On Π-flat Modules

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Abstract :

We investigate the strongly **Π**-regularity of rings whose simple singular right R-module are **Π**-flat . Next we give the following notion , a ring R is said to be right (left) **ΠF**-ring , if for any maximal right (left) ideal M of R and any $y \in M$, R/yM (R/My) is **Π**-flat right (left) R-module .Conditions are given for each rings to be regular rings and right Kasch rings .

1-Introduction :

Throughout this paper R denotes an associative ring with identity , and R-modules are unital . For a subset X of a ring R ,the right annihilator of X in R is $r_R(X)=\{r \in R : xr=0$ for all $x \in X\}$ .For any $a \in R$,we write $r_R(a)$ for $r_R(\{a\})$. Some times ,we simply write $r(X)$ for $r_R(X)$ and $r(a)$ for $r_R(a)$. Left annihilators are defined analogously .We use $J(R), Z(R)(Y(R))$, for the Jacobson radical and the left (right) singular ideal of R , respectively .

Following [5] a ring R is called a right (left) SF-ring if all of its simple right (left) R-modules are flat .It is well known that a ring R is regular if and only if every right (left) R-module is flat [6] .As a generalization of this concept Mahmood and Mohammed [3] ,defined right (left) simple **Π**-flat ring, where we call a ring R right (left) simple **Π**-flat if every simple right (left) R-module is **Π**-flat. An element k of a ring R is called left minimal if Rk is a minimal left ideal of R , and an idempotent e of R is said to be left minimal idempotent if e is a left minimal element of R, An idempotent element $e \in R$ is said to be right semicentral element if $ea = eae$ for all $a \in R$ [9] . A ring R is called strongly right min-able if every right minimal idempotent element is left semicentral [9] .The ring R is said to be reduced if R has no nonzero nilpotent elements[13] . The ring R is called right (left) SXM [9] , if for each $0 \neq a \in R$, $r(a)=r(a^n)$ [$l(a)=l(a^n)$] for all positive integer n satisfying $a^n \neq 0$. For example ,reduced rings are right (left)SXM ring .A ring R is called reversible [2] if for $a,b \in R$, $ab=0$ implies $ba=0$.A ring R is said to be regular (strongly regular) if , $a \in aRa$ ($a \in a^2R$)for every $a \in R$,and R is called strongly **Π**-regular if $a^n \in a^{2n}R$, for some positive integer n [8], A ring R is said to be ERT ring if every essential right ideal is a two-sided ideal of R[12]. A ring R is called right MC2-ring if eRa=0 implies aRe=0, where a, $e^2 = e \in R$ and eR is minimal right ideal of R, or equivalently if $K \approx eR$ are minimal, $e^2 = e \in R$; then K=gR for some $g^2 = g \in R$ [11].

In section 2 ,we study first simple right R-module is **Π**-flat .Next ,we show that , R is strongly **Π**-regular if R is strongly right min-able ring and every simple right R-module is **Π**-flat .

In section 3, we introduce the notion of **Π**F-ring .Next , we show that if R is an ERT, SXM , fully right idempotent and **Π**F-ring, then R is regular .

2-Π-Flat Modules

In this section develop meats of **Π-flat** modules with some of its and some basic properties are given. Also we give the connection between simple (simple singular) **Π**-flat modules and strongly regular rings.

Definition 2.1 : [3]

Let I be a right (left) ideal of R .Then R/I is a right (left) **Π-flat** R-modules if and only if for each $a \in I$, there exist $b \in I$ and a positive integer n such that $a^n \neq 0$ and $a^n = ba^n (a^n = a^n b)$. The ring R is called right (left) simple **Π-flat** if every simple right (left) R-module is **Π-flat**. While [3] and [4] took the term **N -flat** for this notion.

Following [3] let I be a right (left) ideal of R .Then R/I is a right (left) generalized flat module if and only if for each $a \in I$,there exists $b \in I$ and a positive integer n such that $a^n = ba^n (a^n = a^n b)$ .The ring R is called right (left) generalized SF-ring if every simple right (left) R-module is right (left) generalized flat .

