A theoretical study for the differential cross sections of Rayleigh and Compton scattering of silver ${}^{108}_{47}Ag$ by employing CSC model

Rafaa Abduala Abbad

Department of physics , College of Science ,Tikrit University, Tikrit, Iraq

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Abstract

In the present study, the differential scattering cross section for the element ${}^{108}_{47}Ag$ has been calculated by using a computer code named Cross Section Calculations (CSC) ,based on programming the Klein-Nashina and Rayleigh scattering equations. Atomic form factors as well as the coherent Functions in Fortran90 Machine language proved very fast and acceptable results and the possibility of application of such model to obtain the total coefficient for any elements or compounds.

Introduction

Elastic and inelastic scattering processes in atoms, gases and elemental solids remain major topics of current research and investigation because scattering is an excellent probe to study various properties of the matter systems [1], reliable values of cross section for elements ,compounds and alloys are of important value, demanded in varity of applications in radiography, tomography ,space physics ,plasma physics etc[2].A review of literature shows that Geswar [3] measured the total cross section for various elements in the energy range of 4.5 to 20keV.Theoretical photoelectric cross section for Z=13 are calculated by Brysk and Zerby [4] in energy range 1 to 150 keV , employing bound-state wave function that's obtained by Cromer and Lieberman [5] .Hubbell and Berger [6] reported pair production cross sections in the fields of atomic nucleus for 11 elements over the range 15 to 100 MeV. These values have been calculated using Born approximation with Bethe –Hitler high energy approximation. In this work, we present the result of cross section calculation (CSC) which based on programming related equations in addition to feed the values of some atomic factors for the energy range (1KeV- 1MeV) through certain subroutine calculations.

Calculation Technique of CSC model

The probabilities of photons interactions with matter are functions of both incident energy and atomic number Z , these probabilities are expressed as a total cross section σ or differential cross sections **d** σ /**dΩ** expressed in barns unit($1barn=10^{-28}$ $meter^2$) the cross section concept is related to the mechanisms of photon interaction described as follow:

I- The Coherent (Rayleigh) Scattering differential cross section:

In Rayleigh scattering event, a photon scattered off atomic electrons where energies of incident and scattered photons are identical. In this process the recoil energy of absorber atom is negligible and the process occurs at small angle. In Rayleigh scattering, photons are elastically scattered by bound electrons with the atom neither ionized nor excited , the incident photon is recoiled by the entire atom, changing its momentum and polarization while its energy remains unaltered. This scattering is called 'coherent' (from different parts of the atomic charge distribution) that gives rise to interference effects. Thus the cross section of Rayleigh depends upon the photon energy E and atomic number of the absorber Z [7,8]:

($Z^{8/3}/ E^2$) ------------------ (1) $\alpha_{Rayleigh}\sigma$ …(2)

The differential cross section is given by [9]:

$$\frac{d\sigma_{Rayleigh}}{d\Omega} = \frac{d\sigma_{Thomson}}{d\Omega}\left[\left(F(x,Z)\right]^2\right.$$

is defined as $\frac{d\sigma_{Thomson}}{d\Omega}$ where $d\Omega$ is the solid angle and the quantity The differential scattering cross section of free electron or Thomson cross section [9]:

$$\frac{d\sigma_{Thomson}}{d\Omega} = \frac{1}{2} r_e^2\left(1+\cos^2\theta\right) \quad ...(3)$$

Were r_e is the classical radius of electron ($r_e = 2.817\times10^{-15}$m) and F(x,Z) is the atomic form factor will be discussed in next section .Equation (3) can be rewritten as [10]:

$$\frac{d\sigma_{Thomson}}{d\Omega} = \frac{r^2{}_e}{2}\left(1+\cos^2\theta\right)\left(F^2(x,Z)\right) \quad ...(٤)$$

