Dr. Muhsin J. Jweeg
Engineering Mechanic Departmaent
Nahrain University

Dr. Wedad I. Alazzawy

A Suggested Analytical Solution For Laminated Closed Cylindrical Shells Using General Third Shell Theory (G.T.T.)

Muhsin J. Jweeg, Ph. D.,
Wedad I. Alazzawy, Ph. D.

▪ Nomenclature

a, b	Dimensions of shell.
A_{mn}, B_{mn}, C_{mn}, D_{mn}, E_{mn}, F_{mn}, J_{mn}	Arbitrary constants
B	constant
C_{ij}	stiffness matrix elements
E_1, E_2, E_3	Elastic Modulus components (GPa)
$f_{mn}(t)$	Generalized force (N)
g_1, g_2	Body forces (N)
G_{12}, G_{13}, G_{23}	Shear modulus components (GPa)
H	Thickness (mm)
K	Kinetic energy
L	Cylinder length (mm)
m, n	indices
N_i ,M_i, P_i, S_i (i=1,2,3,6)	Resultant reactions (N/mm),(N.mm)
m_1, m_2, m_3, m_4	Body moments (N.mm)
Q_i, K_i (i=4,5)	Resultant reactions (N/mm)
Q_{ij}	Elastic stiffness coefficients
q	Distributed transverse load (N/mm^2)
R	Cylinder radius (mm)
R_{11}, R_{22}	Principal radii of curvature of shell (mm)
U	Potential energy
u, v, w, φ_1, φ_2, ψ_1, ψ_2, ψ_3, θ_1, θ_2, θ_3	Displacement components (mm)
z	Distance from neutral axis (mm)
$\varepsilon_{1,2,3,4,5,6}$	Strain components in principal directions
υ_{12}, υ_{13}, υ_{23}	Poison's ratios
ρ	Density (Ns^2/mm^4)
ω	Frequency (rad/s)
$\sigma_{1,2,3,4,5,6}$	Stress components (MPa) in principal direction

Abstract:

Transient solutions will be developed for laminated simply supported closed cylindrical shells subjected to a uniform dynamic pressure at the outer surface of the cylinder. These solutions are obtained by using General Third Shell Theory (G.T.T.). Rectangular pulse, triangular pulse, sinusoidal pulse and (ramp-constant) load-time varying functions are studied and the required equilibrium equations are developed. The central deformation and principle stresses are investigated for different cross-ply laminates.

Keywords: *Laminate, Cylindrical, Shells.*

1.Introduction

With the increasing use of composite materials in many industries and especially in high performance aircraft industry, there is a need for assessing the response of laminated cylindrical shells to dynamic loading.

Analytical description of laminated composite shell is often based on classical laminate shell theory, which is an extension of the Love-Kirchhoff shell theory to composite shells. In Classical Shell Theory

(CST), the transverse strains are neglected under the assumption that straight lines normal to the middle surface are rigid. The neglect of transverse strains in composite laminates could lead to underestimation of deflections and overestimation of natural frequencies and critical buckling loads because of the very high transverse shear modulus compared to the in-plane Young's modulus [1].

High order shell theories are those in which the transverse strains are accounted. Y. Narita et al [2] developed a theoretical method for solving the free vibration angle-ply laminated cylindrical shells. The angle-ply laminated shell is macroscopically modeled as a thin shell of General anisotropy by using the classical lamination theory. The Functional derived from the flugge-type shell theory is minimized by following the Ritz procedure, and arbitrary combinations of boundary conditions at both ends are accommodated by introducing newly developed admissible functions.

Z.C.Xi et al [3] investigated the effects of shear non-linearity on free vibration of laminated composite shells of revolution using a semi-analytical method based on Reissner-Mindlin shell theory. The coupling between symmetric and anti-symmetric vibration modes of the shell is considered in the shear deformable shell element. Aleksandr Korjakin et al [4] used zig-zag model to investigate the free damped vibration of sandwich shells of revolution. As special cases the vibration analysis under consideration of damping of cylindrical, conical and spherical sandwich shells is performed. A specific sandwich shell finite element with 54 degrees of freedom is employed. Werner Hufenbach et al [5] developed an analytical solution for lightweight design using dynamically loaded fiber-reinforced composite shells. The analytic results were fully corroborated by accompanying FE calculations for special lay-ups. Humayun R. H. Kabir [6] investigated analytically the free vibration response of an arbitrarily laminated (crafted with advanced fiber reinforced composite materials)-thin and shallow cylindrical panels on rectangular planform with simply supported boundary conditions, using Kirchhoff-Love theory. J. J. Lee et al [7] used the finite element method based on Hellinger-Reissner principle with independent strain to analyze the vibration problem of cantilevered twisted plates, cylindrical and conical laminated shells. M. Amabili [8] investigated large-amplitude (geometrically non-linear) vibrations of circular cylindrical shells subjected to radial harmonic excitation in the spectral neighborhood of lowest resonance. Young-Shin Lee et al [9] investigated the free vibration analysis of a laminated composite cylindrical shell with an interior rectangular plate by analytical and experimental methods. The frequency equations of vibration of the shell including the plate are formulated by using the reacceptance method. S. C. Pradhan & J. N. Reddy [10] presented an analytical solution of laminated composite shells with embedded actuating layers. The magnetostrictive actuating layers are used to control natural vibration of laminated composite shell panels. The (FSDT) is used to represent the shell kinematics and equations of motion. Ghanim Shaker [11] presented a general content of the classical composite cylindrical shell theory and the first order shell theory for elasto-static and elasto-dynamic analysis of shells of circular cross section, incorporating for the first time the gathered effects of internal (and external) pressure, thermal gradient, axial end loading, and the conventional end condition, on the mechanical behavior of the structure. M. Darvizeh et al [12] presented a calculation of overall dynamic response of thin orthotropic cylindrical shells. Due to the obvious importance of the effects of transverse shear deformation and rotary inertia, these terms are included in the analysis. The exact method is modified to predict the dynamic behavior of an orthotropic circular cylindrical shell. In this work the developed analytical solution includes deriving the equation of motion using GTT for the first time to analyze displacement and stress components for forced vibration of laminated composite cylindrical shells.

2. Equations of motion:

In present study the high-order theory displacement field is:

$$\begin{aligned} u(x,\theta,z,t) &= u_0(x,\theta,t) + z\times\varphi_1(x,\theta,t) + z^2\times\psi_1(x,\theta,t) + z^3\times\theta_1(x,\theta,t) \\ v(x,\theta,z,t) &= v_0(x,\theta,t) + z\times\phi_2(x,\theta,t) + z^2\times\psi_2(x,\theta,t) + z^3\times\theta_2(x,\theta,t) \\ w(x,\theta,z,t) &= w_0(x,\theta,t) + z\times\psi_3(x,\theta,t) + z^2\times\theta_3(x,\theta,t) \end{aligned} \quad (1)$$

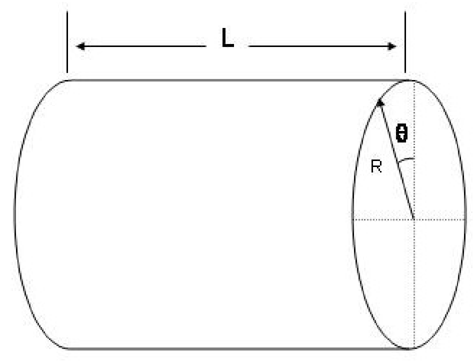


Figure (1) Cylinder geometry (L axis≡ axis 1, θ axis≡ axis 2 & R axis≡ axis 3).

$$\varepsilon_1=\frac{\partial u}{\partial x},\ \varepsilon_2=\frac{1}{R}\times\left(\frac{\partial v}{\partial\theta}\right)+\frac{w}{R},\ \varepsilon_3=\frac{\partial w}{\partial z},\ \varepsilon_6=\frac{1}{R}\times\left(\frac{\partial u}{\partial\theta}\right)+\frac{\partial v}{\partial x}$$
$$\varepsilon_5=\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}$$
$$\varepsilon_4=\frac{\partial v}{\partial z}-\frac{v}{R}+\frac{1}{R}\times\left(\frac{\partial w}{\partial\theta}\right) \tag{2}$$

Assuming that transverse shear stress vanishing at top and bottom of the laminated composite layers, and hence transverse strain also vanishes, so:

$$\varepsilon_5(x,\theta,\pm h/2,t)=\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}=0$$
$$\varepsilon_4(x,\theta,\pm h/2,t)=\frac{\partial v}{\partial z}-\frac{v}{R}+\frac{1}{R}\times\left(\frac{\partial w}{\partial\theta}\right)=0$$

The resulting strain-displacement relations are:

$$\varepsilon_1=\frac{\partial u_0}{\partial x}+z\times\frac{\partial\phi_1}{\partial x}-z^2\times\frac{\partial^2\psi_3}{\partial x^2}-z^3\times\left(\frac{4}{3\times h^2}\right)\times\left[\frac{\partial\phi_1}{\partial x}+\frac{\partial^2 w_0}{\partial x^2}+\left(\frac{h^2}{4}\right)\times\frac{\partial^2\theta_3}{\partial x^2}\right]$$
$$\varepsilon_2=\left(\frac{1}{R}\right)\times\left\{\begin{array}{l}\left(\frac{\partial v_0}{\partial\theta}+w_0\right)+z\times\left(\frac{\partial\phi_2}{\partial\theta}+\psi_3\right)+z^2\times\left(\left(\frac{-1}{2\times R}\right)\times\frac{\partial^2\psi_3}{\partial\theta^2}+\theta_3\right)\\ -z^3\times\left(\frac{4}{3\times h^2}\right)\times\left[\frac{\partial\phi_2}{\partial\theta}-\left(\frac{1}{R}\right)\times\frac{\partial v_0}{\partial\theta}+\left(\frac{1}{R}\right)\times\frac{\partial^2 w_0}{\partial\theta^2}+\left(\frac{h^2}{4R}\right)\times\frac{\partial^2\theta_3}{\partial\theta^2}\right]\end{array}\right\}$$
$$\varepsilon_3=\psi_3+2\times z\times\theta_3$$
$$\varepsilon_6=\frac{\partial v_0}{\partial x}+\left(\frac{1}{R}\right)\times\frac{\partial u_0}{\partial\theta}+z\times\left(\left(\frac{1}{R}\right)\times\frac{\partial\phi_1}{\partial\theta}+\frac{\partial\phi_2}{\partial x}\right)-\left(\frac{z^2}{R}\right)\times\frac{\partial^2\psi_3}{\partial x\partial\theta}$$
$$-z^3\times\left(\frac{4}{3h^2}\right)\times\left[\left(\frac{\partial\phi_2}{\partial x}+\left(\frac{1}{R}\right)\times\frac{\partial\phi_1}{\partial\theta}\right)-\left(\frac{1}{R}\right)\times\frac{\partial v_0}{\partial x}+\left(\frac{2}{R}\right)\times\frac{\partial^2 w_0}{\partial x\partial\theta}+\left(\frac{h^2}{2R}\right)\times\frac{\partial^2\theta_3}{\partial x\partial\theta}\right]$$
$$\varepsilon_4=\left(\phi_2-\frac{v_0}{R}+\left(\frac{1}{R}\right)\times\frac{\partial w_0}{\partial\theta}\right)-z^2\times\left(\frac{4}{h^2}\right)\times\left(\phi_2-\frac{v_0}{R}+\left(\frac{1}{R}\right)\times\frac{\partial w_0}{\partial\theta}\right)$$
$$\varepsilon_5=\phi_1+\frac{\partial w_0}{\partial x}-z^2\times\left(\frac{4}{h^2}\right)\times\left(\phi_1+\frac{\partial w_0}{\partial x}\right) \tag{3}$$

