# Sliding Mode Controller Design for a Crane Container System

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***Abstract-*** Applying automatic control on gantry cranes to move loads with minimal sway angle is considered as a challenge due to the crane system uncertain parameters and a robust automatic control is needed. In this paper a sliding mode controller is applied to overcome the crane system uncertainties and achieve the desired performance. Some labor is spent to transform the system to the regular form and an error function is written depending on the transformed variables then a switching function in terms of the error function is constructed and a sliding mode controller is designed to make the error function reach zero so the crane moves to the specified displacement with minimum sway angle. Stability analyses are provided to show that the system is stable under the control we proposed. Simulations are held by software to validate the effectiveness of our work and prove that the proposed control is successful in giving our system the preferred behavior then Results are discussed and some point view is to be put.

**Keywords –** Sliding Mode Controller, Sliding Surface, Regular Form

## 1. Introduction

The general definition of a crane is a mechanical system designed to lift and move loads through a hook suspended from a movable arm or trolley. Gantry Cranes, Figure 1, are widely used in industrial and civil environments to transfer heavy loads (e.g. containers). Safety and economical constraints require that both the load swing and the transfer time are kept as small as possible.

In the standard industrial practice human operators manually drive the crane, aided in some cases by expensive automatic anti-sway systems. When a human operator attempts to maneuver payloads using an overhead gantry crane like that sketched in Figure 2, the oscillations induced into the payload by the motion of the trolley can be significant. These oscillations make it difficult to manipulate the payload quickly and with positioning accuracy.

Manufacturers are currently trying to develop fully-automated drive systems which would get free from the driver's ability a successful crane operation. That will ensure a repeatable and better service, but affordable commercial products are not yet available.

The usual goal is to achieve zero swing only at the end of the transport, and a two-stage control structure is often used: a “tracking” controller during the load transfer, and a “stabilizing” one to be switched on when a suitable vicinity of the destination point is achieved. However, dangerous load swing may appear during the transport, and this should be certainly avoided.

Many works have been done in controlling the overhead crane. The authors in [1] and [2] adopted input shaping control method but the input shaping must be pre-calculated accurately according to the system model. These approaches lacked robustness to external disturbances and couldn’t damp residual swing well.

Moreover, zero initial condition must be satisfied. Alessandro Giua *et. al*., [3] proposed feedback control methods. Besides needing accurate system model and onerous matrix computation, the above methods were greatly affected by system linearization and system parameters uncertainty.

Inspired from the behavior of expert crane drivers, most of existing approaches consist of a two-stage procedure: off-line trajectory/path planning, carried out in accordance with proper optimization criteria, and on-line tracking by traditional controllers. Optimal control techniques have been widely used to address the path planning problem [4]. Specific paths minimizing traveling time, energy consumption or proper performance indexes linked to the swing angle and its derivative have been proposed in the literature. Nevertheless, due to model uncertainties and many other implementation factors, it often happens that the actual crane behavior significantly differs from the “optimal” desired one [4].

A serious drawback must be faced at the design stage: an overhead crane is an under actuated system, as the tow degrees of freedom (the horizontal load coordinate and the swing angle) must be controlled using only one control actions (the trolley force).

Fuzzy logic control (FLC) is independent of system model and has some robustness. Lee [5] used fuzzy logic only in anti-swing control and applied position servo control for positioning and swing damping. In [6], fuzzy logic adapted to both positioning control and swing damping. However, because of the large number of fuzzy rules, it was difficult to set both rules and parameters of the controller only according to experiences.

Sliding mode control (SMC) is a robust design methodology using a systematic scheme based on a sliding mode surface. The main advantage of sliding mode control is that the system uncertainties and external disturbances can be handled under the invariance characteristics of system's sliding mode state with guaranteed system stability.

In this work the ability of the sliding mode to handle system uncertainties is to be proved and a sliding mode controller is applied to a two dimensional gantry crane which it must reach a desired position with minimum sway angle.

This paper is organized as follows: in section 2 the mathematical model for the crane is derived. Sections 3 and 4 discuss sliding mode control. Section 5 is devoted for stability analysis. In section 6 a simulation is held and in section 7 some remarks and conclusions are took in consideration.

## 2. Mathematical Model

We consider a container crane (Fig. 3) actuated by one DC motor that generate the mechanical forces acting on the trolley. By taking the trolley position $x$, the rope length $l$, the swing angle $\theta$and their time derivatives as the state variables, assuming that the load can be regarded as a material point and that the rope is always stretched (so that the swing angle can be uniquely defined) the following motion equations can be derived using the Euler- Lagrange procedure .

For simplicity, the following assumptions are made:

(a)The trolley and the load can be regarded as point masses [7].

(b)Friction force, which may exist in the trolley, can be neglected.

(c) Elongation of the rope due to tension force is negligible.

(d)The trolley and the load move in the x-y plane [7].

(e) The DC motor parameters are neglected so the horizontal force acting directly on the trolley is proportional to the controller signal [8].

