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Design of a Direct Neural Braking System based on Switching Gains Controller

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Abstract- In this paper, direct neural controller for braking system is proposed. Learning of the presented controller depends on the training data that comes from running the switching gain controller at different conditions of drive. The training data consist of relative velocity error, distance error and braking force. The feed-forward neural network is used to build direct neural controller with two hidden layers and using back-propagation training algorithm. The performance of the presented controller is validated using nonlinear braking model. Simulation results show the presented controller is able to prevent the collision of vehicles at different driving conditions. Also, the results show superiority of the direct neural controller in comparison with the switching gain controller at all drive cases that are tested in this work.

Keywords – Switching Controller, Neural Network, Braking System.

1. Introduction

Many control systems are developed and used in the vehicle field to improve the performance of vehicles, reduce the effort of drivers, and so give more safety for both passengers and the people that use the road.

The antilock braking system (ABS), electronic brake distribution (EBD), electronic traction system (ETS) and cruise control are some control systems that used in the vehicles in the present time [1]. However, these systems are still not optimal and they can be improved using an advanced estimation and control design methods [2].

The control of longitudinal vehicle motion has been pursued at many different levels by researches and automotive manufactures. Common systems involving longitudinal control available on today's passenger vehicles include lock braking systems [3,4,5] cruise control [6,7,8,9] and anti- traction control system[10,11].

The anti collision system which is part of the cruise control, is used to maintain a safe distance between the host vehicle and the leading vehicle to avoid rear end collisions whatever the driving conditions on the road.

Most real-world applications have inherent nonlinear including the behavior of dynamic vehicle. Conventional PID or state feedback controllers are usually not capable of dealing with severe process nonlinearity, variable time delays, time-varying process dynamics and unobservable states [12].

The automatic control based on neural networks can be seen today in the different vehicle systems such as brake, steering and suspension.

The objective of this paper is to develop and simulate braking controller based on neural network structure, which works to reduce the vehicle velocity by braking in order to keep up the safe separation distance between the host vehicle and the leading vehicle to avoid accident even if leading vehicle stopped. The nonlinear braking vehicle model is used for simulations.

Neural Network Controller is trained to produce the required braking force during braking to prevent crashing vehicles. The learning information is collected from behavior of switching gain controller at different conditions of drive that may face the vehicle in the road. Therefore in presented controller design, the identification process, finding the Jacobian of the system, and on line learning are not necessary as done in works [7].

2. Vehicle Model

The equation of vehicle motion for longitudinal direction is formed using figure (1) [7].

$$F_{bf} + F_{br} + f_r mg + R_a \pm Mg\sin(\theta) = Ma_h \quad \ldots(1)$$

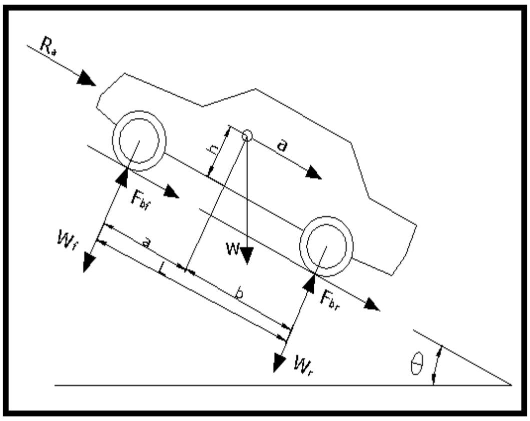

Figure (1) Forces acting on a two-axle vehicle

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The values of the coefficient of rolling resistance (f_r) for passenger car can be calculated using the following equation [6].

$$f_r = f_o + f_s\left(\frac{3.6V_h}{100}\right)^2 \qquad \ldots(2)$$

Where V_h is vehicle velocity in (m/sec.) and f_o , f_s are coefficients which depend on the inflation pressure as shown in Figure (2) [13]. The relations between the f_o , f_s and tires air pressure are found by curve fitting, therefore the effect of tires air pressure is introduced in vehicle model.

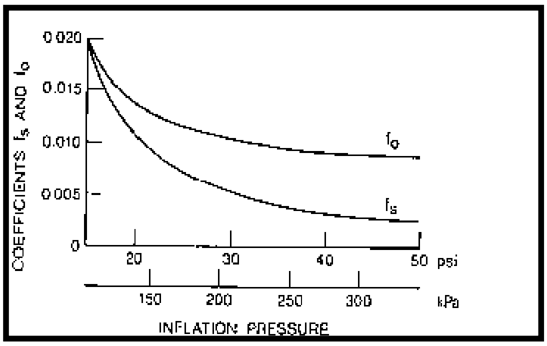

Figure (2) Effect of tire inflation pressure on coefficients f_o and f_s[6]

The aerodynamic resistance can be calculated from equation:

$$R_a = K_a V_h^2 \qquad \ldots(3)$$

Where K_a is the coefficient of aerodynamic resistance, for the passenger car, K_a=0.5 Nsec^2/m^2 [6].

When two vehicles, one of them is in front of the another one, as shown in figure (3), the driver of the host vehicle must left a distance (d) with leading vehicle, this distance is named safe distance which can be calculated from below equation (4).

$$d = d_i + x_2 - x_1 \qquad \ldots(4)$$

where d_i is the initial distance between two vehicles, x_1 is the displacement of the back vehicle and, x_2 is the displacement of the vehicle in front.

