# On 3-prime fuzzy ideal with respect to an element of a near ring

# حول المثالية 3 - الاولية الضبابية بالنسبة لعنصر ما في الحلقة القريبة

Showq Mohammed .E Safaa Mahdi M.
AL-Kufa university College of Education for Girls
Department of Mathematics

## Abstract

This paper introduces the notions of a 3-prime fuzzy ideal with respect to an element of a near ring N, denoted by (x-3-P-F-I) and we give some properties and example and we studied the image and inverse image of (x-3-P-F-I) under epimomorphism .

## الخلاصة

قدمنا في هذا البحث مفهوم المثالية 3- prime الضبابية بالنسبة إلى عنصر ما في الحلقة القريبة والتي يرمـــز لها بالرمــــــز x-3-P-F-I ودرسنا صور المباشرة ومعكوس الصور للمثالية تحت التشاكل الشامل وعرضنا بعض الخواص التي تتعلق بهذه المثالية .

## Introduced

We will refer that all near rings and ideal in this paper are left .In 1905 ,L.E Dickson began the study of a near ring and later in 1930 ,Wieland has investigated it .Furth material about a near ring can be found [1].In 1965,L.A.Zadeh introduced the concept of fuzzy subset [5] .In 1982 W.Liu introduced the notion of a fuzzy ideal of near ring [8] .In 1991,N.J. Groenewal introduce the notion 3- Prime ideal of near ring [3].In 2013 Showq M. introduced the notion x-3-Prime ideal of a near ring [4] . The purpose of this paper is as mention in the abstract.

## Key Word

Near ring, left ideal ,3-prime ideal , x-3-Prime ideal ,fuzzy subset ,fuzzy ideal ,singlton ,t-cut.

## 1. Preliminaries

In this section we give some basic concepts that we need in the second section. In this section we will give some Definitions and theorem that we will need in our paper .

**Definition (1.1)[1]**

Aleft near ring is a set N together with two binary operations."+","." Such that
(1)(N,+) is a group (not necessarity abelian )
(2)(N,.) is a semi group
(3) $n_1.(n_2+n_3)=n_1.n_2+n_1.n_3, \forall n_1,n_2,n_3 \in N.$

**Definition (1.2)[2]**

Let (N,+,.) be a near ring .A normal sub group I of (N,+) is called a left ideal of N if
(1) $N.I \subseteq I$
(2) $\forall n_1, n \in N$ and $i \in I,$ $\text{n.}(n_1+i)-n.n_1 \in I.$

**Definition (1.3)[3]**

An ideal I of a near ring N is called 3-Prime ideal if for all $a,b \in N, a.N.b \subseteq I \rightarrow a \in I \vee b \in I$

**Definition (1.4)[4]**

An ideal I of near ring N is called 3-prime ideal with respect to an element of a near ring denoted by (x-3-prime ideal ) if for all $a,b \in N, x.(a.N.b) \subseteq I \rightarrow x.a \in I \vee x.b \in I.$

**Theorem (1.5)[4]**

Let $(N_1,+,.)$and $(N_2,+',.')$ be two near ring, $f:N_1 \to N_2$ be epimomorphism and I be x-3-prime ideal of $N_1$. Then f(I) is f(x)-3-prime ideal of $N_2$.

**Definition (1.6)[5]**

Let X be a non- empty set .A function $\mu : X \to [0,1]$ is called a fuzzy subset of X ( a fuzzy set in X), where [0,1] is a closed interval of numbers .

**Definition (1.7)[5]**

Let $\mu$ be a fuzzy subset of a non empty set X .If $\mu(y)=0$ ,for every $y \in X$ then $\mu$ is called empty fuzzy set .

**Definition (1.8)[5]**

Let $\mu$ be a non- empty fuzzy subset of a near ring N, that ( $\mu(y) \neq 0$ for some $y \in N$ ).then $\mu$ is said to be fuzzy ideal of N if it satisfies that following conditions :

(1) $\mu(z-y) \geq \min\{\mu(z),\mu(y)\}$;

(2) $\mu(z.y) \geq \min\{\mu(z),\mu(y)\}$ ;

(3) $\mu(y+z-y) \geq \mu(z)$;

(4) $\mu(z.y) \geq \mu(y)$ ,$\forall$ y,z$\in$N.

when the subset of N satisfies 1 ,2 is called fuzzy sub near ring .

