Design of Adaptive Sliding Mode Controller for Uncertain Pendulum System

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A B S T R A C T

This work aims to study and apply the adaptive sliding mode controller (ASMC) for the pendulum system with the existence of the parameters uncertainty, external disturbances, and coulomb friction. The adaptive sliding mode controller has several features over the conventional sliding mode control method. Firstly, the magnitude of the control signal is reduced to the minimally acceptable level defined by special conditions concerned with ASMC algorithm. Secondly, the upper bounds of uncertainties are not necessary to be defined before starting the work. For this reason, the ASMC can be used successfully to control the pendulum system with minimum control effort. These properties of the ASMC are confirming graphically by the simulation results using MATLAB 2019. The ASMC achieves an asymptotically stable system better than the Classical Sliding Mode Controller (CSMC). The unwanted phenomenon is called "chattering", which is appearing in the control action signal. These drawback properties are suppressed by employing a saturation function. Finally, the comparison between the results of the ASMC and CSMC showed that ASMC is the better one.

1. INTRODUCTION

The majority of nonlinear systems are suffering from uncertainty in their dynamic parameters, therefore, they require high-performance and powerful controller design. Today, many strong and modern algorithms are using to design a robust and nonlinear controller that gives the desired performance. The Sliding mode controller (SMC) is one of them. It is a nonlinear and powerful

controller that can be employed with nonlinear systems especially, which is suffering from parameters uncertainty [1]. The sliding mode controller was proposed in 1950. The design procedures of SMC consist of two main steps; reaching phase and sliding phase [2]. The reaching phase is defining as the system trajectory moves from an initial position to the sliding surface, where the sliding phase is defining as the system trajectory moves along the sliding surface until reaches the origin [3]. The SMC is insensitive against parameter uncertainty and external disturbances which have undesired affecting on the system performance [4]. The SMC has a drawback when it has been used. This drawback is called the "chattering phenomenon". The chattering phenomenon is considering an undesirable characteristic that appears in the control action as shown in Figure 1. The chattering phenomenon is appearing because of using the sign function "sign ( )" in the control law. The chattering phenomenon is affecting the system stability; it makes the trajectory motion as zigzag motion, in which the system cannot be stable in presence of this phenomenon [5]. To reduce the chattering, the boundary layer methodology can be utilized instead of the sign function in the control law. Also, many other evaluate technical approaches are using to tune the controllers gain in many other types of research. In the last years, many controllers have been suggested to reduce the chattering, such as a fuzzy sliding mode controller, sliding mode fuzzy controller, adaptive sliding mode controller, and integral sliding mode controller [6]. In this paper, ASMC is proposed to knock down the control chattering. The arrangement of this paper is as follows. In section 2, the SMC is illustrating briefly while the ASMC algorithm is presenting in section 3. In section 4, the pendulum system is explaining with the friction. The simulation result of the pendulum system with friction is showing in section 5, the discussion is presenting in section 6, and finally, the conclusion is explaining in section 7.
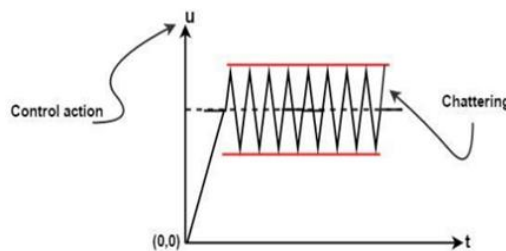

Figure 1: The chattering phenomenon in SMC [2]

2. SMC DESIGN

In modernistic control systems, the SMC is considering a powerful and interesting method. Since 1950 [12, 13], SMC is used successfully with a nonlinear system. It is applying with large different applications such as an electrical servo drive system [2], pendulum system [2, 9], and two link robots [9]. The SMC is a nonlinear controller that can control linear and nonlinear systems. The control action can be classified into two control parts; nominal and discontinuous. The nominal control signal of the SMC is utilized to oblige the trajectory of the state to move from an initial state to the direction of the sliding surface. Where, the discontinuous control action obliges the systems states trajectory to slide along the switching surface till it is reaching the origin [1, 2, 11].

The sliding surface can be written as [1, 2]:

$$s = \lambda e + \dot{e} = 0 \qquad (1)$$

Where λ is constant and it is >0.

