SOME PROPERTIES OF g* COMPLETELY REGULAR SPACES

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Abstract

In this paper, we introduced a new definitions g , $s\ g$, $g\ s$, g^*, $s\ g^*$, g^*s of completely regular spaces respectively and g , s g, g s , g^*,sg^*,g^*s of regular spaces respectively .We study some relations among them and we show the hereditary and topological properties of its .

Introduction

In 1970,N. Levine introduced a new and significant notion in General Topology, namely the notion of a generalized closed set. A subset A of a topological space (X,τ) is called *generalized closed*, (briefly g-closed), if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X,τ) . This notion has been studied extensively in recent years by many topologists. The investigation of generalized closed sets has led to several new and interesting concepts . This notion has been studied extensively in recent years by many topologists because generalized closed sets are not only natural generalizations of closed sets.
(P.Bhattacharya and B.K. Lahiri,1987 and S.P.Arya ,1990), investigated semi g closed set, g semi closed set respectively. (P.Sundaramand A.Pushpalatha, 2001, Al-Ddoury A.F, 2009 and A.I.El-Maghrabi & A.A.Nasef , 2009) introduced and investigated strongly generalized closed sets ,semi strongly generalized closed set and strongly generalized semi closed , Respectively.We study all definitions was in abstract , Relations and some properties.

2. Preliminaries

Definitions 2.1: A subset A of a topological space is said to be :

(1) Generalized closed (briefly g-closed) if cl(A) ⊆ G whenever A ⊆ G and G is open in X.(Jones and Bartlett)

(2) Semi generalized closed (briefly sg-closed) if scl(A) ⊆ G whenever A ⊆ G and G is semi open in X.(N.Levine ,1970)

(3) Generalized semi closed (briefly gs-closed) if scl(A) ⊆ G whenever A ⊆ G and G is open in X.(Alhawez,Z.T.(2008))

(4) strongly generalized closed (briefly g^{*}closed) if cl(A) ⊆ G whenever A ⊆ G and G is g open in X .(Alshamary, A.G. (2008))

(5) semi strongly generalized closed (briefly sg^{*} closed) if scl(A) ⊆ G whenever A ⊆ G and G is sg open in X .(Al-Ddoury A.F.(2009))

(6) strongly generalized semi closed (briefly g*s-closed) if scl(A) ⊆ G whenever A⊆G and G is g-open in X (P.Sundaram, A.Pushpalatha, 2001) .

The complements of the g-closed (sg-closed,gs-closed,g^{*}closed, sg^{*} closed and g*s-closed) sets are called g-open(sg- open,gs-open,$g^{*}-$open, $sg^{*}-$open and g*s- open)sets respectivly .

Definitions 2.2: (J.Dugungji) A topological space (X,Ʈ) is called :

(1) completely regular if for every closed F $\subseteq$X and x $\in$ X \ F, there is a continuous function

f : X $\Rightarrow$ [0,1] , such that f(x) = 0 and f(F) = {1} .

(2) Regular space iff $\forall x \in X$ and $\forall F$ closed in X , $x \notin F$, $\exists U, V \in \tau$, such that $x \in V$ and $F \subseteq V \ni U \cap V = \phi$.

3. Some Properties and Relations:

Definitions 3.1: (J.Dugungji) A topological space (X,Ʈ) is called:

(1) g Complete regular space (briefly g [CR]) if g closed set F in X and $x \in X$, $x \notin F$; Then there exists a continuous mapping $g : X \to [0,1]$ such that $g(F) = \{1\}$ and $g(x) = 0$.

(2) g regular space (briefly g [R]) iff the g closed set A and point $x \notin A$ there exist disjint g open sets $U, V \in \tau$, such that $A \subseteq U$ and $x \in V$ $\ni U \cap V = \phi$.

(3) semi g completely regular (briefy sg [CR]) if for every semi g closed set $F \subseteq X$ and $x \in X \setminus F$, there is a continuous function $f : X \to [0,1]$, such that $f(x) = 0$ and $f(F) = \{1\}$.

(4) semi g regular (briefy sg$[R]$) iff the sg closed set A and point $x \notin A$, There exists disjoint semi g open sets $U, V \in \tau$ such that $A \subseteq U$ and $x \notin V \ni U \cap V = \phi$.

(5) g semi completely regular (briefy gs$[CR]$) if for every g semi closed set $F \subseteq X$ and $x \in X \setminus F$, there exists a continuous function $f : X \to [0,1]$, such that $f(x) = 0$ and $f(F) = \{1\}$.

(6) g semi regular (briefy gs$[R]$) iff the gs closed set A and point $x \notin A$, There exist disjoint g semi open sets $U, V \in \tau$ such that $A \subseteq U$ and $x \notin V$, such that $U \cap V = \phi$.

(7) g^* Complete regular space (briefly $g^*[CR]$) iff g closed set F in X and $x \in X$, such that $x \notin F$, Then there exists a continuous mapping $g : X \to [0,1]$, such that $g(F) = \{1\}$ and $g(x) = 0$.

(8) g^* regular space (briefly $g^*[R]$) if for each g^* closed set A and point $x \notin A$ there exist disjoint g^* open sets $U, V \subseteq X$, such that $A \subseteq U$ and $x \in V$ $U \cap V = \phi$

(9) semi g^* completely regular (briefy sg$^*[CR]$) if for every semi g^*

closed set $F \subseteq X$ and $x \in X \setminus F$, there exists a continuous function $f : X \to [0,1]$, such that $f(x) = 0$ and and $f(F) = \{1\}$.

(10) semi g^* regular (briefy sg$^*[R]$) if for each sg* closed set A and each point $x \notin A$; There exist disjoint semi g^* open sets $U, V \subseteq X$ such that $A \subseteq U$ and $x \notin V$.

(11) g^* semi completely regular (briefy $g^*s[CR]$) if for every g semi closed set $F \subseteq X$ and $x \in X \setminus F$, there is a continuous function $f : X \to [0,1]$, such that $f(x) = 0$ and $f(F) = \{1\}$.

(12) g^* semi regular (briefy $g^*s[R]$) if for each g^*s closed set A and each point $x \notin A$; There exists disjoint g^* semi open sets $U, V \subseteq X$ such that $A \subseteq U$ and $x \notin V$.

Theorem 3.2: Every regular space is g regular space.

Proof: Let (X, τ) be regular space then $\forall x \in X$ and $\forall F$ closed in X, $x \notin F$ $\exists U, V \in \tau$ such that $x \in V$, $F \subseteq V$, $U \cap V = \phi$

But every closed set is g closed set [3].

Then (X, τ) is g regular.

Theorem 3.3: Every $g[CR]$ space is $g[R]$ space.

