

# **Analytical Study about the Behavior of Prestressed Composite Steel Beams**

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Received Date: 29/8/2008 , Accepted Date:25/1/2010

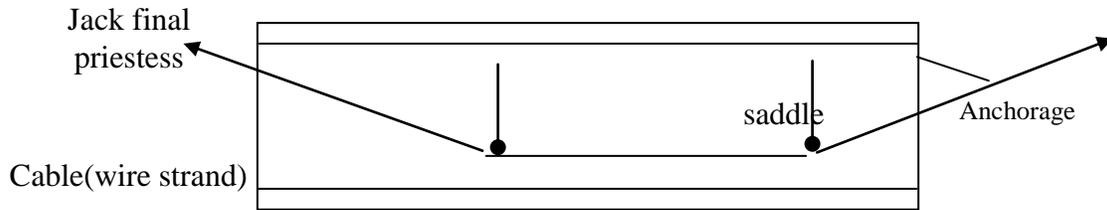
## **Abstract**

prestressing is the deliberate creation of permanent internal stresses in a structure or a system in order to improve its performance under service loads. Such stresses are designed to counter act those induced by external loading. The application of prestressed concrete is widely used and is in away a natural result. Concrete is strong in compression and weak in tension. Prestressing the concrete would produce compressive stresses which will counteract tensile stresses induced by external loading, thus producing a crack-free material during service-steel is strong in both compression and tension. The benefits of prestressing composite steel beam are to increase the elastic strength, to reduce deflection with limited member depth, to reduce steel weight, to increase ductility by redistribution of internal stresses, and to improve fatigue strength of a structural detail by reducing the tensile stress range.

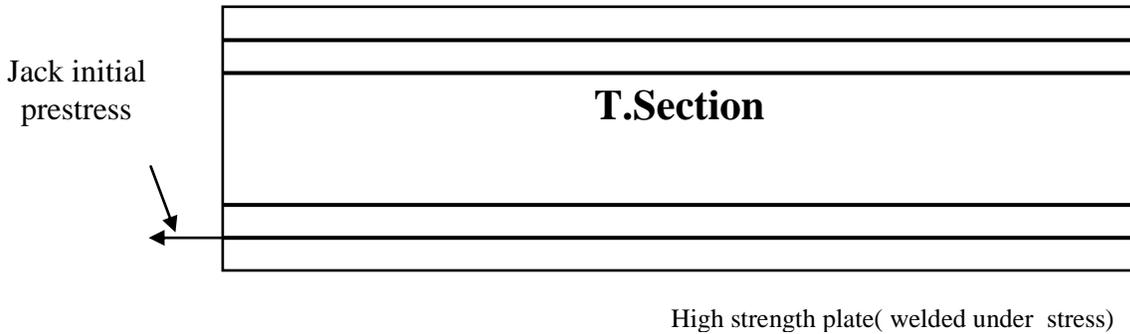
Presteressing mechanisms and bending behaviour of prestressed steel beam are very important to fully utilize the member section. The analysis equations for the elastic state and fully plastic state are developed based on equilibrium of forces and compatibility of deformation. The comparison between bounded and unbounded tendons shows that the bond of the tendons in prestressed steel beams has small effect in improving their behavior. Beams with draped tendons are compared with beams with straight tendons. Draping the tendons reduce the shear stresses in the beam. The magnitude of the reduced shear stresses depends on the depth of the beam and the tendon profile.

## **Introduction**

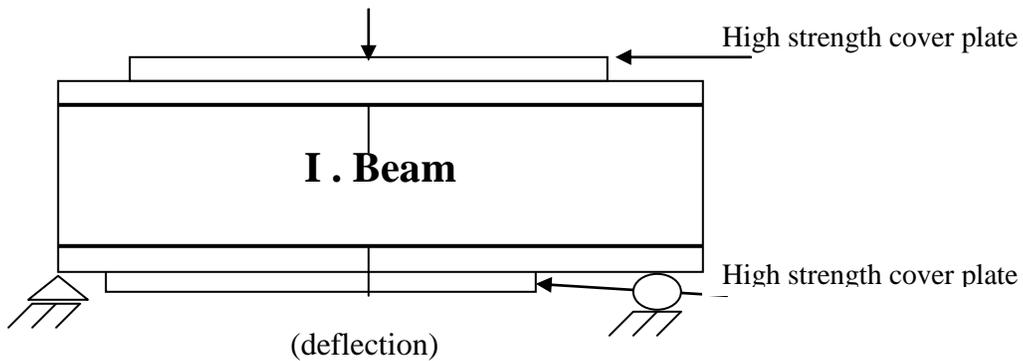
At least three practical ways to prestress steel beams are available, The first method is to use end –anchored high strength wires or bars. The second method is to stress components of hybrid beams, and a third method is to cast a concrete slabs in composite fashion to a deflected beams, fig(1),(2),(3) and (4) show and illustrates the method of prestressing.