Let us assumed that $x_1 = e$ and $x_2 = \dot{e}$, thus the sliding variable surface is going to be re-written as:

$$s = \lambda x_1 + x_2 \qquad (2)$$

when $\lambda = 1$, the sliding variable surface expressed as :

$$s = x_1 + x_2 \tag{3}$$

The complete control law can be specified as below:

$$u = u_n + u_{dis} \tag{4}$$

Where, u_n is the nominal control part, and u_{dis} the discontinuous control part [1, 2].
The discontinuous control part is defining as below:

$$u_{dis} = -k(x)sign(s) \tag{5}$$

Where $k(x)$ is a discontinuous gain. (s) is known as a signum function which is described as in Eq. (6) and Figure 2.

$$sign(s) = \begin{cases} 1 & if\ s > 0 \\ -1 & if\ s < 0 \\ \epsilon[-1.1] & if\ s = 0 \end{cases} \tag{6}$$
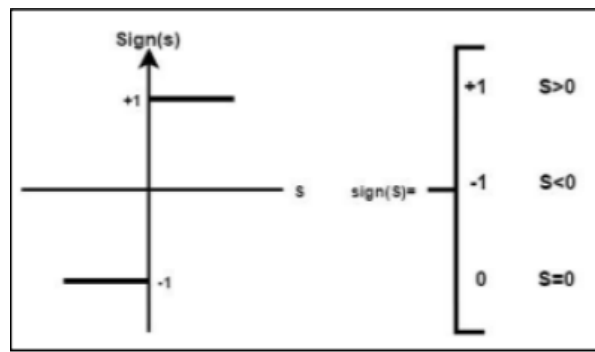
Figure 2: **The signum function.**

Therefore, the equation for the control action can be expressed as below [5]:

$$u = u_n - k(x)sign(s) \tag{7}$$
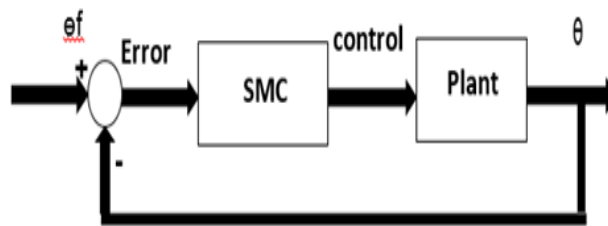
Figure 3. is illustrating SMC within the system.

Figure 3: The Sliding Mode Control system [9]

As mentioned above, the boundary layer is known as the saturation function ($sat\ (s)$) that is shown in Figure 4. It is employed instead of a sign(s) function in Eq. (7) to reduce the chattering. Therefore, Eq. (7) is rewriting as below:

$$u = u_n - k(x)sat(s) \tag{8}$$

Where saturation function can be written as below:

$$sat(s.\varphi) = \begin{cases} sign(s) & if\ |s|\ >\ \varphi \\ \frac{s}{\varphi} & if\ |s|\ \leq\ \varphi \end{cases} \tag{9}$$

where $\varphi=0.01$, which is represented the width of the boundary layer (saturation function).
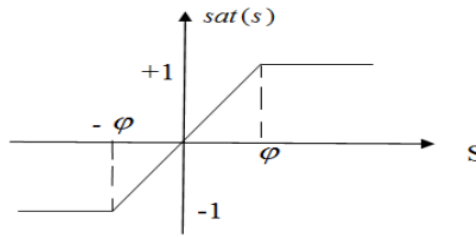

Figure 4: The sat(s) function [2,3]

3. THE ASMC DESIGN

In this section, the ASMC is presenting in detail. The controller gain of the ASMC is continuously reducing until reached the acceptable and minimum value. This acceptable value can able to maintain the system stability and robustness as is in classical SMC. The target is adaptively tuning the controller gain without knowing the maximum bound of the system uncertainty [9,10].

The ASMC has the following structure.

$$u(s.t) = -k(t)\ sign\big((x.t)\big) \tag{10}$$

As mentioned above, for minimizing the chattering, the signum function in Eq. (10) is replaced by the saturation function.

Where $k(t)$ is the gain that would be varying with time, it can be written such :

$$\dot{k(t)} = \begin{cases} p*|s(x.t)|*sign(|s(x.t)|-\epsilon) & if\ k>\mu \\ \mu & if\ k\leq\mu \end{cases} \tag{11}$$

Where p>0 it used to increase or decrease the value of k(t), ϵ >0, and μ>0. Where, μ represent the initial value of k. The value of k (t) must satisfy the condition below:

μ (0) =k (0)

$K_{max} > k(0) > k_{min}$

The simulation for the pendulum system is performed to clarify the effectiveness of the proposed control scheme [14]. The ASMC is more flexible and comfortable in the design than the classical SMC. As well as, the system stability is achieving with a small control effort when using the ASMC [7, 9].

4. PLANT DESCRIPTION

The pendulum system is usually described as a nonlinear system. many studying research uses a pendulum in widely studying for checking the control performance in different control algorithms [4]. In this work, a perturbation term is added to the pendulum system. The perturbation term is containing the disturbance and parameter uncertainty, coulomb friction. The Coulomb friction is assumed as a force that affects the opposite direction of the movement of the pendulum.
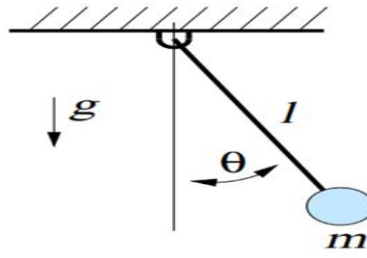
Figure 5: The Simple Pendulum [4]

The equation of pendulum system that can describe the system is written as below:

$$\ddot{\Theta} = -a\,sin(\theta) - b\dot{\theta} + cT + \delta(x.u) \tag{12}$$

Where θ: is the angular position of the link with the vertical axis, and it's measured by (radian) unit. It is defined as the output of the system.