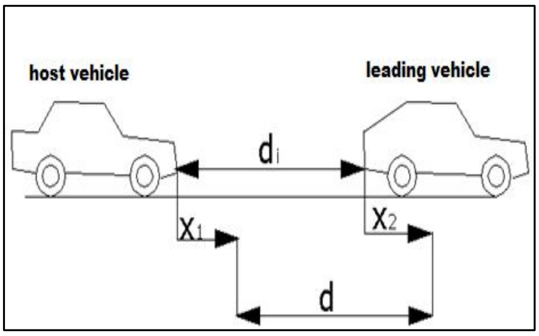

Figure (3) The safe distance between two vehicles

The desired safe distance may be estimated as 1m for 1km/h of the host vehicle velocity.

Practically, the infrared or ultrasound radar is used as a target sensor in the controlled vehicle to determine the distance between the two vehicles and the velocity of the vehicle in front [1].

3. Braking System Model

In this paper, the braking model takes into consideration the saturation effect of ABS controller in which the brake force to the wheels is limited to prescribed wheel slip. The value of saturation effect is different between the front and rear wheels because there is a load transfer from the rear axle to the front axle during braking.

The maximum braking force (saturation effect) on the front and rear axles are given by [7];

$$F_{bf\max} = \mu W_f = \frac{\mu Mg(b + h(\mu + f_r))}{L} \qquad ..(5)$$

$$F_{br\max} = \mu W_r = \frac{\mu Mg(a - h(\mu + f_r))}{L} \qquad ..(6)$$

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The value of coefficient of road adhesion (μ) is taken 0.9 for dry road and 0.5 for wet road.

Also, to prevent lock wheels, the distribution of braking force between the front and rear wheels must not be equal, therefore the proportional of the total braking force on the front and rear axles (K_{bf}, K_{br}) are introduced in the braking model. The (K_{bf} , K_{br}) are determined by [14];

$$\frac{K_{bf}}{K_{br}} = \frac{b + h(\mu + f_r)}{a - h(\mu + f_r)} \qquad \ldots(7)$$

We can see from equation (7) that (K_{bf} , K_{br}) are not constant but they depended on the coefficient of road adhesion (μ) and coefficient of rolling resistance (f_r). The coefficient of road adhesion describes the condition of road, dry or wet and coefficient of rolling resistance including the effect of tires inflation pressure.

4. Controller Design

The working strategy of the presented braking controller is based on the following conditions:

1- The brake force value has a saturation limit to avoid the slip in the tires.

2- The brake force will be applied when the safe distance between the host vehicle and the leading vehicle is less than the recommended distance.

3- The recommended distance is determined as follow: for one km/h of velocity of the host vehicle, the safe distance is one meter. Practically a radar system attached to the front of the host vehicle is used to determine the distance between the host vehicle and the leading vehicle.

4- During the braking process, when the velocity of host vehicle reaches to the value of the leading vehicle, the brake force becomes zero.

5- The controller is adaptive to work when the vehicle mass and coefficient of friction between the tires and road are changing.

4.1. Switching controller

The switching controller that is used in present work is given by the following equation:

$$F_b = \begin{bmatrix} k_1 & k_2 \end{bmatrix} \begin{Bmatrix} e_d \\ e_v \end{Bmatrix} \qquad \ldots(8)$$

Where k_1 and k_2 are the gains of the controller.

The simulation of a closed loop braking system, equations (1 to 7), combined with the switching controller are performed to obtain the training data that will be fed to neural network, see figure (4).

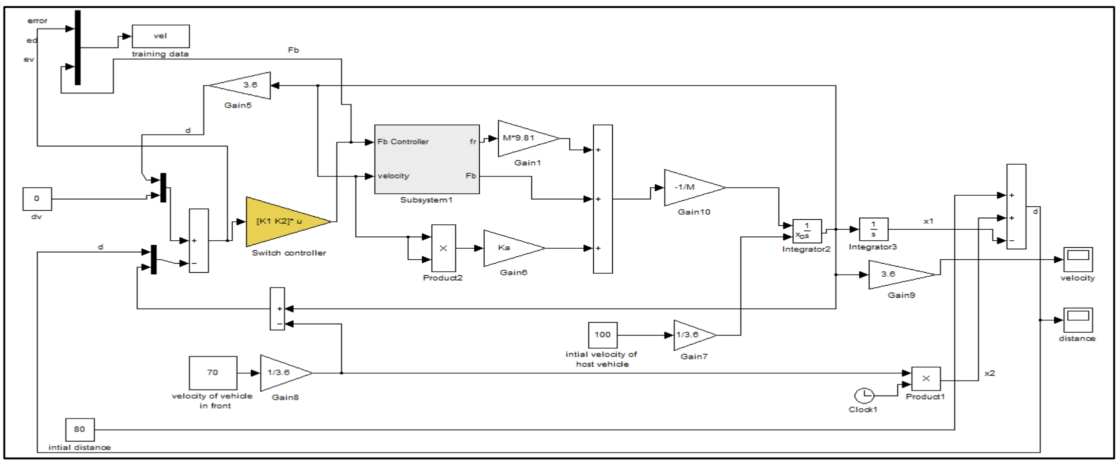

Figure (4) Switching controller with braking system

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The training data is obtained according to the following steps:

1- Change the velocities of the vehicles, the initial distance between them, coefficient of road adhesion and the host vehicle mass.

