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Some Properties of the Odd Weibull Exponential Distribution with Application

Ahmed Ali Ahmed*, Mundher A. Khaleel
College of Computer Science and Mathematics/Tikrit University

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*Corresponding author:
Ahmed Ali Ahmed
College of Computer Science and Mathematics/Tikrit University

Abstract: This paper will suggest a new type of continuous distribution called Odd Weibull Exponential Distribution (OWE) and the purpose of this distribution (OWE) is to apply to two sets of real data. The distribution (OWE) is as a sub-model of NOW-G family and many statistical properties have been derived. Quantile function, Expansion of functions, Moments, Incomplete moments, Probability Weighted Moments, Rényi entropy, as well as estimation of distribution parameters through Maximum Likelihood Estimation (MLE). A modelling study was carried out with data sets from the data includes 30 observation of March precipitation in Minneapolis Paul. Economic data consists of 31 observations on the response variable GDP growth of Egypt. Moreover, the proposed distribution (OWE) has been found to outstrip other existing distributions by basing us on statistical parameters and comparing them.

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بعض خصائص التوزيع ويبل الأسي الفردي مع التطبيق

احمد علي احمد **منذر عبد الله خليل**
كلية علوم الحاسوب والرياضيات/جامعة تكريت

المستخلص

سوف نقترح في هذه الدراسة نوعا جديداً من التوزيعات المستمرة تسمى توزيع ويبل الأسي الفردي (OWE) والهدف من هذا التوزيع (OWE) هو التطبيق على مجموعتين من البيانات الحقيقية. وان التوزيع الذي تم اقتراحه هو حاله فرعية من **(NOW-G family)** وتم الاشتقاق العديد من الخصائص الإحصائية ومنها الدالة الكمية، توسيع الدالة، العزوم، العزوم الناقصة، احتمالية وزن العزوم، ريني انتروبي، وكذلك تقدير معلمات التوزيع من خلال تقدير الاحتمال الأقصى (MLE). وقد تم دراسة النمذجة على مجموعة من البيانات تتضمن 30 ملاحظة من سقوط الامطار في شهر مارس في مينيابوليس بول. وكذلك دراسة البيانات الاقتصادية من 31 ملاحظة حول متغير الاستجابة نمو الناتج المحلي الإجمالي في مصر. وعلاوة على ذلك تبين ان التوزيع (OWE) تتفوق على التوزيعات الأخرى المعمول بها من خلال استنادنا على المعايير الإحصائية ومقارنتها.

الكلمات المفتاحية: توزيع ويبل، دالة الكمية، العزوم، ريني انتروبي، تقدير الامكان الأعظم.

1. Introduction

Statistical distributions can be used to describe and predict real-world events. Several extended distributions have been extensively used in data modeling throughout the last few decades. Recent improvements have focused on developing new families that broaden well-known distributions while simultaneously allowing enormous modeling freedom. To begin with, classical or normal distributions cannot be relied upon only since they are insufficient or inaccurate for obtaining genuine results or modeling real data. One simple method that has gotten a lot of attention in recent years is to add one or more parameters to an existing distribution. Many strategies have been researched in this direction. The MO-G family, as described by (Marshall et al., 1997: 2-3), is shown to be a superior substitute for several prevalent distributions, including the Weibull, gamma, and exponential distributions. Submitted by each of the by (Batsidis al., 2015: 5-7), show A new method for generating new classes of distributions based on the probability-generating function in particular, they focused their interest to the so-called Harris extended family of distributions. Both researchers worked on presenting (Korkmaz et al., 2017: 5), An ordinary-G distribution is a generalization of the ordinary distribution with a cumulative distribution function (CDF) equal to the value of the cdf of the ordinary distribution F with a range of the unit interval at G, denoted as F(G). And many ways and strategies as (Hassan et al., 2016:2-3), (Oluyede et al., 2015: 2-4), (Tahir et al., 2016: 2-5), Submitted by all (Oluyede et al., 2018: 2-4) The Weibull distribution and its related families have been extensively researched in the field of lifetime applications. This work focuses on the Gamma-Weibull G family of distributions (GWG), which is derived from the Weibull-G family of distributions and the exponentiated Weibull distribution.

This proposed new distribution provides great flexibility in data analysis and data modeling

By working on this research paper, we aim to create a new distribution that has very high efficiency in modeling data and interpreting phenomena accurately and with less error than the rest of the distributions that were previously worked on, that is, the distribution that we will present for very high benefit in data analysis.

The remaining components of the research paper will be organized as follows. In Section 2, we will demonstrate how to build the family. A New Odd Weibull-G Family, and then we'll explain how the new distribution the family may be used to generate a new Odd Weibull Exponential Distribution (OWE). The statistical features of the new distribution will be shown in Section 3. Section 4 covers parameter estimation of the distribution (OWE). In Section 5, we offer a simulation analysis using the Monte Carlo technique, and in Section 6, we describe an application of the newly established distribution, end the paper in Section 7.