II-Compton (Incoherent) differential Scattering Cross section

A description of the Compton scattering can be conveniently divided into two aspects :kinematics and cross section .The first relates the energies and angles of the participating particles when a Compton event occurs ; the second predicts the probability that the Compton interaction will occur .In both respects it is customary to assume that the electron struck by the incoming photon is initially unbound and stationery .These assumptions are certainly not rigorous , as much as the electrons occupy all various atomic energy levels, thus are in motion and are bound to the nucleus .Nevertheless the resulting errors remain inconsequential in radiological physics applications, because of the dominance of the competing photoelectric effects under the conditions (high Z and low hv) where electron binding effects are most important in Compton interaction ,the Compton differential cross section scattering is proportional to atomic number Z of the absorber and energy [11]

(Z/ E) ---- (5) $\alpha_{Compton}\sigma$

We are interested particularly in the angular dependence of the differential cross-section and the total cross-section both as a function of the incident photon energy. First the differential cross-section, also known as the Klein-Nishina formula [12]:

$$\frac{d_{Incohernt}}{d\Omega} = \frac{d\sigma_{Klein-Nashina}}{d\Omega} S(q,Z) \quad \text{--- (6)}$$

Where $\frac{d\sigma_{Klein-Nashina}}{d\Omega}$ is defined as:

$$\frac{d\sigma_{Klein-Nashina}}{d\Omega} = Zr^2{}_e\left(\frac{1}{1+\alpha(1-\cos\theta)}\right)\left(\frac{1+\cos^2\theta}{2}\right)\left[1+\frac{\alpha(1-\cos\theta)^2}{(1+\cos^2\theta)(1+\alpha(1-\cos\theta)}\right]$$

This equation gives he probability that a photon is deflected at given angle and transfers some momentum to the electron thought to be free and $\alpha = h\nu/m_oc^2$ (h is Planks constant, f photon frequency and m_oc^2 is the rest mass energy of electron equals to 0.511MeV) and the term S(q, Z)represent the incoherent scattering function . The formula gives the probability of scattering a photon into the solid angle element dΩ= 2π sinθdθ when the incident energy is hv .

IV- Differential Scattering Cross Section concept in terms of Solid angle

The cross section σ is the total area presented by a scattering center to the incident photon, but the deflection of the photon (if it is not absorbed) depends on the distance from the interaction center at which it is incident. The annular area dσ within which the deflection will be within a solid angle dΩ, centered on deflection angle θ, is the differential cross section dσ/dΩ. The relationship between an increment in θ and an increment in Ω is:

$$d\sigma/d\Omega = 1/2\pi\sin\theta \frac{d\sigma}{d\theta} \quad \text{.... (8)}$$

V-The Atomic Form Factor F(x, Z):

The atomic form factor F(x, Z) defined as the amplitude of electron oscillation when interact with incident radiation .Hence the electron will oscillates with the same frequency of the incoming wave train and since an accelerating electric charge emits electromagnetic radiation, scattered photon appears to emerge from the interaction region with the same frequency of the oscillating electron, consequently the scattered amplitude (as function of the scattering angle of Z electrons is not equal to Z times the free electron amplitude; but it is modified by atomic form factor F(x, Z) where x is the momentum transfer parameter measured in units of $1/A^0$ defined as :

$$x = (1/\lambda)\sin(\theta/2) \quad \text{--- (9)}$$

or

$$x = \kappa\alpha(1-\mu)^{1/2} \text{---(10)}$$

Where λ is the incident wavelength and θ is the scattering angle, $\kappa = 10^{-8}$ mc/(h $(2)^{1/2}$) = 29.1445 cm^{-1} and α is the initial photon energy in units of rest mass energy of an electron .The cosine of the scattering polar angle μ, is calculated by using either Kahn's rejection method or Koblinger's direct method which samples the Klein-Nishina formula exactly. The final angle is rejected according to the incoherent scattering function, S(x,Z). The function F(x, Z) is related to the ratio of the amplitude of scattered wave form to scattered amplitude from free electron and given by:

$$\text{...(11)} \quad F(x,Z)^2 = \frac{d\sigma / d\Omega_{Atom}}{d\sigma / d\Omega_{Thomson}}$$

The values of F(x,Z) for atoms up to Z=100 were published by Hubbell et all [12] by employing the relativistic Hartree-Fock wave function to construct a table of atomic form Factor[14].

