

Description of some nuclear properties of ruthenium-94 isotope using Gloeckner model space

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ABSTRACT

In the current research, several nuclear structure features of ⁹⁴Ru isotope were computed by using Nu Shell X@MSU code. These properties were:[levels energy, spin and parity, transition possibilities for electric quadrupole and magnetic dipole, nuclear moments for electric quadrupole and magnetic dipole as well as the distributions of nuclear charge and nuclear mass density]. An active valance nucleon of ⁹⁴Ru nucleus is occupied Gloeckner (Gl) model space that has the orbits:(2p_{1/2}, 1g_{9/2}) with (2d_{5/2}, 3s_{1/2}) for protons and neutrons above the subshell ending of ⁸⁸Sr nucleus respectively. The calculations were achieved by utilizing Gloeckner interaction and bare g-matrix (Glb); appropriate effective nucleon charge and g-effective factor with Skyrme (sly4) potential were chosen for extracting the radial wave functions for the single-particle matrix elements for the isotope mentioned above. The results demonstrated that there have been properly compatible energy levels with the experimental data on the studied nuclear properties; the spins and parity have confirmed some experimental levels. On the other hand, for some experimental energies have assigned the total angular momentum and the parity. Another good agreement was noticed for dipole-magnetic moments of ⁹⁴Ru isotope with the experimental values. These results have expected new values for the above mentioned properties that were not Experimentally specified before.

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وصف بعض الخواص النووية لنظائر الروثينيوم-94 باستخدام فضاء الانموذج جلوكنر

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الكلمات المفتاحية:

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العزوم النووية

الخلاصة

في البحث الحالي تم حساب العديد من خصائص التركيب النووية لنظير ⁹⁴Ru باستخدام كود نيوشيل اكس، وكانت هذه الخصائص هي: [مستويات الطاقة، البرم والتماثل، احتمالات الانتقال لرباعي القطب الكهربائي وثنائي القطب المغناطيسي، العزوم النووية لرباعي القطب الكهربائي وثنائي القطب المغناطيسي وكذلك التوزيعات لكثافة الشحنة والكتلة النووية]. نيوكليونات التكافؤ النشطة لنواة ⁹⁴Ru شغلت فضاء أنموذج جلونكر الذي يحتوي على المدارات: (2p_{1/2}, 1g_{9/2}) مع (2d_{5/2}, 3s_{1/2})

للبروتونات والنيوترونات على التوالي فوق نهاية القشرة الفرعية لنواة ^{88}Sr . تم إجراء الحسابات باستخدام تفاعل جلونكر ومصفوفة g العارية (Glb)؛ تم اختبار شحنة النيوكليون الفعالة المناسبة والعامل الفعال g مع جهد سكايرم (sly4) لاستخراج دوال الموجة الاشعاعية لعناصر المصفوفة الجسيم المنفرد لنظير المذكور أعلاه. وتوصلت النتائج إلى وجود مستويات طاقة متوافقة بشكل صحيح مع البيانات التجريبية للخواص النووية المدروسة؛ وقد أكدت البرم والتماثل لبعض المستويات التجريبية. ومن ناحية أخرى، لبعض الطاقات التجريبية خصص الزخم الزاوي الكلي والتماثل كما تم الحصول على توافق جيد آخر للعزوم الثنائية المغناطيسية لنظير ^{94}Ru مع القيم التجريبية. وقد توقعت هذه النتائج قيمة جديدة للخصائص المذكورة أعلاه والتي لم يتم تحديدها تجريبيا من قبل.

1. INTRODUCTION

Long time ago, shell model calculations were limited only to light nuclei or to heavier ones with a few valance nucleons above closed shells, in addition to the well -known problems related to the determination of a good effective interaction for large model spaces [1]. The nuclear shell model is one of the majority successful theories of nuclear structure [2-3]. It considers the nuclei as complex systems of nucleons that interact with each other in a limited configuration space, usually called valence space, the nuclear interaction is rotationally unvarying, and it is commonly look to be symmetric under proton-neutron exchange. One of the major characteristics of the nuclear interaction is a spin-orbit term accountable for the so-called magic numbers: particular collections of protons(Z) and neutrons(N) construction up particularly stable and spherical nuclei [4]. This vindicates the primary supposition of the shell model that nuclear dynamics can be

approximated by the many-body configurations constructed in a valence space assigned by two magic numbers [5]. the residual interaction offered by the portion of the two-body interaction which is not absorbed by the single-particle potential.

A massive number of studies suggested that nuclei in $A \geq 90$ around ($Z = 38, N = 50$) mass region supplies appropriate objects to examine.

2. Theory

the effects of nucleon excitations in the level structures [6], particularly Johnstone and Skouras who performed shell model calculations of ^{93}Tc nucleus using a ^{88}Sr and ^{66}Ni cores to calculate the excitation energies and the reduced electric quadrupole transition probability, further to the lifetimes [7]. While the researcher D. Bucurescu et al studied the shell model calculations of positive-parity states in the ^{91}Y and ^{95}Nb nuclei with the oxbash code using the model space ($f_{5/2}, p_{3/2}, p_{1/2}, g_{9/2}$) for protons and ($p_{1/2}, g_{9/2}, d_{5/2}, g_{7/2}, s_{1/2}, d_{3/2}$) for ^{95}Nb nucleus, an inactive ^{88}Sr core has been considered; while in the case

of ^{91}Y , up to two holes in the proton $f_{5/2}$ and $p_{3/2}$ orbitals were appropriate [8], the researchers Gloeckner and Serduke also studied the excitation levels and electromagnetic transitions of isotones $N=50$ for $Z>38$ [9] and energy level calculations for $N=50$ nuclei for ^{92}Mo and ^{94}Ru was interpreted by the researcher J. B. Ball [10].