2. A New Odd Weibull – Exponential distribution Start now by explaining how the construction of A New Odd Weibull-G family will be and in clear and simple steps as shown now Work has been done to build this family, which I have proposed by relying on a new way of generating families with continuous distributions by each of them. (Alzaatreh et al., 2013:4-5), (Alzaatreh et al., 2016: 3)

$$f(x) = abx^{b-1}e^{-ax^{b}} \qquad \textbf{(1)}$$

where $x > 0$ is random variable $\alpha, \beta > 0$ & β shape parameters.(papoulis et al., 2002:87)

$$W(F(x,\eta)) = \frac{F(x,\eta)}{1 - F(x,\eta)} \times \mathrm{Log}\frac{1}{1 - F(x,\eta)}$$

where $f(t)$ is the pdf of a continuous random variable T that $[c,d]$, and $F(x)$ is the cdf of X & $W(F(x))$ is a function of the cdf $F(x)$ that satisfy the conditions in (Alzaatreh et al., 2013).

$$G_w(x,\eta) = \int_0^{W(F(x,\eta))} f(t)dt \tag{2}$$

we get

$$G_w(x,\eta) = 1 - \exp\left\{-a\left[\frac{F(x,\eta)}{1-F(x,\eta)} \times \mathrm{Log}\frac{1}{1-F(x,\eta)}\right]^b\right\} \tag{3}$$

By differentiating equation (3) we get the new family pdf from equation (4)

$$\begin{aligned} g_w(x,\eta) = abf(x,\eta)&\left[\frac{F(x,\eta) - \mathrm{Log}(1-F(x,\eta))}{(1-F(x,\eta))^2}\right] \\ &* \left[\frac{F(x,\eta)}{1-F(x,\eta)} \times \mathrm{Log}\frac{1}{1-F(x,\eta)}\right]^{b-1} \\ &* \exp\left\{-a\left[\frac{F(x,\eta)}{1-F(x,\eta)} \times \mathrm{Log}\frac{1}{1-F(x,\eta)}\right]^b\right\} \end{aligned} \tag{4}$$

Take the exponential distribution as an example of baseline distribution. This baseline distribution is applied to both the pdf and the cdf in the following way:

$$f(x,\eta) = \lambda\exp\{-\lambda x\} \tag{5}$$

$$F(x,\eta) = 1 - \exp\{-\lambda x\} \tag{6}$$

Where $x > 0$ and $\lambda > 0$ rate, or inverse scale (Park et al., 2009:11)

Now by substituting (6) in (3) we get the cdf of the Odd Weibull Exponential distribution (OWE) and by differentiating the resulting equation we get the pdf

$$G_w(x,\eta) = 1 - \exp\{-a[\lambda x(\exp\{\lambda x\} - 1)]^b\} \tag{7}$$

$$\begin{aligned} g_w(x,\eta) = ab[\lambda^2 x\exp\{\lambda x\} + \lambda(\exp\{\lambda x\} - 1)] \\ * [\lambda x(\exp\{\lambda x\} - 1)]^{b-1} \\ * \exp\{-a[\lambda x(\exp\{\lambda x\} - 1)]^b\} \end{aligned} \tag{8}$$

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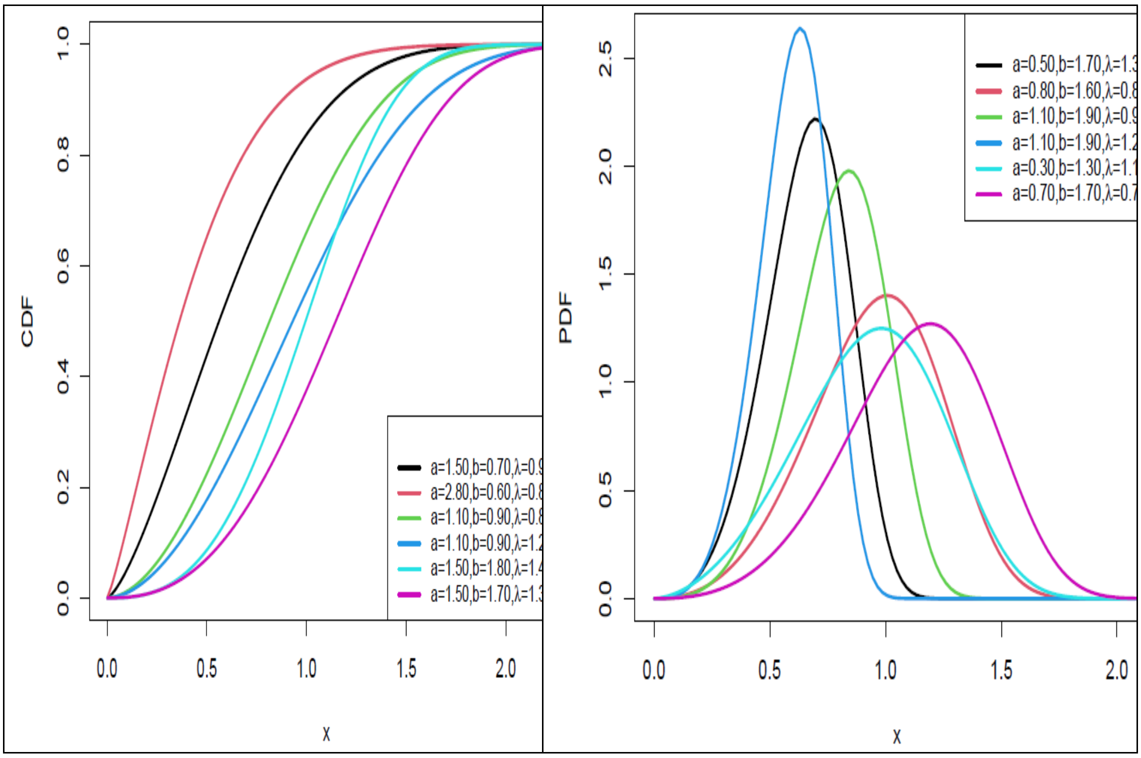

Figure1. cdf and pdf for the OWE distribution.

These Figure were prepared by the researcher and his efforts through working on the R program.