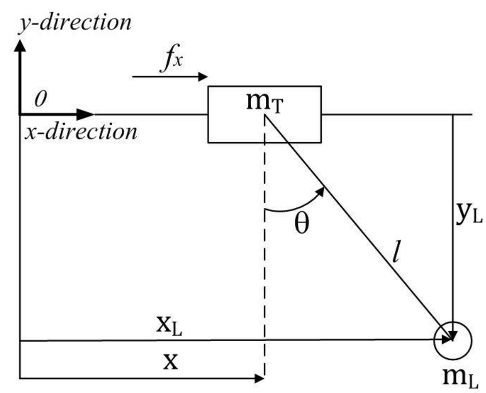



Figure (1): An equivalent diagram for the suspended load

For the system in Fig.3, a generalized coordinate $q$ can be taken as $q = [\theta, x]^T$. Then the kinetic energy function $T$and the potential energy function $V$are given as follows:

$$T = T_{trolley} + T_{load}$$

$$= \frac{1}{2} m_T \dot{x}^2 + \frac{1}{2} m_L (\dot{x}_L^2 + \dot{y}_L^2)$$

$$= \frac{1}{2} m_T \dot{x}^2 + \frac{1}{2} m_L (\dot{x}^2 + 2\dot{x} l \dot{\theta} \cos\theta + l^2 \dot{\theta}^2) \qquad (1)$$

$$V = m_L g l \cos\theta\,(1 - \cos\theta) \qquad (2)$$

Where $m_t$ and $m_l$ are the trolley and the load masses respectively,$x_L$ and $y_L$ are the load Cartesian coordinates.

Constructing the system Lagrangian $L = T - V$ and using Euler-Lagrange's equation defined as:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = Q_i, \quad i = 1{,}2 \qquad (3)$$

Let the generalized input $Q$ be $[0, f_x]^T$.

Applying (1), (2) and (3), we obtain the following equations of motion:

$$(m_T + m_L)\ddot{x} + m_L l\ddot{\theta}\cos\theta - m_L\dot{\theta}^2\sin\theta = f_x \quad (4)$$
$$\ddot{x}\cos\theta + l\ddot{\theta} + g\sin\theta = 0 \quad (5)$$

where $f_x$ is the horizontal force acting on the trolley. Equation (5) can be written also by defining $\tilde{x} = x - x_d$ in the form

$$\ddot{\tilde{x}}\cos(\theta) + l\ddot{\theta} + g\sin(\theta) = 0 \quad (6)$$

Since $f_x$ is generated by a DC motor which is operated due to an electrical voltage $u$ then $f_x$ is written as

$$f_x = k_u u \quad (7)$$

where$k_u$ is the DC motor constant.Substitute for $f_x$ and Rearrange (4) and (5) result in the following form

$$\ddot{\theta} = \frac{-\cos\theta/l}{m_T+m_L(\sin\theta)^2}\Big((m_T + m_L)g\tan\theta + \quad m_L l\dot{\theta}^2 + k_u u\Big) \quad (8)$$

$$\ddot{\tilde{x}} = \frac{1}{m_T+m_L(\sin\theta)^2}\Big(\big((m_T + m_L)g\tan\theta + m_L l\sin\theta\,\dot{\theta}^2 + k_u u\big)\Big) - g\tan\theta \quad (9)$$

## 3. Sliding Mode control

Sliding mode control was first proposed and elaborated by several researchers from the former Russia, starting from the sixties (Emel'yanov and Taran; Utkin,). The ideas did not appear outside of Russia until a survey paper by Utkin [9] was published in English.

Sliding mode control is a discontinuous feedback control forces the system states to reach and remain on a specific surface within the state space (called sliding surface).

The main goal of the control is to move the load to the desired position while ensuring that the sway angle is so small (ideally zero). The first stage of design is the selection of the discontinuity surface such that sliding motion would exhibit desired properties. Before we establish our sliding manifold, the system should be in regular form, it means if we have a system of $(n)$ variables and $(m)$ control actions then $(n-m)$ of the set of differential equations of the system will appear without control action. For our system described by (8) and (9) there are two configuration variables$(x\ and\ \theta)$ and one actuation force $(u)$ which appears in both equations. To transform our system to the regular form we need first to get an output function such that its second derivative contains the following term

$$\ddot{\tilde{x}} + l\sec(\theta)\,\ddot{\theta} \quad (10)$$

with a remainder as a function of $\theta$and $\dot{\theta}$. This task may be accomplished if we consider the following relation (the derivative of the output function):

$$\dot{x}_1 = \dot{\tilde{x}} + l\sec(\theta)\dot{\theta} \quad (11)$$

where $x_1$ is the proposed output that will transform the system model to a regular form. Then when we differentiate $\dot{x}_1$ the first two terms will be equal to those in (11) as can be seen here:

$$\begin{aligned}\ddot{x}_1 &= \ddot{\tilde{x}} + L*sec(\theta)\ddot{\theta} + L*sec(\theta)tan(\theta)\dot{\theta}^2\\ &= sec(\theta)\{\ddot{\tilde{x}}cos(\theta) + L\ddot{\theta} + L*tan(\theta)\dot{\theta}^2\}\\ &= sec(\theta)\{-gsin(\theta) + L*tan(\theta)\dot{\theta}^2\}\\ &= -tan(\theta)\{g - L*sec(\theta)\dot{\theta}^2\}\end{aligned} \quad (12)$$

As a result, the required output is simply computed by integrating (11) as follows:

$$\begin{aligned}x_1 &= \tilde{x} + \int_0^\theta L*sec(\theta)d\theta\\ &= \tilde{x} + l*\ln\big(sec(\theta) + tan(\theta)\big)\end{aligned} \quad (13)$$

From (13) the crane model transforms to the regular form according to the following state transformation rules:

$$x = x_1 + x_d - l * ln(sec(x_2) + tan(x_2)) \\ \theta = x_2 \quad (14)$$

where $x_1$ and $x_2$ are the new coordinate variables that transform the system dynamical model to a regular form. The regular form for
the crane dynamical model is

$$\ddot{x}_1 = -\tan x_2 (g - l \sec x_2 \dot{x}_2) \quad (15)$$

$$\ddot{x}_2 = \frac{-\cos x_2/l}{m_T + m_L(\sin x_2)^2}\left((m_T + m_L)g \tan x_2 + m_L l \dot{x}_2{}^2 + k_u u\right) \quad (16)$$