The objective of this research is the calculation of several nuclear properties includes: the energy spectrum, electromagnetic transition probabilities, nuclear moments of the electric quadrupole and magnetic dipole, in addition to describing the distribution of nuclear density for both charge and mass.

For nuclei, two or more valence nucleons (i.e. nucleons above a closed shell) a residual two-body interaction must be added to the Hamiltonian, so the general nuclear hamiltonian can be written as [11]:

$$H = \sum_{i=1}^A T_i + \sum_{i,j}^A V_{ij} \quad (1)$$

Where (A) nucleons number (T_i) kinetic energy of the single nucleons and (V_{ij}) is the two-body interaction. It is probable to presume the existence, of a single-particle potential (V_i) , so that:

$$\begin{aligned} H &= \sum_{i=1}^A T_i + \sum_i^A V_i + \sum_{i,j}^A V_{ij} - \sum_i^A V_i \\ &= \sum_{i=1}^A H_i^{\text{s.p}} + \sum_{i=1}^A H_{i,j}^{\text{res}} \quad (2) \end{aligned}$$

Where

$$H_i^{\text{s.p}} = \sum_{i=1}^A T_i + \sum_i^A V_i \quad \text{the single-particle Hamiltonian and} \quad H_{i,j}^{\text{res}} = (\sum_{i,j}^A V_{ij} - \sum_i^A V_i)$$

The electromagnetic transition rate is an important parameter that gives structural information. The transition probability or the transition rate is given by [12,13]:

$$T_{fi}(\sigma L) = \frac{1}{\tau} = \frac{8\pi(L+1)}{\hbar L[(2L+1)!!]^2} \cdot \left(\frac{E_\gamma}{\hbar c}\right)^{2L+1} \cdot B(\sigma L: I_i \rightarrow I_f) \quad (3)$$

Where $B(\sigma L)$ a reduced transition probability with multi-polarity σ . Reduced electromagnetic transition probabilities $B(\sigma L)$ between the excited states are one of the most popular measures for quadrupole collectivity and shape changes in isotopes. The probability of an electric or magnetic transition of multipolarity (σ) can be given by [14,15]:

$$B(\sigma L: I_i \rightarrow I_f) = \frac{1}{2I_i+1} | \langle I_f || O(\sigma L) || I_i \rangle |^2 \quad (4)$$

Where $O(\sigma L)$ represents the multipole operator, double bar $(||)$ represents the reduced matrix element.

Electromagnetic moments are very significant in nuclear physics because of their predisposition to nuclear structure. Measuring them give a

hypothesis to gain insight into the way nucleons interact in multiparticle systems. Consequently, the nuclear magnetic moment and the quadrupolar moment are pivotal to comprehend the nature of nuclear states. Deviation of the nuclear charge distribution from a spherical shape leads to the occurrence of a nuclear electric quadrupole moment according to the equation below [16]:

$$Q_s(JK) = \frac{3K^2 - J(J+1)}{(1+J).(3+2J)} Q' \quad (5)$$

Where (J) is the total angular momentum, K is the total angular momentum projection on the nuclear symmetry axis and Q' the intrinsic quadrupole moment is defined according to the equation:

$$Q' = \sqrt{\frac{16\pi}{5e^2}} \cdot (B(E2))^{1/2} \quad (6)$$

If $Q_s > 0$, this indicates that the prolate deformation of isotope, $Q_s < 0$ indicates to the oblate deformation for isotope while $Q_s = 0$ indicates that the isotope has spherical shape [17].

On the one hand, there is a contribution from the positively charged protons orbiting around them in orbital motion which in revolve grant rise to an orbital magnetic moment quite, as in classical mechanics. On the other hand, there is also a contribution from the intrinsic spin of all the

nucleons and the magnetic moments are given by the following formula [18,19]:

$$\mu = \begin{bmatrix} J & 1 & J \\ -J & 0 & J \end{bmatrix} \times \frac{\sqrt{4\pi}}{3} \langle J || \hat{O}(M1) || J \rangle \mu_N \quad (7)$$

where $\langle J || \hat{O}(M1) || J \rangle \mu_N$ magnetic transition operator, $\mu_N = \frac{e\hbar}{2m_p c} = 0.1051$ e.fm, μ_N nuclear magneton, and m_p proton mass. While where $\langle J || \hat{O}(M1) || J \rangle \mu_N$ magnetic transition operator, $\mu_N = \frac{e\hbar}{2m_p c} = 0.1051$ e.fm, μ_N nuclear magneton, and m_p proton mass. While $\begin{bmatrix} J & 1 & J \\ -J & 0 & J \end{bmatrix}$ the 3j refers to the angular momentum factor, the value of which is given by [19]:

$$\begin{bmatrix} J & 1 & J \\ -J & 0 & J \end{bmatrix} = \left[\frac{J(2J-1)}{(2J+1)(J+1)(2J+3)} \right]^{0.5} \quad (8)$$