3. Mathematical Properties: In this section of the research, we're going to find some statistical properties of the Odd Weibull Exponential distribution (OWE) that are very important in distribution studies, as well as the two equations (7) & (8)

3-1 Quantile function: Quantile functions are used in both statistical applications and Monte Carlo methods. The quantile function may now be readily produced by inverting the equation (Oguntunde et al., 2019:4):

$$1 - \exp\{-\lambda x\} = u \qquad \textbf{(9)}$$

u is a uniform random variable continuous in this situation.By solving Equation (9) and Using the exponential distribution baseline, we get the following:

$$x = \frac{-1}{\lambda} \, Ln(1 - u)$$

The quantile function (NOW-G family) is written as follows:

$$Q_{G_w(x,\zeta)}(u) = Q_{F(x,\zeta)}\left(\frac{\sqrt[\beta]{\frac{1}{\alpha}\mathrm{Ln}\left(\frac{1}{1-u}\right)}}{\sqrt[\beta]{\frac{1}{\alpha}\mathrm{Ln}\left(\frac{1}{1-u}\right)} + W_{-1}\left(\sqrt[\beta]{\frac{1}{\alpha}\mathrm{Ln}\left(\frac{1}{1-u}\right)}\right)}\right)$$

As $Q_{F(x,\zeta)}$ Exponential distribution By substituting the above equation for the value of u, we get the following:

$$Q(u) = \frac{-1}{\lambda}\mathrm{Ln}\left[1-\left(\frac{\sqrt[b]{\frac{1}{a}\mathrm{Ln}\left(\frac{1}{1-u}\right)}}{\sqrt[b]{\frac{1}{a}\mathrm{Ln}\left(\frac{1}{1-u}\right)} + W_{-1}\left(\sqrt[b]{\frac{1}{a}\mathrm{Ln}\left(\frac{1}{1-u}\right)}\right)}\right)\right] \quad \textbf{(10)}$$

Note that $0 < u < 1, W_n$ is n (generalized Lambert W function) (Mez˝o and ´Arp´ad Baricz., 2015:6-7)

3-2. Expansion of functions: In this paragraph, we will simplify pdf and cdf and make it more simplified and be clear.

To simplify the exponential function, utilize the Taylor series. (Gradshteyn and Ryzhik., 2014: 26-27)

$$\exp\{-a[\lambda x(\exp\{\lambda x\}-1)]^b = \sum_{k=0}^{\infty}\frac{(-1)^k a^k[\lambda x(\exp\lambda x-1)]^{bk}}{k!} \quad \textbf{(11)}$$

Now by substituting the equation higher in (8) and with some simplification we get the following in (12)

$$g_w(x,\eta) = \sum_{k=0}^{\infty}\frac{(-1)^k a^{k+1} b}{k!}[\lambda^2 x\exp\{\lambda x\} + \lambda(\exp\{\lambda x\}-1)] * [\lambda x(\exp\{\lambda x\}-1)]^{b(k+1)-1} \quad \textbf{(12)}$$

Now using the binomial theorem (Gradshteyn and Ryzhik., 2014:25-26)

$$\frac{1}{(1-x)^s} = \sum_{m=0}^{\infty}\binom{s+m-1}{m}x^m$$

We get to the (13)

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$$g_w(x,\eta) = \sum_{k=0}^{\infty}\sum_{m=0}^{\infty}\frac{(-1)^{b(k+1)-1+k}a^{k+1}b(\lambda x)^{b(k+1)-1}}{k!} \quad \textbf{(13)}$$
$$*\binom{m-b(k+1)}{m}[\lambda^2 x\exp\{\lambda x\}+\lambda(\exp\{\lambda x\}-1)]\exp\{m\lambda x\}$$

Using the Taylor series to simplify the equation above, we will need to get(14)

$$[\lambda^2 x\exp\{\lambda x\}+\lambda(exp\{\lambda x\}-1)] = \sum_{n=0}^{\infty}\frac{(n+2)}{(n+1)!}\lambda^{n+2}x^{n+1} \quad \textbf{(14)}$$

We substitute (14) in (15) we get

$$g_w(x,\eta) = \sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\sum_{k=0}^{\infty}W_{n,m,k}x^{b(k+1)+n}\ exp\{m\lambda x\} \quad \textbf{(15)}$$

Where

$$W_{n,m,k} = \frac{(-1)^{b(k+1)-1+k}a^{k+1}b(n+2)(\lambda)^{b(k+1)+n+1}}{k!\,(n+1)!}\binom{m-b(k+1)}{m}$$

3-3 Moments: Moments are extremely important in any statistical analysis. And the types of moments are expected value, Variance, Skewness, kurtosis ets. The $\boldsymbol{r^{th}}$ moment of the Odd Weibull Exponential Distribution (OWE) are presented in this section. The $\boldsymbol{r^{th}}$ moment of X is denoted by

$$\mu_r = [X^r] = \int_{-\infty}^{\infty} x^r\, g_w(x,\eta)dx \quad \textbf{(16)}$$

Using equation (15) with (16) we get the following and Now in order to get the solution of the integration in the equation above we must make some simplifications in order to get the integration of gamma

$$\mu_r = E[X^r] = \left(\frac{-1}{m\lambda}\right)^{b(k+1)+n+r+1}\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\sum_{k=0}^{\infty}W_{n,m,k}$$
$$*\int_0^{\infty}u^{[b(k+1)+n+r+1]-1}\ exp\{-u\}du$$

Now through the use of the gamma integral, the moment equation will be as follows in (16)

$$\mu_r = E[X^r] = \left(\frac{-1}{m\lambda}\right)^{b(k+1)+n+r+1} \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} \sum_{k=0}^{\infty} W_{n,m,k} * \Gamma(\mathrm{b(k+1)+n+r+1})$$

Where

$$W_{n,m,k} = \frac{(-1)^{b(k+1)-1+k} a^{k+1} b(n+2)(\lambda)^{b(k+1)+n+1}}{k!\,(n+1)!} \binom{m-b(k+1)}{m}$$