Thus the system described by (8) and (9) with $x$ and $\theta$ variables are transformed to regular form described in (15) and (16). In these equations the control action $u$ disappears from (15) where $\ddot{x}_1$ is dependent only on $x_2$ while in (16) the control action $u$ appears.
To construct the switching surface, the error function is defined first as follows:

$$e = x_2 - tan^{-1}(c_1 \dot{x}_1 + c_2 x_1) \quad (17)$$

The aim is to make this error function reach zero value by choosing an appropriate switching function, the switching function may be written as:

$$s = \dot{e} + ce \quad (18)$$

The above equation represents our sliding manifold in terms of $e$ and the crane system must reach and remain in this manifold till it reaches the origin and thus a discontinuous controller is needed for this task, in the next step a discontinuous sliding mode controller is discussed and designed in order to carry this task which will drive the system equilibrium point and gain system stability.

## 4. Controller design

The second stage of the design procedure is the selection of discontinuous control enforcing sliding mode in manifold $s(x) = 0$. A condition for the sliding mode to exist is that the switching function and it's time derivative should be opposite in sign, writing that as the following:

$$s\dot{s} < 0 \quad (19)$$

where

$$\dot{s} = \ddot{e} + c\dot{e} \quad (20)$$

Since $\ddot{x}_2$ appears in $\ddot{e}$ that means the control action $u$will appear in $\dot{s}$. Rearrange equations and isolate the terms with $u$ to make $\dot{s}$ in the following form

$$\dot{s} = F + Gu < 0 \quad (21)$$

back to (17)

$$s\dot{s} = s[F + Gu] \quad (22)$$

Assuming that our controller is of the form

$$u = -k * sign(s) \quad (23)$$

where $sign(s)$ is the sign function defined as

$$sign(s) = \begin{cases} 1 & if\ s > 0 \\ -1 & if\ s < 0 \end{cases}$$

then (22) becomes

$$s\dot{s} = s[F - Gk * sign(s)] \quad (24)$$

Using the fact that $s \cdot F \leq |s||F|$ and $s \cdot sign(s) = |s|$ then (24) becomes

$$s\dot{s} \leq |s||F| - Gk|s| \\ = -|s|[Gk - |F|] \quad (25)$$

In the above inequality, $s\dot{s} < 0$ if and only if the bracket$[Gk - |F|] > 0$. Thus we get the following conditions for the gain $k$ of the sliding mode controller (refer to (22))

$$k > \frac{|F|}{G} \text{ For } G > 0 \tag{26}$$

and

$$k < \frac{|F|}{G} \text{ For } G < 0 \tag{27}$$

Hence a controller which is in the form indicated in (23) with the value of $k$ that have been calculated must bring the system trajectory to the surface $s = 0$ and keep it within that surface.

## 5. Stability analysis

When applying the sliding mode control action, the state will reach after a finite time to the surface $s = 0$(utkin 1992[9]). This is ensured via the inequality (25) with gain selection based on (26) and (27).After that it will be convenient to analyze the system stability within this surface

$$\dot{e} + ce = 0 \tag{28}$$

The above equation is a first order differential equation, using calculus the solution may be written as

$$e = e_0 \exp[-ct] \tag{29}$$

where $e_0$ is the error function initial value, thus $e$ decays with a time constant $c$ and note that $c$ >0 or otherwise the error function value will diverge to an infinite value.
Now when the error function value $e = 0$ then (17) can be written as

$$x_2 = tan^{-1}(c_1\dot{x}_1 + c_2x_1) \tag{30}$$

Substituting (30) in (15) results in

$$\ddot{x}_1 = -(c_1\dot{x}_1 + c_2x_1)(g - l \sec x_2 \, \dot{x}_2) \tag{31}$$

In (30), the stability will be considered for the following set

$$\Phi = \left\{(x_2, \dot{x}_2): \dot{x}_2 < \left(\frac{g}{l}\right) * cos(x_2)\right\} \tag{32}$$

pick the following point from set $\Phi$ as follows:

assume that the swing angle $\theta = x_2$ and it's derivative $\dot{\theta} = \dot{x}_2$are sufficiently small i.e. $x_2 \approx 0$and$\dot{x}_2 \approx 0$, then (31) becomes

$$\ddot{x}_1 = -gc_1\dot{x}_1 - gc_2x_1 \tag{33}$$

the solution for (33) is in the form

$$x_1 = A_1e^{p_1t} + A_2e^{p_2t} \tag{34}$$

where$A_1$, $A_2$ are constants depend on the initial condition,$p_1$and$p_2$ are the roots of the characteristic equation for the differential equation in (32)

$$\left.\begin{aligned} p_1 &= \frac{-gc_1 + \sqrt{g^2c_1^2 - 4gc_2}}{2} \\ p_2 &= \frac{-gc_1 - \sqrt{g^2c_1^2 - 4gc_2}}{2} \end{aligned}\right\} \tag{35}$$

To ensure that the system in (34) is asymptotically stable, $p_1$ and $p_2$ must be $< 0$, this can be ensured only if $c_1$ and $c_2$ are $> 0$.

Choosing appropriate values for $c_1$ and $c_2$ then $x_1$ and $\dot{x}_1$ converges to 0 and as a result $x_2$which is the sway angle $\theta$also converges to 0 due to (30), and finally the trolley position $x$ approaches the desired position $x_d$ according to (14).

The analysis that we have gone through shows the ability of the sliding mode controller in regulating the state to the origin by controlling the trolley motion. The system stability was insured by choosing a positive values for $c_1$, $c_2$and $c$, while the dynamic characteristic depends on the selection of appropriate values.