According to Hamilton's′ Principles:

$$\int_{t2}^{t1}(\delta U-\delta K)dt=0 \tag{4}$$

where:

$$\delta U=\iint_{Az}(\sigma_1\delta\varepsilon_1+\sigma_2\delta\varepsilon_2+\sigma_3\delta\varepsilon_3+\sigma_6\delta\varepsilon_6+\sigma_5\delta\varepsilon_5+\sigma_4\delta\varepsilon_4)*Rd\theta dzd$$

$$\delta K=-\int_t\iiint_V R\rho\left(\ddot{u}\,\delta u+\ddot{v}\,\delta v+\ddot{w}\,\delta w\right)dzdxd\,\theta dt$$

From above 7equations of motion the following equations are obtained:

$$\frac{\partial N_1}{\partial x}+\left(\frac{1}{R}\right)\frac{\partial N_6}{\partial\theta}+g_1=I_1\ddot{u}+I_2\ddot{\phi}_2-\left(\frac{I_3}{2}\right)\frac{\partial\ddot{\psi}_3}{\partial x}-\left(\frac{4I_4}{3h^2}\right)\left(\ddot{\phi}_1+\frac{\partial\ddot{w}}{\partial x}\right)-\left(\frac{I_4}{3}\right)\frac{\partial\ddot{\theta}_3}{\partial x} \tag{5}$$

$$\begin{aligned}&\frac{\partial N_2}{\partial\theta}+R\frac{\partial N_6}{\partial x}+Q_4-\left(\frac{4}{h^2}\right)K_4+\left(\frac{4}{3h^2}\right)\left(\left(\frac{1}{R}\right)\frac{\partial S_2}{\partial\theta}+\frac{\partial S_6}{\partial x}\right)+g_2=\left(RI_2-\left(\frac{4RI_4}{3h^2}\right)-\left(\frac{16I_7}{9h^4R}\right)+\left(\frac{4I_5}{3h^2}\right)\right)\ddot{\phi}_2+\\&\left(RI_1+\left(\frac{8I_4}{3h^2}\right)+\left(\frac{16I_7}{9h^4R}\right)\right)\ddot{v}+-\left(\left(\frac{I_3}{2}\right)+\left(\frac{4I_6}{6Rh^2}\right)\right)\frac{\partial\ddot{\psi}_3}{\partial\theta}+\left(\left(\frac{4I_4}{3h^2}\right)-\left(\frac{16I_7}{9h^4R}\right)\right)\frac{\partial\ddot{w}}{\partial\theta}-\left(\frac{I_4}{3}+\left(\frac{4I_7}{9h^2R}\right)\right)\frac{\partial\ddot{\theta}_3}{\partial x}\end{aligned} \tag{6}$$

$$\left(\frac{4}{3h^2}\right)\left(\frac{1}{R}\frac{\partial^2 S_2}{\partial\theta^2}+R\frac{\partial^2 S_1}{\partial x^2}+2\frac{\partial^2 S_6}{\partial x\partial\theta}\right)-N_2-\left(\frac{4}{h^2}\right)\left(\frac{\partial K_4}{\partial\theta}+R\frac{\partial K_5}{\partial x}\right)+\left(\frac{\partial Q_4}{\partial\theta}+R\frac{\partial Q_5}{\partial x}\right)+q=RI_1\ddot{w}+$$
$$\left(-\left(\frac{16I_7}{9h^4R}\right)+\left(\frac{4I_5}{3h^2}\right)\right)\frac{\partial\ddot{\phi}_2}{\partial\theta}+RI_2\ddot{\psi}_3-\left(\frac{4I_6}{6Rh^2}\right)\frac{\partial^2\ddot{\psi}_3}{\partial\theta^2}+\left(\frac{4I_4}{3h^2}\right)-\left(\frac{16I_7}{9h^4}\right)\left(\left(\frac{1}{R}\right)\frac{\partial^2\ddot{w}}{\partial\theta^2}+R\frac{\partial^2\ddot{w}}{\partial x^2}\right)+RI_3\ddot{\theta}_3$$
$$-\left(\frac{4I_7}{9h^2R}\right)\left(R\frac{\partial^2\ddot{\theta}_3}{\partial x^2}+\frac{1}{R}\frac{\partial^2\ddot{\theta}_3}{\partial x^2}\right)+R\left(-\left(\frac{16I_7}{9h^4R}\right)+\left(\frac{4I_5}{3h^2}\right)\right)\frac{\partial\ddot{\phi}_1}{\partial x}+\left(\left(\frac{4I_4}{3h^2}\right)+\left(\frac{16I_7}{9h^4R}\right)\right)\frac{\partial\ddot{v}}{\partial\theta}+\left(\frac{4I_4}{3h^2}\right)\frac{\partial\ddot{u}}{\partial x} \tag{7}$$

$$R\frac{\partial M_1}{\partial x}-\left(\frac{4R}{3h^2}\right)\frac{\partial S_1}{\partial x}+\frac{\partial M_6}{\partial\theta}-\left(\frac{4R}{3h^2}\right)\frac{\partial S_6}{\partial\theta}-RQ_5+\left(\frac{4R}{h^2}\right)K_5=\left(RI_2-\frac{4RI_4}{3h^2}\right)\ddot{u}+\left(RI_3-\frac{8RI_5}{3h^2}+\frac{16RI_7}{9h^4}\right)\ddot{\phi}_1+$$
$$\left(\frac{4RI_6}{6h^2}-\frac{RI_4}{2}\right)\frac{\partial\ddot{\psi}_3}{\partial x}+\left(-\frac{8RI_5}{3h^2}+\frac{16RI_7}{9h^4}\right)\frac{\partial\ddot{w}}{\partial x}+\left(-\frac{RI_5}{3}+\frac{4RI_7}{9h^2}\right)\frac{\partial\ddot{\theta}_3}{\partial x} \tag{8}$$

$$\frac{\partial M_2}{\partial\theta}-\left(\frac{4R}{3h^2}\right)\frac{\partial S_2}{\partial\theta}+R\frac{\partial M_6}{\partial x}-\left(\frac{4R}{3h^2}\right)\frac{\partial S_6}{\partial x}-RQ_4+\left(\frac{4R}{h^2}\right)K_4+m_2=\left(RI_2-\frac{16RI_7}{9h^4}\right)\ddot{v}+\left(RI_3-\frac{8RI_5}{3h^2}+\frac{16RI_7}{9h^4}\right)\ddot{\phi}_2+$$
$$\left(\frac{4I_6}{6h^2}-\frac{I_4}{2}\right)\frac{\partial\ddot{\psi}_3}{\partial\theta}+\left(-\frac{4I_5}{3h^2}+\frac{16I_7}{9h^4}\right)\frac{\partial\ddot{w}}{\partial\theta}+\left(-\frac{I_5}{3}+\frac{4I_7}{9h^2}\right)\frac{\partial\ddot{\theta}_3}{\partial\theta} \tag{9}$$

$$\left(\frac{R}{2}\right)\frac{\partial^2 P_1}{\partial x^2}+\left(\frac{1}{2R}\right)\frac{\partial^2 P_2}{\partial\theta^2}+\frac{\partial^2 P_6}{\partial x\partial\theta}-RN_3-M_2=\left(\frac{RI_3}{2}\right)\frac{\partial\ddot{u}}{\partial x}+\left(\frac{RI_4}{2}-\frac{4RI_6}{6h^2}\right)\frac{\partial\ddot{\phi}_1}{\partial x}+\left(\frac{I_3}{2}+\frac{4I_6}{6Rh^2}\right)\frac{\partial\ddot{v}}{\partial\theta}$$
$$-\left(\frac{RI_5}{4}\right)\frac{\partial^2\ddot{\psi}_3}{\partial x^2}-\left(\frac{I_5}{4R}\right)\frac{\partial^2\ddot{\psi}_3}{\partial\theta^2}+RI_3\ddot{\psi}_3+\left(-\frac{2RI_6}{3h^2}\right)\frac{\partial^2\ddot{w}}{\partial x^2}+\left(-\frac{4I_6}{6Rh^2}\right)\frac{\partial^2\ddot{w}}{\partial\theta^2}+RI_2\ddot{w}+\left(-\frac{RI_6}{6}\right)\frac{\partial^2\ddot{\theta}_3}{\partial x^2}-\left(\frac{I_6}{6R}\right)\frac{\partial^2\ddot{\theta}_3}{\partial\theta^2}$$
$$+RI_4\ddot{\theta}_3+\left(\frac{I_4}{2}-\frac{4I_6}{6h^2}\right)\frac{\partial\ddot{\phi}_2}{\partial\theta} \tag{10}$$

$$\left(\frac{R}{3}\right)\frac{\partial^2 S_1}{\partial x^2}+\left(\frac{1}{3R}\right)\frac{\partial^2 S_2}{\partial\theta^2}+\left(\frac{2}{3}\right)\frac{\partial^2 S_6}{\partial x\partial\theta}-2RM_3-P_2+m_4=\left(\frac{RI_4}{3}\right)\frac{\partial\ddot{u}}{\partial x}+\left(\frac{RI_5}{3}-\frac{4RI_7}{9h^2}\right)\frac{\partial\ddot{\phi}_1}{\partial x}+\left(\frac{I_4}{3}+\frac{4I_7}{9Rh^2}\right)\frac{\partial\ddot{v}}{\partial\theta}$$
$$-\left(\frac{RI_6}{6}\right)\frac{\partial^2\ddot{\psi}_3}{\partial x^2}-\left(\frac{I_6}{6R}\right)\frac{\partial^2\ddot{\psi}_3}{\partial\theta^2}+RI_4\ddot{\psi}_3+\left(-\frac{4RI_7}{9h^2}\right)\frac{\partial^2\ddot{w}}{\partial x^2}+\left(-\frac{4I_7}{9Rh^2}\right)\frac{\partial^2\ddot{w}}{\partial\theta^2}+RI_3\ddot{w}+\left(-\frac{RI_7}{9}\right)\frac{\partial^2\ddot{\theta}_3}{\partial x^2}-\left(\frac{I_7}{9R}\right)\frac{\partial^2\ddot{\theta}_3}{\partial\theta^2}$$
$$+RI_5\ddot{\theta}_3+\left(\frac{I_5}{3}-\frac{4I_7}{9h^2}\right)\frac{\partial\ddot{\phi}_2}{\partial\theta} \tag{11}$$