The computations in eq. (7) necessitate knowing the values of the magnetic moments (g) factors: $g_l^p = 1$, $g_s^p = 5.585$ for protons and $g_l^n = 0$, $g_s^n = -3.826$ for neutrons [2,13]. In this study, the density distribution of a system containing nucleons was calculated using the equation below [20]:

$$\rho_o(r) = \sum_{i=1}^A |\varphi_i(\vec{r})|^2 \quad (9)$$

3. Results and discussion:

Ruthenium isotopes have been the focus of many theoretical and

experimental investigations of nuclear structure. Low nuclear even cores have been successfully interpreted. Ruthenium isotopes have atomic number $Z=44$. In this study, we used the nuclear shell model with the Nu Shell X@MSU code to determine most of the nuclear properties of the ruthenium nucleus in the region $A=94$. Use of the Gloeckner (Gl) space model in orbits: $(2p_{1/2}, 1g_{9/2})$ with $(2d_{5/2}, 3s_{1/2})$ for protons and neutrons above the subshell closure $^{88}_{38}\text{Sr}$ nucleus respectively. Utilizing Gloeckner interaction and bare g -matrix (Glb), it has been estimated the energy levels, spin, parity, transition possibilities and nuclear moments for electric quadrupole and magnetic dipole, as well as the distributions of nuclear charge and nuclear mass density as a radial distance function from the nucleus center (r). In this work, it has been chosen the single-particle states for both protons and neutrons in MeV, NuShellX@MSU Code to find the exact energies, eigenvectors, and spectral overlaps of the low states of the Hamiltonian matrix. Very large fundamental dimension calculations have used a paired proton-neutron basis J and plot matrix dimensions $J(J; \text{is total angular momentum})$ of up to 100

million orders [21]. Skyrme (sly4) capabilities were used to calculate radial wave functions for single-particle matrix elements by applying Nu Shell X@MSUcode with effective nucleon charge and effective factors g , in which (g) is the values of orbital parameters and effective spin factors $(g)\{g_s(p), g_s(n), g_l(p), g_l(n)\}$. The energy values of single particles of the valence nucleons were calculated using the Nu Shell X @MSU Code are equal $\{\epsilon_{2p_{1/2}}(p)=-7.124\text{MeV}, \epsilon_{1g_{9/2}}(p)=-6.248, \epsilon_{3s_{1/2}}(n)=-5.506\text{MeV}$ and $\epsilon_{2d_{5/2}}(n)=-6.338\text{MeV}\}$.

1.3 Energy levels

^{94}Ru isotope is composed of six protons distributed in the orbits $(2p_{1/2}, 1g_{9/2})$ over ^{88}Sr subshell closure nucleus. The comparison between theoretical and practical energy level value [22] have been listed in Table.1 and Figuer.(1;a,b and c) and can be explained as follows :

- 1- A complete agreement has been gained for the ground state (0^+_1) of the theoretical energy level with experimental energy level.
- 2- A reasonable agreement has been observed for the theoretical energy levels $\{1.407, 2.177, 2.459, 2.592, 2.625$ and $3.661\}\text{MeV}$, associated with the

- states $\{2^+_{11}, 4^+_{11}, 6^+_{11}, 8^+_{11}, 5^-_{11}$ and $0^+_{33}\}$ and the experimental energy levels $\{1.430 \pm 20, 2.186 \pm 3, 2.498 \pm 3, 2.644 \pm 4, 2.624 \pm 3$ and $3.770 \pm 8\}$ MeV.
- 3- A confirmation has been expected for the practical states of energy values $\{3.657 \pm 4, 3.930 \pm 4$ and $6.357 \pm 4\}$ MeV through theoretical states $\{7^-_{11}, 8^+_{22}$ and $12^+_{22}\}$, because there is a good agreement between the theoretical energy values $\{3.652, 4.004$ and $6.381\}$ MeV and their experimental counterparts.
- 4- Within the experimental spin values (3,4,5) accompanying the energies $\{3.117 \pm 4$ and $3.254 \pm 4\}$ MeV, a confirmation of the spins $\{4$ and $5\}$ with the identification of negative and positive parity through the values of the theoretical states $\{4^-_{22}$ and $5^+_{11}\}$ has been expected. This is due to a suitable agreement between these energies and the theoretical energies $\{3.147$ and $3.257\}$ MeV.
- 5- A confirmation has been expected for the spins $\{8, 10, 9, 11, 12$ and $13\}$ with the parties' assignment to the experimental energies $\{3.930 \pm 4, 3.991 \pm 4, 4.197 \pm 4, 4.338 \pm 4, 4.489 \pm 4, 4.716 \pm 4$ and $5.567 \pm 4\}$ MeV through our calculations states in $\{8^+_{22}, 10^+_{11}, 9^-_{11}, 9^-_{22}, 11^-_{11}, 12^+_{11}$ and $13^-_{11}\}$.
- 6- It has been predicted that the experimental energies $\{3.520 \pm 7, 3.820 \pm 8$ and $4.000 \pm 8\}$ MeV at the states $\{4^+_{33}, 6^-_{11}$ and $4^+_{55}\}$ through reasonable agreement with the corresponding theoretical energies.
- 7- The theoretical states with their associated energy level values $\{4^+_{22}; 2.376, 0^+_{22}; 2.614, (6^+_{22}; 2.786$ to $4^-_{11}; 3.025), (7^+_{11}; 3.289$ to $3^-_{22}; 3.465), 3^+_{11}; 3.618, 3^-_{33}; 3.676, 2^-_{11}; 3.778, (4^+_{44}; 3.851$ to $2^-_{22}; 3.990), (2^-_{33}; 4.158$ to $6^+_{44}; 4.260), (2^+_{44}; 4.294$ to $1^-_{22}; 4.367), (6^-_{33}; 4.383$ to $5^-_{44}; 4.576), (8^-_{22}; 4.609$ to $4^-_{44}; 4.840), (5^-_{55}; 4.944$ to $9^+_{22}; 5.790), 9^-_{33}; 6.288$ and $12^-_{11}; 7.195\}$ MeV, they do not have corresponding values in the experimental data.
- 8- In experimental data, the levels at states associated with it $\{2.503 \pm 3; (3,4,5), 2.965 \pm 6; (3^-_{11}), 2.995 \pm 6; 0^+, 3.615 \pm 7; 0^+$ and $(25^-, 26^-, 27^-); 8.321 \pm 5\}$ MeV, there has been no values corresponding in our calculations.
- 9- The highest value has been obtained for the energy level (7.195 MeV) associated with the state (12^-_{11}) , while there have been experimental values higher than it