3-4 Incomplete moments: Incomplete income distribution moments are natural building blocks for assessing inequality; for example, the Lorenz and Bonferroni curves, as well as the Pietra and Gini measures of inequality, all rely on incomplete income distribution moments. (Henrion et al.,2022:2), The definition of r^{th} incomplete moment is

$$m_r(z) = \int_{-\infty}^{z} x^r\, g_w(x,\eta) dx \qquad \textbf{(17)}$$

Using equation (15) with (17) we get the following and Now, in order to solve the integral above, we do some algebraic operations and simplifications and we get the following

$$m_r(z) = \left(\frac{-1}{m\lambda}\right)^{b(k+1)+n+r+1} \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} \sum_{k=0}^{\infty} W_{n,m,k} * \int_{0}^{-m\lambda z} y^{[b(k+1)+n+r+1]-1}\ exp\{-y\} dy$$

We can now obtain the result of the integration through the Incomplete gamma function And we get (17)

$$m_r(z) = \left(\frac{-1}{m\lambda}\right)^{b(k+1)+n+r+1} \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} \sum_{k=0}^{\infty} W_{n,m,k} * \gamma(b(k+1)+n+r+1, -m\lambda z)$$

where

$$W_{n,m,k} = \frac{(-1)^{b(k+1)-1+k} a^{k+1} b(n+2)(\lambda)^{b(k+1)+n+1}}{k!\,(n+1)!} \binom{m-b(k+1)}{m}$$

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3-5 Probability Weighted Moments: It is an unbiased way of estimating the characteristics of the distribution estimate in quantities and is sometimes termed the descriptive function, which is provided by the following relation:

$$w(r,y) = E\left(X^r G_w^p(x,\zeta)\right) = \int_{-\infty}^{\infty} x^r\, g_w(x,\zeta) G_w^y(x,\zeta)dx \qquad \textbf{(18)}$$

Now in order to find the integration at the top we need to make some simplifications and get the following

$$\mathrm{g_w(x,\eta)G_w^y(x,\eta)} \quad = \sum_{i=0}^{\infty}\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\sum_{j=0}^{\infty} \Xi_{i,n,m,j} x^{b(n+1)+j} \exp\{m\lambda x\}$$

where

$$\Xi_{i,n,m,j} \quad = \frac{(-1)^{i+n+b(n+1)-1} a^{n+1} b(i+1)^n (j+2)\lambda^{b(n+1)+j+1}}{n!\,(j+1)!} * \binom{p}{i}\binom{m-b(n+1)}{m}$$

Therefore, the probability-weighted moments are given by

$$w(r,y) = \sum_{i=0}^{\infty}\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\sum_{j=0}^{\infty} \pi_{i,n,m,j}\, \Gamma(b(n+1)+j+r+1)$$

where

$$\pi_{i,n,m,j} \quad = \frac{(-1)^{i+n+b(n+1)-1} a^{n+1} b(i+1)^n (j+2)\lambda^{b(n+1)+j+1}}{n!\,(j+1)!} * \left(\frac{-1}{m\lambda}\right)^{b(n+1)+j+r+1} \binom{p}{i}\binom{m-b(n+1)}{m}$$

3-6 Rényi entropy: If $\mathbf{g(x,\zeta)}$ is the new distribution proposed in (19), then: Rényi entropy is defined by

$$\mathrm{I_R(p)} = \frac{1}{1-\mathrm{p}} \log \int_0^{\infty} \mathrm{g_w^p\,(x,\zeta)dx} \qquad \mathrm{p} \neq 0 \quad , \quad \mathrm{p} > 0 \qquad \textbf{(19)}$$

Raised pdf was simplified to powers and obtained the following

$$\mathrm{g_w^p(x,\zeta)} = \sum_{i=0}^{\infty}\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\sum_{k=0}^{\infty}\sum_{a=0}^{\infty} \phi_{i,n,m,k,a} x^{bk+p(b-1)+in+a} \exp\{m\lambda x\}$$

We will write Rényi entropy for the new family as follows:

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$$I_R(p) = \frac{1}{1-p} log \left[\sum_{i=0}^{\infty} \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} \sum_{k=0}^{\infty} \sum_{a=0}^{\infty} \phi_{i,n,m,k,a} \, exp[(1-p) I_{REF}] \right]$$

Where

$$\phi_{m,k,i,a,n} = \frac{(-1)^{bk+p(b-1)+k+p+i} a^{p+k} b^p p^k (\lambda)^{bk+p(b-1)+in+p}}{k!\,n!} * \binom{m-[bk+p(b-1)+1]}{m} \binom{p}{i} \binom{i}{a}$$

$$I_{REG} = \int_0^{\infty} [x^{bk+p(b-1)+in+a} \, exp\{m\lambda x\}] \, dx$$

4. **Maximum Likelihood Estimator:** Among numerous approaches, the most popular method in estimating the parameters of statistical distribution is the method (MLE), which has outstanding qualities and is useful in determining the fixed periods of the model's parameters. Let $\nu = (a, b, \lambda)^T$ is the parameters vector, The Likelihood function for the (OWE) distribution will be, By taking the log of equation (8) we get the following:

$$\ell(\nu) = nloga + nlogb + \sum_{i=1}^{n} log\, [\lambda^2 xexp\{\lambda x\} + \lambda(exp\{\lambda x\} - 1)] \\ +(b-1) \sum_{i=1}^{n} log\, [\lambda x(exp\{\lambda x\} - 1)] - ab \sum_{i=1}^{n} [\lambda x(exp\{\lambda x\} - 1)] \quad \textbf{(20)}$$

Now we will work on finding the partial differentiation of each parameter of the distribution

$$\frac{\partial \ell(\nu)}{\partial a} = \frac{n}{a} - b \sum_{i=1}^{n} [\lambda x(exp\{\lambda x\} - 1)] \quad \textbf{(21)}$$

$$\frac{\partial \ell(\nu)}{\partial b} = \frac{n}{b} + \sum_{i=1}^{n} log\, [\lambda x(exp\{\lambda x\} - 1)] - a \sum_{i=1}^{n} [\lambda x(exp\{\lambda x\} - 1)] \quad \textbf{(22)}$$