The constitutive relations of the kth lamina are:

$$\begin{Bmatrix}\sigma_1\\ \sigma_2\\ \sigma_3\\ \sigma_6\end{Bmatrix}=\begin{bmatrix}Q_{11} & Q_{12} & Q_{13} & Q_{16}\\ Q_{21} & Q_{22} & Q_{23} & Q_{26}\\ Q_{31} & Q_{32} & Q_{33} & Q_{36}\\ Q_{61} & Q_{62} & Q_{63} & Q_{66}\end{bmatrix}\times\begin{Bmatrix}\varepsilon_1\\ \varepsilon_2\\ \varepsilon_3\\ \varepsilon_6\end{Bmatrix} \tag{12}$$

and:

$$\begin{Bmatrix}\sigma_4\\ \sigma_5\end{Bmatrix}=\begin{bmatrix}Q_{44} & Q_{45}\\ Q_{54} & Q_{55}\end{bmatrix}\times\begin{Bmatrix}\varepsilon_4\\ \varepsilon_5\end{Bmatrix} \tag{13}$$

Substituting eqs.(12 and 13) in eq.(4) and then substituting the resultant forces and moments in equations of motion, the equations of motion are then solved by using Navier 's solution [2], which is presented as follows:

$$\begin{aligned}
u_0(x,\theta,z,t)&=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}A_{mn}\cos\alpha x\sin\beta\theta*T_{mn}(t)\\
v_0(x,\theta,z,t)&=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}B_{mn}\sin\alpha x\cos\beta\theta*T_{mn}(t)\\
w_0(x,\theta,z,t)&=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}C_{mn}\sin\alpha x\sin\beta\theta*T_{mn}(t)\\
\phi_1(x,\theta,z,t)&=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}D_{mn}\cos\alpha x\sin\beta\theta*T_{mn}(t)\\
\phi_2(x,\theta,z,t)&=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}E_{mn}\sin\alpha x\cos\beta\theta*T_{mn}(t)\\
\psi_3(x,\theta,z,t)&=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}F_{mn}\sin\alpha x\sin\beta\theta*T_{mn}(t)\\
\theta_3(x,\theta,z,t)&=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}J_{mn}\sin\alpha x\sin\beta\theta*T_{mn}(t)
\end{aligned} \tag{14}$$

where:

$\alpha=\left(\frac{m\pi}{L}\right)$, β=n

Amn, Bmn, Cmn, Dmn, Emn, Fmn, Jmn are arbitrary constants.

The stiffness and mass matrices will be obtained, then natural frequencies and their modes are also computed by solving eignvalue problems shown below:

$$\left|[C]-\omega^2[M]\right|\{A\}=0 \tag{15}$$

The orthogonality condition of principal modes can be established with the result as shown below:

$$\left(\omega_{mn}^2-\omega_{rs}^2\right)\int_0^L\int_0^{2\pi}\left\{\begin{array}{l}
\left[(I_1)A_{mn}+\left(I_2-\frac{4I_4}{3h^2}\right)D_{mn}-\left(\frac{\alpha}{2}\right)I_3J_{mn}-\left(\frac{4\alpha}{3h^2}\right)I_4C_{mn}-\left(\frac{\alpha}{3}\right)I_4F_{mn}\right]A_{sr}+ \\
\left[\begin{array}{l}\left(I_1+\frac{2I_2}{R_2}\right)B_{mn}+\left(I_2+\frac{I_3}{R_2}-\frac{4}{3h^2}\left(I_4+\frac{I_5}{R_2}\right)\right)E_{mn} \\ -\left(\frac{4\beta}{3h^2}\right)\left(I_4+\frac{I_5}{R_2}\right)C_{mn}-\left(\frac{\beta}{3}\right)\left(I_4+\frac{I_5}{R_2}\right)F_{mn}-\left(\frac{\beta}{2}\right)\left(I_3+\frac{I_4}{R_2}\right)J_{mn}\end{array}\right]B_{sr}+ \\
\left[\begin{array}{l}\left(I_1+\frac{16I_7}{9h^2}\left(\alpha^2+\beta^2\right)\right)C_{mn}+\left(I_2+\frac{4I_6}{6h^2}\left(\alpha^2+\beta^2\right)\right)J_{mn} \\ -\left(\frac{4\alpha}{3h^2}\right)I_4A_{mn}-\left(\frac{4\beta}{3h^2}\right)\left(I_4+\frac{I_5}{R_2}\right)B_{mn}-\left(\frac{4\alpha}{3h^2}\right)\left(I_5-\frac{4I_7}{3h^2}\right)D_{mn} \\ -\left(\frac{4\beta}{3h^2}\right)\left(I_5-\frac{4I_7}{3h^2}\right)E_{mn}+\left(I_3+\frac{4I_7}{9h^2}\left(\alpha^2+\beta^2\right)\right)F_{mn}\end{array}\right]C_{sr} \\
+\left[\begin{array}{l}\left(I_2-\frac{4I_4}{3h^2}\right)A_{mn}+\left(I_3-\frac{8I_5}{3h^2}+\frac{16I_7}{9h^4}\right)D_{mn}+\left(-\frac{I_4}{2}+\frac{4I_6}{6h^2}\right)\alpha J_{mn} \\ +\left(-\frac{4I_5}{3h^2}+\frac{16I_7}{9h^4}\right)\alpha W_{mn}+\left(-\frac{I_5}{3}+\frac{4I_7}{9h^2}\right)\alpha F_{mn}\end{array}\right]D_{sr}+ \\
\left[\begin{array}{l}\left(I_2+\frac{I_3}{R}-\frac{4}{3h^2}\left(I_4+\frac{I_5}{R}\right)\right)B_{mn}+\left(I_3-\frac{8I_5}{3h^2}+\frac{16I_7}{9h^4}\right)D_{mn} \\ +\left(-\frac{I_4}{2}+\frac{4I_6}{6h^2}\right)\beta J_{mn}+\left(-\frac{4I_5}{3h^2}+\frac{16I_7}{9h^4}\right)\beta C_{mn}+\left(-\frac{I_5}{3}+\frac{4I_7}{9h^2}\right)\beta F_{mn}\end{array}\right]E_{sr}+ \\
\left[\begin{array}{l}\frac{4I_6}{6h^2}\left(\alpha^2+\beta^2\right)C_{mn}+\frac{I_5}{4}\left(\alpha^2+\beta^2\right)J_{mn}+\frac{I_6}{6}\left(\alpha^2+\beta^2\right)F_{mn}-\left(\frac{\alpha}{2}\right)I_3A_{mn} \\ -\left(\frac{\beta}{2}\right)\left(I_3+\frac{I_4}{R}\right)B_{mn}-\left(\frac{\alpha}{2}\right)\left(I_4-\frac{4I_6}{3h^2}\right)D_{mn}-\left(\frac{\beta}{2}\right)\left(I_4-\frac{4I_6}{3h^2}\right)E_{mn}\end{array}\right]J_{sr}+ \\
\left[\begin{array}{l}\left(\frac{4I_7}{9h^2}\left(\alpha^2+\beta^2\right)+I_3\right)C_{mn}+\frac{I_6}{6}\left(\alpha^2+\beta^2\right)J_{mn}+\left(\frac{4I_7}{9h^2}\left(\alpha^2+\beta^2\right)+I_5\right)F_{mn} \\ -\left(\frac{1}{3}\right)\left(I_4+\frac{I_5}{R}\right)\beta B_{mn}-\left(\frac{\alpha}{3}\right)\left(I_5-\frac{4I_7}{3h^2}\right)D_{mn}-\left(\frac{\beta}{3}\right)\left(I_5-\frac{4I_7}{3h^2}\right)E_{mn}-\left(\frac{I_4}{3}\right)\alpha A_{mn}\end{array}\right]F_{sr}
\end{array}\right\}d\theta dx=0 \tag{16}$$