2- At each condition of drive, the gains of switching controller are tuned using trial and error method [15] in order to achieve the safe distance and the velocity of host vehicle equals the velocity of leading vehicle in a short time.

3- Repeating step-1 and step-2 for different conditions, a set of training data including distance error, relative velocities error and brake force are obtained. These data will be fed to neural network structure to learn the Direct Neural Controller (DNC).

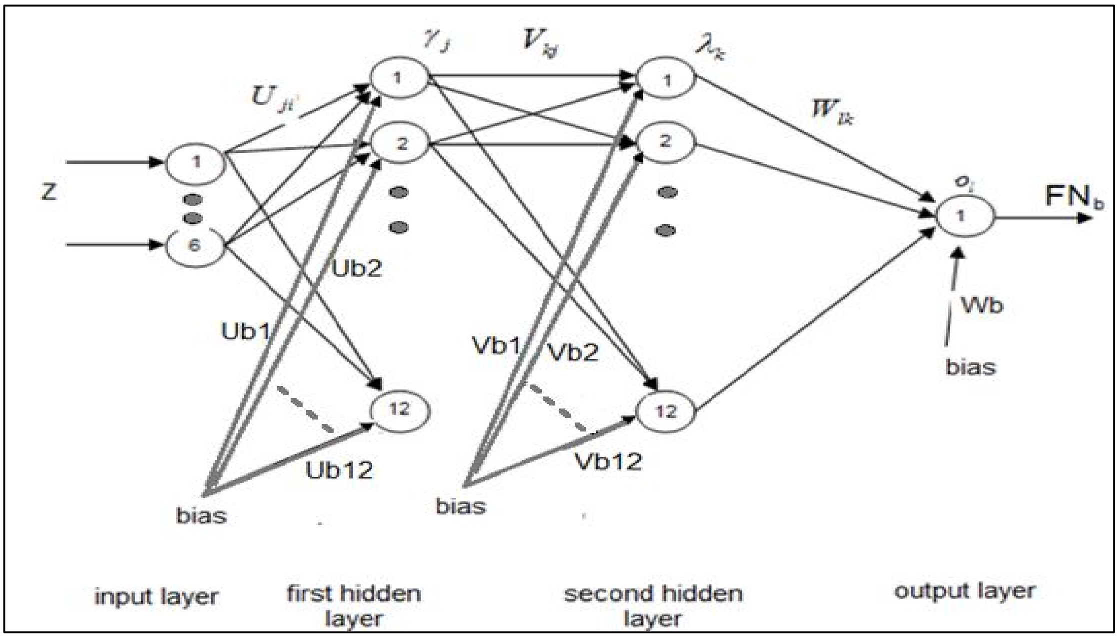

Figure (5) The multi-layer perceptron neural network of the direct neural controller.

4.2. Direct Neural Controller (DNC)

The feed-forward neural is used to build direct neural controller (DNC). The structure of this controller is shown in figure (5), where a multi-layer perceptron with two hidden layers model is used [16].The nodes of input, hidden and output layers are highlighted and the outputs of the direct neural controller represents control action (brake force).

The training of the (DNC) is performed off-line depending on the training data which comes from the work of switching gain controller with system model as shown in figure (6).

The mathematical analysis of the DNC is cleared as follows:

Consider the general j^{th} neuron in the first hidden layer. The inputs to this neuron consist of an i– dimensional vector, where i is the number of the input nodes. Ub_j is the weight vector for the bias of first hidden layer that is set equal to -1 to prevent the neurons quiescent. The output of the first hidden layer is calculated as:

$$net1_j = \sum_{i=1}^{nh} U_{ji} \times Z_i + bias \times Ub_j \qquad \ldots(9)$$

where nh is the number of the hidden nodes, and Z is the input vector.

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$Z = [Fb(m), Fb(m-1), e_v(m);$
$e_v(m-1), e_d(m); e_d(m-1)]$

Next, the output of the neuron γ_j is calculated as the continuous sigmoid function of the net_j as:

$$\gamma_j = \mathrm{H}(net1_j) \qquad \ldots(10)$$

Where;

$$\mathrm{H}(net1_j) = \frac{2}{1+e^{-net1_j}} - 1$$

For the second hidden layer, also the output is calculated as the continuous sigmoid function of the $net2_k$ as;

$$net2_k = \sum_{j=1}^{Nh} V_{kj} \times \gamma_j + bias \times Vb_k \qquad \ldots(11)$$

$$\lambda_k = \mathrm{H}(net2_k) \qquad \ldots(12)$$

Where;

$$\mathrm{H}(net2_k) = \frac{2}{1+e^{-net2_k}} - 1$$

Once the outputs of the hidden layers are calculated, they are passed to the output layer. In the output layer, the linear neuron is used to calculate the weighted sum ($neto_l$) of its inputs.

$$net_l = \sum_{k=1}^{Nh} W_{lk} \times \lambda_k + bias \times Wb_l \qquad \ldots(13)$$

where W_{lk} is the weight between the second hidden neuron λ_k and the output neuron. Wb is the weight vector for the bias of the output neuron. The linear neuron, then, pass the sum (net_l) through a linear function of slope 1 as:

$$O_l = L(neto_l) \qquad \ldots(14)$$

The output of this neural solution is the brake force FN_b.