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$$\frac{\partial \ell(\nu)}{\partial \lambda} = \sum_{i=1}^{n}\left(\frac{\lambda^2 x\exp\{\lambda x\} + 2\lambda x\exp\{\lambda x\} + \lambda\exp\{\lambda x\} + \exp\{\lambda x\} - 1}{\lambda^2 x\exp\{\lambda x\} + \lambda(\exp\{\lambda x\} - 1)}\right) + (b-1)\sum_{i=1}^{n}\left(\frac{\lambda x\exp\{\lambda x\} + x\exp\{\lambda x\} - x}{\lambda x(\exp\{\lambda x\} - 1)}\right) - ab\sum_{i=1}^{n}(\lambda x\exp\{\lambda x\} + x\exp\{\lambda x\} - x) \quad \textbf{(23)}$$

Now after equalizing the three equations with zero, you can find an estimate of the distribution parameters, but through the use of the relevant application because it is difficult to find them manually by R programs or MATLAB

5. Simulation: In this section, we use a Monte Carlo experiment to investigate the asymptotic behavior of MLEs for OWE distribution parameters. The study examines four sets of parameter values: (a =0.1, b =0.4, ג =1.4), (a =0.3, b =0.5, ג =0.9), (a =0.4, b =0.5, ג =0.9), and (a =0.4, b =0.5, ג =1.2). We consider four sample sizes (n = 40, 80, 120, and 180), and the experiment is performed 1,000 times. Table 1 displays the MLEs' mean estimates and root mean squared errors (RMSEs). As expected, the MLEs converge to the correct parameters, and the RMSEs drop as the sample size (n) grows.

Table (1): Monte Carlo Simulation Results for the WE distribution.

	(a =0.1, b =0.4, ג =1.4)				(a =0.3, b =0.5, ג =0.9)		
parameter	**Sample Size**	**Mean**	**RMSE**	**bias**	**Mean**	**RMSE**	**bias**
a	40	0.4654	1.4673	0.3654	1.6669	1.9422	1.3669
	80	0.3114	1.3092	0.2114	0.6953	1.1415	0.3953
	120	0.1139	0.0549	0.0139	0.4349	0.8144	0.1349
	180	0.1084	0.0361	0.0084	0.3718	0.3119	0.0718
b	40	0.4271	0.1780	0.0271	0.5011	0.1466	0.0011
	80	0.4141	0.1254	0.0141	0.4970	0.1025	-0.0029
	120	0.4110	0.099	0.0110	0.4965	0.0830	-0.0034
	180	0.4057	0.0848	0.0057	0.4950	0.0712	-0.0049
ג	40	1.4801	0.8253	0.0801	0.9949	0.5936	0.0949
	80	1.4483	0.6056	0.0483	0.9381	0.4222	0.0381
	120	1.4313	0.5191	0.0313	0.9275	0.3503	0.0275
	180	1.4242	0.4090	0.0242	0.9171	0.2886	0.0171

		(a =0.4, b =0.5, ℷ =0.9)			(a =0.4, b =0.5, ℷ =1.2)		
parameter	**Sample Size**	**Mean**	**RMSE**	**Abias**	**Mean**	**RMSE**	**Abias**
a	40	1.6113	2.4665	1.2113	1.7788	2.0315	1.3788
	80	1.2783	1.9093	0.8783	1.3214	1.1191	0.9214
	120	0.6475	1.8395	0.2475	0.6458	0.8826	0.2458
	180	0.4949	0.4009	0.0949	0.5079	0.5617	0.1079
b	40	0.5034	0.1391	0.0034	0.5088	0.1400	0.0088
	80	0.4981	0.0967	-0.0018	0.4967	0.09424	0.0032
	120	0.4967	0.0786	-0.0032	0.4971	0.0771	0.0028
	180	0.4942	0.0663	-0.0057	0.4936	0.0657	-0.0063
ℷ	40	1.0252	0.6387	0.1252	1.3993	0.9065	0.1993
	80	0.9388	0.4528	0.0388	1.2767	0.5960	0.0767
	120	0.9326	0.3740	0.3265	1.2419	0.4948	0.0419
	180	0.9227	0.3027	0.0227	1.2345	0.4023	0.0345

These Table were prepared by the researcher and his efforts through working on the R program.

6. Application: We will apply the (OWE) distribution in this part on a real-life data collection. The Akaike information criterion (AIC), the Corrected Akaike information criterion (CAIC), the Bayesian information criteria (BIC), the Hannan-Quinn information criterion (HQIC), the Cramer-von Mises statistic (W), and the Anderson-Darling statistic (A) define the standards for judging the efficiency of the models.

We compare the performance of the (OWE) with other distributions Like Lomax Exponential (LOE) Distribution (Khalaf et al., 2024: 52-53), Truncated Inverse Weibull Exponential (TIWE) Distribution (Khubbaz et al., 2023:2-3), Burr XII Exponential (BXIIE) Distribution (NEW), Beta Exponential (BeE) Distribution (NADARAJAH et al.,2006:4), Kum araswamy Exponential (KuE) Distribution (NEW), Gompertz Exponential (GoE) Distribution (NEW), Weibull (We) Distribution.

The results Table 3 below sums the findings. Our model is regarded as the best fit in this case as the first row of this table reveals that the (OWE) distribution has the lowest values for all the metrics used in the research. Figure 2 also shows many facets of this research related to analysis.