Examples and Remarks :

1-Let Z_6 be the ring of integers modulo 6 and I={0,2,4} ,J={0,3}. Then Z_6/I ,and Z_6/J are **Π**-flat .Therefore Z_6 is **Π**-flat ring .

2- Every SF-ring is simple **Π**-flat ring .

3- It is clear that in case of reduced rings , generalized flat modules
coincides with **Π-flat.**

4-Obviously right **Π-flat** modules are right **generalized flat modules** , but the converse is not true .

5-Let Z_9 be the ring of integers modulo 9 and K={0,3,6}.Then Z_9/K is generalized flat which is not **Π-flat**.

The following proposition gives the relation between right and left **Π**-flat modules .

Proposition 2.2 :

If R is reversible ring and I is any ideal of R , then R/I is a right **Π**-flat R- module if and only if R/I is left **Π**-flat R-module.

Proof:

Assume that R/I is a right **Π**-flat module, then for each $a \in I$, there exists $b \in I$ and a positive integer n such that $a^n \neq 0$ and $a^n = ba^n$ implies that $1-b \in l(a^n) = r(a^n)$ (R is reversible) . So $a^n = a^n b$. Therefore R/I is left **Π**-flat R-modules. Similarly we prove the convers . ■

The following results are given in [4] and [11] respectively .

Lemma 2.3 :

Let R be a strongly right min-able ring satisfy condition $l(a) \subseteq r(a)$,if every simple singular right R-module is **Π-flat** .Then R is a semiprime ring.

Lemma 2.4 :

If for any $a \in R$ and right minimal idempotent $e \in R$ with $eaR = 0$ implies $a\mathrm{Re} = 0$, then R is right $MC2$ ring .

Recall that , a ring R is called fully left (right) idempotent ring if every left (right) ideal of R is idempotent [12] .

Theorem 2.5 :

Let R be a left SXM .Then The following conditions are equivalent :

1-R is a fully left idempotent ring .

2-R is ERT and R/N is **Π**-flat where N is essential right ideal .

Proof :

(1)→(2) ,let E be an essential right ideal of R which is an ideal of R .For any $y \in E$ and there exists a positive integer n such that $y^n \neq 0$, $Ry^n = (Ry^n)^2$ which implies that $y^n = uy^n$ for some $u \in Ry^n R \subseteq E$. Therefore $y^n \in Ey^n$ for each $y^n \in E$. This proves that R/E is right **Π**-flat .

(2) →(1) : For any $a \in R$. Set $T = RaR + l(RaR)$. Let K be a complement right ideal of T in R then $T \oplus K$ is an essential right ideal of R.Now $KRaR \subseteq K \cap RaR \subseteq K \cap T = 0$ implies that $K \subseteq l(RaR)$,whence $K \subseteq K \cap T = 0$. This show that T is essential right ideal of R ,which is an ideal of R.By hypothesis R/T is right **Π**-flat .So for every $a \in T$, there exists a positive integer n such that $a^n \neq 0$ and $a^n = da^n$ for some $d \in T$ implies that $(1-d) \in l(a^n) = l(a)$ (R is left SXM) and so $a = da$. If $d = u + v, u \in RaR, v \in l(RaR)$,then $da = a = ua + va = ua$,which implies that $a \in (Ra)^2$ where $Ra = (Ra)^2$.Hence R is fully left idempotent ring. ■

Theorem 2.6 :

Let R be left SXM ring . Then R is regular ring if and only if every cyclic singular right R-module is **Π**-flat .

Proof :

Let $b \in R$. Then there exists a right ideal K such that $L = bR \oplus K$ is essential in R. Now , the cyclic singular right R-module, R/L is **Π**-flat ,then there exists a positive integer n such that $b^n \neq 0$ and $b^n = cb^n$, with some $c \in L$. So $b = cb$ (R is SXM) .Setting $c = ba + k$ $(a \in R, k \in K)$ implies that $b = cb = bab + kb$ we have $b - bab = bk \in bR \cap K = 0$. Therefore $b = bab$.

The converse direction is clear . ■

The following lemma is proved in [14] .