The actual output pattern is compared with the desired output pattern and the weights are adjusted by the supervised back-propagation training algorithm until the pattern matching occurs, i.e., the cost function (E) becomes acceptably small, see figure (6).

The cost function (E) is the sum of the square of the differences between the desired output F_b and neural network output FN_b and given by equation (15) [17]:

$$E = \frac{1}{2}\sum_{i=1}^{np}(F_b - FN_b)^2 \qquad \ldots(15)$$

Where np is the number of the patterns.

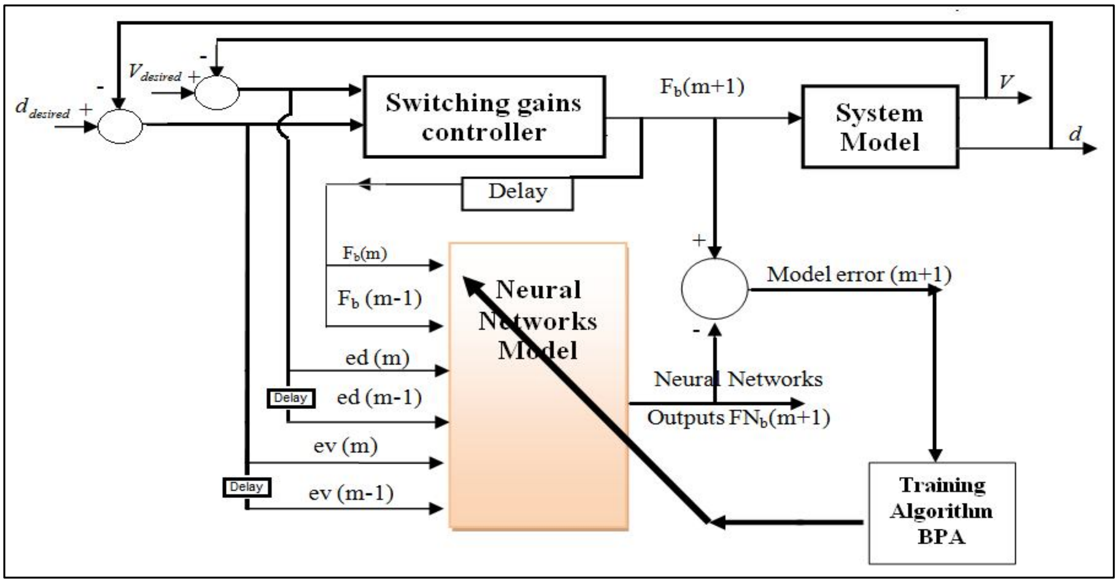

Figure (6) The direct neural controller learning structure

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The adaptation equations of the direct neural controller's weights are shown below:

$$\Delta W_{lk}(m+1) = -\eta \frac{\partial E}{\partial W_{lk}} \qquad \text{...(16)}$$

$$\frac{\partial E}{\partial W_{lk}} = \frac{\partial E}{\partial FN_b(m+1)} \frac{\partial FN_b(m+1)}{\partial o_l} \frac{\partial o_l}{\partial net_l} \frac{\partial net_l}{\partial W_{lk}} \qquad \text{...(17)}$$

$$\Delta W_{lk}(m+1) = \eta \times \lambda_k \times e_l \qquad \text{...(18)}$$

$$W_{lk}(m+1) = W_{lk}(m) + \Delta W_{lk}(m+1) \qquad (19)$$

$$\Delta V_{kj}(m+1) = -\eta \frac{\partial E}{\partial V_{kj}} \qquad \text{...(20)}$$

$$\frac{\partial E}{\partial V_{kj}} = \frac{\partial E}{\partial FN_b(m+1)} \frac{\partial FN_b(m+1)}{\partial o_l} \frac{\partial o_l}{\partial net_l} \frac{\partial net_l}{\partial \lambda_k} \frac{\partial \lambda_k}{\partial net2_k} \frac{\partial net2_k}{\partial V_{kj}} \qquad \text{...(21)}$$

$$\Delta V_{kj}(m+1) = \eta \times f(net_k)' \times \gamma_j \sum_{l=1}^{No} e_l W_{lk} \qquad \text{...(22)}$$

$$V_{kj}(m+1) = V_{kj}(m) + \Delta V_{kj}(m+1) \qquad \text{...(23)}$$

$$\Delta U_{ji}(m+1) = -\eta \frac{\partial E}{\partial U_{ji}} \qquad \text{...(24)}$$