## 6. Simulation results

To validate our work the system with the suggested control is simulated using MATLAB. The simulation parameters given below are taken from an experimental pendulum system built by Feedback Instruments Ltd [11].

Trolley mass $m_T = 2.4\ kg$
Load mass $m_L = 0.23\ kg$
Rope length $l = 0.4\ m$
DC motor constant $k_u = 8\ N/volt$
Desired trolley position $x_d = 1\ m$
$c_1$, $c_2$ and $c$ are chosen to be 0.5, 0.5, 1 respectively.

The above values are used in calculation controller gain $k$ (A8) which is found to be $-2$. The figures from (4) to (10) show the system variables and performance when the controller we designed in section 4 is applied to the crane system. Switching function versus time and error function are displayed in figures (4) and (5) respectively. Trolley position is shown in figure (6) and approach the desired distance in about 5 sec. while the swing angle do not exceed $\pm 4$ deg. as shown in figure (7). The sway angle velocity is shown in figure (8). The controller signal is shown in figure (10) and note how it chatters in a high frequency in order to keep the system states inside the switching manifold.

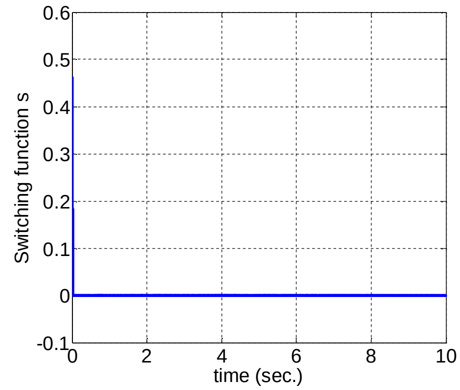



Figure (4): Switching function vs. time.

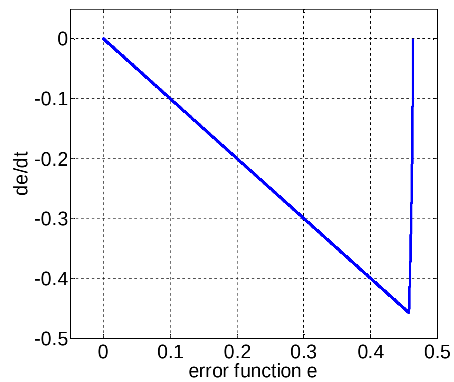



Figure (5):phase plot for error function.

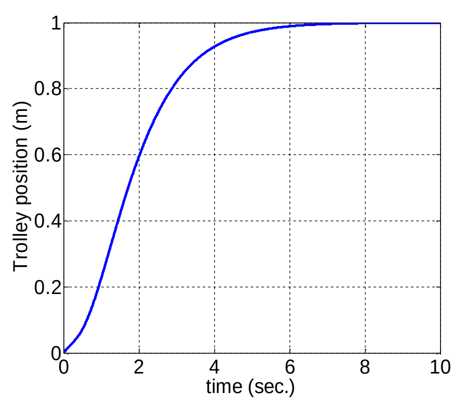



Figure (6):Trolley position vs. time.

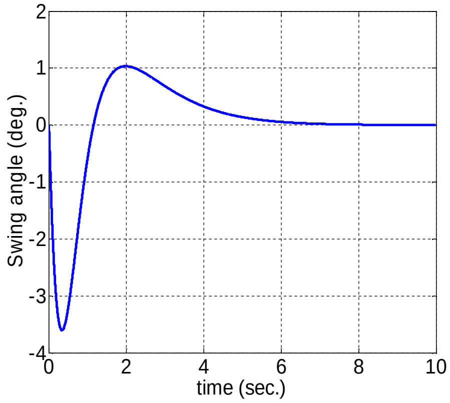



Figure (7): Sway angle vs. time.

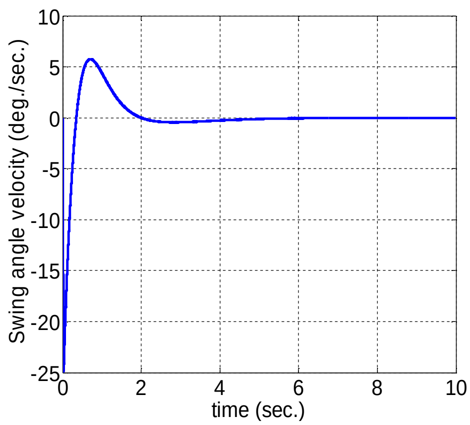



Figure (8): Sway angle velocity.

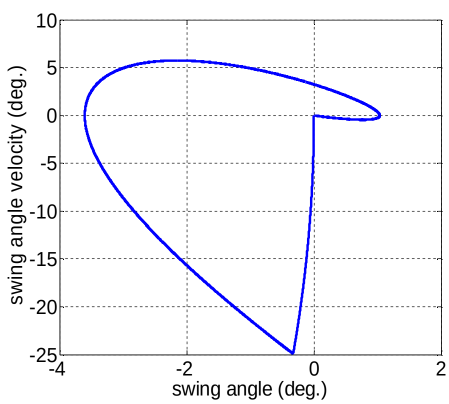



Figure (9): Phase plot for sway angle.

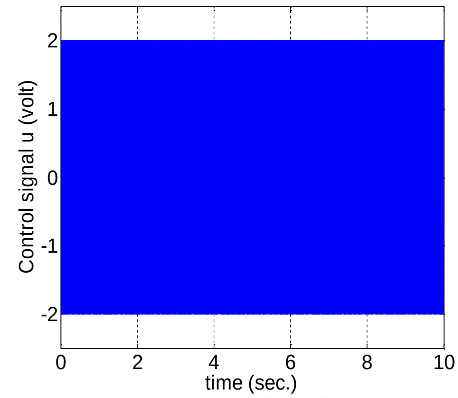



Figure (10): control signal vs. time.