Data I: The data includes 30 observation of March precipitation in Minneapolis Paul. The following values were noted: (Bhat et al., 2023:14-15)

0.77, 1.74, 0.81, 1.20, 1.95, 1.20, 0.47, 1.43, 3.37, 2.20, 3.00, 3.09, 1.51, 2.10, 0.52, 1.62, 1.31, 0.32, 0.59, 0.81, 2.81, 1.87, 1.18, 1.35, 4.75, 2.48, 0.96, 1.89, 0.90, 2.05

Table (2): Descriptive statistics for Data I.

Statistic	Var.	N	Mean	Sd	Median	Min	Max	skew	kurtosis
Data I	x	30	1.68	1	1.47	0.32	4.75	1.03	0.93

These Table were prepared by the researcher and his efforts through working on the R program.

Table (3): Goodness of fit statistics and information for the OWE models

Models	MLEs	-LL	AIC	CAIC	BIC	HQIC	W	A	K-S	p-value
OWE	$\hat{a}$:0.1067 $\hat{b}$:1.6105 $\hat{\lambda}$:2.2315	38.05	82.108	83.031	86.311	83.453	0.0139	0.1055	0.0626	0.9997
LOE	$\hat{a}$:0.7130 $\hat{b}$:5.6708 $\hat{\lambda}$:1.2305	38.39	82.793	83.716	86.997	84.138	0.0167	0.1216	0.0655	0.9995
TIWE	$\hat{a}$:0.7031 $\hat{b}$:3.0503 $\hat{\lambda}$:0.9825	38.20	82.417	83.340	86.620	83.762	0.0260	0.1677	0.0968	0.9412
BXIIE	$\hat{a}$:1.2675 $\hat{b}$:1.0107 $\hat{\lambda}$:0.6648	38.38	82.772	83.695	86.975	84.117	0.0196	0.1349	0.0749	0.9959
BeE	$\hat{a}$:3.6783 $\hat{b}$:0.8082 $\hat{\lambda}$:1.3927	38.10	82.218	83.141	86.422	83.563	0.0163	0.1138	0.0691	0.9987
KuE	$\hat{a}$:4.5976 $\hat{b}$:0.5877 $\hat{\lambda}$:1.7930	38.15	82.318	83.241	86.522	83.663	0.0200	0.1336	0.0778	0.9933
GoE	$\hat{a}$:0.7288 $\hat{b}$:1.1798 $\hat{\lambda}$:0.4164	41.07	88.152	89.075	92.355	89.497	0.0581	0.4134	0.1149	0.8229

These Table were prepared by the researcher and his efforts through working on the R program.

Doi: www.doi.org/10.25130/tjaes.20.68.2.27

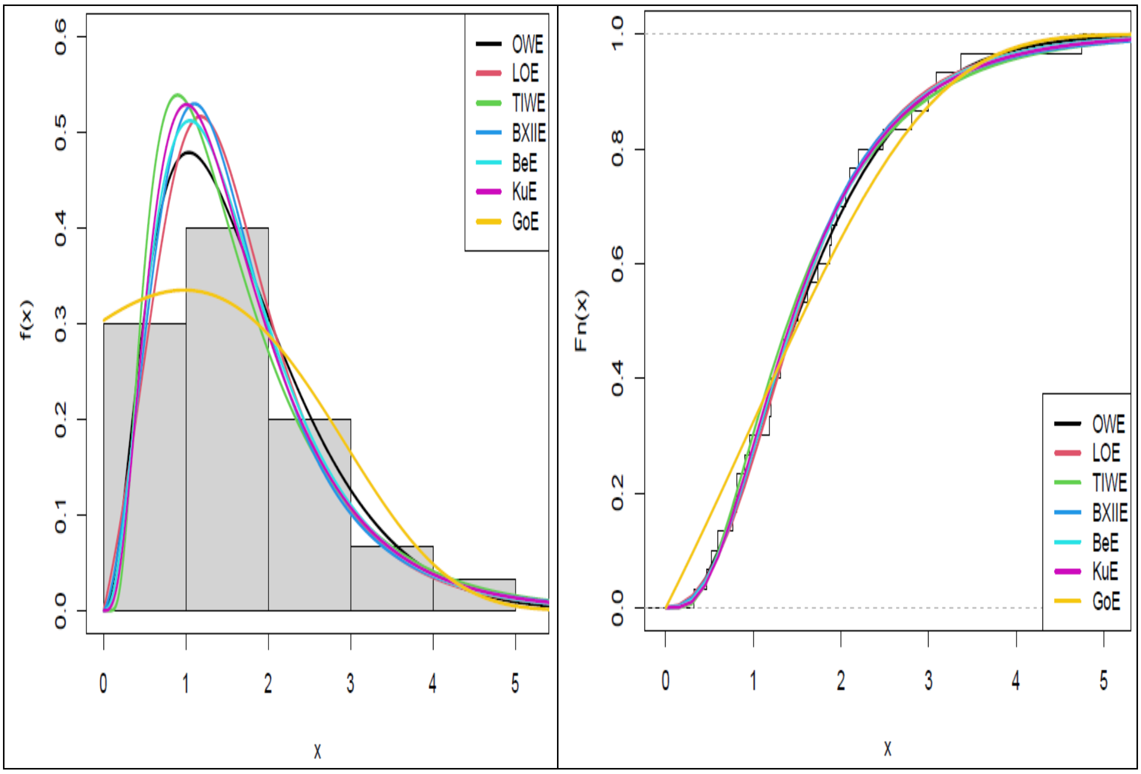

Figure 2: Estimated pdf, and cdf for Data I.

These Figure were prepared by the researcher and his efforts through working on the R program.