$$\frac{\partial E}{\partial U_{ji}} = \frac{\partial E}{\partial FN_b(m+1)} \frac{\partial FN_b(m+1)}{\partial o_l} \frac{\partial o_l}{\partial net_l} \frac{\partial net2_l}{\partial \lambda_k} \frac{\partial \lambda_k}{\partial net2_k} \frac{\partial net2_k}{\partial \gamma_j} \frac{\partial \gamma_j}{\partial net1_j} \frac{\partial net1_j}{\partial U_{ji}} \qquad \text{...(25)}$$

$$\Delta U_{ji}(m+1) = \eta \times f(net1_j)' \times Z_i \times \sum_{k=1}^{Nh} f(net2_k)' \times V_{kj} \times \sum_{l=1}^{No} e_l W_{lk} \qquad \text{...(26)}$$

$$U_{ji}(m+1) = U_{ji}(m) + \Delta U_{ji}(m+1) \qquad \text{...(27)}$$

The algorithm of the (DNC) is carried out using MATLAB program version 2012.

A training set of 486 patterns has been used with a learning rate of 0.1 at different drive conditions (velocity of host and leading vehicle, and initial distance).

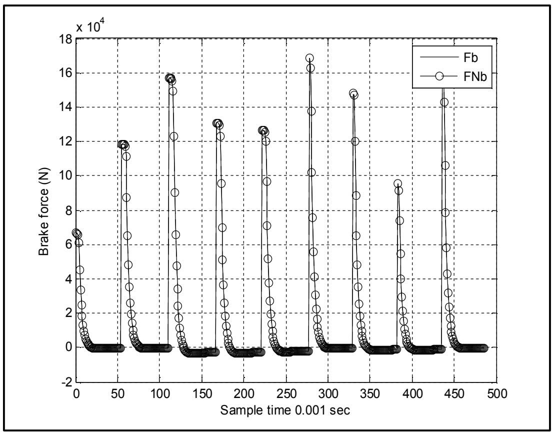

Figure (7) The response of the neural network brake force with the actual brake force for the learning set

After 112 epochs, the output of the neural network is approximated to the actual output (brake force) as shown in figure (7).

The cost function (E) is equal to $7.7e^{-6}$ for excellent learning of DNC as shown in figure (8). The weight marries of neural network are listed in Appendix (B).

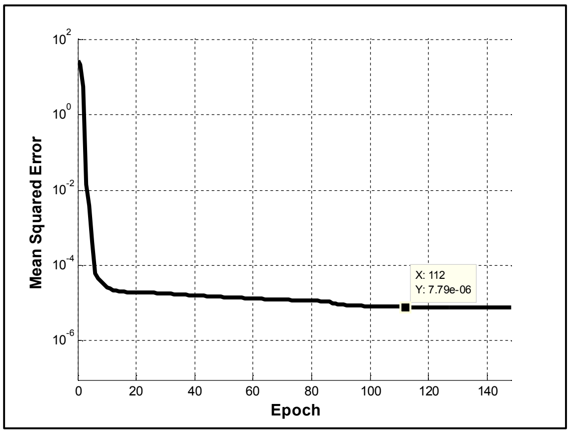

Figure (8) Mean square error vs. epoch

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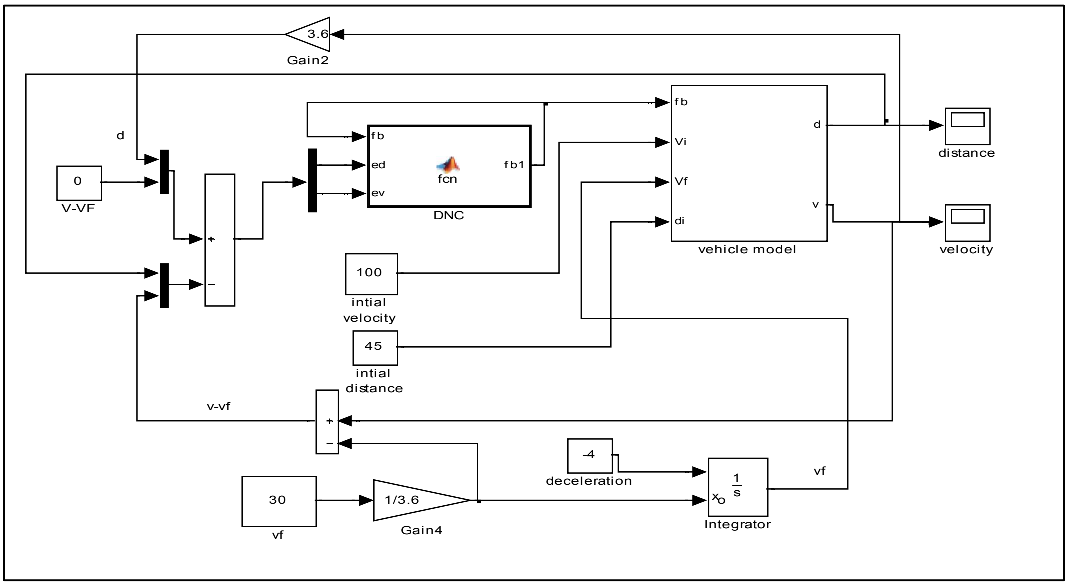

Figure (9) Direct neural controller structure

5. Simulations

The performance of the suggested controller is evaluated using the closed loop step which is response for nonlinear braking system as giving by equations (5 to 7) including the road-tire interaction and variation in vehicle mass, see figure (9). The numerical values of constants vehicle in appendix (A) are used.