The general distributed loads are expanded in a series of principal modes as follows:

$$g_1=\sum_{m=n=1}^{\infty}f_{mn}(t)\times\left[(I_1)A_{mn}+\left(I_2-\frac{4I_4}{3h^2}\right)D_{mn}-\left(\frac{\alpha}{2}\right)I_3J_{mn}-\left(\frac{4\alpha}{3h^2}\right)I_4C_{mn}-\left(\frac{\alpha}{3}\right)I_4F_{mn}\right] \tag{17a}$$

$$g_2=\sum f_{mn}(t)\times\left[\begin{array}{l}\left(I_1+\frac{2I_2}{R_2}\right)B_{mn}+\left(I_2+\frac{I_3}{R_2}-\frac{4}{3h^2}\left(I_4+\frac{I_5}{R_2}\right)\right)E_{mn} \\ -\left(\frac{4\beta}{3h^2}\right)\left(I_4+\frac{I_5}{R_2}\right)C_{mn}-\left(\frac{\beta}{3}\right)\left(I_4+\frac{I_5}{R_2}\right)F_{mn}-\left(\frac{\beta}{2}\right)\left(I_3+\frac{I_4}{R_2}\right)J_{mn}\end{array}\right] \tag{17b}$$

$$q=\sum f_{mn}(t)\times\left[\begin{array}{l}\left(I_1+\frac{16I_7}{9h^2}\left(\alpha^2+\beta^2\right)\right)C_{mn}+\left(I_2+\frac{4I_6}{6h^2}\left(\alpha^2+\beta^2\right)\right)J_{mn} \\ -\left(\frac{4\alpha}{3h^2}\right)I_4A_{mn}-\left(\frac{4\beta}{3h^2}\right)\left(I_4+\frac{I_5}{R_2}\right)B_{mn}-\left(\frac{4\alpha}{3h^2}\right)\left(I_5-\frac{4I_7}{3h^2}\right)D_{mn} \\ -\left(\frac{4\beta}{3h^2}\right)\left(I_5-\frac{4I_7}{3h^2}\right)E_{mn}+\left(I_3+\frac{4I_7}{9h^2}\left(\alpha^2+\beta^2\right)\right)F_{mn}\end{array}\right] \tag{17c}$$

$$m_1=\sum f_{mn}(t)\times\left[\begin{array}{l}\left(I_2-\frac{4I_4}{3h^2}\right)A_{mn}+\left(I_3-\frac{8I_5}{3h^2}+\frac{16I_7}{9h^4}\right)D_{mn}+\left(-\frac{I_4}{2}+\frac{4I_6}{6h^2}\right)\alpha J_{mn} \\ +\left(-\frac{4I_5}{3h^2}+\frac{16I_7}{9h^4}\right)\alpha W_{mn}+\left(-\frac{I_5}{3}+\frac{4I_7}{9h^2}\right)\alpha F_{mn}\end{array}\right] \tag{17d}$$

$$m_2 = \sum f_{mn}(t)\times\left[\begin{aligned}&\left(I_2+\frac{I_3}{R}-\frac{4}{3h^2}\left(I_4+\frac{I_5}{R}\right)\right)B_{mn}+\left(I_3-\frac{8I_5}{3h^2}+\frac{16I_7}{9h^4}\right)D_{mn}\\&+\left(-\frac{I_4}{2}+\frac{4I_6}{6h^2}\right)\beta J_{mn}+\left(-\frac{4I_5}{3h^2}+\frac{16I_7}{9h^4}\right)\beta C_{mn}+\left(-\frac{I_5}{3}+\frac{4I_7}{9h^2}\right)\beta F_{mn}\end{aligned}\right]. \qquad \textbf{17e}$$

$$m_3 = \sum f_{mn}(t)\times\left[\begin{aligned}&\frac{4I_6}{6h^2}\left(\alpha^2+\beta^2\right)C_{mn}+\frac{I_5}{4}\left(\alpha^2+\beta^2\right)J_{mn}+\frac{I_6}{6}\left(\alpha^2+\beta^2\right)F_{mn}-\left(\frac{\alpha}{2}\right)I_3A_{mn}\\&-\left(\frac{\beta}{2}\right)\left(I_3+\frac{I_4}{R}\right)B_{mn}-\left(\frac{\alpha}{2}\right)\left(I_4-\frac{4I_6}{3h^2}\right)D_{mn}-\left(\frac{\beta}{2}\right)\left(I_4-\frac{4I_6}{3h^2}\right)E_{mn}\end{aligned}\right] \qquad \textbf{17f}$$

$$m_4 = \sum f_{mn}(t)\times\left[\begin{aligned}&\left(\frac{4I_7}{9h^2}\left(\alpha^2+\beta^2\right)+I_3\right)C_{mn}+\frac{I_6}{6}\left(\alpha^2+\beta^2\right)J_{mn}+\left(\frac{4I_7}{9h^2}\left(\alpha^2+\beta^2\right)+I_5\right)F_{mn}\\&-\left(\frac{1}{3}\right)\left(I_4+\frac{I_5}{R}\right)\beta B_{mn}-\left(\frac{\alpha}{3}\right)\left(I_5-\frac{4I_7}{3h^2}\right)D_{mn}-\left(\frac{\beta}{3}\right)\left(I_5-\frac{4I_7}{3h^2}\right)E_{mn}-\left(\frac{I_4}{3}\right)\alpha A_{mn}\end{aligned}\right] \qquad \textbf{17g}$$

The generalized forces $f_{mn}(t)$ are determined by making use of orthogonality condition. Multiplying eq. ((17)-a) by Amn, eq.((17)-b) by Bmn, eq.((17)-c) by Cmn, eq.((17)-d) by Dmn, eq.((17)-e) by Emn, eq.((17)-f) by Jmn and eq.((17)-g) by Fmn, and adding the results, integrating over the plane area, and taking into account eq. (16) leads to the following result:

$$f_{mn}(t) = \frac{\int_0^{2\pi}\int_0^{L}\left(g_1A_{mn}+g_2B_{mn}+qC_{mn}+m_1D_{mn}+m_2E_{mn}+s_1J_{mn}+s_2F_{mn}\right)dx\,d\theta}{N_{mn}} \qquad \textbf{18}$$

Substituting eq.(14) into equations of motion, taking into account eq.(15), gives:

$$\ddot{T}_{mn}+\omega^2_{mn}T_{mn} = f_{mn} \qquad \textbf{19}$$

For any (m, n).The solution to above equation is given by:

$$T_{mn}(t) = \frac{1}{\omega_{mn}}\int_0^t f_{mn}(\tau)\sin\omega_{mn}(t-\tau)d\tau \qquad \textbf{20}$$

For sinusoidal spatial distribution of load, q(x,θ,t)=q0sinαxsinβθF(t), (m=n=1), the formal solution to the unknown functions may be expressed as:

$$\begin{pmatrix}u\\v\\w\\\phi_1\\\phi_2\\\psi_3\\\theta_3\end{pmatrix} = \sum_{k=1}^{7}\frac{q_0}{N_{mn}(k)\omega_{mn}(k)}\begin{pmatrix}A_{mn}(k)f_1(x,\theta)\\B_{mn}(k)f_2(x,\theta)\\1\times f_3(x,\theta)\\D_{mn}(k)f_4(x,\theta)\\E_{mn}(k)f_5(x,\theta)\\J_{mn}(k)f_6(x,\theta)\\F_{mn}(k)f_7(x,\theta)\end{pmatrix}\int_0^t F(\tau)\sin\omega_{mn}(k)(t-\tau)d\tau \qquad \textbf{21}$$

It is noted that the solution in eq.(21) is normalized with respect to Cmn(k), the coefficients in expansion of w.

where:

$$
\mathrm{N}_{mn}=\int_0^L\int_0^{2\pi}\left\{\begin{aligned}
&\left[(I_1)A_{mn}+\left(I_2-\frac{4I_4}{3h^2}\right)D_{mn}-\left(\frac{\alpha}{2}\right)I_3J_{mn}-\left(\frac{4\alpha}{3h^2}\right)I_4C_{mn}-\left(\frac{\alpha}{3}\right)I_4F_{mn}\right]A_{mn}+\\
&\left[\begin{aligned}&\left(I_1+\frac{2I_2}{R_2}\right)B_{mn}+\left(I_2+\frac{I_3}{R_2}-\frac{4}{3h^2}\left(I_4+\frac{I_5}{R_2}\right)\right)E_{mn}\\&-\left(\frac{4\beta}{3h^2}\right)\left(I_4+\frac{I_5}{R_2}\right)C_{mn}-\left(\frac{\beta}{3}\right)\left(I_4+\frac{I_5}{R_2}\right)F_{mn}-\left(\frac{\beta}{2}\right)\left(I_3+\frac{I_4}{R_2}\right)J_{mn}\end{aligned}\right]B_{mn}+\\
&\left[\begin{aligned}&\left(I_1+\frac{16I_7}{9h^2}\left(\alpha^2+\beta^2\right)\right)C_{mn}+\left(I_2+\frac{4I_6}{6h^2}\left(\alpha^2+\beta^2\right)\right)J_{mn}\\&-\left(\frac{4\alpha}{3h^2}\right)I_4A_{mn}-\left(\frac{4\beta}{3h^2}\right)\left(I_4+\frac{I_5}{R_2}\right)B_{mn}-\left(\frac{4\alpha}{3h^2}\right)\left(I_5-\frac{4I_7}{3h^2}\right)D_{mn}\\&-\left(\frac{4\beta}{3h^2}\right)\left(I_5-\frac{4I_7}{3h^2}\right)E_{mn}+\left(I_3+\frac{4I_7}{9h^2}\left(\alpha^2+\beta^2\right)\right)F_{mn}\end{aligned}\right]C_{mn}\\
&+\left[\begin{aligned}&\left(I_2-\frac{4I_4}{3h^2}\right)A_{mn}+\left(I_3-\frac{8I_5}{3h^2}+\frac{16I_7}{9h^4}\right)D_{mn}+\left(-\frac{I_4}{2}+\frac{4I_6}{6h^2}\right)\alpha J_{mn}\\&+\left(-\frac{4I_5}{3h^2}+\frac{16I_7}{9h^4}\right)\alpha W_{mn}+\left(-\frac{I_5}{3}+\frac{4I_7}{9h^2}\right)\alpha F_{mn}\end{aligned}\right]D_{mn}+\\
&\left[\begin{aligned}&\left(I_2+\frac{I_3}{R}-\frac{4}{3h^2}\left(I_4+\frac{I_5}{R}\right)\right)B_{mn}+\left(I_3-\frac{8I_5}{3h^2}+\frac{16I_7}{9h^4}\right)D_{mn}\\&+\left(-\frac{I_4}{2}+\frac{4I_6}{6h^2}\right)\beta J_{mn}+\left(-\frac{4I_5}{3h^2}+\frac{16I_7}{9h^4}\right)\beta C_{mn}+\left(-\frac{I_5}{3}+\frac{4I_7}{9h^2}\right)\beta F_{mn}\end{aligned}\right]E_{mn}+\\
&\left[\begin{aligned}&\frac{4I_6}{6h^2}\left(\alpha^2+\beta^2\right)C_{mn}+\frac{I_5}{4}\left(\alpha^2+\beta^2\right)J_{mn}+\frac{I_6}{6}\left(\alpha^2+\beta^2\right)F_{mn}-\left(\frac{\alpha}{2}\right)I_3A_{mn}\\&-\left(\frac{\beta}{2}\right)\left(I_3+\frac{I_4}{R}\right)B_{mn}-\left(\frac{\alpha}{2}\right)\left(I_4-\frac{4I_6}{3h^2}\right)D_{mn}-\left(\frac{\beta}{2}\right)\left(I_4-\frac{4I_6}{3h^2}\right)E_{mn}\end{aligned}\right]J_{mn}+\\
&\left[\begin{aligned}&\left(\frac{4I_7}{9h^2}\left(\alpha^2+\beta^2\right)+I_3\right)C_{mn}+\frac{I_6}{6}\left(\alpha^2+\beta^2\right)J_{mn}+\left(\frac{4I_7}{9h^2}\left(\alpha^2+\beta^2\right)+I_5\right)F_{mn}\\&-\left(\frac{1}{3}\right)\left(I_4+\frac{I_5}{R}\right)\beta B_{mn}-\left(\frac{\alpha}{3}\right)\left(I_5-\frac{4I_7}{3h^2}\right)D_{mn}-\left(\frac{\beta}{3}\right)\left(I_5-\frac{4I_7}{3h^2}\right)E_{mn}-\left(\frac{I_4}{3}\right)\alpha A_{mn}\end{aligned}\right]F_{mn}
\end{aligned}\right\}d\theta dx \qquad (22)
$$

3. Numerical results

Two types of cross-ply and isotropic closed cylindrical shells are analyzed and their transient responses are evaluated numerically. Also a comparative study with a shallow shell of [1] is obtained analytically and numerically by using ANSYS (5.4) program.