VI-The Coherent Scattering function S(q,Z):

The function S(q ,Z) represent the probability that an atom will be raised to the excited or ionized state when a photon imparts a recoil momentum (q) to any one of the atomic electrons , in other words, the Incoherent scattering factor, is used to account for the effects of bound electrons in an atom on the angular distribution of the scattered photon. Values of incoherent scattering factors for various elements are published in tabular form by Hubbell [13] as a function of x (the inverse length) and Z. Also Cromer and Mann[15] have calculated the incoherent scattering function S(q,Z) for all spherically symmetric free electrons by using Hartree –Fock -Slater wave function with exchange term . For any Z, the function S(x,Z) increases from zero until Z at Z=infinity .The values of S(q,Z) and F(x, Z) are fed into series subroutine of Fortran 90 program were all the equations are written in Fortran language which gives the results in a very short time after running.

VII-Results and Discussion

The coherent differential cross section has been calculated from equation (2) and the incoherent differential cross section which obtained from equation (6) and the result is given in table (1). By observing the table 1 we deduce the following:

1- The probability of Compton scattering per atom depends on the number of electrons available as targets and therefore $\sigma_{incoherent}$ linearly increases with atomic number Z, and inversely with incident photon energy E.

2- Noting the values given in the table 1 as energy E increases, the values of $\sigma_{coherent}$ decreases in accordance with equation (1).

3-In this works ,no comparison were made of present results with any experimental works for similar elements ,this gives that in scattering process the atoms are assumed to be isolated from the influences of the neighbor atoms .But actually there are unavoidable interactions between various atoms as the molecular and chemical effects that are not taken into account .For example in Compton scattering ,the electrons is assumed to be bound and not free also initially at rest however in reality the bound electrons in material their momentum gives raise to range of possible energies which is referred as Doppler Broadening and the quantum theory refers that even at zero Kelvin the atoms vibrate in their equilibrium Positions(zero point energy). Also Hubbell and Berger [16] estimated the magnitude of the discrepancy between theoretical and experimental Kedge cross section for various elements and compound from Ti to Zn to be in the range of 3% to 12%.

4-The Coherent scattering differential cross section of Rayleigh never dominates the total cross section σ_{total} but at small scattering angles ((θ less than 10^0)) and at photon energies less than 100 keV because the scattering angle become greater proportional to the atomic Z and the photon incident energy.

Table (1): The values of differential cross section of Coherent and Incoherent for silver $^{108}_{47}Ag$ (atomic units).

E (MeV)	Coh(barn/atom)	Incoh(barn/atom)
0.001	1403	0.8922
0.015	1335	1.642
0.002	1275	2.41
0.003	1095	3.892
0.004	946.6	5.24
0.005	819.9	6.488
0.006	315.7	7.643
0.008	562.8	9.715
0.01	459.1	11.44
0.015	300.6	14.51
0.02	208.6	16.51
0.03	117.2	18.92
0.04	76.57	20.17
0.05	54.57	20.82
0.06	40.02	21.12
0.08	24.44	21.13
0.1	16.54	20.78
0.15	8.026	19.51
0.2	4.726	18.25
0.3	2.193	16.19
0.4	1.26	14.36
0.5	0.8165	13.43
0.6	0.5715	12.47
0.8	0.3243	10.99
1.00	0.2085	9.894

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دراسة نظرية لحساب مساحة المقطع العرضي التفاضلي لاستطارتي رايلي و كومبتن لعنصر الفضة $^{108}_{47}Ag$ باستخدام النموذج CSC .

رافع عبدالله عباد

قسم الفيزياء ، كلية العلوم ، جامعة تكريت ، تكريت ، العراق

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الملخص

في الدراسة الحالية ، تم حساب مساحات المقطع العرضي التفاضلي لعنصر والفضة من خلال ستخدام النموذج الرياضي (حساب مساحات المقطع العرضي) وبالاعتماد على برمجة معادلتي استطارة كلين–ناشينية و رايلي وقيم معاملات التركيب الذري و الدوال المتشاكهة بلغة فورتران ٩٠ والذي اثبت دقة وسرعة عالية في الحصول على النتائج مقبولة وامكانية تطبيقه لحصول على معاملات التوهين الكلية لاي عنصر اوسبيكة