Direct Neural Controller (DNC) must achieve two aims whatever the driving conditions on the road. The difference in the relative velocity of the two vehicles must be zero and give a required safe distance between the vehicles.

Five cases are taken here to exam the performance of DNC.

Case-1

The host vehicle is traveling in 50km/h on dry road and suddenly a vehicle appears in front with 20km/h velocity. The separation distance is 30m; this distance is less than the safe distance of 50m.

The DNC reduces the speed of the host vehicle to 20 km/h at 1.2 second as shown in figure (10) and the distance between the vehicles become 24.8m as shown in figure (11), this is an acceptable safe distance.

Also the case-1 is implemented when the road is wet and the controller gives an acceptable performance as shown in figures (10) and (11). The distance is 22m and this larger than the recommended distance of 20m.

The SGC gives the distances of 15.4 and 12.8 meter on the dry and wet road respectively.

Case-2

In this case the difference between the velocities of vehicles is very large and the initial separation distance is small. The host vehicle and the vehicle in front are traveling in 90km/h and 10 km/h respectively on dry and wet road. The initial distance between them is 50 m which is less than the safe distance of 90m.

10 km/h at 2.5 second as shown in figure (12) and the distance between the vehicles become 22 m, figure (13). While

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for the wet road, the distance of 15m is still larger than the recommended distance of (10m). It can be seen that the SGC fails to prevent the crashing of the vehicles at wet road condition.

Case-3

This case tests the performance of DNC when the mass of vehicle is changing. The initial conditions are: the velocity of host vehicle is 100km/h, velocity of the vehicle in front is 50km/h and the separation distance is 80m.

The responses of the velocity and the distance are shown in figures (14) and (15) respectively when the mass is 1200kg and 1500kg. From these figures; we can see the DNC achieved the safe distance and velocity with respect to the results of SGC. These results mean that the performance of DNC is more robust when the parameters of the vehicle changed.

The common cases that may be faced the host vehicle during drive when the leading vehicle reduces its velocity are shown in Cases 4 and 5.

Case-4, the leading vehicle reduces its velocity from 45 to 40 km/h at constant deceleration of 2 m/s^2. The velocity of the host vehicle is 140 km/h and the initial distance is 80m. Two controllers are succeeded to prevent the crash but the DNC which gives longer safe distance than the distance that generated from SGC, as shown in figure (16).

Case-5, in this case, the velocity of the leading vehicle is reduced to zero (stop condition) at constant deceleration of 4 m/s^2. The host vehicle velocity is 100 km/h and the initial distance 45m which is considered minimum safe distance to achieve the stopping in this driving condition. From figure (17), it can be seen that DNC is successful to stop the host vehicle at 4.8 meter, while SGC fail to stop the vehicle. This case is one of the most difficult cases that may be faced the controller.