$\dot{\theta}$:is the angular velocity (radian/second).

T: defines as the torque (control action), which is applying at the mass of the pendulum system in order to make it swing. It is measured in (Newton. Meter) unit.

$\delta(x.u)$: is the perturbation term, which includes the coefficients uncertainty, external disorders, and the Coulomb friction.

The existence of an external disturbance and parameter uncertainty as mentioned in the perturbation term is considered as a general problem in the plant.

The nominal value of coefficients a_n=10, b_n=1 and c_n=10, The uncertainties values of the coefficients are $\delta a = \mp 10\%*a_n$, $\delta b = \mp 10\% *b_n$, $\delta c = \mp 10\% *c_n$.

The purpose is to move the Pendulum from an initial position to the desired position (θ_f).

The error equation is the difference between the desired position and angular position, that is written in the below [4]:

Assume the error equation as below.

$$e = \theta_f - \theta$$

$$\dot{e} = \dot{\theta} \tag{13}$$

By using the state-space representation to define the error and its derivatives, then

$$x_1 = e \text{ and } x_2 = \dot{e} \tag{14}$$

Then

$$\dot{x}_1 = \dot{e} = x_2 \tag{15}$$

$$\dot{x}_2 = \ddot{e} = -a_n\,sin\left(x_1 + \theta_f\right) - b_n x_2 + C_n u + \delta(x.u)$$

Where

$$\delta(x.u) = -\delta a\,sin\left(x_1 + \theta_f\right) - \delta b x_2 + \delta c u + (c + \delta c)d - m\,sign(x_2).$$

$$a = \delta a \mp a_n$$

$$b = \delta b \mp b_n.$$

$$c = \delta c \mp c_n. \tag{16}$$

Equation (16) represents the perturbation term which is explained on the last page, that is mean sudden variation happens for the system as disturbance or friction or uncertainty parameter.

The values of a, b and c either increase or decrease. For the maximum value, the parameters increase, while it decreasing for minimal value. m is the Coulomb friction about 1.2 (N.m). d is disturbance is equaled 1 (N.m). θ_f is $\pi/4$ as the desired position.

TABLE I: The parameter values vary with uncertainty.

Parameter value	Nominal value	Minimal value	Maximal Value
a	10	9	11
b	1	0.9	1.1
c	10	9	11

Because the pendulum system that is adopting in this paper is containing a perturbation term, therefore, the nominal control part will be rejecting and the overall control action is consisting only of the discontinuous control part.

In this paper, there are two cases for calculating the controller gain k(x) depending on the type of sliding mode controller as is discussing below:

I. . The design of the Classical Sliding Mode Controller (CSMC).

The design of discontinues control law for CSMC for uncertain values of the pendulum system is written as in below:

$$u = u_{dis} = -k(x)sign(s) \tag{17}$$

This equation modifies by using a saturation function to eliminate the chattering in the control action.

$$u = u_{dis} = -k(x)sat(s) \tag{18}$$

Where the sliding variable surface can be described as below:

$$s = x_1 + x_2 \tag{19}$$

The appropriate discontinuous gain k(x) is computing from using the following procedure:

$$\dot{s} < 0 \tag{20}$$

By substituting Eq. (19) in Eq. (20).

$$\dot{x}_1 + \dot{x}_2 < 0$$

$$x_2 - a\sin(x_1 + \theta_f) - bx_2 + cu + \delta(x.u) < 0$$

And by using $u = -k(x) * sign(s)$ in the above equation, and then finding $k(x)$

$$k(x) > \frac{(\delta a + \delta b|x_2| + (c_n + \delta c)d + m)}{C_{min}}$$

$$k(x) = k_0 * \frac{\delta a + \delta b|x_2| + (c_n + \delta c)d + m}{C_{min}} \tag{21}$$

Where, k_0 is a constant and its value is greater than one ($k_0 = 10$). $C_{min} = 9$.

II. . *The design of the Adaptive Sliding Mode Controller (ASMC).*

The design of the Adaptive Sliding Mode Controller (ASMC).

As it is illustrated previously in this paper the control law of the ASMC is as presented in Eq. (10) and Eq. (11).

To reduce the chattering the sign ( ) function in Eq. (10) will be replaced by the sat ( ) function as it is presenting in Eq. (18):

Where, k(t) in ASMC, is calculating from Eq. (11).

5. SIMULATION RESULTS

The initial value of $x_1(0) = x_2(0) = \pi /4$.