**Remark (1.9) [2]**

If $\mu$ is a fuzzy ideal of near ring N then

(1) $\mu(z+y) = \mu(y+z)$

(2) $\mu(0) \geq \mu(z)$ ,$\forall$ y,z$\in$N.

**Definition (1.10) [6]**

Let f : $(N_1,+,.) \to (N_2,+',.')$ be a function. For a fuzzy set μ in $N_2$, we define

$(f^{-1}(\mu))(x) = \mu(f(x)$ for every x $\in N_1$. For a fuzzy set $\lambda$ in X, f($\lambda$) is defined

by

$$(f(\lambda))(y) = \begin{cases} \sup \lambda(x) & \text{if } f(x)=y, y \in N_1 \\ 0 & \text{otherwise } y \end{cases}$$

**Definition (1.11) [6]**

Let $\mu$ be fuzzy ideal of a near ring N and f be a function from the near ring $N_1$ into a near ring $N_2$ . Then we call $\mu$ is f-invariant if and only if for all $y,z \in N$, $f(z)=f(y)$ implies $\mu(z)=\mu(y)$.

**Definition (1.12) [7]**

Let $\mu$ be a fuzzy ideal of a near ring N then $\mu^*$ is a fuzzy subset in N defined by $\mu^*(y) = \mu(y) + 1 - \mu(0), \forall y \in N$ .

**Definition (1.13) [8]**

Let $\mu$ be a fuzzy subset of a near ring N and $t \in [0,1]$ defined $\mu_t = \{n \in N : \mu(n) \geq t\} is$ called $t$-cut .

**Definition (1.14) [9]**

The fuzzy subset $n_t$ of anear ring defined by $\begin{cases} t & y = n \\ 0 & y \neq n \quad \forall y \in N \end{cases}$ is called a fuzzy singleton , where $t \in [0,1]$ .

**Theorem (1.15) [3]**

Let $f : (N_1, +, .) \to (N_2, +', .')$ be ahomomorphism

(i) if I is an ideal of a near ring $N_1$.,then f(t) is an ideal of a near ring $N_2$.

(ii) If J is an ideal of a near ring $N_2$.,then $\overset{-1}{f}(J)$ is an ideal of a near ring $N_1$.

**Remark (1.16) [3]**

Let $\{I_j\}_{j \in J}$ be afamily of ideals of near ring N, then

(i) $\bigcap_{j \in J} I_j$ is an ideal of N.

(ii) if $\{I_j\}_{j \in J}$ is a chain ,then $\bigcup_{j \in J} I_j$ is an ideal of N.

**Definition (1.17) [10]**

let $\mu$ be a fuzzy ideal of N . then the set $\mu_*$ is defined by $\mu_* = \{y \in N : \mu(y) = \mu(0)\}$ where 0 is the zero element of N .

**Remark [1.18] [3]**

let $\mu$ is a fuzzy ideal of N if and only if $\mu_*$ is an ideal of N .

## 2. The main Results

In this section we study 3-prime fuzzy ideal with respect to the element x .

**Definition (2.1)**

A fuzzy ideal $\mu$ of a near ring N is called 3- prime fuzzy ideal with respect to an element of anear ring N denoted by x-3-P–F-I for all $a, b \in N$ $\max\{\mu(x.a), \mu(x.b)\} \geq \inf \mu(x.a.n.b)$.

**Example (2.2)**

Consider the nar ring N={0,a,b,c} with addition and multiplication defind by the following tabes .

| + | 0 | a | b | c |
|---|---|---|---|---|
| 0 | 0 | a | b | c |
| a | a | 0 | c | b |
| b | b | c | 0 | a |
| c | c | b | a | 0 |

| . | 0 | a | b | c |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| a | 0 | a | 0 | a |
| b | 0 | b | 0 | b |
| c | 0 | c | 0 | c |

The fuzzy ideal $\mu$ of N denoted by $\mu(y) = \begin{cases} 1 & \text{if } y \in \{0,b\} \\ 0 & \text{if } y \in \{c,a\} \end{cases}$ is an c-3-P-F-I .