3.2. Reduced electromagnetic transition probabilities B (E2), B (M1):

The necessary inputs for calculating reduced electromagnetic transitions are the effective charges of the proton and neutron as well as the magnetic factors. For ^{94}Ru isotope, we rely on the values of the charges $e_p=1.313e, e_n= 0.6e$ to calculate the electric quadrupole transition and the orbital and spin magnetic factors of the nucleon (g) factors $g_s(p), g_s(n), g_l(p)$, and $g_l(n)$ while $\{5.583, -3.826, 0.825$ and $0.0\}$ are for the magnetic factors to calculate the magnetic dipole transition. In the current calculations, Skyrme (sly4) potential has been applied for the extraction the radial wave functions for the single-particle matrix elements of ruthenium-94 isotope. Table.2 describes the comparison between theoretical calculations and experimental data [23] for the electromagnetic transitions of isotope ^{94}Ru .

Through this comparison, We can be explained: The value $2.971 \text{ e}^2\text{fm}^4$ for the electric quadrupole transition $4^+_1 \rightarrow 6^+_1$ is in perfect agreement with the experimental value $2.97 \pm 1.71 \text{ e}^2\text{fm}^4$, while for the electric quadrupole transition $11^-_1 \rightarrow 9^-_1$ at the value $126 \text{ e}^2\text{fm}^4$, an acceptable agreement with the experimental data $111.19 \pm 5.33 \text{ e}^2\text{fm}^4$ has been found. It has also been found that there are some

values for electromagnetic transitions that have no values in the experimental information.3.3.

Electric quadrupole and magnetic dipole moments:

Table.3 shows the comparison of theoretical and experimental results [23] of electric quadrupole and magnetic dipole moments of ^{94}Ru isotope, and it has been noticed the following:

- the quadrupole electrical moment of the ^{94}Ru isotope at $\{2^+_1, 4^+_1$ and $5^+_1\}$ states, has exhibited positive signs representing the dominance of the prolate shape, while $\{6^+_1, 8^+_1, 7^+_1, 3^+_1, 10^+_1, 9^+_1$ and $12^+_1\}$ states of the ^{94}Ru isotope has seemed with negative marks to show the dominance of the oblate shape.
- A perfect agreement was seen for the theoretical magnetic dipole moment in the value $8.119 (\mu_N)$ at the state 6^+ with the experimental value $(8.12 \pm 5 (\mu_N))$ while the theoretical states $\{8^+, 12^+$ and $11^-\}$ at the values $\{10.829, 16.242$ and $13.835\} (\mu_N)$ have been in a good agreement of the magnetic dipole moments with the available experimental data $\{11.10 \pm 4, 12.4 \pm 17$ and $14.1 \pm 17\} (\mu_N)$.
- A number of values was found for quadrupolar electric moments in

addition to dipole magnetic moments for which no experimental values have been found yet.

3.4. Distributions of charge and mass nuclear density:

The nuclear density distribution for both, the charge and the nuclear mass in the ^{94}Ru isotope have been computed and listed in the Figures.2. For the ruthenium isotope with mass number 94, it has been noticed that the beginning of the distribution of the nuclear charge and mass are in the centre at the values $\{ (\rho_{\text{ch}} = 0.06955 \text{ Ze/fm}^{-3}; \rho_{\text{m}} = 0.1482 \text{ nuclei/fm}^{-3})$, these values are stabilized at the distance $r = 0.1 \text{ fm}$, then this distance has begun to increase gradually from the distance $r = 0.2 \text{ fm}$ in values $\{ \rho_{\text{ch}} = 0.06983 \text{ Ze/fm}^{-3}; \rho_{\text{m}} = 0.1488 \text{ nuclei/fm}^{-3} \}$ to reach the values for $\rho_{\text{ch}} = 0.07847 \text{ Ze/fm}^{-3}$ at the distance $r = 1.8 \text{ fm}$, while at $r = 1.6 \text{ fm}$ of the $\rho_{\text{m}} = 0.1651 \text{ nuclei/fm}^{-3}$ then decreased at the distance $r = 1.9 \text{ fm}$, the nuclear charge distribution continues the gradual decrease until stabilizes at zero at the distance $r = 7.6 \text{ fm}$, while the distribution of the nuclear charge decrease starts from the distance $r = 1.7 \text{ fm}$ at the value $\rho_{\text{m}} = 0.1650 \text{ nuclei/fm}^{-3}$ and continues gradually until it stabilizes at zero in the distance

$r = 7.5 \text{ fm}$. There has no experimental data that can compare the distribution values of the nuclear density of the mass charge for the isotope under this study.

