Sufficient Conditions for Conditional Stability of the Zero Solution of Systems of Impulsive Functional Differential Equations

Muayyad Mahmood Khalil
Dept. of Mathematics , College of Education, University of Tikrit , Tikrit , Iraq
math.muayyad@gmail.com

(**Received:** 3 / 1 / 2012 ---- **Accepted:** 2 / 4 / 2012)

Abstract
In this paper we have use vector Lyapunov functions and the comparison principle and we give sufficient conditions for conditional stability of the zero solution of systems under consideration.

Keywords: zero solution , conditional stability.

1. Preliminary

Let $t_0 \in \mathbb{R}$, $r > 0$. Let $\|x\| = |x_1| + |x_2| + \cdots + |x_n|$ be the norm of $x \in \mathbb{R}^n$. Consider the system

$$\begin{cases} \dot{x}(t) = f(t, x_t), t \neq t_k \\ \Delta x(t) = I_k(x(t)), t = t_k, k = 1,2,\dots \end{cases} \quad (1.1)$$

Where $f: [t_0, \infty) \times PC[[-r,0], \mathbb{R}^n] \to \mathbb{R}^n$; $I_k: \mathbb{R}^n \to \mathbb{R}^n$, k=1,2,… ; $t_0 < t_1 < t_2 < \cdots < t_k < t_{k+1} < \cdots$ and $\lim_{k\to\infty} t_k = \infty$.

Let $\varphi_0 \in PC[[-r,0], \mathbb{R}^n]$. Denote by $x(t) = x(t; t_0, \varphi_0)$ the solution of system (1.1), satisfying the initial condition

$$\begin{cases} x(t; t_0, \varphi_0) = \varphi_0(t - t_0), t_0 - r \le t \le t_0 \\ x(t_0 + 0; t_0, \varphi_0) = \varphi_0(0), \end{cases} \quad (1.2)$$

Let $M(n-l), l < n$ be a $(n-1)$-dimensional manifold in $\mathbb{R}^n$, containing the origin.

We set
$M_0(n-l) = \{\varphi: \varphi \in PC[[-r,0], M(n-l)]\}$.

We shall give the following definitions of conditional stability of the zero solution of system (1.1) with respect to the manifold $M(n-l)$.[1][2][3]

Definition 1.1[5] **:** The zero solution of system (1.1) is said to be :

(a) Conditionally stable with respect to the manifold $M(n-l)$,if
$(\forall t_0 \in \mathbb{R})(\forall \varepsilon > 0)(\exists \delta = \delta(t_0, \varepsilon) > 0)$
$(\forall \varphi_0 \in \overline{S_\delta}(PC_0) \cap M_0(n-l))(\forall t \ge t_0): x(t; t_0, \varphi_0) \in S_\varepsilon$;

(b) Conditionally uniformly stable with respect to $M(n-l)$, if the function δ in (a) is independent of t_0;

(c) Conditionally globally equi-attractive with respect to $M(n-l)$, if
$(\forall t_0 \in \mathbb{R})(\forall \alpha > 0)(\forall \varepsilon > 0)(\exists T = T(t_0, \alpha, \varepsilon) > 0)$
$(\forall \varphi_0 \in \overline{S_\alpha}(PC_0) \cap M_0(n-l))(\forall t \ge t_0 + T): x(t; t_0, \varphi_0) \in S_\varepsilon$;

(d) Conditionally uniformly globally attractive with respect to $M(n-l)$,if the number T in(c) is independent of t_0;

(e) Conditionally globally equi-asymptotically stable with respect to $M(n-l)$, if it is conditionally stable and conditionally globally equi-attractive with respect to $M(n-l)$;

(f) Conditionally uniformly globally asymptotically stable with respect to $M(n-l)$, if it is conditionally uniformly stable and conditionally uniformly globally attractive with respect to $M(n-l)$;

(g) Conditionally unstable with respect to the manifold $M(n-l)$, if (a) fails to hold.

<u>Remark 1.2[2][3]</u> If $M(n-l) = \mathbb{R}^n$, then the definitions (a)-(g) are reduced to the usual definitions of stability by Lyapunov for the zero solution of system (1.1).

