

## **Determining Energy Levels of $^{56}\text{Co}$ Using ( MSDI) of The Nuclear Shell Model**

**تحديد مستويات الطاقة لنواة  $^{56}\text{Co}$  باستخدام جهد دلتا السطحي المعدل لنموذج القشرة النووي**

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### **ABSTRACT**

The nuclear energy levels for nucleus  $^{56}\text{Co}$  have been calculated within Nuclear Shell Model (N.S.M.) using Modified Surface Delta Interaction ( MSDI ) as the residual interaction between two nucleon as finding spectra of  $^{50}\text{Sc}$  we obtain the energy levels of the  $^{56}\text{Co}$  energy levels of configuration (particle-hole) in model space  $(0f_{7/2}^{-1}1p_{3/2})$ .

Computer programs is apply to calculate the Clebsch-Gordan , Racah coefficient and matrix elements which use to get the energy levels for all allowable total angular momentum . After comparing the theoretical results by using (MSDI) and considering the experimental results a fine identification was found between them.

### **الخلاصة**

تم تحديد مستويات الطاقة النووية لنواة  $^{56}\text{Co}$  بتطبيق نموذج القشرة النووي (NSM) Nuclear Shell Model باستخدام جهد دلتا السطحي المعدل (MSDI) Modified Surface Delta Interaction كتفاعل متبقي بين النيوكليونات وبايجاد طيف  $^{50}\text{Sc}$  تم الحصول على مستويات الطاقة لنواة  $^{56}\text{Co}$  للترتيب (جسيمة - فجوة) في فضاء الانموذج  $(0f_{7/2}^{-1}1p_{3/2})$ .

تم استخدام برامج حاسوب لحساب بعض المعاملات المهمة مثل معامل كليش - كوردين , معاملات راكاه وعنصر المصفوفة التي تدخل بشكل اساسي في حساب مستويات الطاقة لجميع قيم الزخم الزاوي الكلي . بعد مقارنة النتائج النظرية باستخدام جهد دلتا السطحي المعدل مع النتائج العملية لأحضا تطابق جيد ومقبول بينهما .

### **Introduction**

The shell model has played an indispensable role in the study nuclear structure ,since it was conceived by Mayer and Jensen . The shell model several important and basic features such as the independence of model assumption ,the usage realistic nucleon- nucleon interaction ,and the common Hamilton for various types of eigenstates ,and for different nuclei [ 1] . the shell model continues to provide the main theoretical tool for understanding all properties of nuclei [2] .in this paper the shell model is applied by using (MSDI) to calculate the energy levels for  $^{56}\text{Co}$  nucleus which contain one particle and one hole outside the closed shell  $^{56}\text{Ni}$  .

### **Theory**

The shell model uses two principle assumption are there exists an inet core ,model of close shell which acts with central force on valence nucleons and there exists a residual interaction caused by two body fore action between the valence nucleons [3]. The shell model Hamiltonian contains the single particle energies two particle matrix elements describing the residual interaction between the particles (proton or neutron , particle or hole) . we employ in this work the (MSDI) which is given by [4,5,6] :

$$V^{MSDI}(1,2) = -4\pi A_i' \delta(r(1) - r(2))\delta(r(1) - R_0) + B'(\tau(1).\tau(2)) + C' \dots\dots\dots(1)$$

Where ,  $r(2), r(1), \tau(1)$  and  $\tau(2)$  the position vectors ,  $R_0$  radius of nucleus and  $A_T$  strength of interaction .

Atypical two body matrix element of the  $V^{MSDI}(1,2)$  is [4,7,8] :

$$\begin{aligned} \langle j_a j_b | V^{MSDI}(1,2) | j_a j_b \rangle_{JT} = & -A_T \frac{(2j_a+1)(2j_b+1)}{2(2J+1)(1+\delta_{ab})} * \left\{ \left\langle j_b - \frac{1}{2} j_a \frac{1}{2} \middle| J0 \right\rangle^2 \left[ 1 - (-1)^{l_a+l_b+J+T} \right] + \right. \\ & \left. \left\langle j_b \frac{1}{2} j_a \frac{1}{2} \middle| J1 \right\rangle^2 \left[ 1 + (-1)^T \right] \right\} + [2T(T+1) - 3] + B + C \dots \dots \dots (2) \end{aligned}$$