To examine the validity of the derived equations for forced vibration response for composite laminated shells, a comparison study is done with a shallow spherical shell of [1]. By using the present analysis and finite element method in ANSYS (5.4), which shows good agreement between the results for the central deflection of two layer (0/90) cross ply laminate shell, which are (16.287(mm) using GTT, 17.978(mm) using ANSYS), while it was (≈16 .537 (mm) taken from graph in [1]), its obvious that the difference between the published results and the present work is (1.533%), these results are shown in Fig. (1).

Maximum central displacements (W) for antisymmetric cross ply (0/90) under different load-time functions are listed in Table (1), from which its obvious that maximum displacement of this cylindrical shell occurs when it is under the rectangular-pulse, the variations of these displacements as function of time are plotted in Fig.(2), as a result then stresses (σ1), (σ2), also have their maximum values under this pulse and their variation with time are shown in Figs.(3 and 4) respectively, in all figures the plotted stress component are taken as: $\sigma_1=\dfrac{\sigma_1\left(\left(\frac{L}{2}\right),\frac{\pi}{2},\left(\frac{H}{2}\right)\right)}{q_0}$

$\sigma_2=\dfrac{\sigma_2\left(\left(\frac{L}{2}\right),\frac{\pi}{2},\left(\frac{H}{2}\right)\right)}{q_0}$

The maximum central displacements for symmetric cross ply (0/90/0) laminated cylindrical shells are developed in Table (2), rectangular-pulse, here also gives maximum central displacement and therefore maximum stress components to the cylindrical shells. The variation of central displacements under these load-time functions are shown in Fig. (5), while the variation of stress components are shown in Figs. (6 and 7). Further, the amplitudes are smaller for symmetric cross ply than that for antisymmetric cross ply laminates, (22.57 and 22.36 mm) respectively.

Similar results are presented for central displacements (W) and normal stress ($\sigma 1$), ($\sigma 2$), ($\sigma 3$) and transverse shear stress ($\sigma 4$), ($\sigma 5$), ($\sigma 6$) for isotropic cylindrical shells in Table (3). Rectangular-pulse dynamic load also causes the maximum central displacement for this shell (its variation with time under different load functions are shown in Fig.(8), but it is smaller than that for both types of cross-ply cylindrical shells. Therefore, the stress components are also smaller, the variation of the isotropic shell stress components with time are as shown in Figs.(9 and 10).

Geometrical dimensions for the worked cases are, for spherical shell: (a=b=20, R11=R22=5a, H=2), for laminated closed cylindrical shell: (R=L=20 H=2), while load amplitude qo=2000MPa for cylindrical shells and qo=13.788 MPa for spherical shell, time duration for all load-time functions TD=.003sec.. Also in all calculations, material properties of the shells are listed in Table (4).

Table (1): Central displacement and stress components for two- layered (0/90) closed cylindrical shell and four types of pulses.

Load-time Function	W (mm)	(σ_1/q_o)	(σ_2/q_o)	(σ_3/q_o)	(σ_4/q_o)	(σ_5/q_o)	(σ_6/q_o)
Rectangular-pulse	22.57	45.12	-175.62	-17.91	1.22	9.33e-1	13.32
Triangular-pulse	17.78	37.00	-107.22	-11.18	1.00	8.24e-1	9.97
Sine-pulse	12.09	23.02	-94.60	-10.14	6.60e-1	5.47e-1	5.96
Ramp-Constant	12.11	23.05	-94.83	-10.17	6.61e-1	5.48e-1	5.97

Table (2): Central displacement and stress components for three- layered (0/90/0) closed cylindrical shell and four types of pulses.

Load-time Function	W (mm)	(σ_1/q_o)	(σ_2/q_o)	(σ_3/q_o)	(σ_4/q_o)	(σ_5/q_o)	(σ_6/q_o)
Rectangular-pulse	22.36	112.53	-253.86	-38.93	3.37e-1	14.22	34.54
Triangular-pulse	20.14	104.36	-201.58	-30.21	2.58e-1	12.88	32.05
Sine-pulse	11.45	52.45	-120.91	-20.16	1.76e-1	7.39	14.66
Ramp-Constant	11.44	52.40	-120.84	-20.15	1.76e-1	7.38	14.65

Table (3): Central displacement and stress components for isotropic (steel) closed cylindrical shell and four types of pulses.

Load-time Function	W (mm)	(σ_1/q_o)	(σ_1/q_o)	(σ_3/q_o)	(σ_4/q_o)	(σ_5/q_o)	(σ_6/q_o)
Rectangular-pulse	4.16	-34.05	-99.15	-61.52	5.38e-1	1.86	13.52
Triangular-pulse	3.79	-29.48	-85.786444	-54.10	4.65e-1	1.61	11.43
Sine-pulse	2.09	-17.42	-49.89	-31.15	2.75e-1	9.47e-1	5.70
Ramp-Constant	2.10	-17.51	-50.14	-31.31	2.76e-1	9.52e-1	5.74

Table (4): Material properties.

Material properties	Present work	Steel	[1]
E_1	132.38e3MPa	20.6e4MPa	132.38e3MPa
$E_2= E_3$	10.75e3MPa	20.6e4MPa	10.75e3MPa
$G_{12}=G_{13}$	5.653e3MPa	8e4MPa	5.653e3MPa
G_{23}	3.608e3MPa	8e4MPa	3.608e3MPa
$\nu_{12} = \nu_{13}$	.24	.3	.24
ν_{23}	.49	.3	.49
ρ	1.32288e-9(N-s^2/mm^4)	7.85e-9(N-s^2/mm^4)	1.32288e-9(N-s^2/mm^4)

4. Conclusions:

The prominent points in this work are as follows:

1- A general third order shell theory (GTT) is developed to derive the governing equations for forced vibration of simply supported cylindrical shells, for first time, and these equations are solved using Navier´s method.
2- Good agreement between the results obtained by using GTT in present work with those obtained by other researchers using FEM for analyzing the dynamic behavior of laminated spherical shells, maximum percentage of error is (1.533%).
3- The response due to the rectangular loading has largest amplitude than that of other types of loading.
4- Symmetric cross-ply laminates have smaller amplitudes than that for antisymmetric one.

5. References:

1. J.N. Reddy & A.A. Khdeir,'Dynamic response of cross- ply laminated shallow shells according to a refined shear deformation theory', J. Acous. Soc. Am., June, 1988.
2. 2-Y. Narita, Y.Ohta, G. Yamada and Y.Kobayashi, ' Analytical method for vibration of angle ply cylindrical shells having arbitrary edges', AIAA, vol.30, no.3, 1992.
3. Z.C.Xi, L.H. Yam and T.P. Leung, 'Free vibration of composite laminated shell of revolution: Effects of shear non-linearity',Int. J. Mech. Sci., vol.41, 1999.
4. Aleksandr Korjakin , Rolands Rikards, Holm Altenbach and Andris Chate, 'Free damped vibrations of sandwich shells of revolution', J. of Sandwich Stru. & Mater., vol.3, 2001.
5. Werner Hufenbach, Carsten Holste and Lothar Kroll, 'Vibration and damping behavior of multi-layered composite cylindrical shells', Comp.Stru., vol.58, 2002.
6. Humayun R. H. Kabir, 'Application of linear shallow shell theory of Reissner to frequency response of thin cylindrical panel with arbitrary lamination', Comp.Stru.,vol.56, 2002.
7. J. J. Lee, C.H. Yeom and I. Lee, ' Vibration analysis of twisted cantilevered conical composite shells', J. of Sound and vibration, vol. 255, no.5, 2002.
8. M. Amabili, 'A comparison of shell theories for large-amplitude vibrations of circular cylindrical shells: Lagrangian approach', J. of Sound and vibration, vol.264, 2003.
9. Young-Shin Lee, Myung-Hwan and Jae-Hoon Kim, 'Free vibrations of laminated composite cylindrical shells with an interior plate', J. of Sound and vibration, vol.265, 2003.

10. S. C. Pradhan & J. N. Reddy, 'Vibration control of composite shells using embedded actuating layers', Smart Mater. Struct., vol.13, 2004.
11. Ghanim Shaker, 'Static and dynamicanalysis of composite thin cylindrical shells', Ph.D. Thesis, Baghdad university, (2004)
12. M. Darvizeh, H. Haftchenari, A. Darvizeh, R. Ansari, C.B. Sharma, 'The effect of boundary conditions on the dynamic stability of orthotropic cylinders using a modified exact analysis', Composite Structures, article in press, 2005.
13. J.N. Reddy, 'Mechanics of laminated composite plates and shells', CRC Press Press 2004.