Together with the system (1.1), we shall consider the following system of impulsive ordinary differential equations:

$$\begin{cases} \dot{u}(t) = D(t)u(t), t \neq t_k, t \ge t_0 \\ \Delta u(t_k) = D_k u(t_k), k = 1,2,\dots, t_k > t_0 \end{cases} \quad (1.3)$$

Where $u: [t_0, \infty) \to \mathbb{R}_+^m$; $D(t)$ is an $(m \times m)$-matrix valued function; $D_k; k = 1,2,\dots$ are $(m \times m)$-constant matrices.

Let $u_0 \in \mathbb{R}_+^m$, We denote by $u(t) = u(t; t_0, u_0)$ the solution of system (1.3), which satisfies the initial condition $u(t_0) = u_0$, and by $J^+(t_0, u_0)$ the maximal interval of type $[t_0, \beta)$ in which the solution $u(t; t_0, u_0)$ is defined.

Let $e \in \mathbb{R}_+^m$ be the vector (1,1,……,1). We introduce the sets:

$B(\alpha) = \{u \in \mathbb{R}_+^m: 0 \le u < \alpha e\}$,
$\bar{B}(\alpha) = \{u \in \mathbb{R}_+^m: 0 \le u \le \alpha e\}, \alpha = const > 0$,
$R(m-l) = \{u = (u_1, \dots, u_m \in \mathbb{R}^m: u_1 = u_2 = \cdots = u_l = 0\}, l < m$.

Introduce the following conditions:

H1: The matrix-valued $(m \times m)$-function D(t) is continuous for $t \in [t_0, \infty)$.

H2: The functions $\psi_k: \mathbb{R}_+^m \to \mathbb{R}_+^m, \psi_k(u) = u + D_k u, k = 1,2,\dots$, are non-decreasing in $\mathbb{R}_+^m$.

H3: $J^+(t_0, u_0) = [t_0, \infty)$.

H4: $t_0 < t_1 < t_2 < \cdots < t_k < t_{k+1} < \cdots$

H5: $\lim_{k\to\infty} t_k = \infty$.

H6: $f(t, 0) = 0, t \ge t_0$.

H7: $I_k(0) = 0, k = 1,2,\dots$.

H8: The function f is continuous on $[t_0,\infty)\times PC[[-r,0],\mathbb{R}^n]$.

H9: The function f is locally Lipschitz continuous with respect to its second argument on $[t_0,\infty)\times PC[[-r,0],\mathbb{R}^n]$.

H10: There exist a constant $P>0$ such that $\|f(t,x_t)\|\le P<\infty\ for\ (t,x_t)\in[t_0,\infty)\times PC[[-r,0],\mathbb{R}^n]$

H11: $I_k\in[\mathbb{R}^n,\mathbb{R}^n], k=1,2,\dots.$

We shall consider such solutions $u(t)$of the system (1.2) for which $u(t)\ge 0$. That is why the following definitions on conditional stability of the zero solution of this system will be used.

Definition 1.3[2][3] The zero solution of system (1.2) is said to be:

(a) Conditionally stable with respect to the manifold $R(m-l)$, if

$(\forall t_0\in\mathbb{R})(\forall\varepsilon>0)(\exists\delta=\delta(t_0,\varepsilon)>0)$
$(\forall u_0\in\bar{B}(\delta)\cap R(m-l))(\forall t\ge t_0):u^+(t;t_0,u_0)\in B(\varepsilon);$

(b) Conditionally uniformly stable with respect to $R(m-l)$, if the function δ from (a) does not depend on t_0;

(c) Conditionally globally equi-attractive with respect to $R(m-l)$, if

$(\forall t_0\in\mathbb{R})(\forall\alpha>0)(\forall\varepsilon>0)(\exists T=T(t_0,\alpha,\varepsilon)>0)$
$(\forall u_0\in\bar{B}(\alpha)\cap R(m-l))(\forall t\ge t_0+T):u^+(t;t_0,u_0)\in B(\varepsilon);$

(d) Conditionally uniformly globally attractive with respect to $R(m-l)$, if the number T in (c) does not depend on t_0;

(e) Conditionally globally equi-asymptotically stable with respect to $R(m-l)$, if it is conditionally stable and conditionally globally equi-attractive with respect to $R(m-l)$,

(f) Conditionally uniformly globally asymptotically stable with respect to $R(m-l)$, if it is conditionally uniformly stable and conditionally uniformly globally attractive with respect to $R(m-l)$,

(g) Conditionally unstable with respect to the manifold $R(m-l)$, if (a) fails to hold.