4. Conclusions

- A perfect agreement was found for the ground level of the ^{94}Ru isotope in its ground state 0^+_1 as compared to the experimental value.
- For ^{94}Ru isotope, a reasonable agreement has been observed for most of the calculated energies with the value in the experimental data
- The spin and parity of most experimental values in ^{94}Ru isotope have been confirmed by our calculations.
- The total angular momentum of some experimental levels of ^{94}Ru isotope were confirmed.
- Calculation of ^{94}Ru isotope has been identified for some experimental levels that have not been originally specified in the experimental data.
- A good agreement was found for some reduced electromagnetic transition probabilities and some magnetic dipole moments for ^{94}Ru isotope.
- The values of the quadrupole moments of ^{94}Ru isotope was found in which the prolate shape has been

dominated for some cases, while in other cases the dominant shape has been oblate. This has led to conclude that the shape of some regions of this isotope is affected by structural influences and could change from one isotope to another neighbor. In addition, it was found that the shape changes with the number of neutrons can be changed with the excitation energy or state within the same nucleus. These variances occur due to the rearrangement of the structure space of the valance particles or to

the dynamic response.

- The nuclear distribution of charge and mass have specific values at the center, then has started to increase at certain values of the nuclear distance, it then decreases at other values and continues to decrease until it stabilizes at zero at specific values on the x axis.
- New values were obtained for most of the properties for which there are no practical values yet and this adds new information to the theoretical knowledge of this isotope under current study.

Table (1): Comparison of experimental and calculated energy spectra values of ^{94}Ru isotope using glb interaction The experimental data are taken from ref.[22]

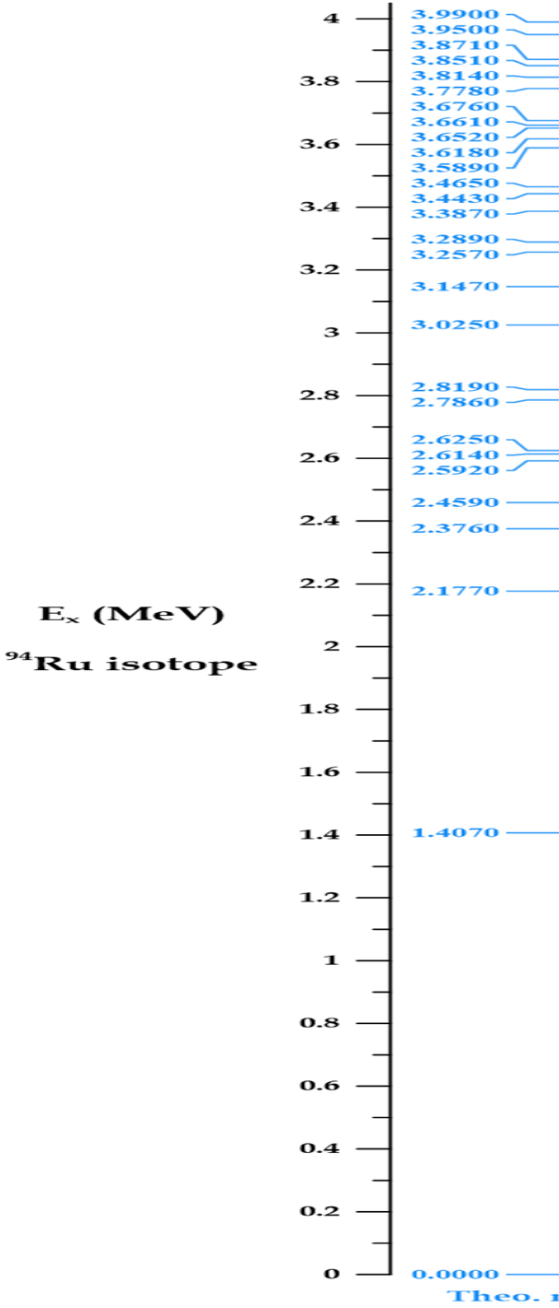
J^π	Ex thero.(MeV)	J^π	Ex exp.(MeV)
0^+ 1	0 theoretical energy eigenvalue (-38.88MeV)	0^+	0 experimental energy eigenvalue (-38.88MeV)
2^+ 1	1.407	2^+	1.430 ± 20
4^+ 1	2.177	4^+	2.186 ± 3
4^+ 2	2.376	-- ---	-----
6^+ 1	2.459	6^+	2.498 ± 3
--- --	-----	(3 ,4,5)	2.503 ± 3
8^+ 1	2.592	8^+	2.644 ± 4
0^+ 2	2.614	-- ---	-----
5^- 1	2.625	5^-	2.624 ± 3