Nomenclature

a, b	Dimensions of shell.
A_{mn}, B_{mn}, C_{mn}, D_{mn}, E_{mn}, F_{mn}, J_{mn}	Arbitrary constants
B	constant
C_{ij}	stiffness matrix elements
E_1, E_2, E_3	Elastic Modulus components (GPa)
$f_{mn}(t)$	Generalized force (N)
g_1, g_2	Body forces (N)
G_{12}, G_{13}, G_{23}	Shear modulus components (GPa)
H	Thickness (mm)
K	Kinetic energy
L	Cylinder length (mm)
m, n	indices
N_i, M_i, P_i, S_i (i=1,2,3,6)	Resultant reactions (N/mm),(N.mm)
m_1, m_2, m_3, m_4	Body moments (N.mm)
Q_i, K_i (i=4,5)	Resultant reactions (N/mm)
Q_{ij}	Elastic stiffness coefficients
q	Distributed transverse load (N/mm^2)
R	Cylinder radius (mm)
R_{11}, R_{22}	Principal radii of curvature of shell (mm)
U	Potential energy
u, v, w, φ_1, φ_2, ψ_1, ψ_2, ψ_3, θ_1, θ_2, θ_3	Displacement components (mm)
z	Distance from neutral axis (mm)
$\varepsilon_{1,2,3,4,5,6}$	Strain components in principal directions
υ_{12}, υ_{13}, υ_{23}	Poison´s ratios
ρ	Density (Ns^2/mm^4)
ω	Frequency (rad/s)
$\sigma_{1,2,3,4,5,6}$	Stress components (MPa) in principal direction

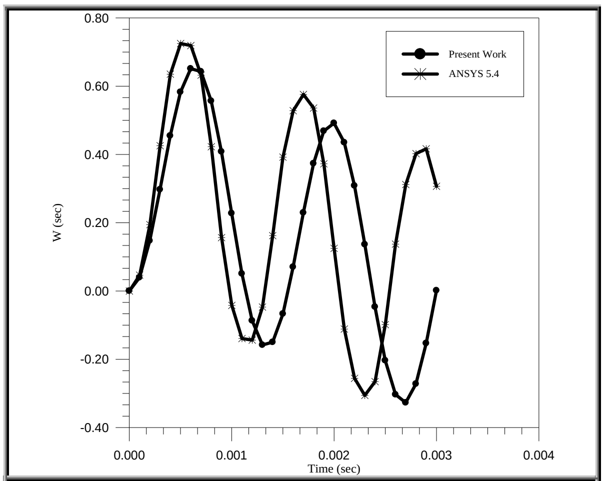
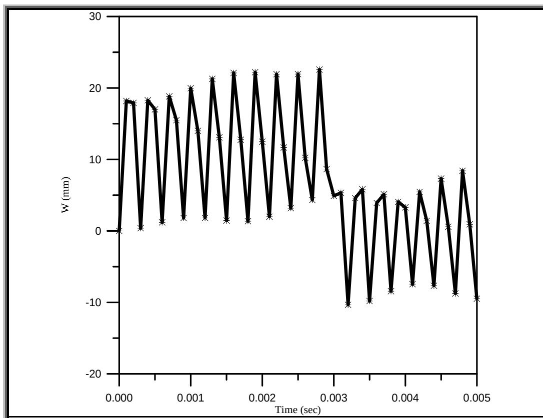
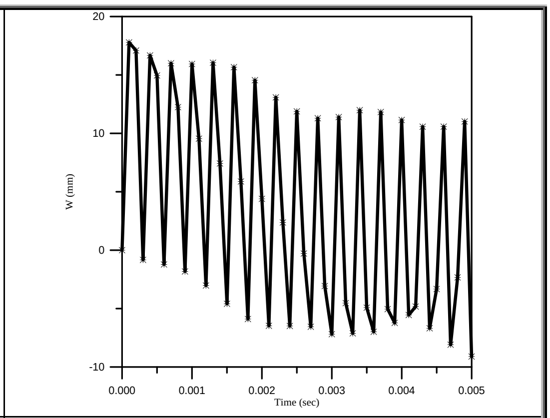


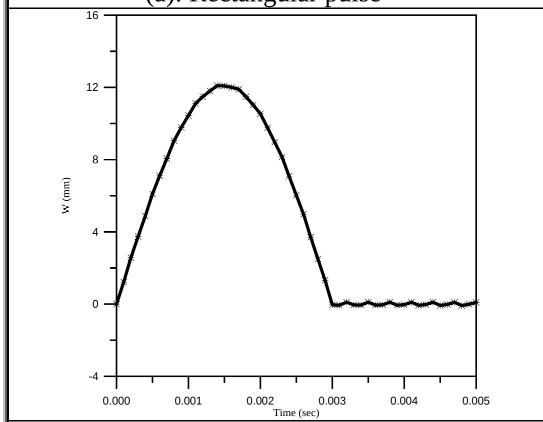
Figure (1) Variation of center deflection as a function of time, for two- layered (0/90) shallow spherical shell under triangular pulse.



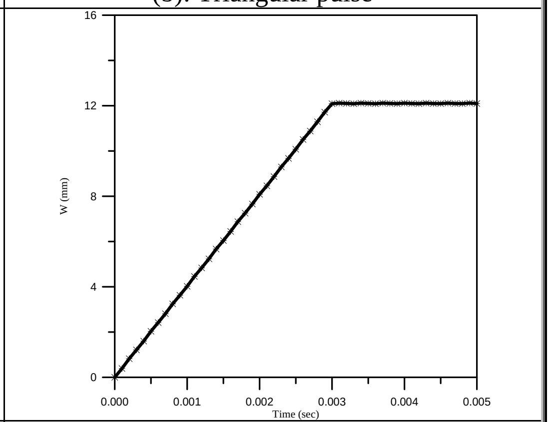
(a): Rectangular pulse



(b): Triangular pulse

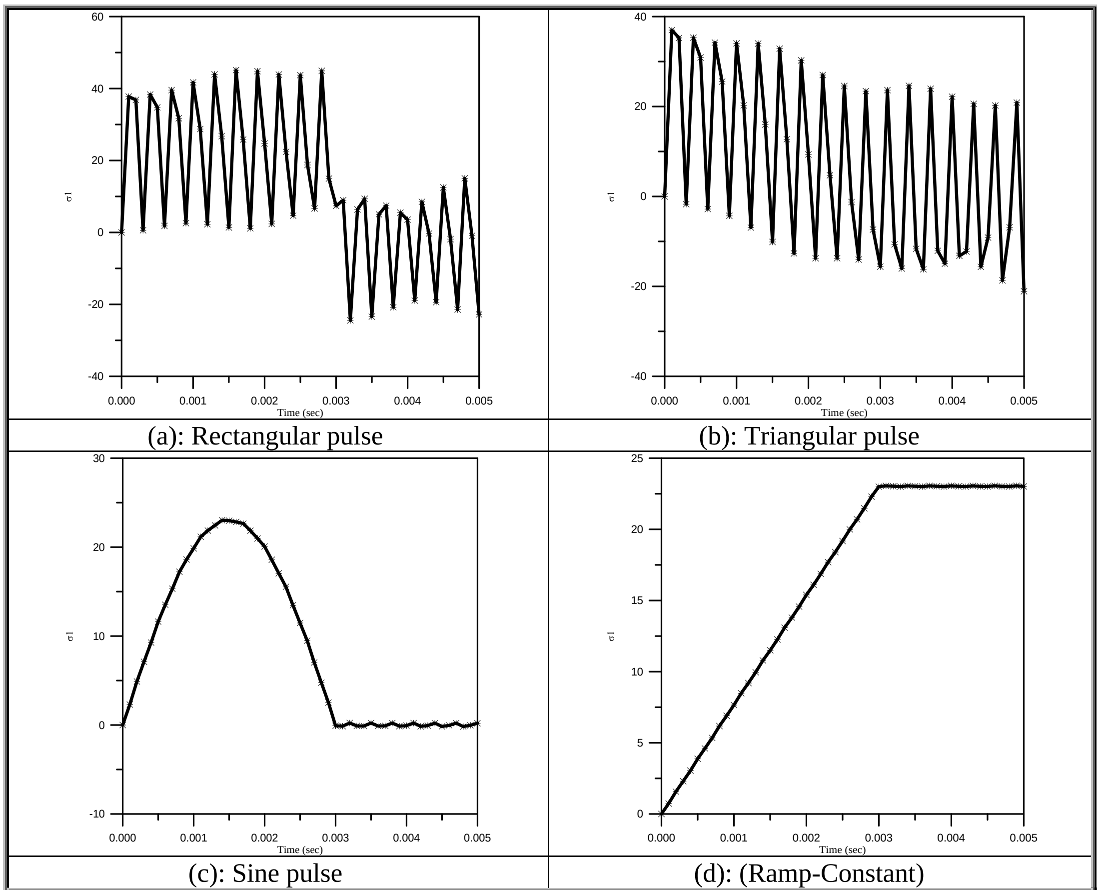


(c): Sine pulse



(d): (Ramp-Constant)

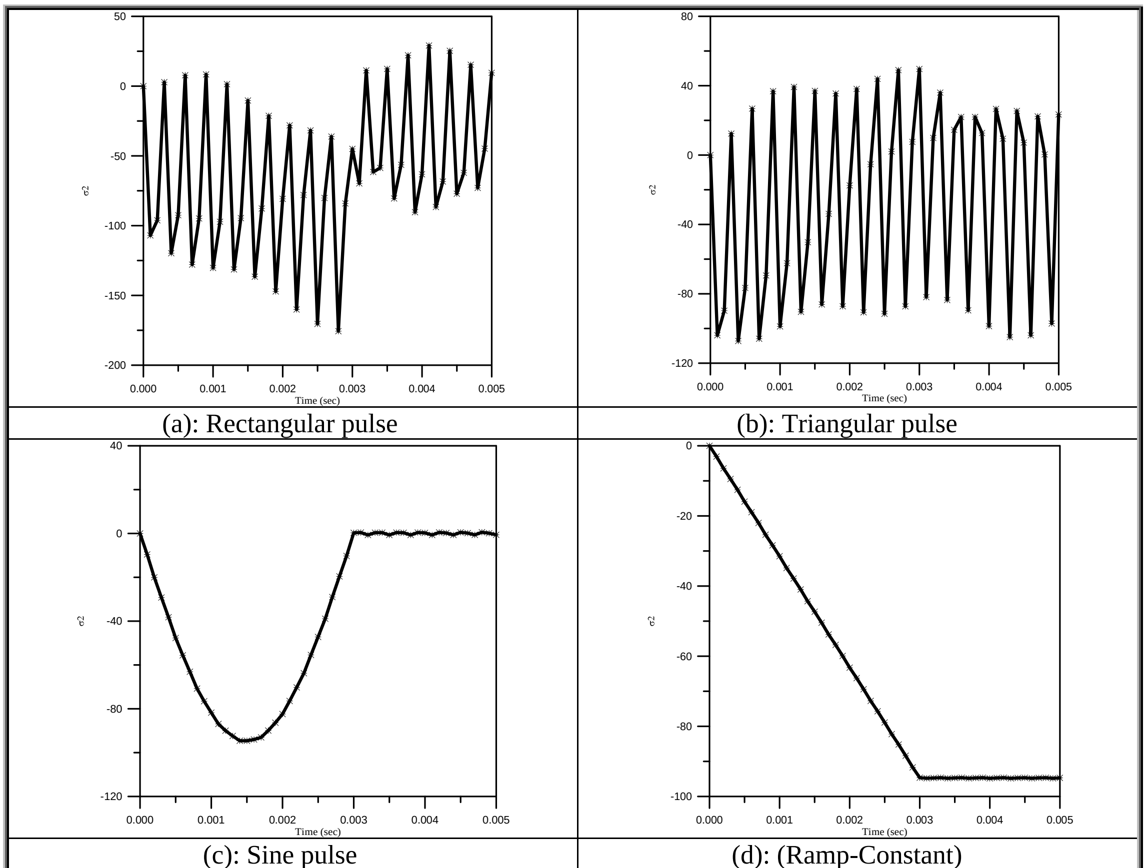
Figure (2) Variation of central deflection as a function of time, for two- layered (0/90) closed cylindrical shell and four types of pulses.