Proof : Let (X,τ) is $g[CR]$ space. Let F is g closed set in X and let $x \in X$ be a point of X not in F

, That is $x \in X - F$. By g completely, there exists a continuous mapping $f^* : X \to [0,1]$ such that $f^*(F) = \{1\}$ and $f^*\{x\} = 0$

Since $[0,1]$ is T_2 – space then there exists two disjoint g open sets H and G such that $G \cap H = \phi$, But f^* is a continuous map then $f^{*-1}(H)$ and $f^{*-1}(G)$ are disjoint g open sets such that $f^{*-1}(G) \cap f^{*-1}(H) = \phi$

$\because f^*(x) = 0 \in G \Rightarrow x \in f^{*-1}(G)$ and $f^*(F) = \{1\} \in H \Rightarrow F \subseteq f^{*-1}(H)$

Now are g open sets containing x and F respectively

It follows that (X,τ) is $g[R]$. The converse is not truo as in (3.4)

Exampel3.4: Let $X = \{a,b,c\}$, and $\tau = \{\phi, X, \{a\}\{b,c\}\}$.

The only closed sets are $X, \phi, \{a\}, \{b,c\}$ Let the point $a \in \{a\}$ and the closed set $F = \{b,c\} \subseteq \{b,c\}$ open and $\{a\} \cap \{b,c\} = \phi$, Then (X,τ) is $[R]$. But every closed set is (g closed, sg close gs closed, g^*closed, g^*s closed, sg^* *closed*) (Al-Ddoury A.F.(2009), (Alhawez,Z.T.(2008))and (Alshamary, A.G. (2008)) .Then (X,τ) is $g[R], sg[R]\, gs[R], g^*[R], g^*s[R], sg^*[R]$ respectively .Now to show (X,τ) is not $g[CR]$ its very easy to show that because there is no continuous mapping from X to $[0,1]$.

Theorem 3.5: A topological space (X,τ) is $g[CR]$ iff $\forall x \in X$ and $\forall$ g open set G containing x, there exists a continuous mapping $f^* : X \to [0,1]$, such that $f^*(x) = 0$ and $f^*(y) = \{1\}, \forall y \in X - G$.

Proof : Let X be $g[CR]$ and G is g open set containing $x \in X$, $x \in G$, Then $X - G$ is g closed set of X such that $x \notin X - G$. From definition of $g[CR]$, there exists a continuous mapping f^* from X into $[0,1]$ such that $f^*(x) = 0$ and $f^*(G - X) = \{1\}$.

$\Leftarrow$ Let F be g closed subset of X, x any point such that $x \notin F \Rightarrow X - F$ is g^* containing x, By hypothesis there exists a continuous mapping f^* from X into $[0,1]$ such that $f^*(x) = 0$ and $f^*(y) = \{1\} \forall \; y \in X - (X - F) = F$

Then (X,τ) is $g[CR]$.

Theorem 3.6: *Let* (X,τ) be g $[CR]$ and (Y,τ^*) is a subregular space of (X,τ), Then a subset A is g closed in Y iff there exists a g closed set F in X such that (1) $A = F \cap Y$ (2) for every $A \subset Y, cl_Y(A) = cl_X(A \cap Y)$

Proof :

(1) $\Leftrightarrow$ Y − A is g open in Y

$\Leftrightarrow Y - A = G \cap Y$ (from some g open subset G of X)

$\Leftrightarrow A = Y - (G \cap Y) = (Y - G) \cup (Y - Y)$

$\Leftrightarrow A = Y - G$ since $(Y - Y = \phi)$

$\Leftrightarrow A = Y \cap G'$ (Where G' denoted the complement of G in X)

$\Leftrightarrow A = Y \cap F$ (Where $F = G'$ is g closed in X since G is g open in X)

(2) $cl_Y(A) = \bigcap\{k : k \text{ is g closed in X and } A \subset k\}$

$= \bigcap\{F \cap Y : F \text{ is g closed in X and } A \subset F \cap Y \text{ by } (1)\}$

$= \bigcap\{F \cap Y : F \text{ is g closed and } A \subset F\}$

$= [\bigcap\{F : F \text{ is g closed in X and } A \subset F\} \cap Y] = cl_X(A) \cap Y$.

Theorem 3.7: g Completey regular is a hereditary property. Proof : Let (Y,τ^*) be a subspace of g completely regular (X,τ). To show that (Y,τ^*) is also g completely regular. Let F^* *be* g closed subset of τ^* and y be a point of Y such that $y \notin F^*$. Since F^* *is* g closed set of τ^* $\exists$ g closed set F of X such that $F^* = Y \cap F$, Also

$y \notin F^* \Rightarrow y \notin Y \cap F \Rightarrow y \notin F$ $(\because y \in Y)$. And $y \in Y \Rightarrow y \in X$.

Thus F *is a* g closed subset of X and y is a point of X such that $y \notin F$.

$\because (X,\tau^*)$ g completely regular, hence there exists a continuous mapping f of X into $[0,1]$ such that $f(y) = 0$ and $f(F) = \{1\}$. Let g_r denote the restriction of f to Y.

(the restriction of continuous function is continuous[6]) g_r is a continuous mapping of Y into $[0,1]$. Now by definition of g_r $g_r(x) = f(x)\ \forall x \in Y$

Hence $f(y) = 0 \Rightarrow g_r(y) = 0$ and since $f(x) = 1\ \forall x \in F$ and $F^* \subset F$, we have $g_r(x) = f(x) = 1\ \forall x \in F^*$ So that $g_r[F^*] = \{1\}$..

Thus we have shown that for each g closed set subset F^* of Y and each point $y \in Y - F^*$, $\exists$ a continuous mapping g_r of Y into $[0,1]$ such that $g_r(y) = 0$ and $g_r(F^*) = \{1\}$,

Hence the space (Y,τ^*) is g completely regular.

Theorem 3.8: g completely regular is a topological property.

Proof : *Let* (X,τ) be a g completely regular space and let (Y,τ^*) be a homeomorphic to (X,τ) under a homeomorphism f . To show that (Y,τ^*) is also g completely regular . Let F be a g closed set into Y and let y be a point of Y such that $y \notin F$. Since f is one to one, there exists a point $x \in X$ such that $f(x) = y \Leftrightarrow x = f^{-1}(y)$. Again since f is a continuous mapping, $f^{-1}[F]$ is g closed set of X Farther $y \notin F \Rightarrow f^{-1}(y) \notin f^{-1}[F] \Rightarrow x \notin f^{-1}[F]$.