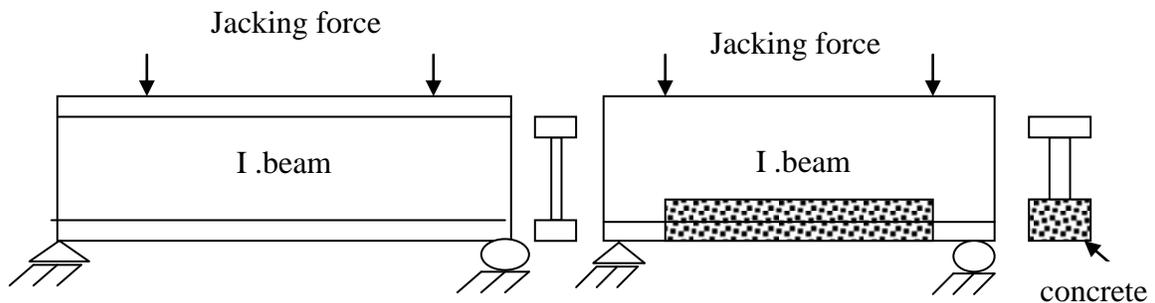
**Fig (1) : prestresses beam with draped cable(wirestrand)**



**Fig(2): prestressing by applying direct tention to high strength plats**



**Fig(3): prestressing by deflecting A beam and attaching Cover Plate**



**Step(1): Jacking force are applied to beam furnished by mill with predetermined camber**

**Step(2): concrete is placed while jacking force are maintained**

**Fig(4): Diagrams showing pre flex Technique**

The concept of using prestressing in composite steel beams is not new. An important for steel girder prestressed by means of cables and this prestressing cables out side the cross-section of the beam (Naillon, 1961). The experimental difficulties of the prestressed composite steel beam testing; include slip of prestressing cables a loss of prestress occurred during the handling and costing operation (Strass, 1964). In (Reagan, 1966) studied the behavior of prestressed composite beam under effects of the variation of prestressed force and tendon size on the load capacity of beam (load causing allowable steel stress, load causing yielding of steel beam and ultimate load).

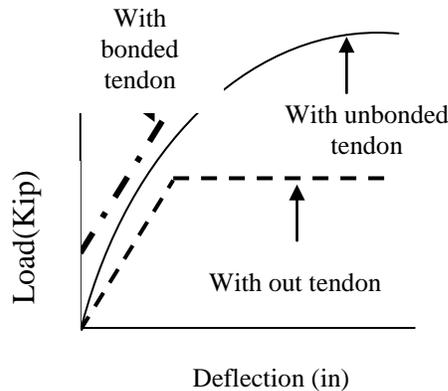
In (Saad Atmanesh&Etal, 1986) studied the behavior of prestressed composite steel-concrete beam experimentally, and they concluded that the steel weight can be reduced, fatigue strength is increased deformation is reduced, internal stresses are redistributed in favorable manner.

In (Son 1987,) in his experimental study about prestressed composite steel-concrete beams, he concluded that the ultimate strength is dependent on the quantity, geometry and strength of the material used in the prestressed composite beam. In (PCI design Handbook, 1999) show that , to prevent the horizontal and vertical cracks, the value of maximum applied factored shear force must be:

$$\leq 0.2\lambda^2 fc'A_{cr} \quad \dots(1)$$

(when  $\lambda =1$ ,  $A_{cr}$ : area of crack plane). For interface condition of concrete to steel the maximum effective shear friction coefficient=2.4.

(Nawy, 2006) said that all connections should be designed for a minimum horizontal tensile force of 0.2 times the vertical dead load. Also must be consider the factors, Load transfer mechanism, durability in addition to the economics of the details of the connection.( Mattock & Etal 1971 ) made a good comparison between the deflection with properties of section of beam with and without banded tendons. as shown in fig.(5) of load deflection relationship for beam.



**Fig (5) Load – deflection Relation Ship for composite beam**

(Abrams, 1973) suggest the following bond stresses after a comprehensive series of bond tests.

$$u = \frac{V}{J_d \cdot b_o} \quad \dots(2)$$

Where  $u$ =bond unit stress (MPa),  $V$ =total external shear (N),  $J_d$ =distance between centers of tension and compression (mm),  $b_o$ =sum of perimeters of all horizontal tension bars (mm) . And the working unit bond stresses as given in ref (1)

$$u_w = 0.04fc' \quad \text{for plain bar} \quad \dots(3)$$

$$u_w = 0.05fc' \quad \text{for deformed bar} \quad \dots(4)$$

Fig.(6) shows the comparison between prestressed composite beams with straight and draped tendons, in which the angle of inclination ( $\Phi$ ) depend on ( $\frac{span}{depth}$ ) ratio of the beam

$$\tan \phi = \frac{d/2}{L/2} = \frac{d}{L} = \frac{1}{20} \quad \dots(5)$$

and the increase in shear resistance ( $v$ ) is:  $v = T \sin \phi$  where  $T$ =prestress force of tendon.