I. *The outcomes of designing CSMC with sign function.*
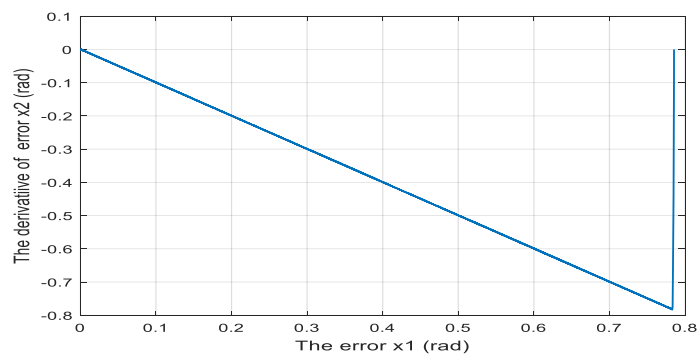
Figure 6: The relationship between x_1 & x_2
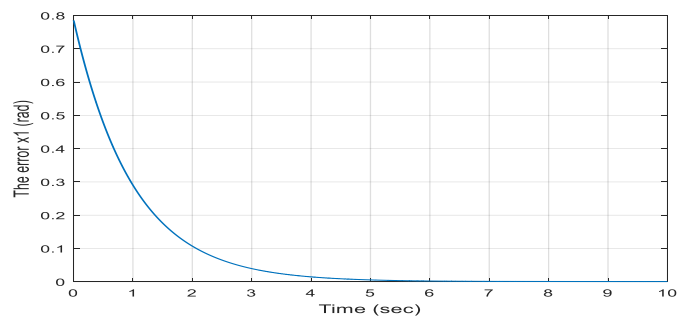
Figure 7: The relationship of error (x_1) with time.
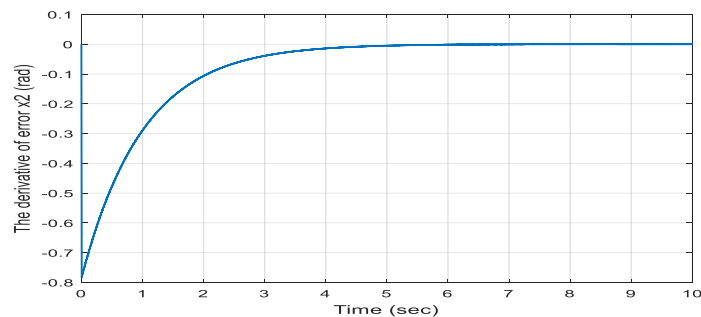
Figure 8: The relationship of derivative of error (x_2) with time.
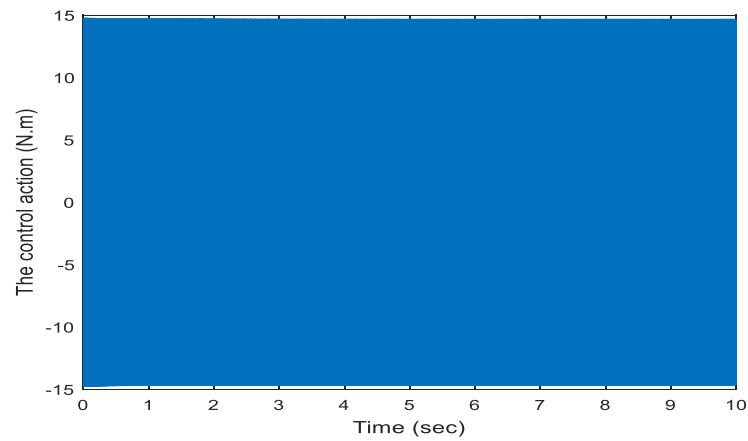
Figure 9: The relationship of control action U with time.
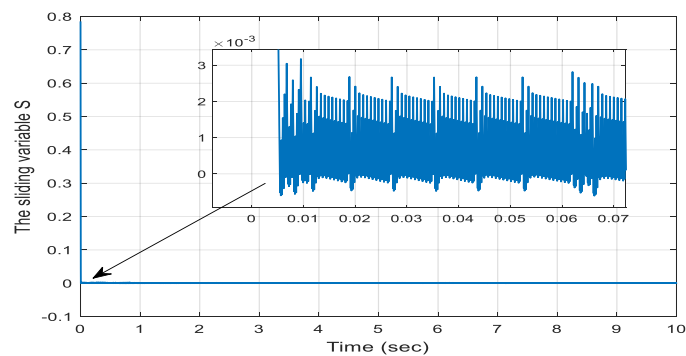
Figure 10: The relationship of sliding variable surface S with time.
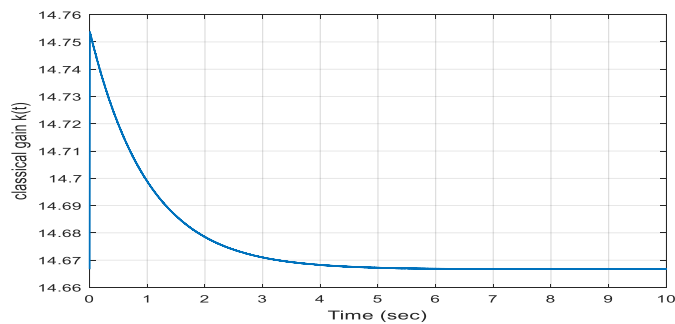
Figure 11: The relationship of classical gain K with time.