To validate that the controller designed in our work is robust to the variations we repeated the simulation with a change of 100% in load mass, i.e. $m_L = 2m_{L0} = 0.46\ kg$ where $m_{L0}$ is the nominal load mass value used in designing the sliding mode controller here. The results are shown below through figures (11) to (17).

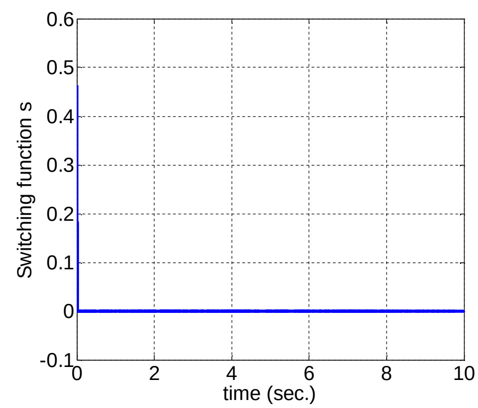
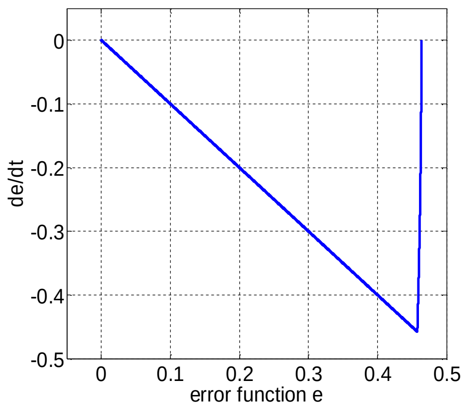
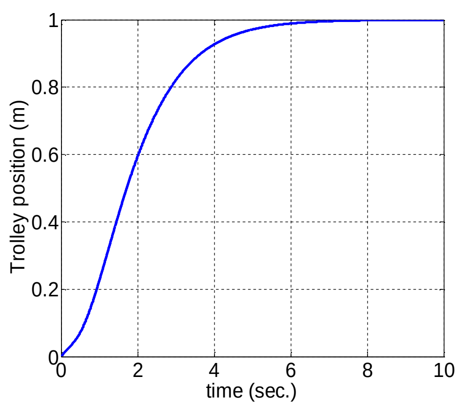


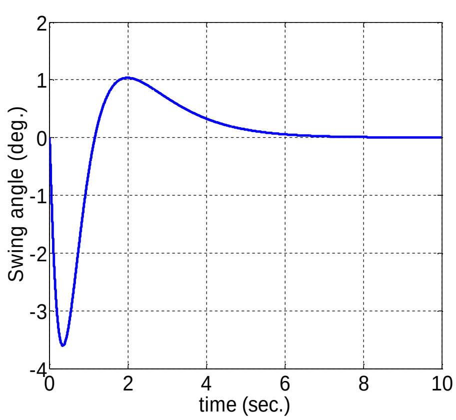

Figure (11): Switching function vs. time for $m_L = 0.46\ kg$.



Figure(12): phase plot for error functionfor$m_L$ = 0.46 $kg$ .



Figure(13) : Trolley position vs. time for $m_L$ = 0.46 $kg$ .



Figure(14): Sway angle vs. time for $m_L$ = 0.46 $kg$ .

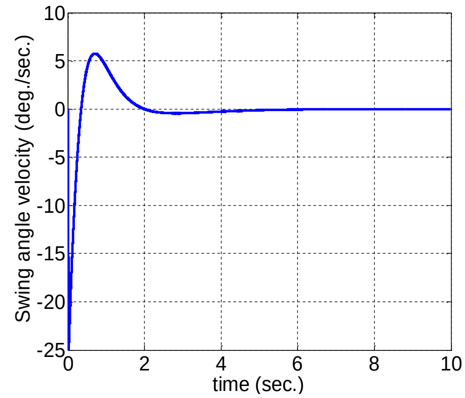


Figure (15): sway angle velocity vs.time for$m_L$ = 0.46 $kg$ .

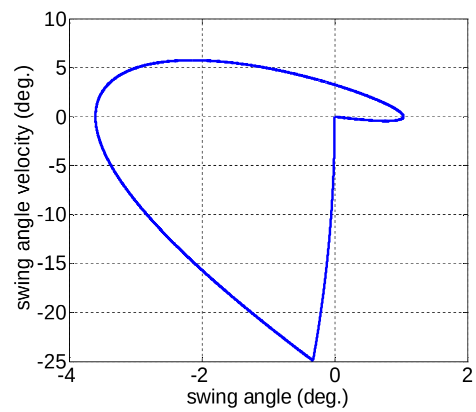


Figure (16): Phase plot for sway angle for $m_L$ = 0.46 $kg$.

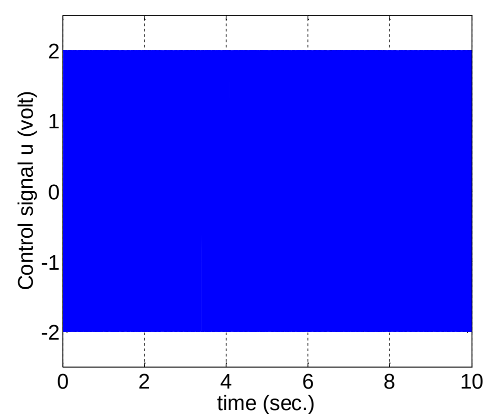


Figure (17): Control signal vs. time for $m_L$ = 0.46 $kg$ .