(a): Rectangular pulse (b): Triangular pulse

(c): Sine pulse (d): (Ramp-Constant)

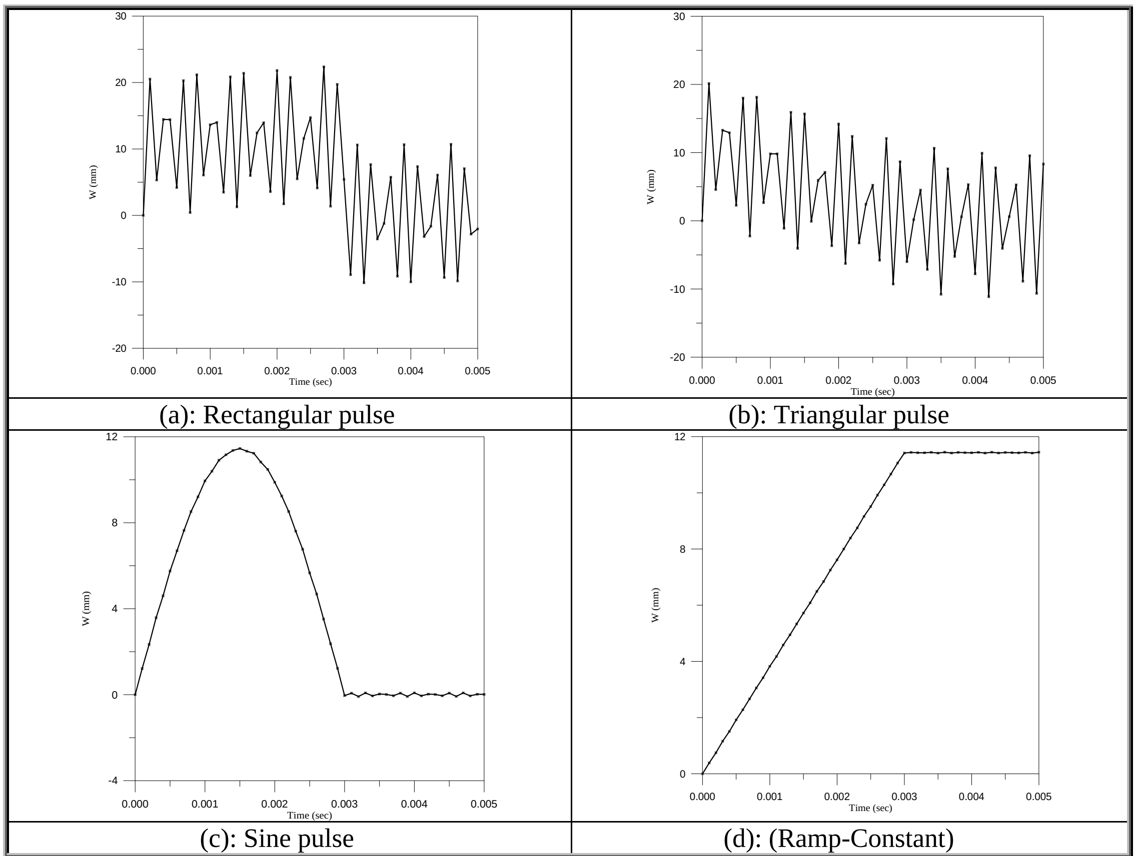
Figure (3) Variation of (σ1) as a function of time, for two- layered (0/90) closed cylindrical shell and four types of pulses.



(a): Rectangular pulse (b): Triangular pulse

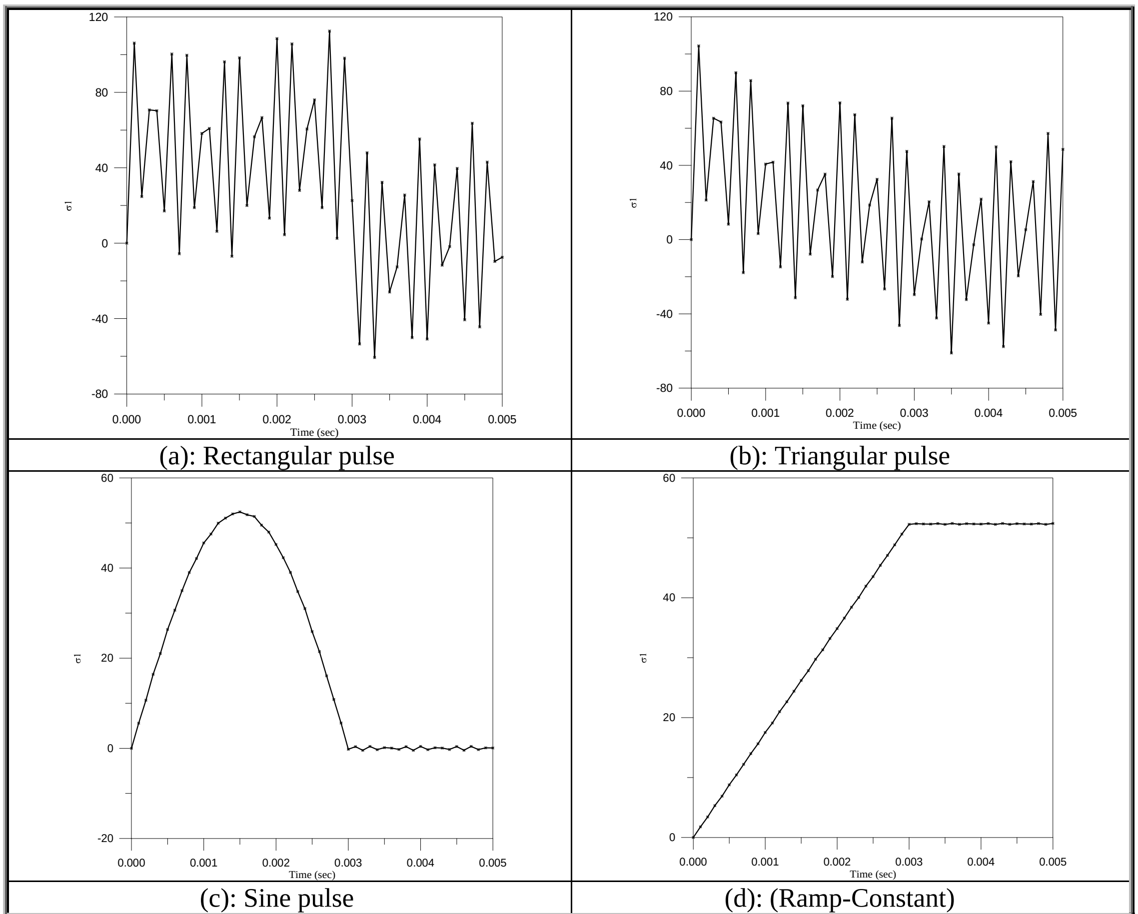
(c): Sine pulse (d): (Ramp-Constant)

Figure (4) Variation of (σ_2) as a function of time, for two- layered (0/90) closed cylindrical shell and four types of pulses.



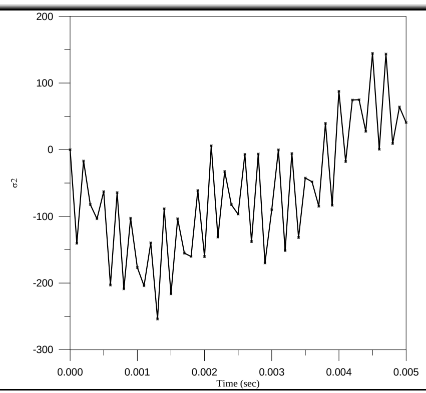
(a): Rectangular pulse | (b): Triangular pulse | (c): Sine pulse | (d): (Ramp-Constant)

Figure (5) Variation of central deflection as a function of time, for three- layered (0/90/0) closed cylindrical shell and four types of dynamic load.

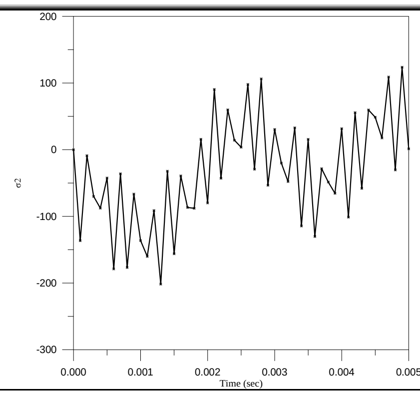


(a): Rectangular pulse | (b): Triangular pulse | (c): Sine pulse | (d): (Ramp-Constant)

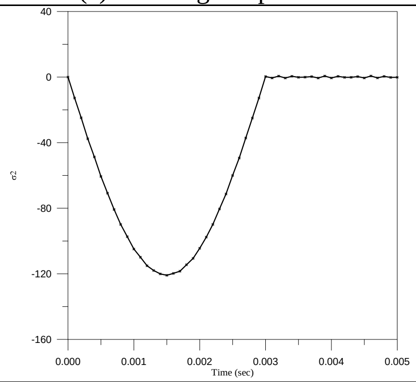
Figure (6) Variation of (σ1) as a function of time, for three- layered (0/90/0) closed cylindrical shell and four types of dynamic load.