In the successive investigations, we shall use piecewise continuous auxiliary vector functions $V:[t_0,\infty)\times\mathbb{R}^n\to\mathbb{R}^m_+, V=col(V_1,\dots,V_m)$ such that $V_j\in V_0$, $j=1,2,\dots,m.$

Theorem 1.4[5] assume that

(1) The conditions

H1: The function f is continuous in $[t_0,\infty)\times PC[[-r,0],\Omega]$

H2: there exist a constant $P>0$ such that $\|f(t,x_t\|\le P<\infty\ for\ (t,x_t)\in[t_0,\infty)\times PC[[-r,0],\Omega]$

H3: $(E+I_k):\Omega\to\Omega, k=1,2,\dots, where\ E$ is the identity in Ω.

H4: $I_k\in C[\Omega,\Omega], k=1,2,\dots$

H5: : $t_0<t_1<t_2<\dots<t_k<t_{k+1}<\cdots$

H6: $lim_{k\to\infty}t_k=\infty.$

(2) The function F is quasi-monotone increasing, continuous in the sets $(t_k,t_{k+1}]\times\mathbb{R}^m_+, k\in N\cup\{0\}\ and\ for\ k=1,2,\dots\ and\ v\in\mathbb{R}^m_+$ there exists the finite limit

$$\lim_{\substack{(t,u)\to(t,v)\\ t>t_k}} F(t,u)$$

(3) The maximal solution $u^+:J^+(t_0,u_0)\to\mathbb{R}^m_+$ of the system

$$\begin{cases}\dot{u}(t)=F(t,u(t)), t\ne t_k\\ \Delta u(t_k)=u(t_k+0)-u(t_k)=J_k(u(t_k)), t_k>t_0\end{cases}$$

Is defined for $t\ge t_0$.

(4) The functions $\psi_k:\mathbb{R}^m_+\to\mathbb{R}^m_+,\psi_k(u)=u+J_k(u), k=1,2,\dots$ are non-decreasing in $\mathbb{R}^m_+$.

(5) The function $V:[t_0,\infty)\times\Omega\to\mathbb{R}^m_+, V=col(V_1,\dots,V_m)$ $V_j\in V_0$, $j=1,2,\dots,m.$ such that

$V(t_0+0,\varphi_0(0))\le u_0$

$V(t+0,x+I_k(x))\le\psi_k(V(t,x)), x\in\Omega, t=t_k, k=1,2,\dots$

And the inequality

$D^+_{(*)}V(t,x(t))\le F(t,V(t,x(t))), t\ne t_k, k=1,2,\dots,$

Where * is the system

$$\begin{cases}\dot{x}(t)=f(t,x_t), t\ne t_k\\ \Delta x(t_k)=x(t_k+0)-x(t_k)=I_k(x(t_k)), t_k>t_0\end{cases}$$

Is valid for $t\in[t_0,\infty), x\in\Omega_P$.

Then

$V(t,x(t;t_0,\varphi_0))\le u^+(t;t_0,u_0) for\ t\in[t_0,\infty)$

2.Main Results

Theorem 2.1[2][5] Assume that

(1) Conditions H1-H11in Remark 1.2 are hold.

(2) There exist a function $V:[t_0,\infty)\times\mathbb{R}^n\to\mathbb{R}^m_+, m\le n, V=col(V_1,\dots,V_m)$, $V_j\in V_0, j=1,2,\dots,m.$ such that

$sup_{[t_0,\infty)\times\mathbb{R}^n}\|V(t,x)\|=K\le\infty,$

$V(t,0)=0, t\ge t_0$

$a(\|x\|)e\le V(t,x), a\in K, (t,x)\in[t_0,\infty)\times\mathbb{R}^n$

(2.1)

$V(t+0,x+I_k(x))\le\psi_k(V(t,x)), x\in\mathbb{R}^n, t=t_k, k=1,2,\dots,$

And the inequality

$D^+_{(1.1)}V(t,x(t))\le D(t)V(t,x(t)), t\ne t_k, k=1,2,\dots,$

Is valid for $t\ge t_0$ and $x\in\Omega_1$.

(3) The set $M(n-l)=\{x\in\mathbb{R}^n: V_k(t+0,x)\equiv 0, k=1,2,\dots,l\}$ is

an $(n-l)$-dimensional manifold in $\mathbb{R}^n$, containing the origin, $l < n$.