Where is  $\left\langle j_b - \frac{1}{2} j_a \frac{1}{2} \middle| J0 \right\rangle$  Clebsch-Gordan Coefficient [9] , T isospin ,  $A_T$  ,B and C are parameters obtain form fits to experimental data in various mass region , from such calculation one can obtain the empirical estimate of parameters  $A_T$  ,B and C as a function of mass number A . The result can be summarized by the approximate evaluations [4-8]:

$$A_0 \approx A_T \approx B \approx \frac{25}{A} \qquad C \approx 0$$

The binding energy of the nucleus with two particles outside the core in the orbit  $j_a$  ,  $j_b$  and isospin T is given [4] :

$$\begin{aligned} E_J^b = & E_J(p,n) + e_{j_a} + e_{j_b} \\ E_J^b = & \frac{1}{2} \{ \langle j_a j_b | V^{MSDI}(1,2) | j_a j_b \rangle_{J,T=0} + \langle j_a j_b | V^{MSDI}(1,2) | j_a j_b \rangle_{J,T=1} \} \\ & + e_{j_a} + e_{j_b} \dots \dots \dots (3) \end{aligned}$$

Where  $e_{j_a}$  and  $e_{j_b}$  single particle energy .

The hole-particle interaction energy in term the particle-particle matrix element which represented energy level is given by [10] :

$$E_I(j' j'^{-1}, j' j'^{-1}) = - \sum_J (2J+1) W(jj' j' j; II) E_J(j' j, j' j) \dots \dots \dots (4)$$

Where W ( ; ) Racah Coefficient [11], I and J angular momentum .

We have adopt this theorem [6,7,8,10] :

For two particle in the state  $j_1$  and  $j_2$  ( $j_1 \neq j_2$ ) allowable angular momentum values are :

$$J = j_1 + j_2, j_1 + j_2 - 1, j_1 + j_2 - 2, \dots \dots \dots |j_1 - j_2| \dots \dots \dots (5)$$

## Calculations

In this work ,the energy levels have been calculation for  $^{56}\text{Co}$  :

The nuclei  $^{56}\text{Co}$  is consist of Z=27 and N=29 , which has one particle in shell ( $1p_{3/2}$ ) , one hole in shell ( $0f_{7/2}$ ) and outside close shell  $^{56}\text{Ni}$  for description the energy levels we have the single

particle energy which are [10]:  $e_{0f_{7/2}} = -9.11522\text{Mev}$  and  $e_{1p_{3/2}} = -5.147485\text{Mev}$  In order to determine the values of all angular momentum allowable by using eq (5) , as :

$$I^\pi = 2^+, 3^+, 4^+, 5^+ \quad \text{for } ^{56}\text{Co}$$

$$J^\pi = 2^+, 3^+, 4^+, 5^+ \quad \text{for } ^{50}\text{Sc}$$

The  $^{56}\text{Co}$  levels are get by eq (3) and eq (4) , which is calculated the spacing of levels applying while  $^{50}\text{Sc}$  is described as core  $^{48}\text{Ca}$  plus one proton in the  $(0f_{7/2})$  shell , one neutron in the  $(1p_{3/2})$  shell ,the matrix element  $\langle 0f_{7/2}1p_{3/2} | V^{MSDI}(1,2) | 0f_{7/2}1p_{3/2} \rangle_{J,T}$  is calculate from eq (2) by using the parameters  $A=0.55$  ,  $B=0.533$  and  $C=0.0$  table (1) .

and shown with respect to ground state in table (2) with the experimental values [12] . By application the nuclear shell model using (MSDI) to calculate energy levels for nucleus  $^{56}\text{Co}$  and shown with respect to ground state in table (3) with the experimental values [12] as shown in figure( 1 )

## Results and Discussion

From the study of nuclear shell model using modified surface delta interaction and through the values obtained for the energy and total angular momentum we got four values of all levels of energy as the user interaction depends on parameter one is the strength of interaction and the chosen works match with the situation the ground state as the reason for the emergence of some differences of these results we conclude from the above:

1. Obtained a value level 1.901 MeV angular momentum and parity of 3+ and practical value that has been compared with the value of this level is equal to 1.58 MeV in the same angular momentum and parity.
2. Angular momentum and parity 5+ got on the value of this level that 6.29 MeV compared with the value for which the same angular momentum and parity
3. Identified the value angular momentum and parity 2+ which found practically the same value 9.7 MeV with determining the value of the total angular momentum and parity

Table (1) : values of two body matrix element for  $^{50}\text{Sc}$  by using (MSDI)