In the simulations we done previously we assumed the controller as a sign function $(-ksign(s))$ , because this function can have only the values $-1$ or $+1$ this causes the control signal to chatter in a high speed between these two values in order to keep the system having the desired performance, this phenomenon is frequently not preferred and may have a drawbacks on the physical system devices. A solution is to use a smooth function to eliminate the chattering phenomenon; we suggested using the Arctangent function defined as

$$Arctan(s) = tan^{-1}(s) \quad (36)$$

which have a smooth response, defined at $s = 0$and have a maximum value ranges between$\pm\frac{\pi}{2}$, then the controller becomes in the following form

$$u = -\frac{2}{\pi}k\tan^{-1}(\gamma s) \quad (37)$$

where $\gamma$ is an arbitrary number with an appropriate value used to make the arctan function as close as possible to the sign function. Substitute for $u$ in (23) then

$$s\dot{s} = s\left[F - Gk\frac{2}{\pi}\tan^{-1}(\gamma s)\right] \quad (38)$$

knowing that $s\tan^{-1}\gamma s = |s|\tan^{-1}\gamma|s|$and repeating the steps in (24) and (25) as the following

$$s\dot{s} \le |s||F| - Gk\frac{2}{\pi}|s|\tan^{-1}\gamma|s|$$

$$= -|s|\left[Gk\frac{2}{\pi}\tan^{-1}\gamma|s| - |F|\right] \quad (39)$$

Equation (38) induce that

$$k > \frac{\pi}{2\tan^{-1}\gamma|s|}\frac{|F|}{G}\text{For } G > 0 \quad (40)$$

and

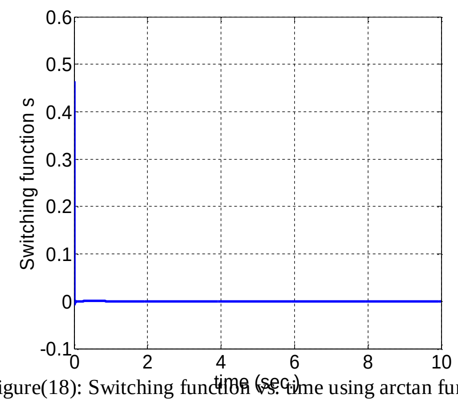
$$k < \frac{\pi}{2\tan^{-1}\gamma|s|}\frac{|F|}{G}\text{For } G < 0 \quad (41)$$

For $|s| \le 0.01$ and taking a suitable value for $\gamma$, say $60$ then (40) yields that $k = -6$. In this case the state reaches the switching manifold asymptotically and finally it reaches a compact set around the origin but no longer reaches the origin since the disturbance is of a non-vanishing type at the origin (one can refer to [12] for the definition of the non-vanishing disturbances). In [13] the stability was proved but with an ultimate bond. This bound is given by

$$\Delta = \{(e,\dot{e}) \in \mathcal{R}^2 : |e| < 0.01, |s| \le 0.01\}$$

where $\Delta$ is a compact invariant set.

The system is simulated again using the controller defined in (36) and the results are plotted in the following figures with $m_L = 0.23\,kg$ .




Figure(18): Switching function vs. time using arctan function.

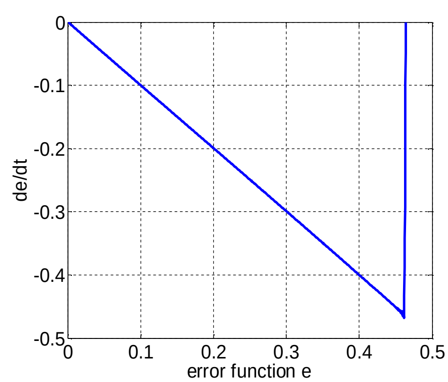
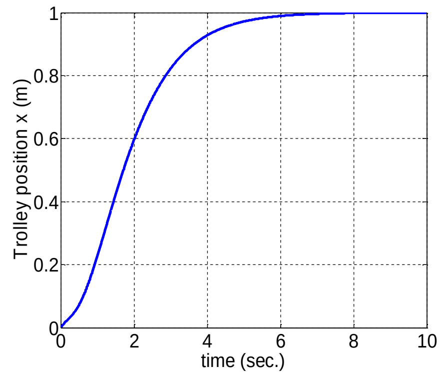



Figure (19): phase plot for error function using arctan function.




Figure(20): Trolley position vs. time using arctan function.

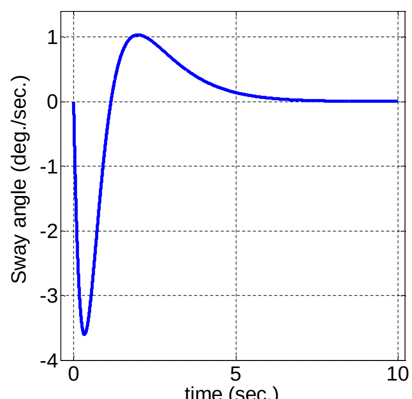



Figure (21): Sway angle vs. time using arctan function.

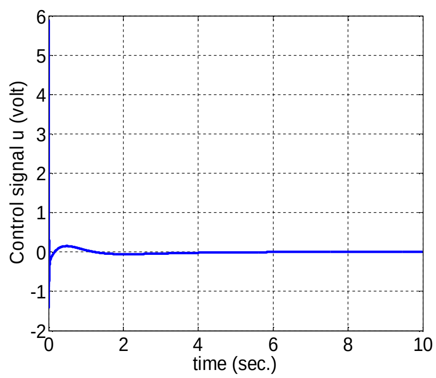



Figure (22): Control signal vs. time using arctan function.

To show that the controller using arctan function is also robust to model variations the load mass will be doubled to become 0.46 kg.

The simulations will be repeated to show the time response of the system.