**Proposition (2.3)**

Let $\mu$ be a fuzzy ideal of a near ring N ,and $\mu$ is a x-3-P-F-I of N then $\mu_t$ is x-3-prime ideal of N for all $t \in [0, \mu(0)]$.

**Proof**

Since $\mu_t = \{n \in N : \mu(n) \geq t\}$ ,x(a.N.b)$\subseteq \mu_t \rightarrow$, x.a.n.b $\in \mu_t$ , $\forall x, a, b, n \in N$ , $\mu(x.a.n.b) \geq t$

Since $\mu$ is x-3-P-F-I of N $\max\{\mu(x.a), \mu(x.b)\} \geq \inf \mu(x.a.n.b)$

$\max\{\mu(x.a), \mu(x.b)\} \geq t$

either $\mu(x.a) \geq t \rightarrow x.a \in \mu_t$

or $\mu(x.b) \geq t \rightarrow x.b \in \mu_t$

$\Rightarrow$ $\mu_t$ is x-3-prime ideal of N.

**Remark (2.4)**

let f be an aepimomorphisem from the near ring $N_1$ onto the near ring $N_2$ and let $\mu$ be x-3-P-F-I of $N_1$ ,then f($\mu_t$) is a f(x) -3-prime ideal of $N_2$

**proof**

by proposition (2.3) we have $\mu_t$ is x-3-P- I of $N_1$ .By theorem (1.5) we get f($\mu_t$) is a f(x) -3-prime ideal of $N_2$

**Theorem (2.5)**

Let $\mu$ be fuzzy subset of a neare ring N , if $\mu$ is x-3-prime ideal of N, then $\mu_*$ is x-3-prime ideal of N.

**Proof**

Let $a, b, c, x \in N$ such that $\mu_*$ is an ideal of N $x.a.N.b \subseteq \mu_*$

$\rightarrow x.a.n.b \in \mu_* \Rightarrow \mu(x.a.n.b) = \mu(0)$ by using defintion $\mu_*$ since $\mu$ is x-3-P-F-I of N

$\max\{\mu(x.a), \mu(x.b)\} \geq \inf \mu(x.a.n.b)$

$\max\{\mu(x.a), \mu(x.b)\} = \mu(0)$

$\text{since } \mu(0) \geq \mu(y), \forall y \in N, \text{since } \mu \; fuzzy\, ideal$

$either\; \mu(x.a) = \mu(0) \rightarrow x.a \in \mu_*$

$or\;\; \mu(x.b) = \mu(0) \rightarrow x.b \in \mu_*$

$\Rightarrow \mu_*$ is x - 3 - prime ideal.

**Theorem (2.6)**

Let I be an x-3-prime ideal of a near ring N,for $t \in [0,1)$ ,there exists an x-3-P-F-I $\mu$ of N, such that $\mu_t = I$.

**Proof**

Let $\mu$ be a fuzzy ideal subset of N definsd by

$$\mu(y) = \begin{Bmatrix} t & \text{t} \in \text{I} \\ 0 & \text{t} \in N/I \end{Bmatrix}$$

to prove $\mu$ is fuzzy ideal of N let $x, y, z \in N$ . Then we have the following case.

1. $if\;$ y - z $\in$ I

$\Rightarrow \mu(y - z) = t \geq \min\{\mu(y), \mu(z)\}$

$if\;$ y - z $\notin$ I $\rightarrow$ either y $\notin$ I $\vee$ z $\notin$ I

$\mu(y - z) = 0 = \min\{\mu(y), \mu(z)\}$.

2. $if\ \ y.z \in I$
$\Rightarrow \mu(y.z) = t \geq \min\{\mu(y), \mu(z)\}$
$if\ y.z \notin I \rightarrow z \notin I$
$\mu(y.z) = 0 = \min\{\mu(y), \mu(z)\}.$
3. $if\ \ y + z - y \in I$
$\Rightarrow \mu(y + z - y) = t \geq \mu(z)$
$if\ y + z - y \notin I \rightarrow z \notin I$
$\mu(y + z - y) = 0 = \mu(z).$