II. The outcomes of designing (ASMC) with sign function.

In this work, $\epsilon = 0.3$. $\mu(0) = 2.2$. $K_{max}=3.1$ and $k_{min}=2$, p=100.

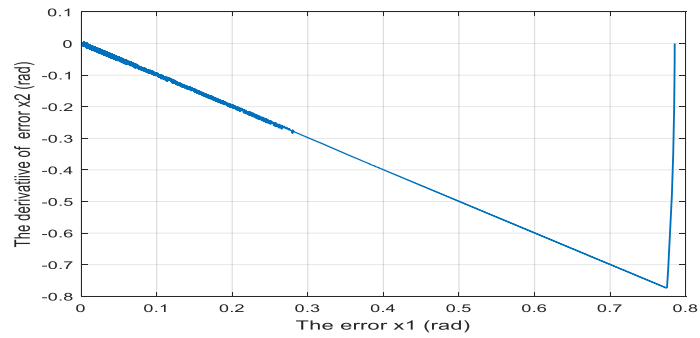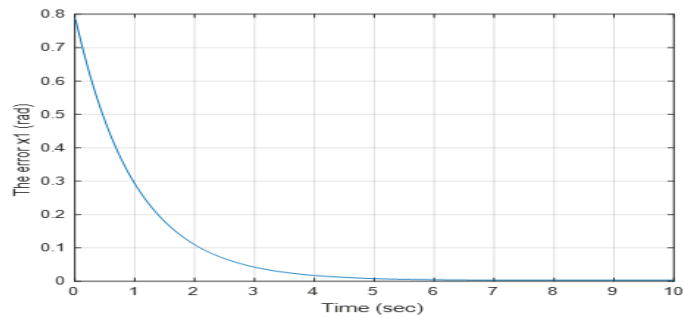**Figure 12: The relationship between x_1 & x_2 in ASMC.**

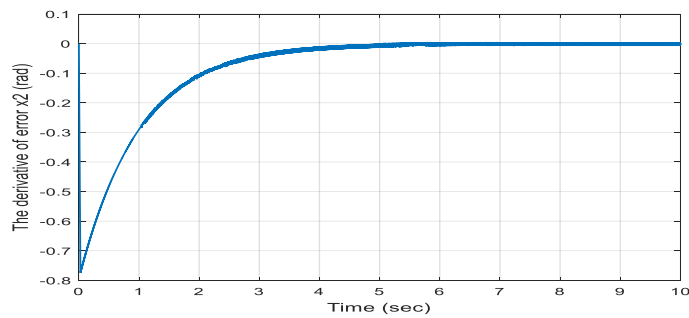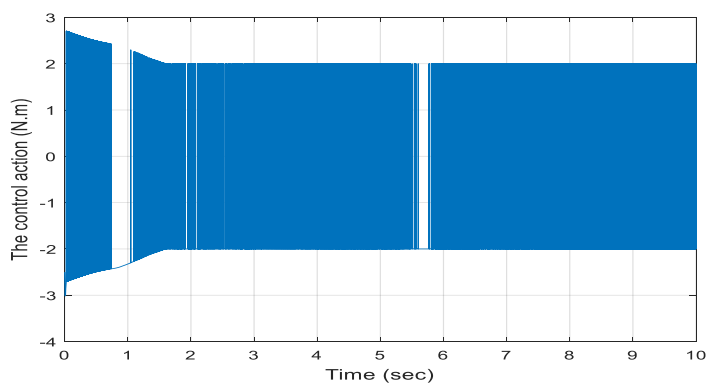**Figure 13: The relationship of error (x_1) with time in ASMC.**

Figure 14: The relationship of derivative of error (x_2) with time in ASMC.

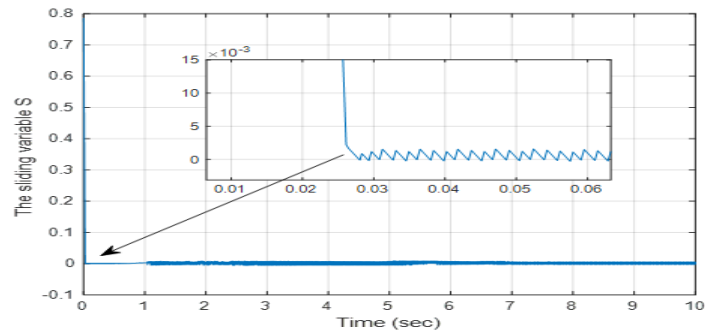**Figure 15: The relationship of control action U with time in ASMC.**

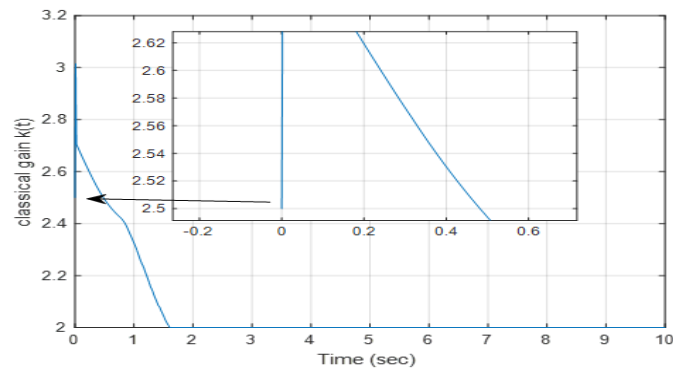**Figure 16: The relationship of sliding variable surface S with time in ASMC.**

Figure 17: The relationship of gain K vs time in ASMC.