Hence by g completely regular *of X*, there exists a continuous mapping f^* of X into $[0,1]$ such that $f^*[f^{-1}(y)] = f^*(x) = 0$ and $f^*[f^{-1}[F]] = \{1\}$

That is $(f^* \circ f^{-1})(y) = 0$ and $(f^* \circ f^{-1})[(F)] = \{1\}$, Since f is homeomorphism, f^{-1} is a continuous mapping of Y onto X . Also f^* is a continuous mapping of X into $[0,1]$. it follows from theorem (The composition of continuous function is also continuous[6]) that $f^* \circ f^{-1}$ is a continuous mapping of Y into $[0,1]$. Thus we have shown that for each g closed set F of Y and each point $y \in Y - F$, there exists a continuous mapping $h = f^* \circ f^{-1}$ of Y into $[0,1]$ such that $h(y) = 0$ and $h(F) = \{1\}$. Then (Y,τ^*) is g completely regular space and hence g completely regular is a topological property.

Theorem 3.9: Every regular space is sg regular space.

Proof :

Let (X,τ) be regular space then $\forall x \in X$ and $\forall F$ closed in X, $x \notin F$, there exist $U, V \in \tau$, such that $x \in V$ $F \subseteq V$ such that $\ni U \cap V = \phi$.

But every closed set is sg closed set, [3].Then (X,τ) is sg regular .

Theorem 3.10: Every $sg[CR]$ space is $sg[R]$ space.

Proof :

Let (X,τ) *is* $sg[CR]$ space, then F is semi g closed set in X and $x \in X$ $\ni x \notin F$, Then there exists a continuous function $f^* : X \to [0,1]$ such that $f^*(F) = \{1\}$ and $f^*\{x\} = 0$. Since $[0,1]$ is T_2 – space

Then there exists a two disjoint semi g open sets G and H such that $1 \in H$ and $0 \in G$, $\ni G \cap H = \phi$. But f^* is continuous then $f^{*-1}(H)$ and $f^{*-1}(G)$ are disjoint semi g open sets, such that $f^{*-1}(G) \cap f^{*-1}(H) = \phi$.

$\because f^*(x) = 0 \in G \Rightarrow x \in f^{*-1}(G)$ And $f^*(F) = \{1\} \in H \Rightarrow F \subseteq f^{*-1}(H)$

Now $f^{*-1}(H)$, $f^{*-1}(G)$ are semi g open sets containing x and F respectively

It follows that (X,τ) is $sg[CR]$. The converse is not truo see (3.4).

Theorem 3.11: A topological space (X,τ) is sg $[CR]$ iff $\forall x \in X$ and $\forall$ semi g open G containing x there exists a continuous mapping f^* from X into $[0,1]$ such that $f^*(x)=o$ and $f^*(y)=1, \forall y \in X-G$

Proof :

Let (X,τ) is sg $[CR]$ space and G is semi g open set continaing x ,X-G is semi g closed set of X such that $x \notin X-G$ from defention of sg $[CR]$
$\Rightarrow$ there exists a continuous mapping f^* from X into $[0,1]$ such that $f^*(x)=0$ and $f^*(G-X)=\{1\}$

$\Leftarrow$ Let F be semi g closed subset of X, x any point such that $x \notin F$
$\Rightarrow X-F$ is semi g open set containing x , By hypothesis there exists a continuous mapping f^* From X into $[0,1]$ such that $f^*(x)=0$ and $f^*(y)=\{1\}, \forall y \in X-(X-F)=F$, Then (X,τ) is sg $[CR]$.

Theorem 3.12: Let (X,τ) be sg $[CR]$ and (Y,τ^*) is a sub sg $[CR]$ of (X,τ) Then a subset A of (Y,τ^*) is semi g closed set in Y iff there exists a set F in (X,τ) is semi g closed in X such that

$(1)\ A=F\cap Y \quad (2)$ for every $A\subset Y, cl_Y(A)=cl_X(A\cap Y)$

Proof :

$(1) \Leftrightarrow Y-A$ is semi g open in Y
$\Leftrightarrow Y-A=G\cap Y$ (for some semi g open subset G of X)
$\Leftrightarrow A=Y-(G\cap Y)=(Y-G)\cup(Y-Y)$
$\Leftrightarrow A=Y-G$ since $(Y-Y=\phi)$
$\Leftrightarrow A=Y\cap G'$ (Where G' denoted the complement of G in X)
$\Leftrightarrow A=Y\cap F$ (Where $F=G'$ is semi g closed in X since G is semi g open in X)

(2) $cl_Y(A)=\cap\{k : k$ is semi g closed in X and $A\subset k\}$
$=\cap\{F\cap Y : F$ is semi g closed in X and $A\subset F\cap Y\}$ by(1)
$=\cap\{F\cap Y : F$ is semi g closed and $A\subset F\}$
$=[\cap\{F : F$ is semi g closed in X and $A\subset F\}\cap Y] = cl_X(A)\cap Y$.

Theorem 3.13: semi g completely regular is a hereditary property.

Proof :

Let (Y,τ^*) be a subspace of semi g completel regular (X,τ). To show that (Y,τ^*) is also semi g completely regular .

Let F^* *be* semi g closed subset of τ^* and y be a point of Y such that $y\notin F^*$.

Since F^* *is* semi g closed of τ^* then there exists semi g closed set F of X such that $F^*=Y\cap F$, Also $y\notin F^* \Rightarrow y\notin Y\cap F \Rightarrow y\notin F\ (\because y\in Y)$.

And $y \in Y \Rightarrow y \in X$. Thus F is a semi g closed subset of X and y is a point of X such that $y \notin F$. Hence by semi g completely regular of X, there exists a continuous mapping f of X into $[0,1]$ such that $f(y)=0$ and $f(F)=\{1\}$. *Let* g_r denote the restriction of f to Y.
(the restriction of a continuous function is continuous[6])
g_r is a continuous is mapping of Y into $[0,1]$. Now by definition of g_r
$g_r(x)=f(x)\ \forall x \in Y$.
Hence $f(y)=0 \Rightarrow g_r(y)=0$ and since $f(x)=1\ \forall x \in F$ and $F^* \subset F$, we have $g_r(x)=f(x)=1\ \forall x \in F^*$. So that $g_r[F^*]=\{1\}$
Thus we have shown that for each semi g closed of τ^* subset F^* of Y and each point $y \in Y - F^*$ there exists a continuous mapping g_r of Y into $[0,1]$ such that, $g_r(y)=0$ and $g_r(F^*)=\{1\}$
Hence the space (Y,τ^*) is semi g completely regular.

Theorem 3.14: semi g completely regular is a topological property.