4. $if\ \ y.z \in I$
$\Rightarrow \mu(y.z) \geq \mu(z)$
$if\ y.z \notin I \rightarrow z \notin I$
$\mu(z) = 0 \rightarrow \mu(y.z) = 0 = \mu(z)$
from 1,2,3,and 4 we have $\mu$ is fuzzy ideal of N .Now suppose $\mu$ is not x-3-P-F-I of N then there exists $x, y, z \in N$ ,such that $\max\{\mu(x.y), \mu(x.z)\} \prec \inf \mu(x.y.n.z)$
we have $\mu_t$ is x-3-prime ideal of N,that mean
$x.y.N.z \subseteq \mu_t \rightarrow x.y \in \mu_t \vee x.z \in \mu_t$
$x.y.n.z \in \mu_t \rightarrow x.y \in \mu_t \vee x.z \in \mu_t$
$\max\{\mu(x.y), \mu(x.z)\} \prec \inf \mu(x.y.n.z) \rightarrow t \prec t$ contradiction then $\mu$ is x-3-P-F-I of N.

**Theorem (2.7)**
Let f be an a epimorphism from the near ring $N_1$ onto the near ring $N_2$ .Then $\mu$ is f(x)-3-P-F-I of $N_2$ if $f^{-1}(\mu)$ is x-3-prime fuzzy ideal of $N_1$,for all $x \in N$.
**Proof**
Let $x, y, z \in N$ such that
$x.y.N_1.z \subseteq f^{-1}(\mu)$
$f(x.y.N_1.z) \subseteq \mu$ since $\mu$ is f(x)-3-P-F-I of $N_2$
$f(x).f(y).f(N_1).f(z) \subseteq \mu$
$\max\{\mu(f(x).f(y)), \mu(f(x).f(z))\} \leq \inf \mu(f(x).f(y).f(n).f(z))$
$\max\{\mu(f(x.y)), \mu(f(x.z))\} \geq \inf \mu(f(x.y.n.z))$
$\max\{f^{-1}\mu(x.y), f^{-1}\mu(x.z)\} \geq \inf f^{-1}\mu(x.y.n.z)$
$\Rightarrow f^{-1}(\mu)$ is x-3-P-F-I of $N_1$

**Corollary (2.8)**
Let f be an a epimorphism from the near ring $N_1$ onto the near ring $N_2$ .Then the mapping $\mu \rightarrow f(\mu)$ defines a onto correspondence between the set of all f-invariant x-3-P-F-I of $N_1$ the set of all f(x)-3-P-F-I of $N_2$.
**Proof**
Directy from theorem (2.7)

**Corollary (2.9)**
Let f be an a epimorphism from the near ring $N_1$ onto the near ring $N_2$ .Then $\mu^*$
Is f(x)-3-P-F-I of $N_2$ if and only if $f^{-1}(\mu)$ is an x-3-P-F-I of $N_1$.
**Proof**
Directy from theorem (2.7)

**proposition (2.10)**
let $\mu$ be a fuzzy subset of a near ring N ,then $\mu$ is x-3-P-F-I of N if and only if $\mu^*$ is x-3-P-F-I of N.
**proof**
$\rightarrow$ let $x, y, z \in N$ since $\mu$ is x-3-P-F-I of N
$\max\{\mu(x.y), \mu(x.z)\} \geq \inf \mu(x.y.n.z)$
since $\max\{\mu(x.y), \mu(x.z)\} + 1 - \mu(0) \geq \inf \mu(x.y.n.z) + 1 - \mu(0)$
$\max\{\mu(x.y) + 1 - \mu(0), \mu(x.z) + 1 - \mu(0)\} \geq \inf \mu(x.y.n.z) + 1 - \mu(0)$ ,
$\max\{\mu^*(x.y), \mu^*(x.z)\} \geq \inf \mu^*(x.y.n.z)$
$\Rightarrow \mu^*$ is x-3-P-F-I of N.
$\leftarrow$ let $\mu^*$ is x-3-P-F-I of N.
$\max\{\mu^*(x.y), \mu^*(x.z)\} \geq \inf \mu^*(x.y.n.z)$
by using definition of $\mu^*$
$\max\{\mu(x.y) + 1 - \mu(0), \mu(x.z) + 1 - \mu(0)\} \geq \inf \mu(x.y.n.z) + 1 - \mu(0)$
$\max\{\mu(x.y), \mu(x.z)\} + 1 - \mu(0) \geq \inf \mu(x.y.n.z) + 1 - \mu(0)$
$\max\{\mu(x.y), \mu(x.z)\} \geq \inf \mu(x.y.n.z) \Rightarrow \mu$ is x-3-P-F-I of N.