Then :

a) If the zero solution of system (1.3) is conditionally stable with respect to the manifold $R(m-l)$, then the zero solution of system (1.1) is conditionally stable with respect to the manifold $M(n-l)$.

b) If the zero solution of system (1.3) is conditionally globally equi-attractive with respect to the manifold $R(m-l)$, then the zero solution of system (1.1) is conditionally globally equi-attractive with respect to the manifold $M(n-l)$.

Proof of a : Let $t_0 \in \mathbb{R}$ and $\varepsilon > 0$ $(a(\varepsilon) < K)$ be given. Let the zero solution of system (1.3) be conditionally stable with respect to $R(m-l)$. Then, there exists a positive function $\delta_1 = \delta_1(t_0, \varepsilon)$ which is continuous in t_0 for given ε and is such that, if $u_0 \in \bar{B}(\delta_1) \cap R(m-l)$, then $u^+(t; t_0, u_0) < a(\varepsilon)e$ for $t \geq t_0$.

It follows, from the properties of the function V, there exists $\delta = \delta(t_0, \varepsilon) > 0$ such that if $x \in \overline{S_\delta}$ then $V(t_0+0, x) \in \bar{B}(\delta_1)$.Let $\varphi_0 \in \overline{S_\delta}(PC_0) \cap M_0(n-l)$. Then $\varphi_0(0) \in \overline{S_\delta}$ and therefore, $V(t_0+0, \varphi_0(0)) \in \bar{B}(\delta_1)$.moreover, $V_k(t_0+0, \varphi_0(0)) = 0 \; for \; k = 1,2,\dots,l$ i.e. $V(t_0+0, \varphi_0(0)) \in R(m-l)$. Thus,

$$u^+(t; t_0, V(t_0+0, \varphi_0(0))) < a(\varepsilon)e, \quad t \geq t_0 \quad (2.2)$$

Let $x(t) = x(t; t_0, \varphi_0)$ be the solution of the initial value problem (1.1),(1.2) then the function V satisfies all condition of theorem (1.4) for $u_0 = V(t_0+0, \varphi_0(0))$ and by (2.1) and (2.2), we arrive at

$$a(\|x(t)\|)e \leq V(t, x(t)) \leq u^+(t; t_0, V(t_0+0, \varphi_0(0))) < a(\varepsilon)e$$

For $t \geq t_0$. Hence, $x(t; t_0, \varphi_0) \in S_\varepsilon$ for $t \geq t_0$, i.e. the zero solution of system (1.1) is conditionally stable with respect to the manifold $M(n-l)$.

Proof of b: Let $t_0 \in \mathbb{R}$ and $\varepsilon > 0$ $(a(\varepsilon) < K)$ be given.

It follows, from the properties of the function V, that there exists $\alpha_1 = (t_0, \alpha) > 0$ such that if $x \in \overline{S_\alpha}$ then $V(t_0+0, x) \in \bar{B}(\alpha_1)$.If the zero solution of system (2.1) is conditionally globally equi-attractive with respect to $R(m-l)$, there exists a number $T = T(t_0, \alpha, \varepsilon) > 0$ such that if $u_0 \in \bar{B}(\alpha_1) \cap R(m-l)$, then $u^+(t; t_0, u_0) < a(\varepsilon)e$ for $t \geq t_0 + T$.

Let $\varphi_0 \in \overline{S_\alpha}(PC_0) \cap M_0(n-l)$. Then $\varphi_0(0) \in \overline{S_\alpha}$ and, $V(t_0+0, \varphi_0(0)) \in \bar{B}(\alpha_1) \cap R(m-l)$ therefore

$$u^+(t; t_0, V(t_0+0, \varphi_0(0))) < a(\varepsilon)e, \quad t \geq t_0 + T \quad (2.3)$$

If $x(t) = x(t; t_0, \varphi_0)$ be the solution of the initial value problem (1.1),(1.2) then it follows from theorem (1.4) that

$$V(t, x(t)) \leq u^+\left(t; t_0, V(t_0+0, \varphi_0(0))\right), t \geq t_0$$

The last inequality , (2.1) and (2.3) imply the inequalities

$$a(\|x(t)\|)e \leq V(t, x(t)) \leq u^+(t; t_0, V(t_0+0, \varphi_0(0))) < a(\varepsilon)e$$

For $t \geq t_0 + T$.