Proof :

Let (X,τ) be a semi g completely regular space and let (Y,τ^*) be a homeomorphic to (X,τ) under a homeomorphism f.
To show that (Y,τ^*) is also semi g completely regular. Let F be a semi g closed set into Y and let y be a point of Y such that $y \notin F$. Since f is one to one, there exists a point $x \in X$ such that $f(x)=y \Leftrightarrow x=f^{-1}(y)$. Again since f is a continuous mapping $f^{-1}[F]$ is semi g closed set of X. Farther $y \notin F \Rightarrow f^{-1}(y) \notin f^{-1}[F] \Rightarrow x \notin f^{-1}[F]$,
Hence by semi g completely regular of X, there exists a continuous mapping f^* of X into $[0,1]$, such that $f^*[f^{-1}(y)]=f^*(x)=0$ and $f^*[f^{-1}[F]]=\{1\}$, That is $(f^* \circ f^{-1})(y)=0$ and $(f^* \circ f^{-1})[(F)]=\{1\}$ Since f is homeomorphism, f^{-1} is of Y onto X. *Also* f^* is a continuous mapping of X into $[0,1]$. it follows theorem (The composition of continuous map is also continuous[6]) that is $f^* \circ f^{-1}$ is a continuous mapping of Y into $[0,1]$. Thus we have shown that for each semi g closed F of Y and each point $y \in Y - F$,
There exists a continuous mapping $h = f^* \circ f^{-1}$ of Y into $[0,1]$ such that $h(y)=0$ and $h(F)=\{1\}$. Then (Y,τ^*) is semi g completely regular space and hence semi g completely regular is a topological property.

Theorem 3.15: Every regular space is g s regular space.

Proof :

Let (X,τ) be regular space then $\forall x \in X$ and $\forall F$ closed in X, $x \notin F$, such that $U, V \in \tau$, such that $x \in V$, $F \subseteq V$ such that $U \cap V = \phi$.

But every closed set is g s closed set [6]. Then (X,τ) is g s regular.

Theorem 3.16: Every gs $[CR]$ space is gs $[R]$ space.

Proof :

Let is (X,τ) gs $[CR]$ space, then F is g semi closed set *in X and* $x \in X$ such that $x \notin F$, Then there exists a continuous function $f^* : X \to [0,1]$ such that $f^*(F) = \{1\}$ and $f^*\{x\} = 0$

Since $[0,1]$ is T_2 – space Then there exists two disjoint g semi open sets G and H $\ni 1 \in H$ and $0 \in G$ such that $G \cap H = \phi$ But f^* is continuous map then $f^{*-1}(H)$ and $f^{*-1}(G)$ are disjoint g semi open sets, such that $f^{*-1}(G) \cap f^{*-1}(H) = \phi$

$\because f^*(x) = 0 \in G \Rightarrow x \in f^{*-1}(G)$ And $f^*(F) = \{1\} \in H \Rightarrow F \subseteq f^{*-1}(H)$

Now $f^{*-1}(H)$, $f^{*-1}(G)$ are g semi open sets containing x and F, respectively

It follows that (X,τ) is gs $[R]$. The converse is not truo as in (3.4)

Theorem 3.17: A topological space (X,τ) is gs $[CR]$ iff $\forall$ $x \in X$ and $\forall$ g semi open G containing x, there exists a continuous mapping f^* from X into $[0,1] \ni f^*(x) = o$ and $f^*(y) = \{1\}, \forall y \in X - G$.

Proof:

Let (X,τ) is gs $[CR]$ space and G is g semi open set, such that $x \in X$, $X - G$ is g semi closed set of X such that $x \notin X - G$. From defention of gs $[CR]$

$\Rightarrow$ there exists g^* continuous mapping f^* from X into $[0,1]$

$f^*(x) = 0$ and $f^*(G - X) = \{1\}$

$\Leftarrow$ Let F be g semi closed subset of X, x any point, such that $x \notin F$

$\Rightarrow X - F$ is g semi open set containing x By hypothesis there exists a continuous mapping f^* From X into $[0,1] \ni f^*(x) = 0$ and $f^*(y) = \{1\}$, $\forall y \in X - (X - F) = F$. Then (X,τ) is gs $[CR]$.

Theorem 3.18: Let (X,τ) *be* gs $[CR]$ and (Y,τ^*) *is a sub* gs $[CR]$ of (X,τ) Then a subset A of (Y,τ^*) is g semi closed set in Y iff there exists a g semi closed set F in X such that

(1) $A = F \cap Y$ (2) for every $A \subset Y, cl_Y(A) = cl_X(A \cap Y)$

Proof:

(1) $\Leftrightarrow Y - A$ *is g semi open in Y*

$\Leftrightarrow \mathrm{Y} - \mathrm{A} = \mathrm{G} \bigcap \mathrm{Y}$ (for some g semi open subset G of X).

$\Leftrightarrow A = Y - G$ since $(Y - Y = \phi)$ $\Leftrightarrow \mathrm{A} = \mathrm{Y} - (\mathrm{G} \bigcap \mathrm{Y}) = (\mathrm{Y} - \mathrm{G}) \bigcup (\mathrm{Y} - \mathrm{Y})$

$\Leftrightarrow \mathrm{A} = \mathrm{Y} \bigcap \mathrm{G}'$ (Where G' denoted the complement of G in X)

$\Leftrightarrow \mathrm{A} = \mathrm{Y} \bigcap \mathrm{F}$ (Where $\mathrm{F} = \mathrm{G}'$ is g semi closed in X since G is g semi open in X)

(2) $\mathrm{cl}_{\mathrm{Y}}(\mathrm{A}) = \bigcap\{\mathrm{k} : \mathrm{k}$ is g semi closed in X and $\mathrm{A} \subset \mathrm{k}\}$

$= \bigcap\{\mathrm{F} \bigcap \mathrm{Y} : \mathrm{F}$ is g semi closed in X and $\mathrm{A} \subset \mathrm{F} \bigcap \mathrm{Y}$ by $(1)\}$

$= \bigcap\{\mathrm{F} \bigcap \mathrm{Y} : \mathrm{F}$ is g semi closed and $\mathrm{A} \subset \mathrm{F}\}$

$= [\bigcap\{\mathrm{F} : \mathrm{F}$ is g semi closed in X and $\mathrm{A} \subset \mathrm{F}\} \bigcap \mathrm{Y}] = \mathrm{cl}_{\mathrm{X}}(\mathrm{A}) \bigcap \mathrm{Y}$.

Theorem 3.19: g semi completely regular is a hereditary property.

Proof :

Let (Y, τ^*) be a subspace of g semi completely regular (X, τ). To show that (Y, τ^*) is also g semi completely regularity. Let F^* *be* g semi closed subset of τ^* and y be a point of Y such that $\mathrm{y} \notin \mathrm{F}^*$. Since F^* *is* g semi closed of τ^* then there exists a g semi closd set F of X such that $F^* = Y \bigcap F$, Also $\mathrm{y} \notin \mathrm{F}^* \Rightarrow \mathrm{y} \notin \mathrm{Y} \bigcap \mathrm{F} \Rightarrow \mathrm{y} \notin \mathrm{F}$ $(\because \mathrm{y} \in \mathrm{Y})$. And $\mathrm{y} \in \mathrm{Y} \Rightarrow \mathrm{y} \in \mathrm{X}$.