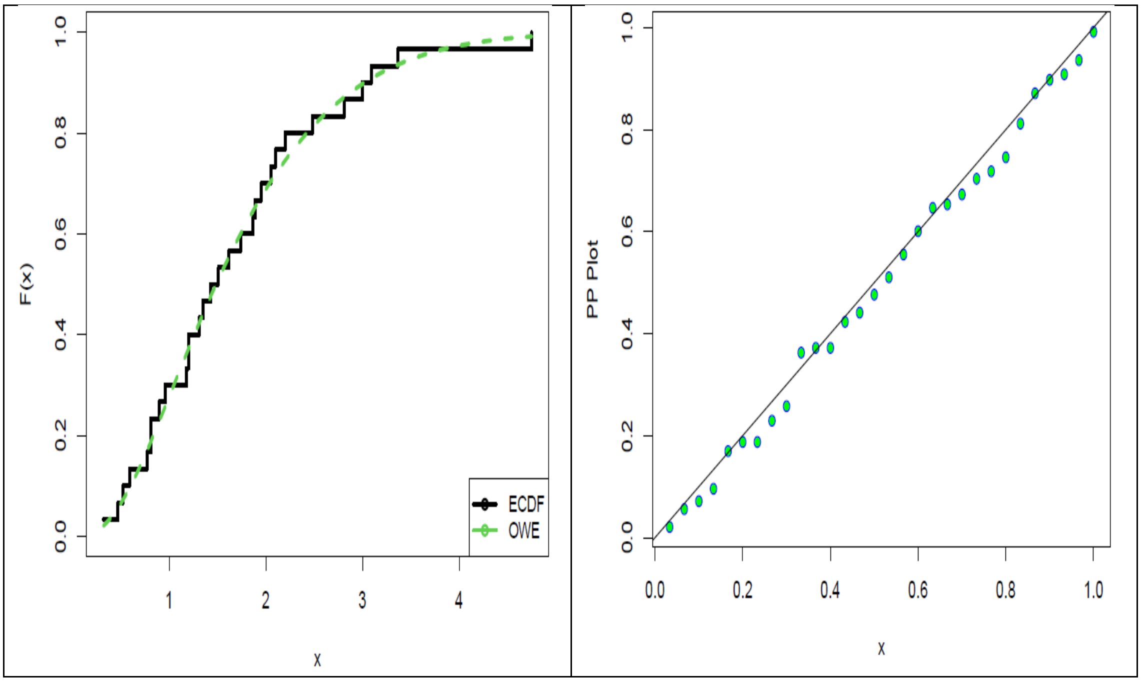

Figure (3): Empirical cdf, and PP-Plot for data I.

These Figure were prepared by the researcher and his efforts through working on the R program.

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Data II: economic data consists of 31 observations on the response variable GDP growth of Egypt. The data are:(El-Sherpieny et al., 2020:9)
10.01, 3.76, 9.91, 7.40, 6.09, 6.60, 2.65, 2.52, 7.93, 4.97, 5.70, 1.08, 4.43, 2.90, 3.97, 4.64, 4.99, 5.49, 4.03, 6.11, 5.37, 3.54, 2.37, 3.19, 4.09, 4.48, 6.85, 7.09, 7.16, 4.67, 5.15

Table (4): Descriptive statistics for Data II.

Statistic	Var.	N	Mean	Sd	Median	Min	Max	skew	kurtosis
Data II	x	35	5.13	2.08	4.97	1.08	10.01	0.48	-0.11

These Table were prepared by the researcher and his efforts through working on the R program.

Table (5): Goodness of fit statistics and information for the OWE models

Models	MLEs	-LL	AIC	CAIC	BIC	HQIC	W	A	K-S	p-value
OWE	$\hat{a}$:1.4569 $\hat{b}$:1.7453 $\hat{\lambda}$:0.2058	65.53	137.06	137.95	141.36	138.46	0.0163	0.1488	0.0516	0.9999
LOE	$\hat{a}$:3.3372 $\hat{b}$:6.8263 $\hat{\lambda}$:0.2739	65.89	137.79	138.68	142.10	139.20	0.0220	0.1827	0.0770	0.9860
TIWE	$\hat{a}$:0.1643 $\hat{b}$:1.7574 $\hat{\lambda}$:0.4965	66.82	139.64	140.52	143.94	141.04	0.0394	0.2996	0.0947	0.9190
BXIIE	$\hat{a}$:1.8870 $\hat{b}$:1.0375 $\hat{\lambda}$:0.1963	66.03	138.07	138.96	142.37	139.47	0.0203	0.1688	0.0557	0.9998
BeE	$\hat{a}$:4.8317 $\hat{b}$:1.5070 $\hat{\lambda}$:0.3257	66.86	139.76	140.65	144.06	141.17	0.0265	0.2153	0.0987	0.8941
KuE	$\hat{a}$:4.0006 $\hat{b}$:2.0985 $\hat{\lambda}$:0.2707	66.56	139.14	140.03	143.44	140.54	0.0208	0.1778	0.0962	0.9097
GoE	$\hat{a}$:0.1941 $\hat{b}$:1.6996 $\hat{\lambda}$:0.2225	68.20	142.40	143.29	146.71	143.81	0.0865	0.6296	0.1204	0.7145
We	$\hat{a}$:0.0430 $\hat{b}$:1.8518	68.65	141.33	141.76	144.20	142.26	0.0167	0.1525	0.1677	0.3117

These Table were prepared by the researcher and his efforts through working on the R program.