Lemma 2.7 :

Let $Y(R) \neq 0$, then there exists $0 \neq x \in Y(R)$ such that $x^2 = 0$.

A consequence of the Lemmas (2.4 and 2.7), we have the following theorem :

Theorem 2.8 :

Let R be a ring with every minimal idempotent element of R be a left semiceneral and $l(a) \subseteq r(a)$ for every $a \in R$.If every simple singular right R-module is **Π-flat ,** then Z(R) =0 .

Proof :

If $eRa = 0$ where $a \in R$ and $e \in R$ is right minimal idempotent .By hypothesis ,e is left semicentral of R ,thus $a\mathrm{Re} = ea\mathrm{Re} = 0$. By Lemma (2.4). R is right MC2 ring .Now suppose that $Z(R) \neq 0$,then there exists $0 \neq a \in Z(R)$,such that $a^2 = 0$, Lemma (2.7) .We claim that $Z + r(a) = R$. Otherewise there exists a maximal right ideal M such that $Z + r(a) \subseteq$ M .If M is not essential , then M= $r(e)$, $e^2 = e \in R$.Hence $ea = 0$,because $a \in r(a) \subset r(e)$. If $eRa \neq 0$, then $eRaR = eR$ because eR is a minimal right ideal of R .Since $a \in Z(R), eRaR \subseteq Z(R)$,then $e \in Z(R)$,which is a contradiction. Hence $eRa = 0$ and so $a\mathrm{Re} = 0$, (R is MC2 ring) $e \in r(a) \subset r(e)$ which is a contradiction .Hence M is essential in R. Thus R/M is **Π-flat** , so there exists a positive integer n such that $a^n \neq 0$ and $a^n = ba^n$ .Since $a^2 = 0$,then $a = ba$ this implies that $(1-b) \in l(a) \subseteq r(a) \subseteq M$, so $1 \in M$,which is a contradiction .Hence $Z + r(a) = R$ .Write 1=x+y , $x \in Z(R), y \in r(a)$,then a=ax. Since $x \in Z(R)$ and $l(x) \cap l(1-x) = 0$, $l(1-x) = 0$. Thus $a = 0$,because $a \in l(1-x)$,which is a contradiction . Therefore $Z(R) = 0$ ■

Following [9] , a right R-module M is called YJ-injective if for any $0 \neq a \in R$ there exists a positive integer n such that $a^n \neq 0$ and every right R-homomorphism from $a^n R$ to M extends to one from R_R to M ; R is called right YJ-injective if the right R-module R_R is YJ-injective .

Proposition 2.9 :

Let R be a ring with every simple right R-module is either YJ-injective or Π-flat .Then $J(R) = 0$ if and only if $J(R)$ is a reduced ideal of R .

Proof :

Suppose that $J(R)$ is a non zero reduced ideal of R . If $0 \neq b \in J$, Set $L = bR + r(b)$. If we suppose that $L = R$, then $1 = ba + c$, $a \in R$, $c \in r(b)$, which implies that $b = b^2 a$. since $b \in J$, $(b - bab) \in J$ and $(b - bab)^2 = 0$ yields $b = bab$. Therefore $b = eb$, where $e = ba$ is idempotent , Since J can not contain a non zero idempotent , then $b = 0$. This proves that $L \neq R$. Let M be a maximal right ideal of R containing L . If R/M is YJ-injective , then there exists a positive integer n such that $b^n \neq 0$ and any right R-homomorphism define the map $f : b^n R \to R/M$ by $f(b^n a) = a + M$ for all $a \in R$. Then $f(b^n) = cb^n + M$ for some $c \in R$ and therefore $1 + M = cb^n + M$, which implies that $1 - cb^n \in M$, whence $1 \in M$ (because $cb^n \in J / cM$), contradicting $R \neq M$ .If R/M is Π-flat .Since $b \in M$ then there exists a positive integer n such that $b^n \neq 0$ and $b^n = db^n$ for some $d \in M$. Now $1 - d \in l(b^n) = r(b) \subseteq M$ (J reduced) which implies that $1 \in M$, again a contradiction .This proves that if J is reduced ,then $J = 0$.

Conversely : It is trivial .