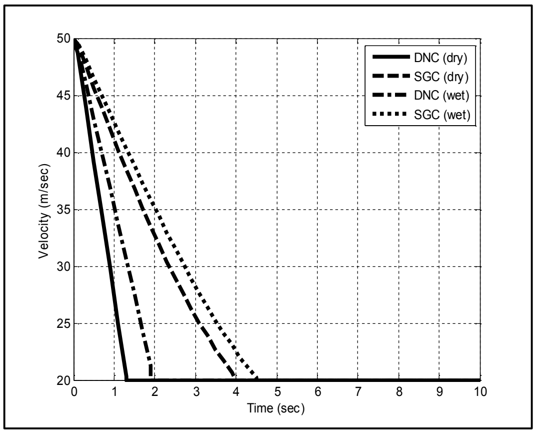

Figure (10) Velocity response for case-1

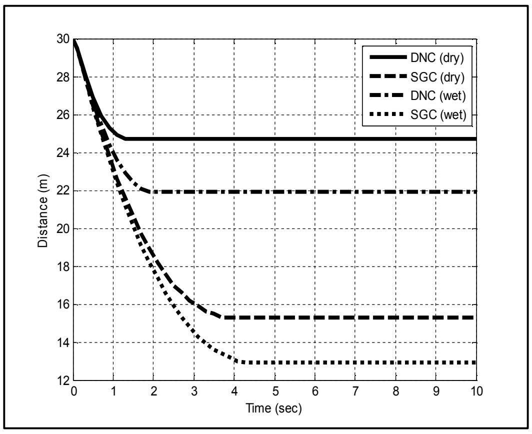

Figure (11) Distance response for case-1

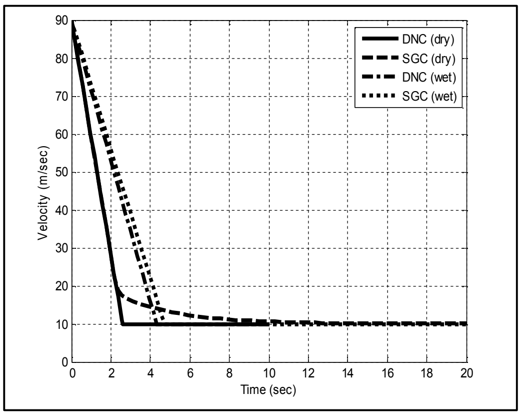

Figure (12) Velocity response for case-2

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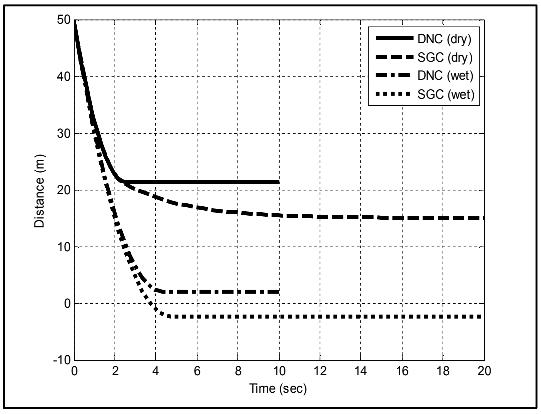

Figure (13) Distance response for case-2

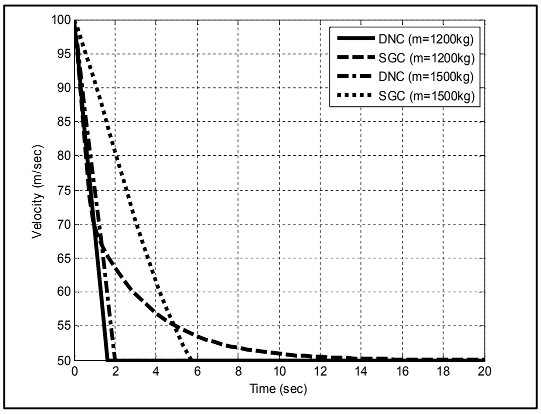

Figure (14) Velocity response for case-3

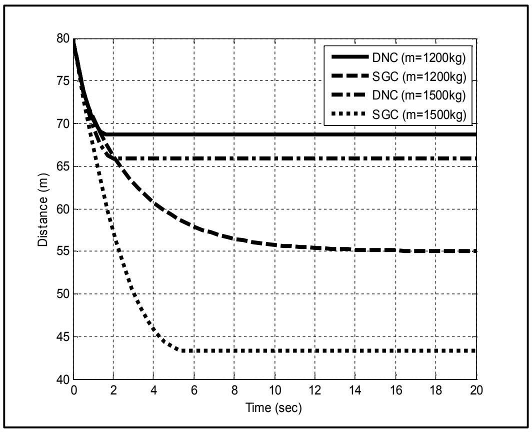

Figure (15) Distance response for case-3

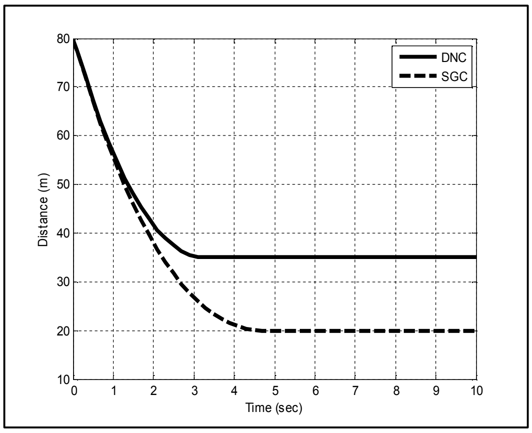

Figure (16) Distance response for case-4

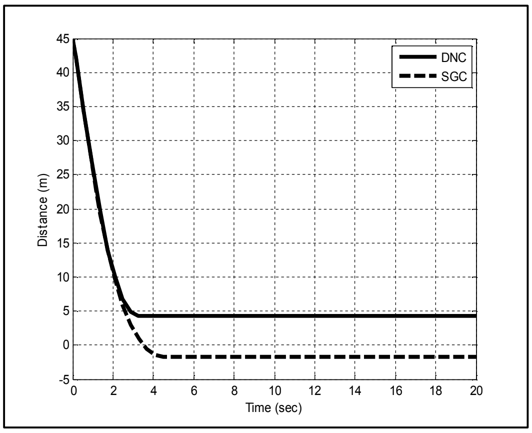

Figure (17) Distance response for case-5

6. Conclusions

The proposed controller (direct neural controller) in this paper was implemented using the neural network theory and the training data come from the switch controller for various driving cases.

This approach makes the presented controller having a high ability to deal with different driving and the road conditions without the need to change its gains at each condition as it gets in the switching controller. Also, the proposed control system can deal with all conditions lie within the same training range.

When there is a nonlinear model with the presence of changes in the parameters of

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the system, the use of control systems is preferred like the proposed controller in this research. Because of the presence of the intrinsic characteristics of neural network in having internal memory, they are capable of modeling nonlinear dynamic system.
Direct neural controller is tested with a nonlinear braking vehicle model. The results give acceptable responses for velocity and safe distance for different cases even in difficult cases in comparison with the switching controller results.

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Nomenclatures

a=length from mass center to front axle (m)
a_h= deceleration (m/sec^2)
b=length from mass center to rear axle (m)
d= distance between two vehicles (m)
d_i= initial distance between two vehicles(m)
e_d=distance error (m)
e_v=relative velocity error (m/sec)
E= cost function
F_{bf} =front brake force (N)
F_{br} =rear brake force (N)
FN_b=neural network brake force (N)
f_r = coefficient of rolling resistance.