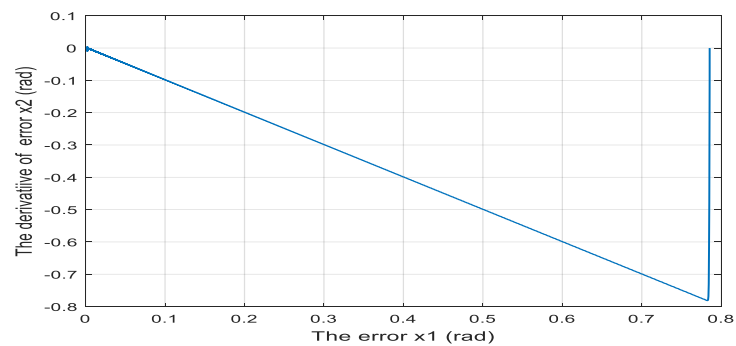*III. The outcomes of designing (CSMC) with saturation function.*

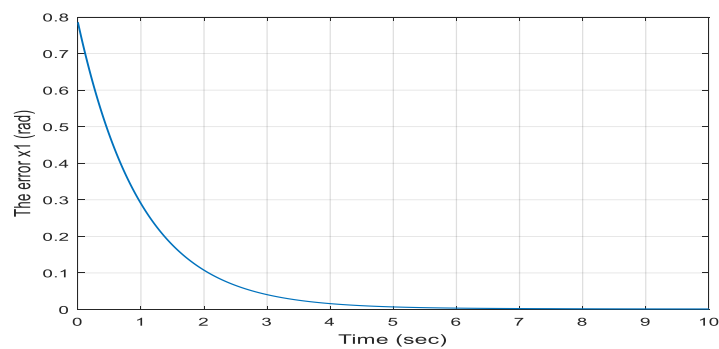

Figure 18: The relationship of phase plane between x_2 & x_1.

Figure 19: The relationship of error (x_1) with time.
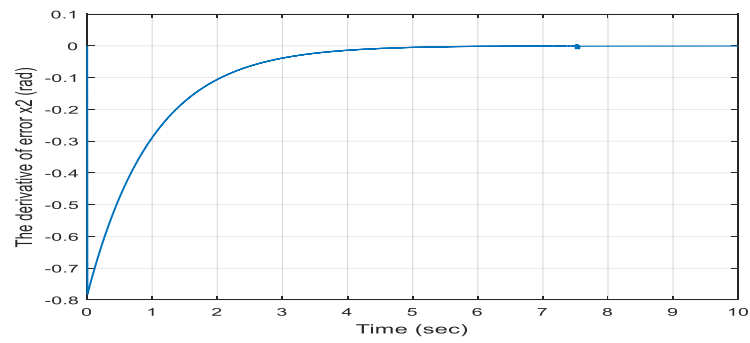
Figure 20: The relationship of derivative of error (x2) with time.
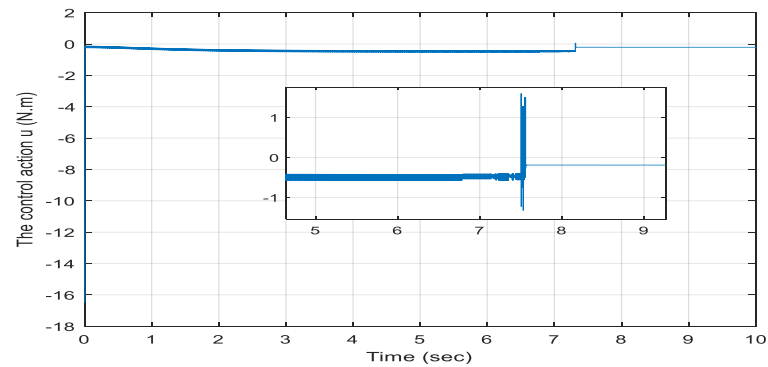