**Corollary (2.11)**
Let f:$N_1 \rightarrow N_2$ be an a epimorphism $\mu^*$ is x-3-P-F-I of $N_2$ if and only if $f^{-1}(\mu^*)$ is an x-3-P-F-I of $N_1$.
**Proof**
Directy from theorem (2.7)

**Theorem (2.12)**
A fuzzy ideal $\mu$ of a near ring N is an x-3-P-F-I of N if and only if $x_t.y_t.n_t.z_t \in \mu$ implies $x_t.y_t \in \mu$ or $x_t.z_t \in \mu$ for all fuzzy singlton $xt, yt, nt, zt \in N$.
**Proof**
$\rightarrow$ Let $xt, yt, nt, zt \in N$ , $t \in [0, \mu(0)]$, $xt.yt.nt.zt \in \mu \rightarrow (x.y.n.z)_t \in \mu \rightarrow x.y.n.z \in \mu_t$ ,
$\mu(x.y.n.z) \geq t$ since $\mu$ is x-3-P-F-I of N , $\max\{\mu(x.y), \mu(x.z)\} \geq \inf \mu(x.y.n.z)$
$\max\{\mu(x.y), \mu(x.z)\} \geq \inf \mu(x.y.n.z) \geq t \rightarrow \max\{\mu(x.y), \mu(x.z)\} \geq t$ either
$\mu(x.y) \geq t$ or $\mu(x.z) \geq t \rightarrow (x.y)_t \in \mu$ or $(x.z)_t \in \mu \rightarrow x_t.y_t \in \mu$ or $x_t.z_t \in \mu$
$\leftarrow$ let $xt, yt, nt, zt \in N$ suppose that
$x_t.y_t.n_t.z_t \in \mu \rightarrow x_t.y_t \in \mu \vee x_t.z_t \in \mu, \rightarrow x_t.y_t.n_t.z_t \in \mu$
$(x.y.n.z)_t \in \mu \Rightarrow \mu(x.y.n.z) \geq t = \inf \mu(x.y.n.z)$
$x_t.y_t \in \mu \vee x_t.z_t \in \mu, \rightarrow x_t.y_t.n_t.z_t \in \mu$
$\mu(x.y) \geq t \vee \mu(x.z) \geq t$
$\max\{\mu(x.y), \mu(x.z)\} \geq t$
$\max\{\mu(x.y), \mu(x.z)\} \geq \inf \mu(x.y.n.z)$
$\Rightarrow \mu$ is x-3-P-F-I of N.

**Theorem (2.13)**

Let $\{\mu_i\}_j \in J$ be a family of is x-3-P-F-I of N, then $\bigcap_{j\in J}\mu_j$ is x-3-P-F-I of N.

**Proof**

Let x, y, z, n $\in$ N and $\{\mu_i\}_j \in J$ be a famaly of x-3-P-F-I of N where $\bigcap_{j\in J} \mu_j(x.y) = \inf \mu_j(x.y)$

$\max\{\mu_j(x.y), \mu_j(x.z)\} \geq \inf_{j\in J} \mu_j(x.y.n.z)$

$\max\{\inf_{j\in J} \mu_j(x.y), \inf_{j\in J} \mu_j(x.z)\} \geq \inf(\inf_{j\in J} \mu_j(x.y.n.z))$

$\max\{\bigcap_{j\in J} \mu_j(x.y), \bigcap_{j\in J} \mu_j(x.z)\} \geq \inf(\bigcap_{j\in J} \mu_j(x.y.n.z))$

$\max\{\mu(x.y), \mu(x.z)\} \geq t$

$\max\{\mu(x.y), \mu(x.z)\} \geq \inf \mu(x.y.n.z)$

$\Rightarrow \bigcap_{j\in J}\mu_j$ is x-3-P-F-I of N.

**Remark (2.14)**

Let $\{\mu_i\}_{j\in J}$ be chain of a x-3-P-F-I of N.,then $\bigcup_{j\in J}\mu_j$ is x-3-P-F-I of N.

**Proof**

By using Remark (1.16).

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