Theorem 2.10 :

Let R be a ring with the following properties :

1- $J(R) = 0$

2- every maximal left ideal M of R is an ideal such that R/M is either a left or right Π-flat R-module .Then R is strongly regular .

Proof :

Let $a \in R$ such that $a^2 = 0$ .Suppose there exists a maximal left ideal T not containing a . Then $l(a) \not\subset T$ implies $R = T + l(a)$ and if $1 = y + b$, $y \in T$, $b \in l(a)$, when $a = ya \in T$ (an ideal of R) which is a contradiction . This proves that $a \in J = 0$ which implies that R is reduced .

Now suppose that R is not strongly regular .Then there exists $c \in R$ such that $Rc + l(c) \neq R$. Let M be a maximal left ideal containing $Rc + l(c)$. If R/M is a left Π-flat ,Since $c \in M$ then there exists a positive integer n such that $c^n \neq 0$ and $c^n = c^n d$ for some $d \in M$ and since R is reduced , then $c = dc$ ($l(c^n = l(c)$). The same result holds if R/M is right Π-flat . Now $(1-d) \in l(c) \subseteq M$ implies $1 \in M$, contradicting $M \neq R$ .Therefore $Rc + l(c) = R$ and R is strongly regular ring .

Now, we recall the following result which are due to Wei [9]

Lemma 2.11 :

Let R be a ring .Then R is a reduced if and only if R is semiprime reversible ring .

The next result is considered a necessary and sufficient condition for rings whose simple singular right R-module is **Π-flat** to be strongly ring .

Theorem 2.12:

Let R be a reversible ring . Then R is strongly **Π-regular** if R is strongly right min-able ring and every simple singular right R-module is **Π-flat .**

Proof :

From Lemma (2.3 and 2.11) R is reduced . Next , we shall show that $a^n R + r(a^n) = R$ for all $a \in R$ and a positive integer n . Suppose that there exsits $b \in R$ such that $b^n R + r(b^n) \neq R$. Then there exists a maximal right ideal M of R containing $b^n R + r(b^n)$. Observe that M must be an essential right ideal of R . If not , then M is a direct summand of R .So we can write $M = r(e)$ for some $0 \neq e^2 = e \in R$ and hence $eb^n = 0$ .Because eR is a minimal right ideal of R .Since R is a strongly right min-able ring, $b^n e = eb^n e = 0$ .Thus $e \in r(b^n) \subseteq r(e)$. whence $e = 0$ .This is a contradiction . Therefore M must be an essential right ideal of R . Thus

R/M is **Π**-flat ,hence there exists a positive integer m such that $(b^n)^m \neq 0$ and $(b^n)^m = c(b^n)^m$ for some $c \in M$ implies that $(1-c) \in l(b^n)^m = r((b^n))$ (R is reduced) and so $1 \in M$, which is a contradiction . Therefore $a^nR + r(a^n) = R$. In particular $a^n x + y = 1, x \in R, y \in r(a^n)$ .Thus $a^{2n}x = a^n$. Therefore R is strongly **Π-regular** . ■

3. ΠF- rings :

In this section we introduces the notion of **ΠF**-ring . We study such ring and give some of its basic properties . Also we give the connection of **ΠF**-rings and other rings .

Definition 3.1 :
A ring R is said to be right(left)**ΠF**-ring ,if for any maximal right (left) ideal M of R and any $y \in M, R/yM$ (R/My) is **Π**-flat right (left) R-module .

Examples

1-Let Z_{10} be a ring of integer module 10 and $M_1 = (2)$, $M_2 = (5)$ are both maximal ideals of Z_{10}. Clearly for every $a \in M_1$ and $b \in M_2$, Z_{10}/aM_1 and Z_{10}/bM_2 are **Π**-flat . Therefore Z_{10} is **ΠF**-ring .

