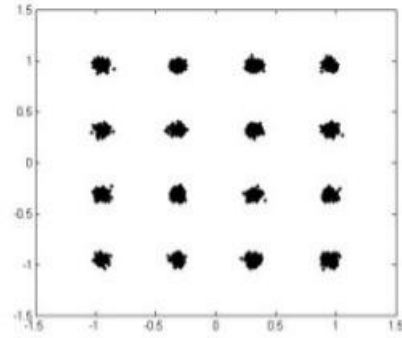
(a)



(b)



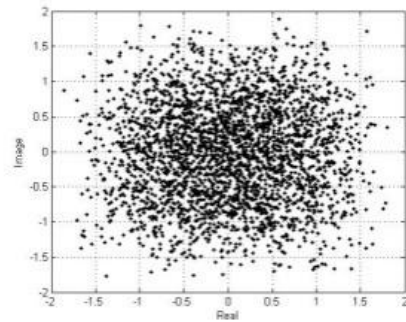
(c)



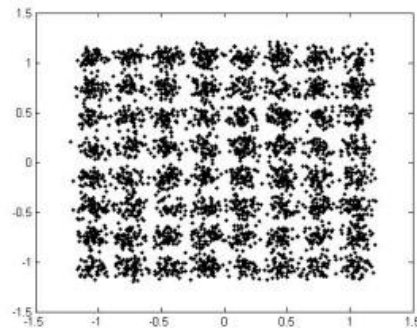
(d)

Figure. 4. Constellations of 16-QAM signals with (a) equalizer input (b) MCMA (c) VSS-MCMA and (d) CIAE-MCMA

While the equalizer input and using three algorithms with 64-QAM is shown in figure 5.



(a)



(b)

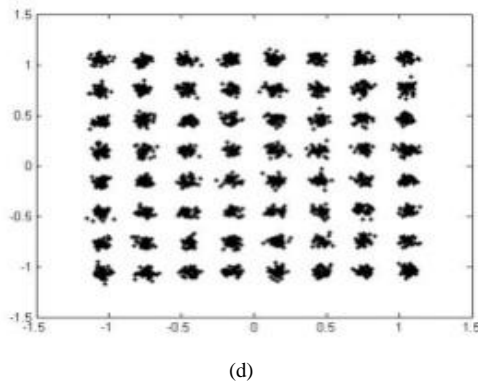
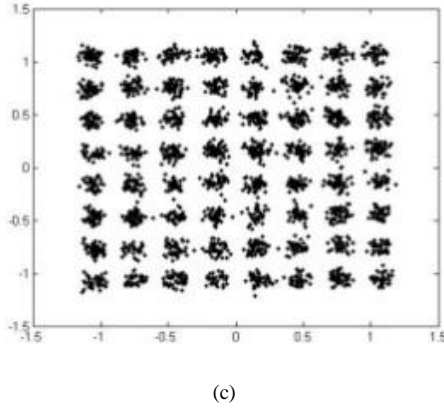


Figure 5. Constellations of 64-QAM signals with (a) equalizer input (b) MCMA (c) VSS-MCMA and (d) CIAE-MCMA

The results in figures 4 and 5, show that the proposed algorithm has the best ability for estimating the original signal.

C. BER Performance

To carry out the BER performance of different algorithms for 16-QAM and 64-QAM, the signal to noise ratio at the primary input for all simulation is alternated between 0, and 30 dB with an increment step equal to 2 dB. These results are illustrated in Fig.6. From these figures, we see that BER performance is improved with the proposed algorithm.

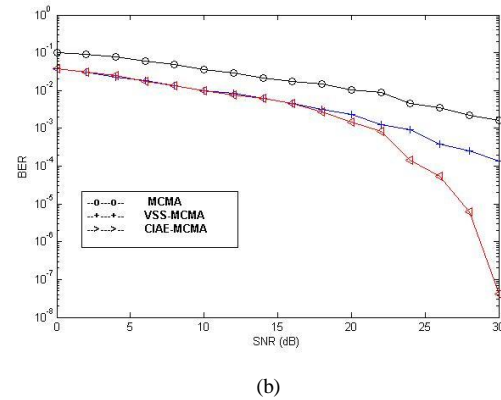
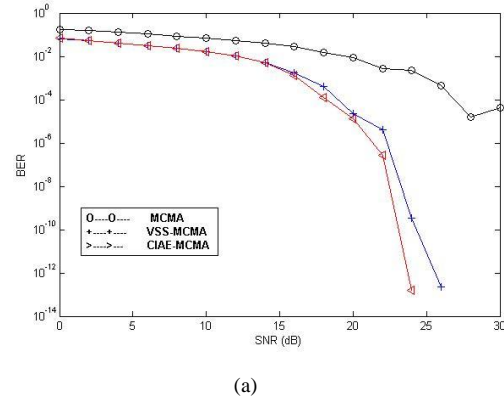


Figure 6. BER performance for (a) 16-QAM scheme (b) 64-QAM scheme

5. Conclusion

In this paper, a variable step size modified constant modulus algorithm (CIAEMCMA) is proposed. The step size of the algorithm is adjusted according to the combined absolute difference error with iteration number. The (CIAEMCMA) can obtain both fast convergence rate, and a small steady state MSE compared with traditional MCMA and other variable step size MCMA. The simulation results for 16-QAM and 64-QAM signals demonstrate the effectiveness of the (CIAEMCMA) in the equalization performance.

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