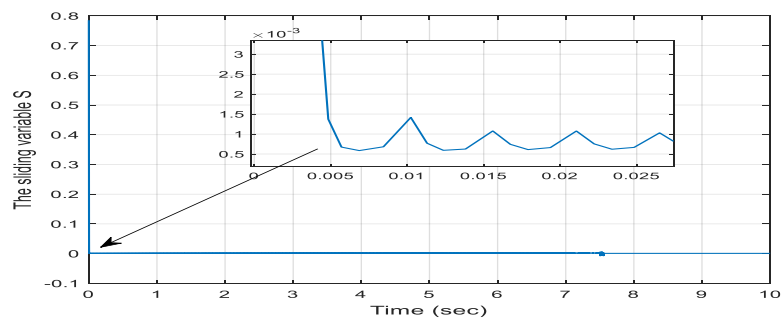**Figure 21: The relationship of control action U with time.**

Figure 22: The relationship of sliding variable surface S with time.
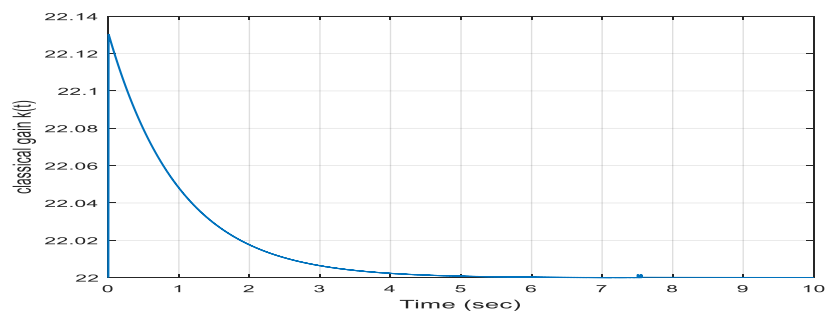

Figure 23: The relationship of classical gain K value with time.

IV. The outcomes of designing (ASMC) with saturation function

In this work, $\epsilon = 0.3$. $\mu(0) = 20.2$. $K_{max}=21.2$ and $k_{min}=20$, p=100.
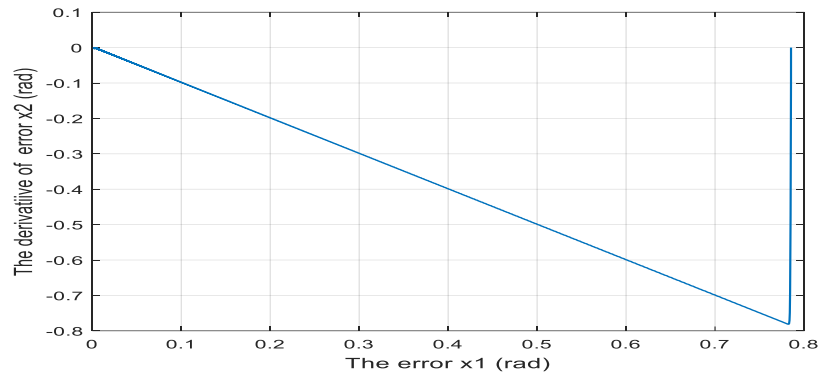

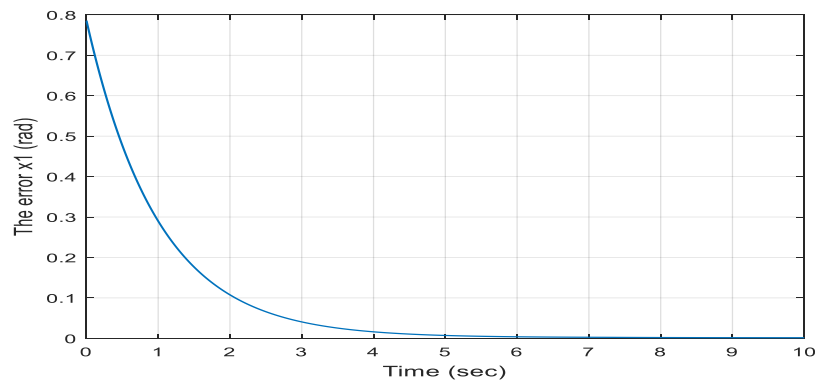**Figure 24: The relationship of x_2 & x_1 in ASMC.**

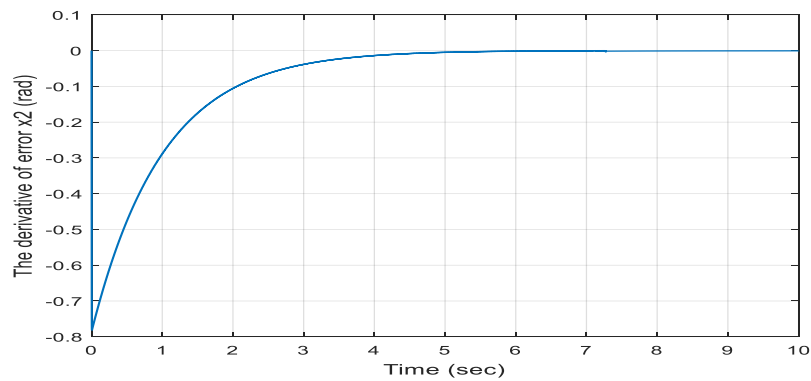**Figure 25: The relationship of error (x1) with time in ASMC.**