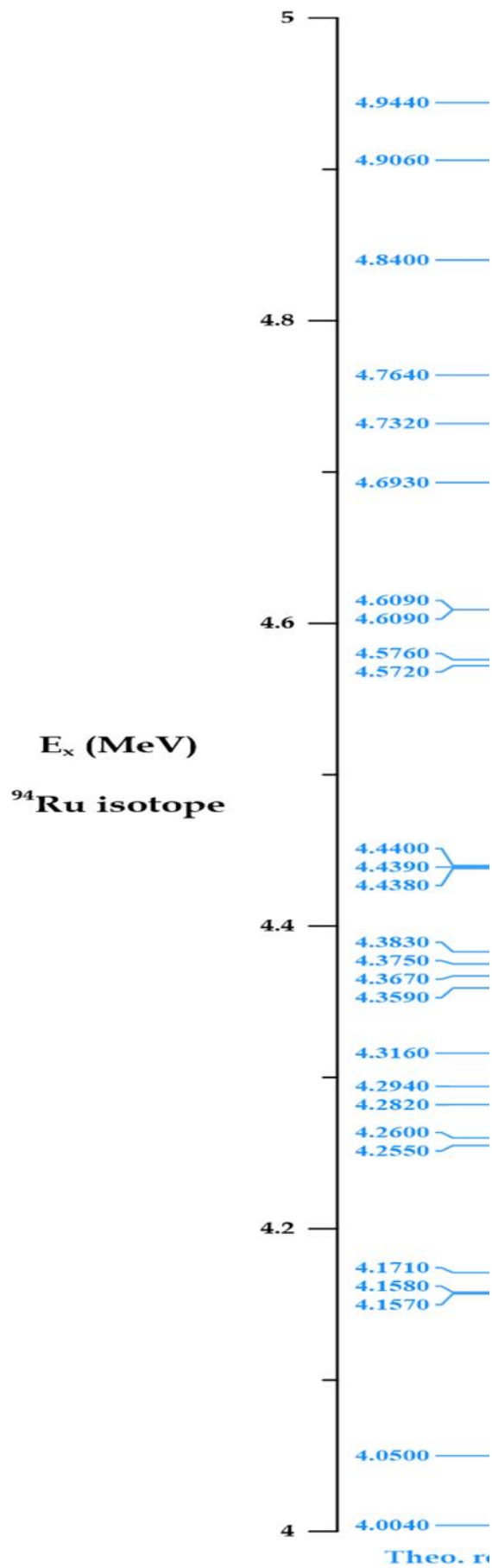
---	----	(3 -)	2.965±6
---	----	0 ⁺	2.995±6
6 ⁺ 2	2.786	-- ---	----
2 ⁺ 2	2.819	-- ---	----
4 ⁻ 1	3.025	-- ---	----
4 ⁻ 2	3.147	(3 ,4,5)	3.117±4
5 ⁺ 1	3.257	(3 ,4,5)	3.254±4
7 ⁺ 1	3.289	-- ---	----
2 ⁺ 3	3.387	-- ---	----
3 ⁻ 1	3.443	-- ---	----
3 ⁻ 2	3.465	-- ---	----
4 ⁺ 3	3.589	-- ---	3.520±7
---	----	0 ⁺	3.615±7
3 ⁺ 1	3.618	-- ---	----
7 ⁻ 1	3.652	(7 -)	3.657±4
0 ⁺ 3	3.661	0 ⁺	3.770±8
3 ⁻ 3	3.676	-- ---	----
2 ⁻ 1	3.778	-- ---	----
6 ⁻ 1	3.814	-- ---	3.820±8
4 ⁺ 4	3.851	-- ---	----
6 ⁺ 3	3.871	-- ---	----
6 ⁻ 2	3.950	-- ---	----

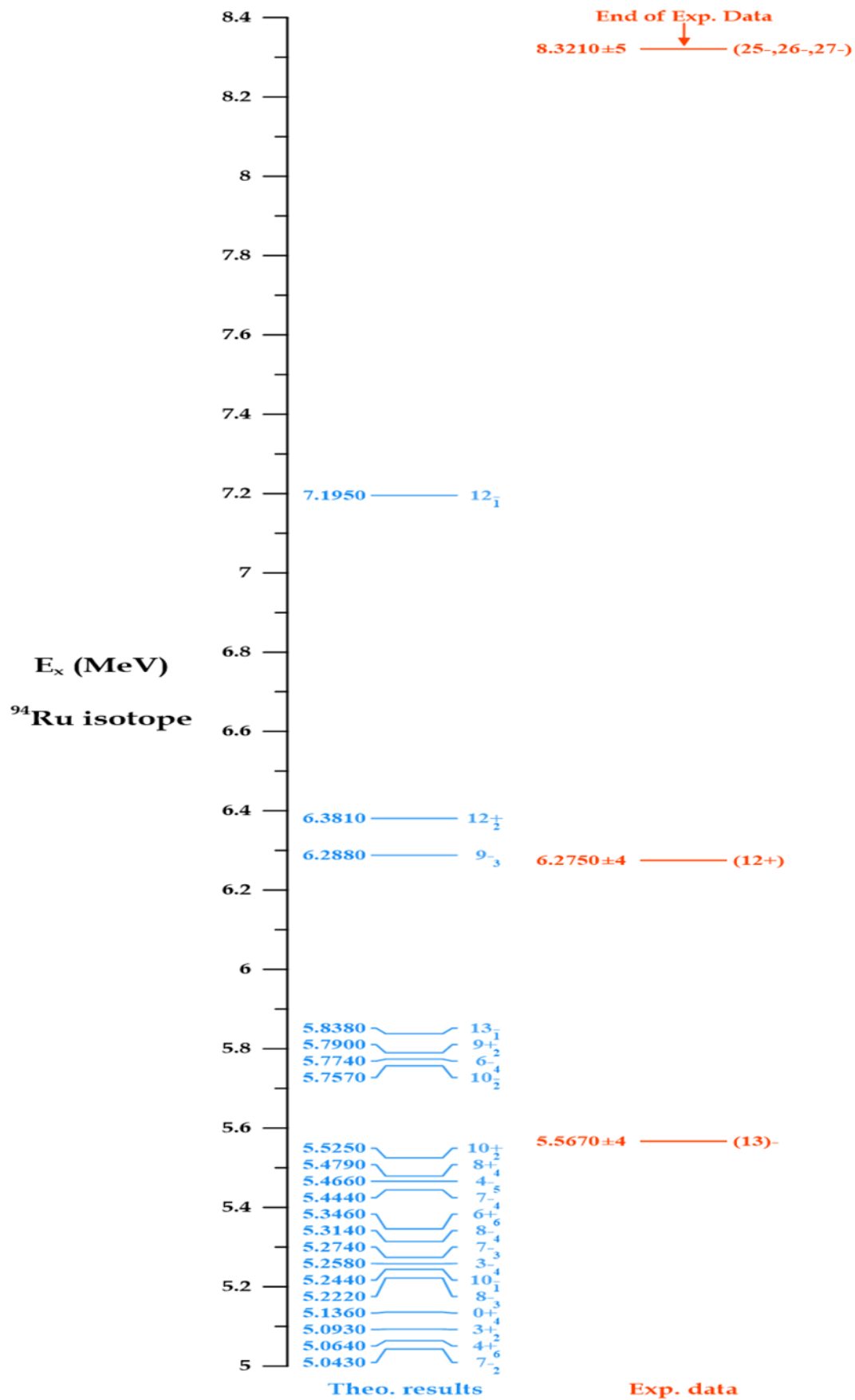
2^- 2	3.990	-- ---	-----
8^+ 2	4.004	(8 +)	3.930±4
10^+ 1	4.050	(1 0) ⁺	3.991±4
4^+ 5	4.157	-- ---	4.000±8
2^- 3	4.158	-- ---	-----
5^- 2	4.171	-- ---	-----
1^- 1	4.255	-- ---	-----
6^+ 4	4.260	-- ---	-----
9^- 1	4.282	(9) ⁻	4.197±4
2^+ 4	4.294	-- ---	-----
9^+ 1	4.316	-- ---	-----
0^- 1	4.359	-- ---	-----
1^- 2	4.367	-- ---	-----
9^- 2	4.375	(9) ⁻	4.338±4
6^- 3	4.383	-- ---	-----
8^- 1	4.438	-- ---	-----
6^+ 5	4.439	-- ---	-----
5^- 3	4.440	-- ---	-----
8^+ 3	4.572	-- ---	-----
5^- 4	4.576	-- ---	-----
11^- 1	4.609	(1 1) ⁻	4.489±4
8^- 2	4.609	-- ---	-----
4^- 3	4.693	-- ---	-----
5^+ 2	4.732	-- ---	-----