Thus F is a g semi closed subset of X and y is a point of X such that $\mathrm{y} \notin \mathrm{F}$. Hence by g semi completely regular of X , there exists a continuous mapping f of X into $[0,1]$ such that $\mathrm{f}(\mathrm{y}) = 0$ and $\mathrm{f}(\mathrm{F}) = \{1\}$. Let g_{r} denote the restriction of f to y (the restriction of continuous function is continuous[6]) g_{r} is continuous mapping of Y into $[o,1]$. Now by definition of g_{r}, $\mathrm{g}_{\mathrm{r}}(\mathrm{x}) = \mathrm{f}(\mathrm{x})$ $\forall \mathrm{x} \in \mathrm{Y}$. Hence $\mathrm{f}(\mathrm{y}) = 0 \Rightarrow \mathrm{g}_{\mathrm{r}}(\mathrm{y}) = 0$ and since $\mathrm{f}(\mathrm{x}) = \{1\} \forall \mathrm{x} \in \mathrm{F}$ and $\mathrm{F}^* \subset \mathrm{F}$, we have $\mathrm{g}_{\mathrm{r}}(\mathrm{y}) = \mathrm{f}(\mathrm{x}) = \{1\} \forall \mathrm{x} \in \mathrm{F}^*$

So that $\mathrm{g}_{\mathrm{r}}[\mathrm{F}^*] = \{1\}$. Thus we shown that for each g semi closed subset F^* *of Y and each point* $y \in X - F^*$ *there exists a continuous* mapping g_{r} of Y into [0,1] such that $\mathrm{g}_{\mathrm{r}}(\mathrm{Y}) = 0$ and $\mathrm{g}_{\mathrm{r}}(\mathrm{F}^*) = \{1\}$

Hence the space (Y, τ^*) is g semi completely regular.

Theorem 3.20: g semi completely regular is a topological property.

Proof :

Let (X, τ) be a g semi completely regular space and let (Y, τ^*) be a homeomorphism of (X, τ) under a homeomorphism f . To show that (Y, τ^*) is also g semi completely regular. Let F be a g semi closed set into Y and let y be a point of Y such that $\mathrm{y} \notin \mathrm{F}$. Since f is one to one, there exists a point $x \in X$ such that $f(x) = y \Rightarrow x = f^{-1}(y)$. Again since f is a continuous mapping, $f^{-1}[F]$ is g semi closed set of X. Farther $y \notin F \Rightarrow f^{-1}(y) \notin f^{-1}[F] \Rightarrow x \notin f^{-1}[F]$ Hence by g semi completely regular of X, there exists a continuous mapping f^* of X into $[0,1]$ such that $f^*[f^{-1}(y)] = f^*(x) = 0$ and $f^*[f^{-1}[F]] = \{1\}$ That is $(f^* \circ f^{-1})(y) = 0$ and $(f^* \circ f^{-1})[(F)] = \{1\}$. Since f is homeomorphism, f^{-1} is a continuous mapping of Y onto X. Also f^* is a continuous mapping of X into $[0,1]$. it follows from theorem (The composition of continuous map is also continuous [6]) that $f^* \circ f^{-1}$ of Y into $[o,1]$. Thus we have shown that for each g semi closed set F of Y and each point $y \in Y - F$, There exists a continuous mapping $h = f^* \circ f^{-1}$ of Y into $[0,1]$ such that $h(y) = 0$ and $h(F) = \{1\}$. Then (Y, τ^*) is g semi completely regular space and hence g semi completely regular is a topological property.

, $x \notin F$, there exists $U, V \in \tau$, $x \in V$, $F \subseteq V$ such that $U \cap V = \phi$

But every closed set is g^* closed set.

Theorem 3.21: Every regular space is g^* regular space.

Proof:

Let (X, τ) be regular space then $\forall x \in X$ and $\forall F$ closed in X, $x \notin F, \exists U, V \in \tau$ $x \in V, F \subseteq U$ such that $V \cap U = \phi$. But every closed set is g^* closed set [2] Then (X, τ) is g^* regular.

Theorem 3.22: Every g^* [CR] space is g^* $[R]$ space.

Proof:

Let (X, τ) g^* [CR] space then F is g^* closed set in X and $x \in X$ such that $x \notin F$ Then there exists a continuous mapping $f^*: X \to [0,1]$ such that $f^*(F) = \{1\}$ and $f^*\{x\} = 0$. Since $[0,1]$ is T_2 – space. Then there exists two disjoint g^* open sets H and G, such that $1 \in H$ and, $0 \in G$, such that $G \cap H = \phi$. But f^* *is* a continuous then $f^{*-1}(H)$ and $f^{*-1}(G)$ are disjoint g^* open such that $f^{*-1}(G) \cap f^{*-1}(H) = \phi$,

$\because f^*(x) = 0 \in G \Rightarrow x \in f^{*-1}(G)$ And $f^*(F) = \{1\} \in H \Rightarrow F \subseteq f^{*-1}(H)$, Now $f^{*-1}(H)$, $f^{*-1}(G)$ are g^* open sets containing x and F respectively it follows that (X, τ) is $g^*[R]$.

The converse is not truo see (3.4)

Theorem 3.23: A topological space (X,τ) is g^* $[CR]$ if and only if $\forall x \in X$ and $\forall g^*$ openset G contaning x, there exists a continuous mapping f^* from X into $[0,1]$ such that $f^*(x)=0$ and $f^*(y)=\{1\}$, $\forall y \in X-G$.

Proof :

Let X be g^* $[CR]$ and G is g^* open set, $x \in X$, $X-G$ is g^* closed set of X, and $x \notin X-G$ from definition of g^* $[CR]$

$\Rightarrow$ there exists a continuous mapping f^* from X into $[0,1]$ such that $f^*(x)=0$ and $f^*(G-X)=\{1\}$.

$\Leftarrow$ Let F be g^* closed subset of X, x any point $\ni x \notin F \Rightarrow X-F$ is g open containing x, By hypothesis there exists a continuous mapping f^* From X into $[0,1] \ni f^*(x)=0$ and $f^*(y)=\{1\}$ $\forall y \in X-(X-F)=F$ Then (X,τ) is g^* $[CR]$.