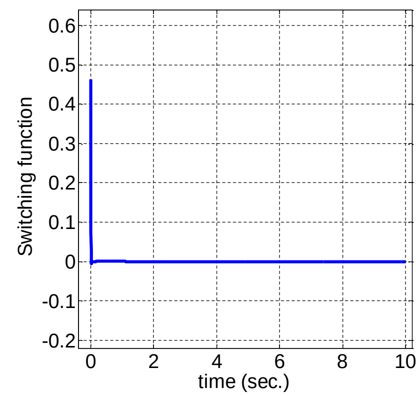



Figure (23): Switching function.

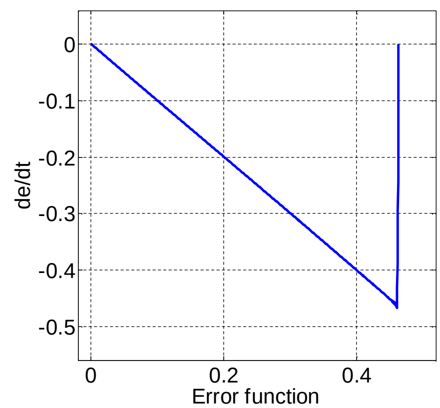



Figure (24): Phase plot for error function.

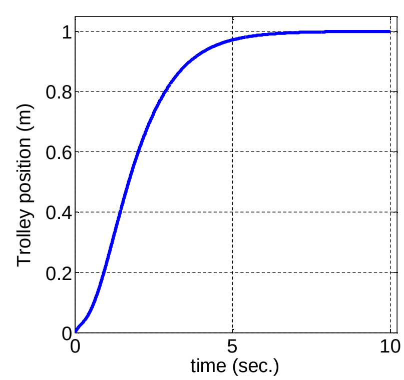



Figure (25): Trolley position.

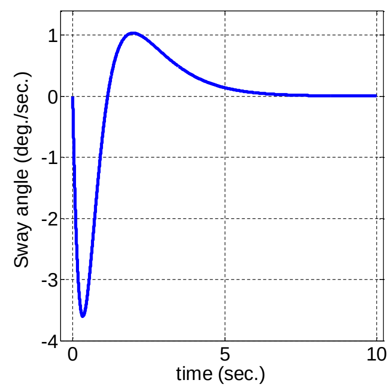



Figure (26): Sway angle vs. time.

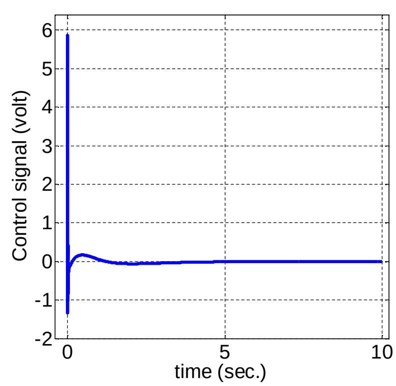



Figure (25): Control signal.

## 7. Conclusions

In this paper a sliding mode control have been proposed for a two dimensional overhead crane to move the load to the desired position with minimum sway angle. In section2we first modeled the crane system where a full nonlinear model for the gantry crane is derived and represented by (8) and (9) after considering some assumptions. In section 3 we succeeded in transforming the gantry crane model to the regular form then an error function is written using the new system variables and a switching function is constructed in terms of that error function.

A sliding mode controller is designed in section 4 depending on the switching function which will force the state trajectory to reach the sliding surface $s = 0$ and remain on that surface. In section 5 stability analyses are made to prove that the system is asymptotically stable within the sliding surface. The proposed discontinuous controller assigned in (23) with suitable value of $k$ was able to transfer the load to the desired position (1m in our case) within time interval approximately 5*sec.*while the sway angle did not exceed $\pm 4 deg.$ which considered very acceptable value and the oscillation vanishes as the load reaches the desired position with minimum sway angle.

The swing angle velocity is found to be no more than $\pm 25$deg. /sec. which is in the range we specified for them in appendix, this shows that our controller was able to keep the system variables in ranges specified for them.

In addition the proposed control has proved it's robustness by giving the desired performance in spite of change in the load mass which has been doubled. The problem of chattering is took in consideration and a solution to this problem has been put by using the arctan function and see how the control signal obtained by simulation is smooth compared by the signal obtained from the signum function.

The work and results we had gone through proves the sliding mode control ability to control the system we proposed in spite of system uncertainties by choosing an appropriate stable switching function and calculating the value of $k$ that makes the controller able to bring the switching function to origin.

**Appendix (A)**. Calculating the value of *k*:
From (17), (18) and (20) we have

$$e = x_2 - tan^{-1}(c_1\dot{x}_1 + c_2x_1)$$
$$s = \dot{e} + ce$$
$$\dot{s} = \ddot{e} + c\dot{e}$$

Derive (16) yields

$$\dot{e} = \dot{x}_2 - \frac{(c_1\ddot{x}_1+c_2\dot{x}_1)}{(c_1\dot{x}_1+c_2x_1)^2+1} \quad (A1)$$

Derive the above equation

$$\ddot{e} = \ddot{x}_2 - \left[\frac{(c_1\dddot{x}_1+c_2\ddot{x}_1)}{(c_1\dot{x}_1+c_2x_1)^2+1} + (c_1\ddot{x}_1 + c_2\dot{x}_1)^2 \frac{-2(c_1\dot{x}_1+c_2x_1)}{((c_1\dot{x}_1+c_2x_1)^2+1)^2}\right] \quad (A2)$$