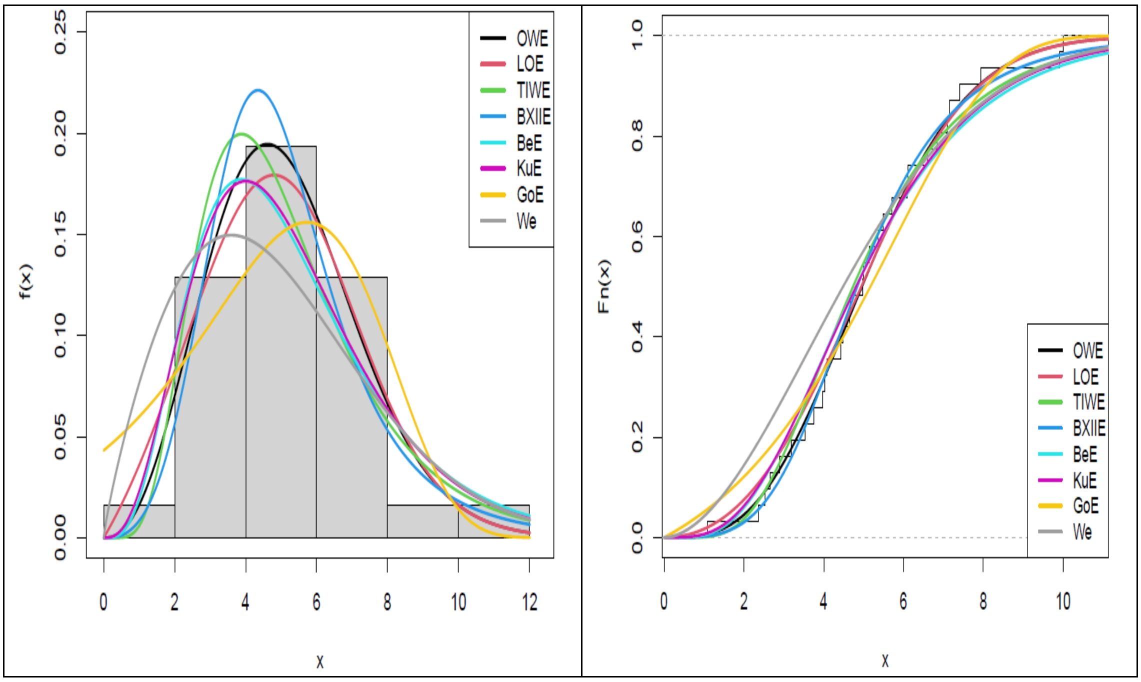

Figure (4): Estimated pdf, and cdf for Data II.

These Figure were prepared by the researcher and his efforts through working on the R program.

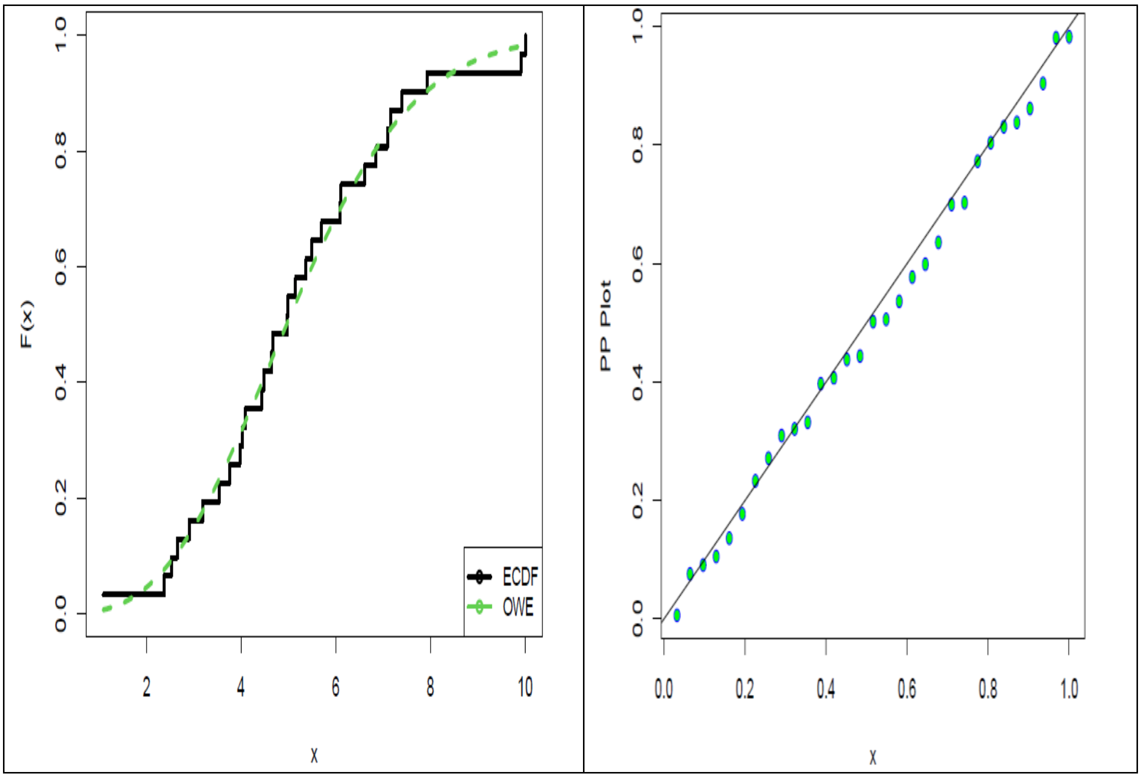

Figure (5): Empirical cdf, and PP-Plot for data II.

These Figure were prepared by the researcher and his efforts through working on the R program.

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7. CONCLUDING REMARKS: This study introduces the new Odd Weibull Exponential Distribution (OWE). This study specifies the statistical properties of the proposed distribution (OWE), such as density function expansion, quantile function, moments, variance, first and second moments, incomplete moments, and maximum likelihood estimation for parameters in the new distributions. The performance of the parameters is further investigated using a Monte Carlo simulation analysis. The data includes 30 observations of March precipitation in Minneapolis Paul. Economic data consists of 31 observations on the response variable GDP growth of Egypt. We find that the distribution (OWE) is the best and outperforms many other distributions.

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