Theorem 3.24: Let (X,τ) be g^* $[CR]$ and (Y,τ^*) is a subregular space of (X,τ), $\exists$ a set F is g – closed set in X Then a subset A is g^* closed in Y iff such that

$(1)\ A=F\cap Y \quad (2)$ for every $A\subset Y, cl_Y(A)=cl_X(A\cap Y)$

Proof:

(1) $\Leftrightarrow Y-A$ is g^*open in Y (from some g^* open subset G of X)

$\Leftrightarrow Y-A=G\cap Y$

$\Leftrightarrow A=Y-(G\cap Y)=(Y-G)\cup(Y-Y)$

$\Leftrightarrow A=Y-G$ since $(Y-Y=\phi)$

$\Leftrightarrow A=Y\cap G'$ (Where G' denoted the complement of G in X)

$\Leftrightarrow A=Y\cap F$ (Where $F=G'$ is g^* closed in X since G is g^* open in X)

(2) $cl_Y(A)=\cap\{k : k$ is g^* closed in X and $A\subset k\}$

$=\cap\{F\cap Y : F$ is g^* closed in X and $A\subset F\cap Y$ by $(1)\}$

$=\cap\{F\cap Y : F$ is g^* closed and $A\subset F\}$

$=[\cap\{F : F$ is g^* closed in X and $A\subset F\}\cap Y]=cl_X(A)\cap Y$.

Theorem 3.25: g^* Completely regular is a hereditary property.

Proof :

Let (Y,τ^*) be a subspace of g^*completely regular (X,τ). To show that (Y,τ^*) is also g^* completely regular. Let F^* be g^* closed subset of τ^* and y be a point of Y such that $y \notin F^*$. Since F^* is g^* closed set of τ^* then

there exists g^*closed set F of X such that $F^* = Y \cap F$, Also $y \notin F^* \Rightarrow y \notin Y \cap F \Rightarrow y \notin F$ $(\because y \in Y)$. And $y \in Y \Rightarrow y \in X$. Thus F is a g^* closed subset of X and y is a point of X such that $y \notin F$. Hence by g^*completely regular of X, there exist a continuous mapping f of X into $[0,1]$ such that $f(y) = 0$ and $f(F) = \{1\}$. Let g_r denote the restriction of f to Y. (the restriction of continuous function is continuous[6]) g_r is a continuous mapping of Y into $[0,1]$. Now by definition of g_r,
$g_r(x) = f(x) \;\; \forall x \in Y$.
Hence $f(y) = 0 \Rightarrow g_r(y) = 0$ and since $f(x) = 1 \; \forall x \in F$ *and* $F^* \subset F$, we have $g_r(x) = f(x) = 1 \; \forall x \in F^*$ So that $g_r[F^*] = \{1\}$.
Thus we have shown that for each g^*closed set F^* of Y and each point $y \in Y - F^*$, there exists a continuous mapping g_r of Y into $[0,1]$ such that $g_r(y) = 0$ and $g_r(F^*) = \{1\}$,
Hence the space (Y, τ^*) is g^* completely regular.

Theorem 3.26: g^* completely regular is a topological property.

Proof :

Let (X, τ) be a g^*completely regular space and let (Y, τ^*) be a homeomorphic to (X, τ) under a homeomorphism f. To show that (Y, τ^*) is also g^* completely regular. Let F be a g^* closed set into Y and let y be a point of Y such that $y \notin F$. Since f is one to one, there exists a point $x \in X$ such that $f(x) = y \Leftrightarrow x = f^{-1}(y)$. Again since f *is* a continuous mapping, $f^{-1}[F]$ *is* g^*closed set *of* X. Farther
$y \notin F \Rightarrow f^{-1}(y) \notin f^{-1}[F] \Rightarrow x \notin f^{-1}[F]$
Hence by g^* completely regular of X, $\exists$ a continuous mapping f^* of X into $[0,1]$ such that $f^*[f^{-1}(y)] = f^*(x) = 0$ and $f^*[f^{-1}[F]] = \{1\}$ That is $(f^* \circ f^{-1})(y) = 0$ and $(f^* \circ f^{-1})[(F)] = \{1\}$ Since f is homeomorphism, f^{-1} is a continuous mapping of Y onto X. Also f^* is a continuous mapping of X into $[0,1]$. it follows from theorem (the composition of continuous maps is continuous[6]) that $f^* \circ f^{-1}$ is a continuous mapping of Y into $[0,1]$. Thus we have shown that for each g^*closed set F of Y and each point $y \in Y - F$, There exists a continuous mapping $h = f^* \circ f^{-1}$ of Y into $[0,1]$ such that

$h(y)=0$ and $h(F)=\{1\}$. Then (Y,τ^*) is g^* completely regular space and hence g^*completely regular is a topological property.

Theorem 3.27: Every g^*closed set is s g^*closed set.

Proof :

Let A be a g^*closed set that is $cl(A)\subseteq U$, $A\subseteq U$ and U is g open in X,
But, $scl(A)\subseteq cl(A) \Rightarrow scl(A)\subseteq U$ and s g open $\subseteq$ g open $\Rightarrow scl(A)\subseteq U$
, $A\subseteq U$ and U is s g open set in X .Then A is s g^*open set

Theorem 3.28: Every regular space is sg^* regular space.

Proof :

Let (X,τ) be regular space then $\forall x\in X$ and $\forall F$ closed in X, $x\notin F$, there exists $U,V\in\tau$, $x\in V, F\subseteq U$, such that $U\cap V=\phi$, But every closed set is sg^* closed set [3] by theorem 3.27 .we get (X,τ) is sg^* regular .

Theorem 3.29: Every $sg^*[CR]$ space is $sg^*[R]$ space.

Proof :

Let (X,τ) is $sg^*[CR]$ space, then F is semi g^* closed set in X and $x\in X$ such that $x\notin F$, Then there exists a continuous function $f^*: X\to[0,1]$ such that $f^*(F)=\{1\}$ and $f^*\{x\}=0$ Since $[0,1]$ is T_2-space Then there exists two disjoint semi g^*open sets G and H such that $1\in H$ and $0\in G$, such that $G\cap H=\phi$ But f^* is continuous then $f^{*-1}(H)$ and $f^{*-1}(G)$ are disjoint semi g^* open sets such that $f^{*-1}(G)\cap f^{*-1}(H)=\phi \because f^*(x)=0\in G\Rightarrow x\in f^{*-1}(G)$ And $f^*(F)=\{1\}\in H$ $\Rightarrow F\subseteq f^{*-1}(H)$ Now $f^{*-1}(H), f^{*-1}(G)$ are semi g^*opensets containing x and F, respectively It follows that (X,τ) is s $g^*[CR]$. The converse is not truo see(3.4).

Theorem 3.30: A topological space (X,τ) is $sg^*[CR]$ if and only if $\forall x\in X$ and $\forall$ semi g^* open G containing x there exists a continuous mapping f^* from X into $[0,1] \ni f^*(x)=0$ and $f^*(y)=1$, $\forall y\in X-G$

Proof :

Let (X,τ) is $sg^*[CR]$ space and G is semi g^* openset such that $x\in X$, $X-G$ is semi g^* closed set of X, such that $x\notin X-G$ From defention of $sg[CR]\Rightarrow$ ther exists a continuous mapping f^* from X into $[0,1]$ $f^*(x)=0$ and $f^*(G-X)=\{1\}$.