Substitute (A1) and (A2) in (19) yields

$$\dot{s} = \ddot{x}_2 - \left[\frac{(c_1\dddot{x}_1+c_2\ddot{x}_1)}{(c_1\dot{x}_1+c_2x_1)^2+1} + (c_1\ddot{x}_1 + c_2\dot{x}_1)^2 \frac{-2(c_1\dot{x}_1+c_2x_1)}{((c_1\dot{x}_1+c_2x_1)^2+1)^2}\right] + c\left[\dot{x}_2 - \frac{(c_1\ddot{x}_1+c_2\dot{x}_1)}{(c_1\dot{x}_1+c_2x_1)^2+1}\right] \quad (A3)$$

Derive (10) then we have

$$\dddot{x}_1=\{-sec^2(x_2)[g - lsec(x_2)\dot{x}_2^2] + [-lsec(x_2)\tan(x_2)\dot{x}_2^2]\}\dot{x}_2 + ((-\tan(x_2))(-lsec(x_2)2\dot{x}_2))\frac{-\cos x_2/l}{m_T+m_L(\sin x_2)^2}[(m_T + m_L)g\tan x_2 + m_L l\sin x_2\dot{x}_2^2 + k_u u] \quad (A4)$$

Substitute $\dddot{x}_1$, $\ddot{x}_2$ and $\dddot{x}_1$ from (14), (15) and (A4) respectively in (A3) we get

$$\dot{s} = \frac{-\cos x_2/l}{m_T + m_L(\sin x_2)^2}[(m_T + m_L)g\tan x_2 + m_L l\sin x_2\dot{x}_2^2 + k_u u]$$
$$-c_1\Bigg[\{-sec^2(x_2)[g - lsec(x_2)\dot{x}_2^2] + [-lsec(x_2)\tan(x_2)\dot{x}_2^2]\}\dot{x}_2 + ((-\tan(x_2))(-lsec(x_2)2\dot{x}_2))\frac{-\cos\frac{x_2}{l}}{m_T + m_L(\sin x_2)^2}[(m_T + m_L)g\tan x_2 + m_L l\sin x_2\dot{x}_2^2 + k_u u] + c_2(-tan(x_2) * \{g - l * sec(x_2)\dot{x}_2^2\})\Bigg]$$
$$\overline{(c_1\dot{x}_1 + c_2x_1)^2 + 1}$$
$$+(c_1(-tan(x_2) * \{g - l * sec(x_2)\dot{x}_2^2\}) + c_2\dot{x}_1)^2\frac{2(c_1\dot{x}_1 + c_2x_1)}{((c_1\dot{x}_1 + c_2x_1)^2 + 1)^2}$$
$$+c\dot{x}_2 - \frac{c(c1(-tan(x_2) * \{g - l * sec(x_2)\dot{x}_2^2\}) + c_2\dot{x}_1)}{(c_1\dot{x}_1 + c_2x_1)^2 + 1}$$

(A5)

Then

$$F = \frac{-\cos x_2/l}{m_T + m_L(\sin x_2)^2}[(m_T + m_L)g\tan x_2 + m_L l\sin x_2\dot{x}_2^2]$$
$$-c_1\Bigg[\{-sec^2(x_2)[g - Lsec(x_2)\dot{x}_2^2] + [-lsec(x_2)\tan(x_2)\dot{x}_2^2]\}\dot{x}_2 + ((-\tan(x_2))(-lsec(x_2)2\dot{x}_2))\frac{-\cos\frac{x_2}{l}}{m_T + m_L(\sin x_2)^2}[(m_T + m_L)g\tan x_2 + m_L l\sin x_2\dot{x}_2^2] + c_2(-tan(x_2) * \{g - l * sec(x_2)\dot{x}_2^2\})\Bigg]$$
$$\overline{(c_1\dot{x}_1 + c_2x_1)^2 + 1}$$
$$+(c_1(-tan(x_2) * \{g - l * sec(x_2)\dot{x}_2^2\}) + c_2\dot{x}_1)^2\frac{2(c_1\dot{x}_1 + c_2x_1)}{((c_1\dot{x}_1 + c_2x_1)^2 + 1)^2}$$
$$+c\dot{x}_2 - \frac{c(c1(-tan(x_2) * \{g - l * sec(x_2)\dot{x}_2^2\}) + c_2\dot{x}_1)}{(c_1\dot{x}_1 + c_2x_1)^2 + 1}$$

(A6)

And

$$G = k_u\frac{-\cos x_2/l}{m_T + m_L(\sin x_2)^2}[1 + (\tan(x_2))(lsec(x_2)2\dot{x}_2)] \quad (A7)$$

To evaluate $F$and $G$ these assumptions are taken in consider:
The sway angle $\theta$and it's derivative$\dot{\theta}$ with time are supposed to range between the following values

$$-\pi/18 \leq \theta \leq \pi/18$$
$$-\pi/6 \leq \dot{\theta} \leq \pi/6$$

The desired travelling distance$x_d$ is specified as$x_d = 1m$ and so$-1.1 \leq x_1 \leq 1.1$.
The trolley velocity $\dot{x}$is considered to be as follows

$$-1m/sec \leq \dot{x} \leq 1m/sec$$

So that

$$-1.25 \leq \dot{x}_1 \leq 1.25 .$$

Using the values suggested above the maximum value that $F$ can have is approximately $15.6$ and the minimum value for $G$ is approximately $-8$. Since $G$ is negative in sign then (26) is used

$$k < \frac{|F|}{G}$$

$$\therefore k < -\frac{15.6}{8} \quad \text{or} \quad k < -1.95 \qquad \text{(A8)}$$

This value for $k$ was obtained under the assumption that the sway angle and it's derivative are approximately zero. In our work we will take $k = -2$ to get sure that the controller will be able to bring the system to the origin even if the sway angle and/or it's derivative are little bit more or less than values that have been suggested.

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