Therefore , $\|x(t; t_0, \varphi_0)\| < \varepsilon$ for $t \geq t_0 + T$, that leads to the conclusion that the zero solution of system (1.1) is conditionally globally equi-attractive with respect to the manifold $M(n-l)$.

Corollary 2.2. let the conditions of theorem 2.1 be fulfilled then conditional global equi-asymptotic stability of the zero solution of system (1.3) with respect to the manifold $R(m-l)$ implies the conditional global equi-asymptotic stability of the zero solution of system (1.1) with respect to the manifold $M(n-l)$.

Theorem 2.3. let the conditions of theorem 2.1. be fulfilled, and let a function $b \in K$ exist such that $V(t,x) \leq b(\|x\|)e$ for $(t,x) \in [t_0, \infty) \times \mathbb{R}^n$.

Then

(1) If the zero solution of system (1.3) is conditionally uniformly stable with respect to the manifold $R(m-l)$, then the zero solution of system (1.1) is conditionally uniformly stable with respect to the manifold $M(n-l)$.

(2) If the zero solution of system (1.3) is conditionally uniformly globally attractive with respect to the manifold $R(m-l)$, then the zero solution of system (1.1) is conditionally uniformly globally attractive with respect to the manifold $M(n-l)$.

The proof of above theorem is analogous to the proof of theorem 2.1. we shall note that in this case the function δ and the number T can be chosen independently of t_0.

Corollary 2.4. let the conditions of theorem 2.3. be satisfied then conditional uniform global asymptotic stability of the zero solution of system (1.3) with respect to the manifold $R(m-l)$ implies the conditional uniform global asymptotic stability of the zero solution of system (1.1) with respect to the manifold $M(n-l)$.

3. Examples

Example 3.1 we shall apply theorem 2.1 to the system

$$\begin{cases} \dot{x}(t) = (1+t^2)x(t-r(t)) + (1-t^2)y(t-r(t)) + (t^2-1)z(t-r(t)), t \neq t_k \\ \dot{y}(t) = (1-e^{-t})x(t-r(t)) + (1+e^{-t})y(t-r(t)) + (e^{-t}-1)z(t-r(t)), t \neq t_k \\ \dot{z}(t) = (t^2-e^{-t})x(t-r(t)) + (e^{-t}-t^2)y(t-r(t)) + (e^{-t}+t^2)z(t-r(t)), t \neq t_k \\ \Delta x(t_k) = a_{1k}x(t_k) + b_{1k}[y(t_k) - z(t_k)], k = 1,2,\dots \\ \Delta y(t_k) = a_{2k}y(t_k) + b_{2k}[z(t_k) - x(t_k)], k = 1,2,\dots \\ \Delta z(t_k) = a_{3k}z(t_k) + b_{3k}[x(t_k) - y(t_k)], k = 1,2,\dots \end{cases} \quad (3.1)$$

Where $t \geq 0; 0 \leq r(t) \leq r;$

$a_{1k} = \frac{1}{2}(\sqrt{1+d_{1k}} + \sqrt{1+d_{3k}} - 2),$

$a_{2k} = \frac{1}{2}(\sqrt{1+d_{2k}} + \sqrt{1+d_{1k}} - 2),$

$a_{3k} = \frac{1}{2}(\sqrt{1+d_{3k}} + \sqrt{1+d_{2k}} - 2);$

$b_{1k} = \frac{1}{2}(\sqrt{1+d_{1k}} - \sqrt{1+d_{3k}}),$

$b_{2k} = \frac{1}{2}(\sqrt{1+d_{2k}} - \sqrt{1+d_{1k}}),$

$b_{3k} = \frac{1}{2}(\sqrt{1+d_{3k}} - \sqrt{1+d_{2k}});$

$-1 < d_{ik} \leq 0, i = 1,2,3, k = 1,2,\dots; 0 < t_1 < t_2 < \cdots \text{ and } \lim_{k\to\infty} t_k = \infty$