$\Leftarrow$ Let F be semi g^* closed subset of X, x any point such that $x\notin F$

$\Rightarrow X - F$ is semi g^*open set containing x, By hypothesis there exists a continuous mapping f^* From X into $[0,1] \ni f^*(x) = 0$ and $f^*(y) = \{1\}$, $\forall y \in X - (X - F) = F$ Then (X, τ) is sg^* $[CR]$.

Theorem 3.31: *Let* (X, τ) be sg^* $[CR]$ and (Y, τ^*) is a sub $sg^*[CR]$ of (X, τ), then a subset A of (Y, τ^*) is semi g^* closed set in Y iff there exists a set F in (X, τ) is semi g^* closed in X such that:

(1) $A = F \cap Y$ (2) for every $A \subset Y$, $cl_Y(A) = cl_X(A \cap Y)$

Proof :

(1) $\Leftrightarrow$ *Y − A is semi g^* open in Y*

$\Leftrightarrow Y - A = G \cap Y$ (for some semi g^* open subset G of X)

$\Leftrightarrow A = Y - (G \cap Y) = (Y - G) \cup (Y - Y)$

$\Leftrightarrow A = Y - G$ since $(Y - Y = \phi)$

$\Leftrightarrow A = Y \cap G'$ (Where G' denoted the complement of G in X)

$\Leftrightarrow A = Y \cap F$ (Where $F = G'$ semi g^* closed in X since G semi g^* open in X)

(2) $cl_Y(A) = \bigcap\{k : k$ is semi g closed in X and $A \subset k\}$

$= \bigcap\{F \cap Y : F$ is semi g^* closed in X and $A \subset F \cap Y$ by (1)$\}$

$= \bigcap\{F \cap Y : F$ is semi g^* closed and $A \subset F\}$

$= [\bigcap\{F : F$ is semi g^* closed in X and $A \subset F\} \cap Y] = cl_X(A) \cap Y$.

Theorem 3.32: semi g^*completely regular is a hereditary property.

Proof :

Let (Y, τ^*) be a subspace of semi g^* completely regularity (X, τ).

To show that (Y, τ^*) is also semi g^* completely regularity.

Let F^* *be* semi g^* closed subset *of* τ^* and y be a point of Y such that $y \notin F^*$

Since F^* is semi g^* closed of τ^*, there exists semi g^* closed set F of X such that $F^* = Y \cap F$, Also $y \notin F^* \Rightarrow y \notin Y \cap F \Rightarrow y \notin F$ $(\because y \in Y)$.

And $y \in Y \Rightarrow y \in X$. Thus F is a semi g^* closed subset of X and y is a point of X such that $y \notin F$. *Hence by* semi g^* completely regular of X, there exists a continuous mapping f of X into $[0,1]$ such that $f(y) = 0$ and $f(F) = \{1\}$. Let g_r denote the restriction of f to Y (the restriction of continuous function is continuous[6]) g_r is a continuous mapping of Y into $[0,1]$. Now by definition of g_r

$g_r(x) = f(x)$ $\forall x \in Y$.

Hence $f(y)=0 \Rightarrow g_r(y)=0$ and since $f(x)=1\ \forall x \in F$ and $F^* \subset F$, we have $g_r(x)=f(x)=1\ \forall x \in F^*$. So that $g_r[F^*]=\{1\}$
Thus we have shown that for each semi g^* closed of τ^* subset F^* of Y and each point $y \in Y - F^*$, there exists a continuous mapping g_r of Y into $[0,1]$ such that, $g_r(y)=0$ and $g_r(F^*)=\{1\}$
Hence the space (Y,τ^*) is semi g^* completely regular.

Theorem 3.33: semi g^* completely regular is a topological property.

Proof:

Let (X,τ) be a semi g^* completely regular space and let (Y,τ^*) be a homeomorphism of (X,τ) under a homeomorphism f. To show that (Y,τ^*) is also semi g^* completely regular Let F be a semi g^* closed set into Y and let y be a point *of Y* such that $y \notin F$. Since f is one to one, there exists a point $x \in X$ such that $f(x)=y \Leftrightarrow x=f^{-1}(y)$.
Again since f is a continuous mapping, $f^{-1}[F]$ is semi g closed set of X. Farther $y \notin F \Rightarrow f^{-1}(y) \notin f^{-1}[F] \Rightarrow x \notin f^{-1}[F]$ Hence by semi g completely regular of X, there exists a continuous mapping f^* of X into $[0,1]$ such that $f^*[f^{-1}(y)]=f^*(x)=0$ and $f^*[f^{-1}[F]]=\{1\}$. That is $(f^* \circ f^{-1})(y)=0$ and $(f^* \circ f^{-1})[(F)]=\{1\}$ Since f is homeomorphism, f^{-1} is a continuous mapping of Y onto X. Also f^* is a continuous mapping *of* X into $[0,1]$. it follows from theorem (The composition of continuous map is also continuous[6]) that $f^* \circ f^{-1}$ is a continuous mapping of Y into $[0,1]$. Thus we have shown that for each semi g^* closed set *F of Y* and each point $y \in Y - F$, there exists a continuous mapping $h = f^* \circ f^{-1}$ of Y into $[0,1]$ such that $h(y)=0$ and $h(F)=\{1\}$. Then (Y,τ^*) is semi g^* completely regular space and hence semi g^* completely regular is a topological property.

Theorem 3.34: Every regular space is g^*s regular space.

Proof : Let (X,τ) be regular space then $\forall x \in X$ and $\forall F$ closed in X, $x \notin F$, there exists $U, V \in \tau, x \in V, F \subseteq V$ such that $U \cap V = \phi$.
But every closed set is g^*s closed set [2] Then (X,τ) is g^*s regular.

Theorem 3.35: Every $g^*s[CR]$ space is $g^*s[R]$ space.

Proof : Let (X,τ) is $g^*s[CR]$ space, then F is g^* semi closed set in X and $x \in X$ such that $x \notin F$, Then there exists a

continuous function $f^*: X \to [0,1]$ such that $f^*(F)=\{1\}$ and $f^*\{x\}=0$ Since $[0,1]$ is T_2 – space Then there exists two disjoint g^* semi open sets G and H $\ni 1 \in H$ and $0 \in G$, $\ni G \cap H = \phi$, But f^* is continuous then $f^{*-1}(H)$ and $f^{*-1}(G)$ are disjoint g^* semi open sets, such that $f^{*-1}(G) \cap f^{*-1}(H) = \phi$
$\because f^*(x) = 0 \in G \Rightarrow x \in f^{*-1}(G)$ And $f^*(F) = \{1\} \in H \Rightarrow F \subseteq f^{*-1}(H)$
Now $f^{*-1}(H), f^{*-1}(G)$ are g^* semi open sets containing x and F, respectively
It follows that (X,τ) is $g^*s[R]$. The converse is not truo see (3.4).