7^+ 2	4.764	-- ---	-----
4^- 4	4.840	-- ---	-----
12^+ 1	4.906	(1 2) ⁺	4.716±4
5^- 5	4.944	-- ---	-----
7^- 2	5.043	-- ---	-----
4^+ 6	5.064	-- ---	-----
3^+ 2	5.093	-- ---	-----
0^+ 4	5.136	-- ---	-----
8^- 3	5.222	-- ---	-----
10^- 1	5.244	-- ---	-----
3^- 4	5.258	-- ---	-----
7^- 3	5.274	-- ---	-----
8^- 4	5.314	-- ---	-----
6^+ 6	5.346	-- ---	-----
7^- 4	5.444	-- ---	-----
4^- 5	5.466	-- ---	-----
8^+ 4	5.479	-- ---	-----
10^+ 2	5.525	-- ---	-----
10^- 2	5.757	-- ---	-----
6^- 4	5.774	-- ---	-----
9^+ 2	5.790	-- ---	-----
13^- 1	5.838	(1 3) ⁻	5.567±4
9^- 3	6.288	-- ---	-----
12^+ 2	6.381	(1 2) ⁺	6.275±4

$^{12}_{-1}$	7.195	-- ---	-----
--- --	-----	(2 5-,26- ,27)	8.321±5







Figuer.(1;a ,b and c):Comparison between calculated results and experimental

data[22] of the energy levels in ^{94}Ru isotope using glb interaction.

Table (2): Theoretical comparison of the electromagnetic transition probabilities of first band in the ^{94}Ru isotope with experimental data taken [22]