2- Let Z_{12} be the ring of integers modulo 12 and $M_1 = (2)$, $M_2 = (3)$, are maximal's ideals in Z_{12} . Clearly for every $a \in M_1$, $b \in M_2$, Z_{12}/aM_1 and Z_{12}/bM_2 are **Π**-flat .Therefore Z_{12} is **ΠF**-ring . On the other hand Z_{12} is not **Π**-flat .(Z_{12}/M_1 is not **Π**-flat)

Theorem 3.2:
Let R be a right **Π**F-ring and M a maximal right ideal of R .Then for any $a \in M$, $R = M + l(a^{2n})$, for some positive integer n .

Proof:
Let M be a maximal right ideal of R and $a \in M$. Since R is **Π**F-ring, then R/aM is **Π**-flat and there exists a positive integer n such that $a^{2n} \neq 0$ and $a^{2n} = aba^{2n}$ for some $b \in M$. This implies that $1 - ab \in l(a^{2n})$. Now consider $1 = ab + 1 - ab$. Therefore $R = M + l(a^{2n})$ ■

In [7] proved that :

Lemma 3.3 :
If $0 \neq a \in Y(R)$, then $r(1-a) = 0$.

Next , we shall give several basic properties of right **ΠF**-rings .

Proposition 3.4 :
Let R be a right **ΠF**-ring .Then
1-Any non zero divisor of R is invertible .
2- $Z \cap Y \subseteq J(R)$

.**Proof : (1)**
Let $a \in R$ be a non zero divisor of R. Then there exists a positive integer n such that $l(a^n) = 0$. Suppose that $aR \neq R$ .If M is maximal right ideal containing aR, then R/aM is **Π**-flat this implies that $(a^{2n}) = axa^{2n}$ for some $x \in M$ and $a^{2n} \neq 0$ so $(1-ax)a^{2n} = 0$ .Hence $(1-ax) \in l(a^{2n})$ (because $l(a^n) = 0$), thus $ax = 1$, which is a contradicts , $aR \neq R$. Therefore $aR = R$, and hence a is a right invertible .

Proof : (2)
Let $0 \neq y \in Z(R) \cap Y(R)$ .For any $a \in R$,if $c \in r(1-ya)$, then $c = yac$ and $cR \cap r(ya) = 0$ implies $c = 0$ .Similarly , if $d \in l(1-ya)$,since $ya \in Z(R)$, then $d = 0$ .By (1) ,(1-ya)w=1 ,for some $w \in R$,so $y \in J(R)$. ■

Proposition 3.5 :
Let R be **ΠF**-ring .Then $J(R) = 0$.

Proof :
Let $0 \neq u \in J(R) \subseteq M$ .Since R/uM is **Π**-flat ,then there exists a positive integer n such that $u^{2n} \neq 0$,and $u^{2n} = ubu^{2n}$,for some $b \in M$ .So $(1-ub)u^{2n} = 0$ and this implies that either $(1-ub) = 0$ or $u^{2n} = 0$ .But $u^{2n} \neq 0$ then $1 - ub = 0$,and hence $1 = ub \in J(R)$,a contradiction . Whence $J(R) = 0$ ■

Theorem 3.6 :
Let R is a left SXM and M is a left maximal ideal of R with the following properties :
a- R/M is a left flat and
b- For any $a \in M$, aM is a right ideal such that R/aM is a right **Π**-flat .Then $J(R) = 0$

Proof :
If $b \in J \subseteq M$,then R/bM is a right **Π**-flat ,and for any $c \in M, bc \in bM$ and there exists a positive integer n such that $(bc)^n \neq 0$ implies that $(bc)^n = bd(bc)^n$ for some $d \in M$. So $(1-bd) \in l(bc)^n = l(bc)$ (R is a left SXM) and $bc = bdbc$ .Since $w(1-bd) = 1$ for some $w \in R$ (because $bd \in J$) ,then $bc = w(1-bd)bc = w(bc - bdbc) = 0$, which yields $bM = 0$ .Now R/M is left flat which

implies that $b = bx$ for some $x \in M$ .Therefore $b = bx \in bM = 0$. Whence $J(R) = 0$. ■

Now ,we give the relation between **Π**F-ring and regular rings .

Theorem 3.7:

Let R be **ΠF**-ring and ,$l(a^2) \subseteq r(a)$ for every $a \in R$. Then every maximal ideal of R is **Π**-regular ring .