Theorem 3.36: A topological space (X,τ) is $g^*s[CR]$ iff $\forall x \in X$ and $\forall$ g^* semi open G containing x there exists a continuous mapping f^* from X into $[0,1] \ni f^*(x) = 0$ and $f^*(y) = 1$ $\forall y \in X - G$

Proof: Let (X,τ) is gs[CR] space and G is g^* semi open set containing x, $X - G$ is g^* semi closed set of X, such that $x \notin X - G$ From defention of $g^*s[CR] \Rightarrow$ there exists a continuous mapping $f^*: X \to [0,1]$, $f^*(x) = 0$ and $f^*(G - X) = \{1\}$.

$\Leftarrow$ Let F be g^* semi closed subset of X, x any point $\ni x \notin F \Rightarrow X - F$ is g^* semi open set containing x, By hypothesis, there exists a continuous mapping f^* From X into $[0,1] \ni f^*(x) = 0$ and $f^*(y) = \{1\}$, $\forall y \in X - (X - F) = F$. Then (X,τ) is $g^*s[CR]$.

Theorem 3.37: Let (X,τ) be $g^*s[CR]$ and (Y,τ^*) is a sub $g^*s[CR]$ of (X,τ) Then a subset A of (Y,τ^*) is g^* semi closed set in Y iff there exists a set F in (X,τ) is g^* semi closed in X such that (1) $A = F \cap Y$ (2) for every $A \subset Y$, $cl_Y(A) = cl_X(A \cap Y)$

Proof: (1) $\Leftrightarrow Y - A$ is g^* semi open in Y
$\Leftrightarrow Y - A = G \cap Y$ (for some g^* semi open subset G of X)
$\Leftrightarrow A = Y - (G \cap Y) = (Y - G) \cup (Y - Y)$
$\Leftrightarrow A = Y \cap G'$ (Where G' denoted the complement of G in X)
$\Leftrightarrow A = Y \cap F$ (Where $F = G'$ is g^* semi closed in X since G is g^* semi open in X)
(2) $cl_Y(A) = \cap\{k : k$ is g^* semi closed in X and $A \subset k\}$
$= \cap\{F \cap Y : F$ is g^* semi closed and $A \subset F\}$
$= [\cap\{F : F$ is g^* semi closed in X and $A \subset F\} \cap Y] = cl_X(A) \cap Y$.

Theorem 3.38: g^* semi completely regular is a hereditary property.

Pr*oof* : Let (Y,τ^{*}) be a subspace of g^{*} semi completely regularity (X,τ).
To show that (Y,τ_{Y}) is also g^{*}semi completely regularity. Let F^{*} be g^{*}semi closed subset of τ^{*} and y be a point of Y such that $y\notin F^{*}$. Since F^{*} is g^{*} semi closed of τ^{*}, there exists a g^{*}semi closed set F of X such that $F^{*}=Y\cap F$, Also $y\notin F^{*}\Rightarrow y\notin Y\cap F.\Rightarrow y\notin F\ (\because y\in Y)$.
And $y\in Y\Rightarrow y\in X$. Thus F is a g^{*}semi closed subset of X and y is a point of X such that $y\notin F$. Hence by g^{*}semi completely regular of X, there exists a continuous mapping f of X into $[0,1]$ such that $f(y)=0$ and $f(F)=\{1\}$. Let g_{r} denote the restriction of f to Y (the restriction of continuous function is continuous[6]) g_{r} is a continuous mapping of Y into $[0,1]$. Now by definition of g_{r},
$g_{r}(x)=f(x)\ \forall x\in Y$.
Hence $f(y)=0\Rightarrow g_{r}(y)=0$ and since $f(x)=1\ \forall x\in F$ and $F^{*}\subset F$, We have $g_{r}(x)=f(x)=1\ \forall x\in F^{*}$. So that $g_{r}[F^{*}]=\{1\}$
Thus we shown that for each g* semi closed subset F* of Y and for each $y\in Y-F^{*}$, there exist a continuous mapping g_{r} of Y into $[0,1]$ such that $g_{r}(y)=0$ and $g_{r}(F^{*})=\{1\}$,
Hence the space (Y,τ_{Y}) is g^{*}semi completely regular.

Theorem 3.39 : g^{*}semi completely regular is a topological property.
Proof : Let (X,τ) be a g^{*}semi completely regular space and let (Y,τ^{*}) be a homomorphic to (X,τ) under a homeomorphism f.
To show that (Y,τ^{*}) is also g^{*}semi completely regular. Let F be a g^{*}semi closed set into Y and let y be a point of Y such that $y\notin F$
Since f is one to one, there exists a point $x\in X$, such that $f(x)=y\Leftrightarrow x=f^{-1}(y)$. Again since f is a continuous mapping, $f^{-1}[F]$ is g^{*}semi closed set of X. Farther $y\notin F\Rightarrow f^{-1}(y)\notin f^{-1}[F]\Rightarrow x\notin f^{-1}[F]$
since X is g^{*}semi completely regular, there exists a continuous mapping f^{*} of X into $[0,1]$ such that $f^{*}[f^{-1}(y)]=f^{*}(x)=0$ and $f^{*}[f^{-1}[F]]=\{1\}$
That is $(f^{*}\circ f^{-1})(y)=0$ and $(f^{*}\circ f^{-1})[(F)]=\{1\}$ Since f is homeomorphism, f^{-1} is a continuous mapping of Y onto X. Also f^{*} is a continuous mapping of X into $[0,1]$. it follows from theorem (the composition of continuous maps is continuous[6]) that $f^{*}\circ f^{-1}$ of Y into $[0,1]$. Thus we have shown that for each

g^* semi closd set F of Y and each point $y \in Y - F$, there exists a continuous mapping $h = f^* \circ f^{-1}$ of Y into $[0,1]$ such that $h(y) = 0$ and $h(F) = \{1\}$. Then (Y, τ^*) is g^*semi completely regular space and hence g^*semi completely regular is a topological property.

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بعض خواص فضاءات *g كاملة الانتظام

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الخلاصة

في هذا البحث تم تقديم تعاريف جديدة $g, sg, gs, g^*, sg^*, g^*s$ للفضاء التبولوجي الكامل الأنتظام والمنتظم ودراسة العلاقات بين تلك الفضاءات والتحقق من الصفات الوراثية والصفات التبوولوجية