$J_i \rightarrow J_f$	Theoretical Results		Experimental Results		
	$(BE2\downarrow)(e^2 \text{ fm}^4)$	$(BM1\downarrow)(\mu_N^2)$	multi-polarity	$(BE2\downarrow)(e^2 \text{ fm}^4)$	$(BM1\downarrow)(\mu_N^2)$
$2^+_1 \rightarrow 0^+_1$	123.1	----	----	----	----
$4^+_1 \rightarrow 2^+_1$	4.29	----	----	----	----
$6^+_1 \rightarrow 4^+_1$	2.971	----	E2	2.97 ± 1.71	----
$8^+_1 \rightarrow 6^+_1$	1.192	----	E2	0.119 ± 0.076	----
$5^+_1 \rightarrow 4^+_1$	110.9	----	----	----	----
$5^+_1 \rightarrow 6^+_1$	23.3	----	----	----	----
$7^+_1 \rightarrow 6^+_1$	62.18	----	----	----	----
$7^+_1 \rightarrow 8^+_1$	98.36	----	----	----	----
$7^+_1 \rightarrow 5^+_1$	24.02	----	----	----	----
$3^+_1 \rightarrow 2^+_1$	9.465×10^{-7}	----	----	----	----
$3^+_1 \rightarrow 4^+_1$	48.73	----	----	----	----
$3^+_1 \rightarrow 5^+_1$	17.22	----	----	----	----
$10^+_1 \rightarrow 7^+_1$	93.49	----	----	----	----
$9^+_1 \rightarrow 8^+_1$	22.36	----	----	----	----
$9^+_1 \rightarrow 7^+_1$	26.01	----	----	----	----
$9^+_1 \rightarrow 10^+_1$	31.13	----	----	----	----
$12^+_1 \rightarrow 10^+_1$	52.42	----	E2	$126.9^{+177.7}_{-126.9}$	----
$4^-_1 \rightarrow 5^-_1$	203	9.217×10^{-11}	----	----	----
$3^-_1 \rightarrow 5^-_1$	12.42	----	----	----	----
$3^-_1 \rightarrow 4^-_1$	10.78×10^{-6}	6.004×10^{-2}	----	----	----
$7^-_1 \rightarrow 5^-_1$	125.5	----	----	----	----
$2^-_1 \rightarrow 4^-_1$	29.870	----	----	----	----
$2^-_1 \rightarrow 3^-_1$	26.88	2.132×10^{-8}	----	----	----
$6^-_1 \rightarrow 5^-_1$	68.91	----	----	----	----
$6^-_1 \rightarrow 4^-_1$	7.165×10^{-8}	----	----	----	----
$6^-_1 \rightarrow 7^-_1$	7.165×10^{-8}	----	----	----	----
$1^-_1 \rightarrow 3^-_1$	3.45×10^{-8}	----	----	----	----
$1^-_1 \rightarrow 2^-_1$	3.45×10^{-8}	----	----	----	----
$9^-_1 \rightarrow 7^-_1$	152.9	----	----	----	----
$0^-_1 \rightarrow 1^-_1$	----	8.007×10^{-1}	----	----	----
$8^-_1 \rightarrow 7^-_1$	6.088×10^{-8}	----	----	----	----
$8^-_1 \rightarrow 6^-_1$	6.088×10^{-8}	----	----	----	----
$8^-_1 \rightarrow 9^-_1$	62.32	----	----	----	----
$11^-_1 \rightarrow 9^-_1$	126	----	E2	111.19 ± 5.33	----
$10^-_1 \rightarrow 9^-_1$	4.928×10^{-9}	----	----	----	----
$10^-_1 \rightarrow 8^-_1$	33.29	----	----	----	----
$10^-_1 \rightarrow 11^-_1$	45.14	----	----	----	----
$13^-_1 \rightarrow 11^-_1$	78.4	----	E2	192.9 ± 22.8	----
$12^-_1 \rightarrow 11^-_1$	5.7×10^{-1}	----	----	----	----
$12^-_1 \rightarrow 13^-_1$	----	5.544×10^{-1}	----	----	----

Table (3) :Comparison of experimental [23] and theoretical values for the quadrupole with magnetic dipole moments in ^{94}Ru isotope using glb interactions

Theoretical Results			Experimental Results	
J_1^π	(Q)(efm ²)	μ (μ_N)	(Q)(efm ²)	μ (μ_N)
2^+	3.38	2.708	-----	-----
4^+	15.31	5.415	-----	-----
6^+	-1.35	8.119	-----	8.12 $\pm$ 5
8^+	-6.74	10.829	-----	11.10 $\pm$ 4
5^+	5.49	6.771	-----	-----
7^+	-15.69	9.476	-----	-----
3^+	-4.25	4.061	-----	-----
10^+	-6.5	13.537	-----	-----
9^+	-1.36	12.183	-----	-----
12^+	-13.59	16.242	-----	12.4 $\pm$ 17
5^-	-----	5.712	-----	-----
4^-	-----	4.357	-----	-----
3^-	-----	4.854	-----	-----
7^-	-----	8.418	-----	-----
2^-	-----	3.412	-----	-----
6^-	-----	7.063	-----	-----
1^-	-----	0.296	-----	-----
9^-	-----	11.127	-----	-----
8^-	-----	9.772	-----	-----
11^-	-----	13.835	-----	14.1 $\pm$ 17
10^-	-----	12.481	-----	-----
13^-	-----	16.541	-----	-----
12^-	-----	17.221	-----	-----

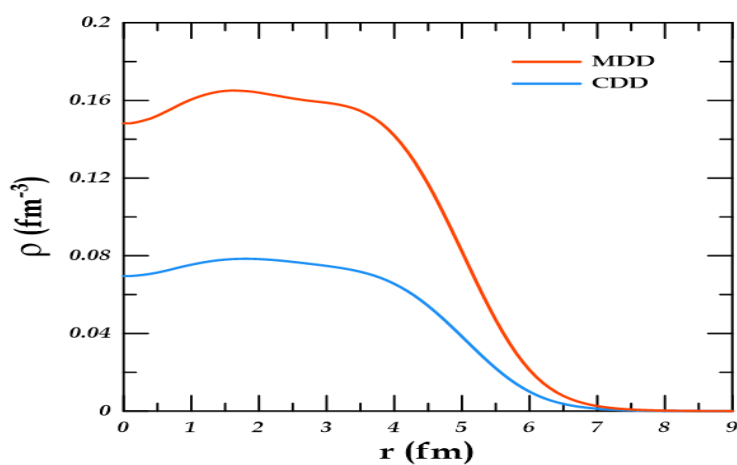


Figure.2: Theoretical values of the nuclear density distributions for charge and mass as a function of the radial distance from the center of the nucleus (r) in

⁹⁴Ru isotope. The red and blue curves represent the density distributions of the mass and nuclear charge, respectively, as a function of the radial distance from the center of the nucleus

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