Consider the manifold $M(2) = \{col(x,y,z) \in \mathbb{R}^3 : x + y = z\}.$

We shall use the vector function

$V(x,y,z) = ((x+y-z)^2, (-x+y+z)^2, (x-y+z)^2)^T.$

Then, the set

$\Omega_1 = \{(x,y,z) \in PC[\mathbb{R}_+, \mathbb{R}^3]: V(x(s),y(s),z(s)) \leq V(x(t),y(t),z(t)), t - r \leq s \leq t\}.$

For $t \geq 0, t \neq t_k \text{ and } (x,y,z)^T \in \Omega_1$, we have

$$D^+_{(3.1)}V(x(t),y(t),z(t)) \leq 2\begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{-t} & 0 \\ 0 & 0 & t^2 \end{pmatrix} V(x(t),y(t),z(t)) + 2\begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{-t} & 0 \\ 0 & 0 & t^2 \end{pmatrix} V(x(t-r(t)), y(t-r(t)), z(t-r(t))) \leq 4\begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{-t} & 0 \\ 0 & 0 & t^2 \end{pmatrix} V(x(t),y(t),z(t))$$

$$V(x(t_k+0), y(t_k+0), z(t_k+0)) = V(x(t_k),y(t_k),z(t_k)) + \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{-t} & 0 \\ 0 & 0 & t^2 \end{pmatrix} V(x(t_k),y(t_k),z(t_k)).$$

Also , for $k = 1,2,\dots$

Since the zero solution of the comparison system

$$\begin{cases} \dot{u}_1(t) = 4u_1(t), t \neq t_k, t \geq 0 \\ \dot{u}_2(t) = 4e^{-t}u_2(t), t \neq t_k, t \geq 0 \\ \dot{u}_3(t) = 4t^2u_3(t), t \neq t_k, t \geq 0 \\ \Delta u_1(t_k) = d_{1k}u_1(t_k), \\ \Delta u_2(t_k) = d_{2k}u_2(t_k) \\ \Delta u_3(t_k) = d_{3k}u_3(t_k), k = 1,2,\dots \end{cases}$$

Is conditionally stable with respect to the manifold $R(2) = \{col(0,u_2,u_3) \in \mathbb{R}^3 : u_2 \geq 0, u_3 \geq 0\}$ [129] and all the conditions of theorem 2.1 are fulfilled, the zero solution of (3.1) is conditionally stable with respect to the manifold $M(2)$.

References

[1] D.D. Bainov and I.M. Stamva, stability of sets for impulsive differential-difference equations with variable impulsive perturbations, Communications on Applied Nonlinear Analysis 5 (1998), pp. 69-81.

[2] D.D. Bainov and I.M. Stamva, Lyapunov functions and asymptotic stability of impulsive differential-difference equations, PanAmerican Mathematical Journal 9 (1999), pp. 87-95.

[3] D.D. Bainov and I.M. Stamva, Vector Lyapunov functions and conditional stability for systems of impulsive differential-difference equations, ANZIAM J. 42 (2001), pp. 341-353.

[4] D.D. Bainov and I.M. Stamva, Stability of the solutions of impulsive functional differential equations by Lyapunov's direct method, ANZIAM J. 43 (2001), pp. 269-278.

[5] I.M. Stamova and G.T. Stamov, On the conditional stability of impulsive functional differential equations, Applied Mathematics Research eXpress 2006 (2006), pp. 1-13.

[6] I.M. Stamova and G.T. Stamov, Lyapunov-Razumikhin method for asymptotic stability of sets for impulsive functional differential equations, Electronic Journal of Differential Equations 48 (2008), pp. 1-10.

[7] V.Lakshmikantham, S. Leela and A.A. Martynyuk, Stability Analysis of Nonlinear Systems, Marcel Dekker, New York, 1989.

الشروط الكافية للاستقرارية المشروطة للحل الصفري لأنظمة المعادلات التفاضلية الدالية النبضية

مؤيد محمود خليل

قسم الرياضيات ، كلية التربية ، جامعة تكريت ، تكريت ، العراق

(تاريخ الاستلام: ٣ / ١ / ٢٠١٢ ---- تاريخ القبول: ٢ / ٤ / ٢٠١٢)

الملخص

درسنا في هذا البحث الاستقرارية المشروطة للحل الصفري لأنظمة المعادلات التفاضلية الدالية النبضية التي فرضناها وذلك باستخدام دوال ليابنوف المتجهة حيث أعطينا الشروط الكافية التي تجعل الحلول الصفرية لتلك الأنظمة مستقرة شرطياً.