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f_o, f_s = coefficients depend on the inflation pressure.
g=acceleration of gravity (m/sec^2)
H= nonlinear node with sigmoidal function
h=height of the center of mass (m)
L=vehicle track (m) and linear node.
K_a= coefficient of the aerodynamic resistance (Nsec^2/m^2)
K_{bf}, K_{br} = ratio of the total braking force on the front and rear axles
K_1,K_2=gain of switching controller
M=vehicle mass (kg)
m= index of time
o= outputs of output layer
R_a=aerodynamic resistance (N)
U_{ji}= Weight of first hidden layer
Ub_i=bias weight for first hidden layer
V_{kj}= Weight of second hidden layer
Vb_j=bias weight for second hidden layer
V_h=host vehicle velocity (m/sec.)
W_{lk}= Weight of output layer
Wb_k=bias weight for output layer
W_f=normal load on the front axle (N)
W_r= normal load on the rear axle (N)
x_1= displacement of the back vehicle(m)
x_2= displacement of the vehicle in front (m)
θ=angle of the slop with the horizontal (deg)
μ= coefficient of road adhesion
γ = outputs of first hidden layer
η = learning rate
λ = outputs of second hidden layer

Appendix –A
Physical parameters of the vehicle

m=1200kg L=2.5 m a=1 m b=1.5m K_a=0.5 Nsec^2/m^2 h=0.6m μ=0.9 for dry road, 0.5 for wet road

Appendix –B
Neural network weights

U=[0.1389 0.9080 -0.1592 -0.7203 0.6975 1.2602
-0.6295 0.1989 0.4057 0.1117 -0.6289 -0.8398
-1.4113 -0.1211 -0.4647 -0.5102 -0.0237 0.5721
0.5239 0.1759 0.6346 0.5791 -1.4004 -0.4002
1.6552 -0.4451 -0.6373 0.8688 0.3752 0.8947
-0.1570 0.4076 0.2630 -1.2988 -1.3316 0.5231
0.5576 1.0042 0.9396 -0.9900 1.1537 0.0424
-0.3794 0.4940 2.1834 0.0830 0.2939 -0.2959
1.6590 -0.1583 -0.9977 -0.4214 0.3472 0.6934
-1.4836 0.0754 -0.7593 0.3737 0.8640 0.6736
-0.6040 -1.2771 -0.8120 0.2403 -0.7598 -0.3224
0.8256 0.7268 -0.4658 -0.9538 0.1284 -0.6531]

Ub=[-2.1853 1.7755 2.6886 -1.4210 0.3850 -0.0245 0.5018 -0.6261 1.3673 -1.1524 0.5543 1.9463]^T

V=[1.012 -0.899 -1.018 0.231 -0.007 0.098 0.985 0.395 0.663 -0.502 -0.165 -0.1563
0.5885 1.1695 0.5242 0.6710 0.780 -0.0055 0.8612 0.7082 1.5435 1.4407 1.4281 -0.7501
-0.3918 -0.5886 0.2133 -0.4089 0.8634 0.3527 0.8293 0.5480 1.0266 -0.1318 -0.2118 0.5631
-0.0936 -0.8699 0.4844 -0.2536 0.8130 0.1091 0.2972 0.1883 1.1175 -0.7918 -0.6890 0.1793
-0.8136 -0.3899 1.2831 0.1137 -0.2354 0.5851 0.4314 0.7297 0.1130 -0.0911 0.9052 -0.6161
-0.3236 0.8228 0.0390 0.8019 -1.2915 1.3540 0.5329 -0.0599 -2.2487 -0.6940 -1.0102 -0.0079
-0.3817 0.3686 -1.4307 0.6251 -0.1923 -0.2680 0.0610 1.0088 1.0625 0.3563 -0.9838 0.4939
-0.5170 0.2385 0.0868 -0.5453 -1.0841 -1.0603 -0.5745 0.7051 -0.2930 -0.6314 -0.9974 0.2727
-0.5261 1.0174 0.5933 -0.0085 0.1454 -0.4957 -0.5302 0.2814 -2.1127 0.8037 -0.1832 -0.4509
-0.6287 0.3891 -0.9206 0.1499 -0.1708 0.1850 -0.4551 -0.5868 -0.0546 0.1380 -1.2542 -0.3001
-0.7531 -0.8381 -0.1040 -0.7869 -0.5756 -0.0892 -0.8384 -0.2025 0.2762 0.1120 -0.1131 0.3710
0.6719 0.4046 1.4512 0.0535 0.1495 -0.0311 -0.5548 -0.8030 0.6168 0.6103 -0.0486 0.5933]

Vb=[-1.7461 -0.7488 0.7074 1.0785 0.6076 0.2491 0.0406 -0.6163 -0.6243 -1.7292 -1.4636 0.2342]^T

W=[2.0489 1.6416 0.8354 1.0331 -0.7401 -0.2857 1.1733 0.0253 -1.7252 0.0827 -0.2391 -1.4594]

Wb= [-0.8904]