Proof :

Let M be a maximal right ideal of R. Since R is **ΠF**-ring then for every $a \in M, R/aM$ is **Π**-flat right R-module .So there exists a positive integer n and $a^{2n} \neq 0$ such that $(a)^{2n} = aba^{2n}$ for some $b \in M$ implies that $(1 - ab) \in l(a^{2n}) \subseteq r(a^n)$.Therefore $a^n = a^{n+1}b$ and so M is **Π**-regular ring . ■

A ring R is called right (left) Kasch ring if every maximal right (left) ideal of R is a right (left) annihilator [13].

Example 3 :

Let Z_3 be a ring of integer module 3 , and let $R = \begin{bmatrix} z_3 & z_3 \\ 0 & z_3 \end{bmatrix}$. Then R is right Kasch ring .

As a parallel result to [1,Th 2.4.5] , the following result was obtained:

Theorem 3.8 :

Let R a left SXM , right **ΠF**-ring and $l(a) \subseteq r(a)$ for every $a \in R$.Then R is right Kasch ring .

Proof :

Let M be a maximal right ideal in R. Suppose that $Y(R)$ is singular right ideal in R ,if $M \cap Y(R) = 0$, then for all $y \in Y(R), y \notin M$ and this implies that $r(y)$ right essential in R . From Lemma [3.3] we get $r(1-y) = 0$. So $l(1-y) = 0$ $(l(a) \subseteq r(a))$. From Proposition (3.4) ,$(1-y)$ is invertible in R .Hence $y \in J(R) \subseteq M$, which is a contradiction . So $M \cap Y(R) \neq 0$. Suppose that $0 \neq a \in M \cap Y(R)$, then $a \in M$ and R/aM is **Π** –flat right module and there exists a positive integer n such that for any $b \in M$ $(ab)^n \neq 0$,and $(a\ b)^n = ac(a\ b)^n$ for some $c \in M$ this implies that $ab = acab$ (left SXM) .We claim that $r(ac) \cap (ab)R = 0$ .If not ,suppose that $d \in r(ac) \cap (ab)R$,then $d = (ab)r$ and $acd = 0$ where $r \in R$,thus $ac(ab)r = 0$,this implies $(ab)r = 0$ hence $d = 0$,then $r(ac) \cap (ab)R = (0)$ .Since $r(ac)$ isessential right ideal then $(ab)R = 0$ thus $(ab) = 0$ and this implies that $b \in r(a)$, hence $M = r(a)$ Therefore R is right Kasch ring ■

The next result will consider some conditions for **ΠF**-rings to be a regular ring .

Theorem 3.9 :

Let R be an ERT, SXM , fully right idempotent and **Π**F-ring. Then
R is regular .

Proof :

Suppose there exists $b \in R$ such that bR is not a direct summand of R .

If K is a complement right ideal such that $bR \oplus K$ is an essential right ideal .Let M be a maximal right ideal containing $bR + K$ .Since R is fully right idempotent and M is an ideal of R ,then R/M is a left flat which implies $b = bd$ for some $d \in M$ .Now , R/bM is **Π**-flat then there exists a positive integer n such that $b^{2n} \neq 0$ and $b^{2n} = bcb^{2n}$ for some $c \in M$,this implies that $b = bcb, (l(b^{2n}) = l(b))$, which proves that bR is generated by the idempotent bc , contradicting our hypothesis.■

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(تاريخ الاستلام: ٩ / ١٠ / ٢٠١١ ---- تاريخ القبول: ١٣ / ٣ / ٢٠١٢)

الملخص

نحن نختبر الحلقات المنتظمة بقوة من النمط− Π عندما يكون فيها كل مقاس بسيط منفرد ايمن مسطح من النمط − Π . كذلك أعطينا صنف من الحلقات تسمى الحلقات من النمط − **ΠF** اليمنى (اليسرى)، لكل مثالي أعظمي أيمن (أيسر) M من R و $y \in M$ ، R/yM(R/My) مقاس مسطح من النمط Π أيمن (أيسر) . كذلك أعطينا الشروط لهذه الحلقات لكي تكون حلقات منتظمة وحلقات كاش .