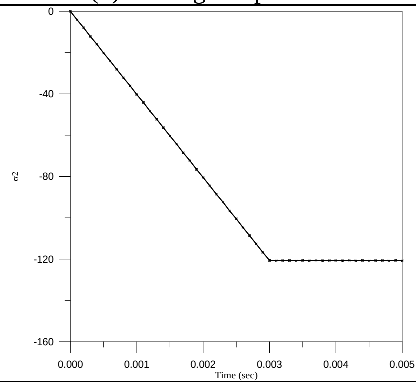
(a): Rectangular pulse



(b): Triangular pulse

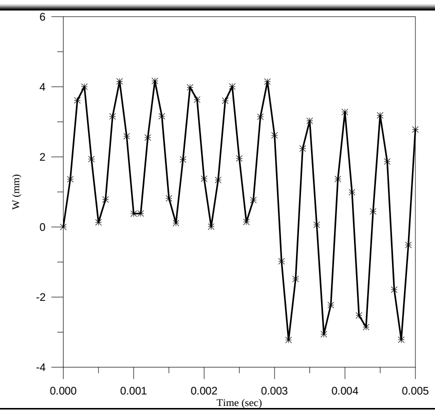


(c): Sine pulse

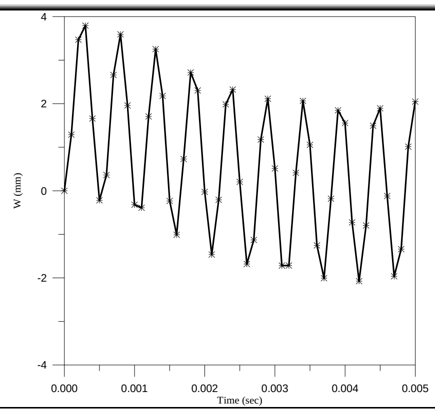


(d): (Ramp-Constant)

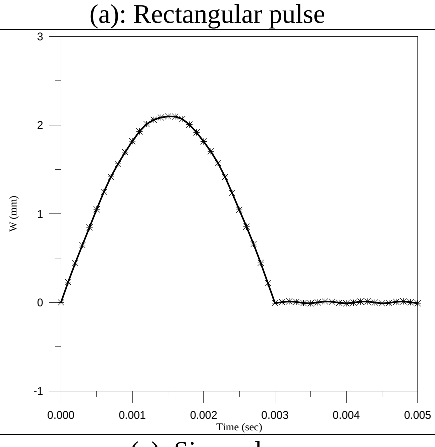
Figure (7) Variation of (σ_2) as a function of time, for three- layered (0/90/0) closed cylindrical shell and four types of dynamic load.



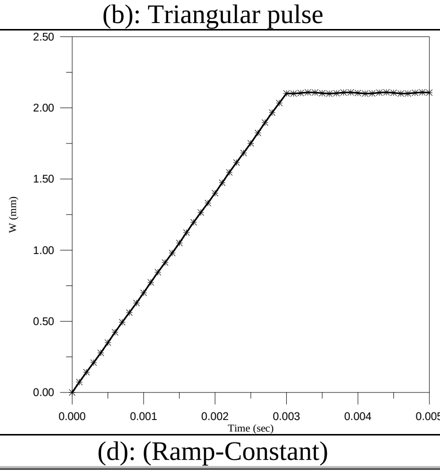
(a): Rectangular pulse



(b): Triangular pulse

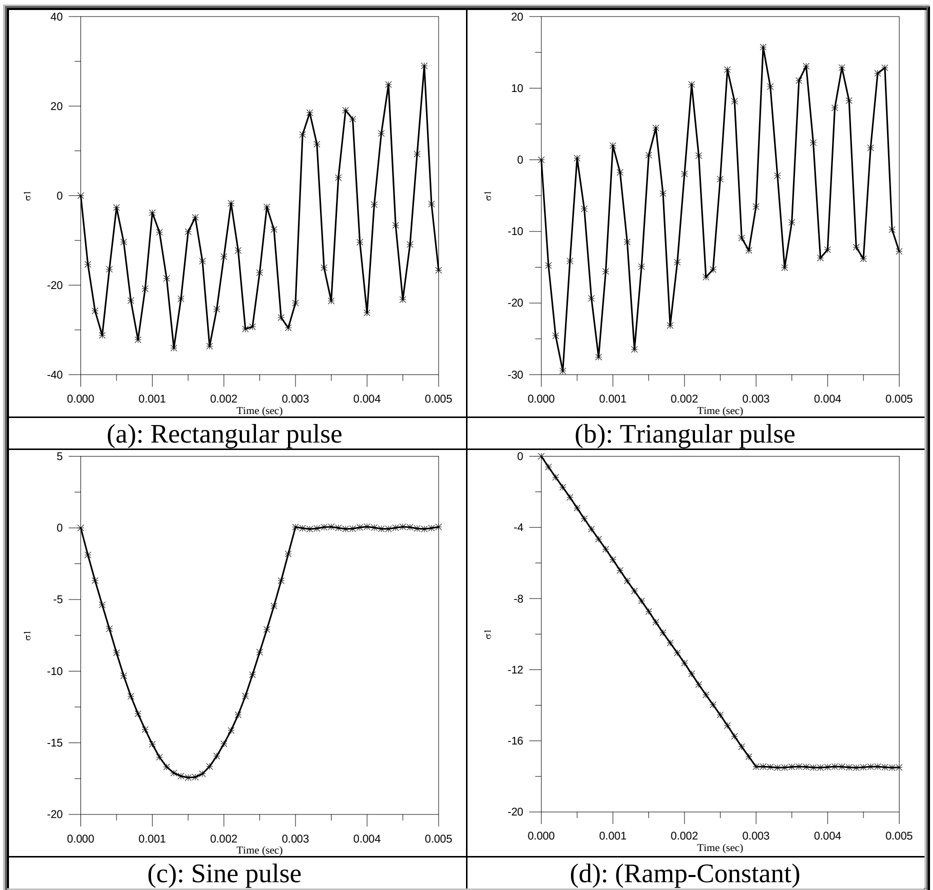


(c): Sine pulse



(d): (Ramp-Constant)

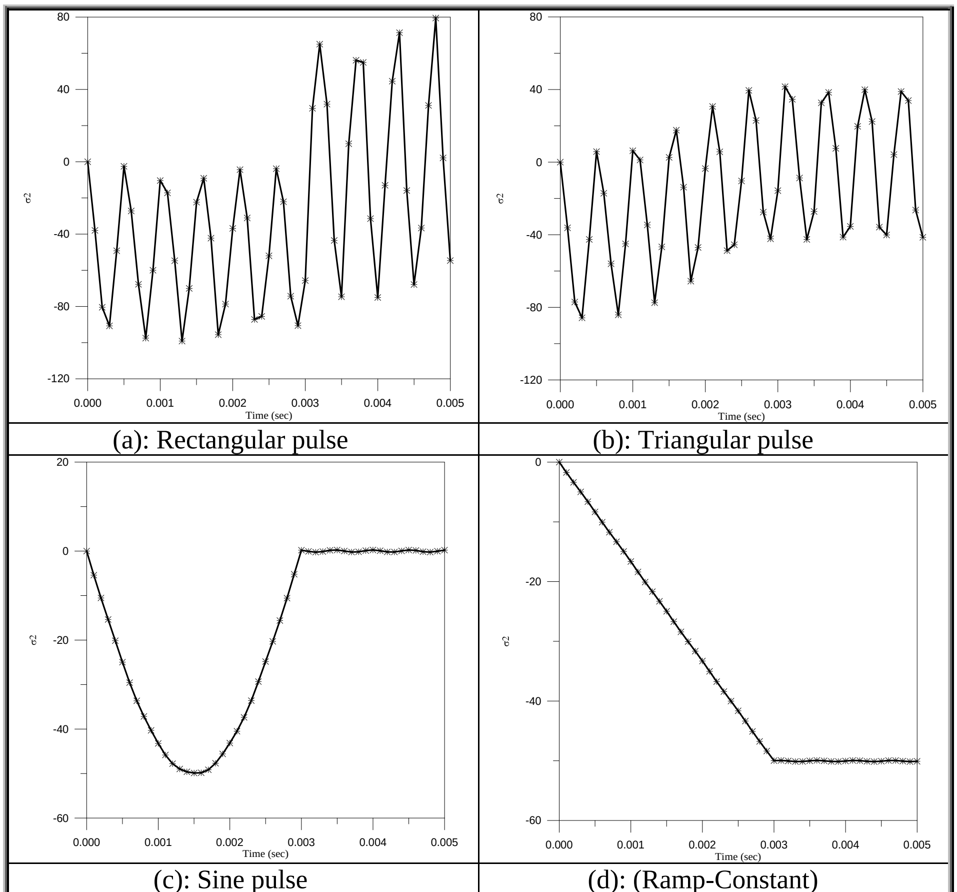
Figure (8) Variation of central deflection as a function of time, for isotropic (Steel) closed cylindrical shell and four types of dynamic load.



(a): Rectangular pulse | (b): Triangular pulse

(c): Sine pulse | (d): (Ramp-Constant)

Figure (9) Variation of (σ_1) as a function of time, for isotropic (steel) closed cylindrical shell and four types of dynamic load.



(a): Rectangular pulse | (b): Triangular pulse

(c): Sine pulse | (d): (Ramp-Constant)

Figure (10) Variation of (σ_2) as a function of time, for isotropic (steel) close cylindrical shell and four types of dynamic load.

حل تحليلي مقترح للاسطوانات الطباقية المغلقة باستخدام نظرية الرقائق العامة الثالثة

د. محسن جبر جويج
قسم الهندسة الميكانيكية
جامعة النهرين

د. وداد ابراهيم العزاوي
قسم الهندسة الميكانيكية
جامعة بغداد

الخلاصة:

تم تطوير حل مشكلة الاستجابة الحظية للرقائق الاسطوانية المغلقة الطباقية ذات الاسناد البسيط المعرضة لضغط موزع بصورة منتظمة على سطحها الخارجي. هذه الحلول تمت باستخدام نظرية الرقائق الثالثة العامة. النبضة المستطيلة، النبضة المثلثة، النبضة الجيبية ونبضة (التغير الخطي- ثابت) كدوال لتغير الحمل مع الزمن تم دراستها وتحصيل المعادلات الاتزان اللازمة لها. الازاحة ا لمركزية والاجهادات الاساسية تم بحثها لمختلف الطبقات المتقاطعة.