Figure 26: The relationship of derivative of error (x2) with time in ASMC.
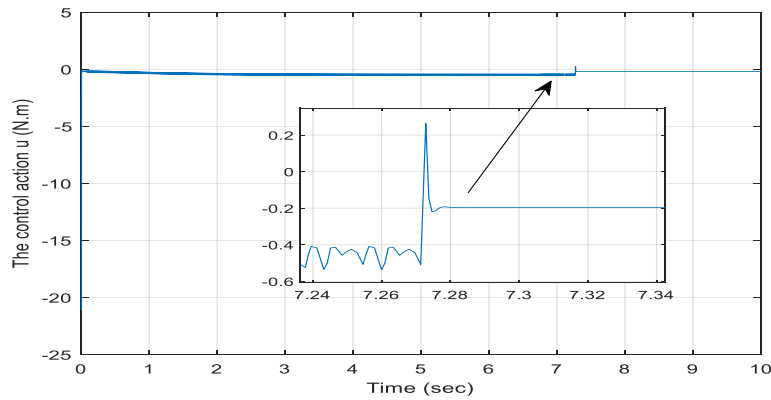
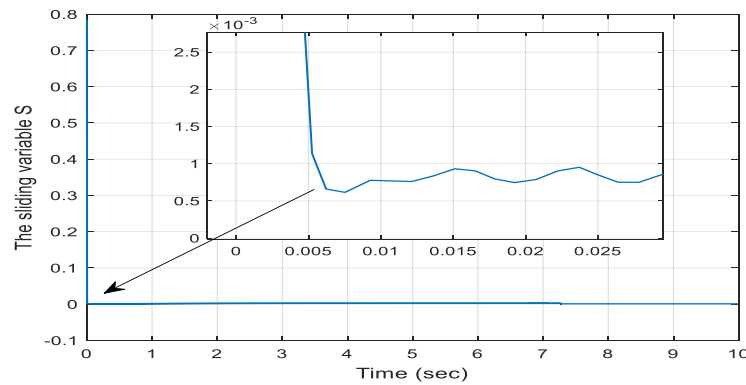**Figure 27: The relationship of control action U with time in Adaptive SMC.**

Figure 28: The relationship of sliding variable surface S with time in ASMC.
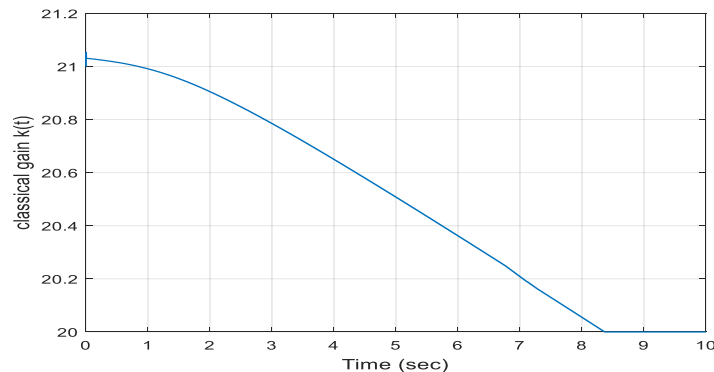
Figure 29: The relationship of adaptive gain K with time in ASMC.

6. DISCUSSION

In this paper, the position of the pendulum system is controlling by using two types of controllers with the existence of the perturbation term. Both controllers, CSMC and ASMC, have the ability for making the system asymptotically stable by making the error and derivative of error approaching zero value as it is shown in Figures 6- 12- 18 - 24 respectively.

Both controllers; the CSMC and ASMC are suffering from the chattering phenomenon that appears in the control action due to the applying of the signum function in controller law as shown in Figures 9-15 respectively. This chattering is knockdown by using the saturation function in control law instead of the signum function as shown in Figures 21- 27 respectively.

7. CONCLUSIONS

The comparison between the ASMC and CSMC, showed that the ASMC can reduce the controller gain to an acceptable and minimum value and as a result, the magnitude of the control action and chattering are reduced.

It is concluded from Table 2, that both CSMC and ASMC are a robust controller, because of their ability to give a good response even in case of the existence of disturbance and parameter uncertainty as clarified in Figures 6- 12- 18- 24.

In figures 23- 29, the values of gain k(t) are increasing to a large value in order to make the steady-state error approximately reaching zero.

It is concluded from Table 2, that the performance of the ASMC is better than the CSMC.

TABLE II: **The performance and the characteristics of the four controllers when using sign (s) and sat(s) function in control law.**

Controllers type			Maximum Control gain k(t)	maximum chattering magnitude in the control action (N.m)	The steady-state error of x_1 (rad.)
1	*CSMC*	*Sign function*	14.75	29.42	0.00234
		Sat function	22.13	2.63	0.00375
2	*ASMC*	*Sign function*	3.01	5.23	0.00477
		Sat function	21.12	0.14	0.00384

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