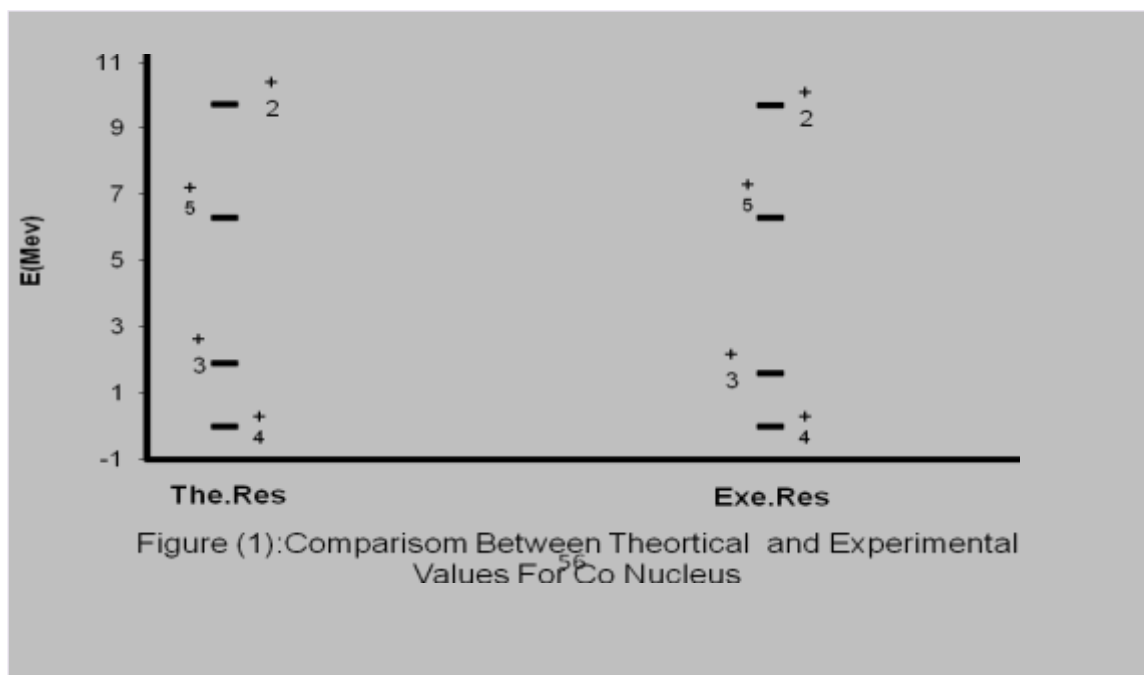
J+	T	$\langle 0f_{7/2}1p_{3/2}   V^{MSDI}(1,2)   0f_{7/2}1p_{3/2} \rangle_{J,T}$
2	1	-0.02514159
3	1	0.8999997
4	1	0.61428564
5	1	0.8999997
2	0	-3.317143
3	0	-3.385714
4	0	-2.75714
5	0	-3.9

Table (2) :Comparison between theoretical values and the experimental values for  $^{50}\text{Sc}$  with respect to ground state by using (MSDI)

J+	Energy (MeV)	
	Th.	Exp.[12]
5	0.0	0.0
4	6.999995	7.6
3	3.2432947	3.28447
2	2.5031056	2.56895

Table (3) : Comparison between theoretical values and the experimental values for  $^{56}\text{Co}$  by using (MSDI) for allowable angular momentum

I+	Energy (MeV)	
	Th.	Exp[12]
4	0	0
3	1.901	1.5838
2	9.7372	9.7023
5	6.2961	6.30019



## Conclusions

The nuclear shell model using (MSDI) to calculate energy level for nucleus  $^{56}\text{Co}$  we founded all calculated levels with allowable angular momentum in good agreement with experimental values. From our results obtained in this work and experimental data shows that the shell model by using (MSDI) is a successful model to describe the energy levels for  $^{56}\text{Co}$  and the agreement with experimental values is fairly good in general. The shell model calculation using the Modified Surface Delta Interaction is successful in introducing the energy spectrum of nuclei.

## References

1. T. Otsuka, Nucl. Phys. A **704**, 21, 2002
2. B. Alex Brown, Nucl. Phys. A **704**, 11, 2002
3. A. K. Hasan and F. M. Hassain Adv. phys, theo. and appl. 2013.
4. P. J. Brussaard and P. W. M. Glavdemans, "Shell Model in Nuclear Spectroscopy" North – Holland publishing company, 1977.
5. S. F. Abualhous, M.Sc Thesis, Kufa University, 2005.
6. D. N. Hamed, M.Sc Thesis, Kufa University, 2004.
7. A. K. Hasan and D. N. Hamed, Journal of Baby. for pure and Appl. Sc 748, 2004.
8. A. K. Hasan and D. N. Hamed, Journal of Kufa. for pure and Appl. Sc. 8, 2, 149, 2007.
9. I. Talmi, Acta Physica Polonica B, **40**, 391, 2009.
10. R. D. Lawson, "Theory of The Nuclear Shell Model", clarendon press, Oxford, 1980.
11. J. Suhonen "From Nucleons to Nucleus Concepts of Microscopic Nuclear Theory" Springer Berlin Heidelberg, New York, 2007
12. J. A. Cameron B. Singh, Nuclear data sheet, **1**, 109, 2008.