Eccentric prestress is usually much more efficient than concentric prestress and variable eccentricity is usually preferable than constant eccentricity, from the view-points of both stress and deflection control (Nilson, 2004) The strength and other characteristics of prestressing wire strands and bars vary some what between manufactures, as do method of grouping tendons anchoring them (Nawy, 2002; Collins & Mitchell, 1991).

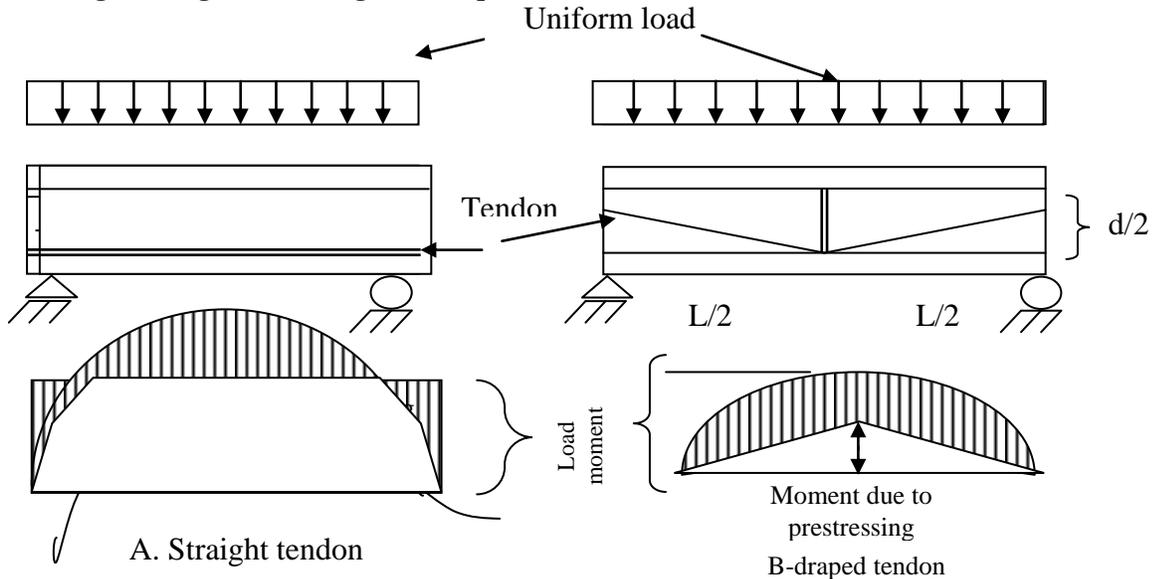
### **Objective of the study**

- 1) To make comparison between the behavior of beams with draped tendon and with straight tendon.
- 2) To compare the behavior of prestressed composite steel beam with the behavior of conventional beam.
- 3) To determine load-deflection characteristics and the ultimate capacity of the prestressed composite steel beams under static load.
- 4) To determine the behavior of prestressed composite steel beams in a positive bending moment region. (tension at bottom fibre stress)

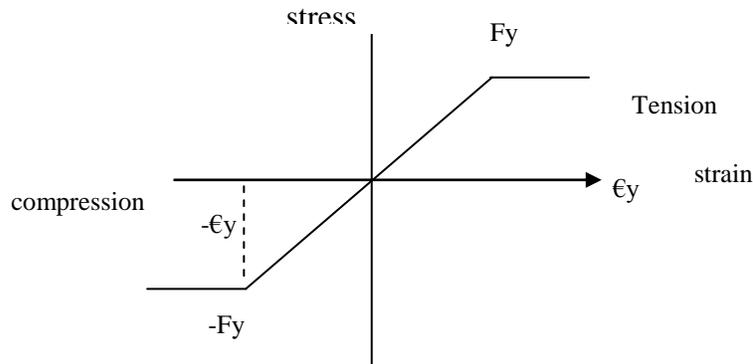
**Theoretical analysis**

The assumption used in the analysis are as follows:

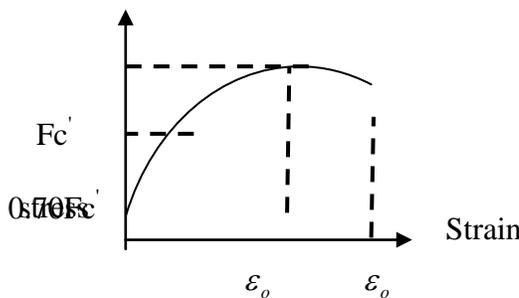
- 1) stress-strain relationship for steel beams concrete and tendons shown in fig.(7),fig(8) and fig.(9) respectively.



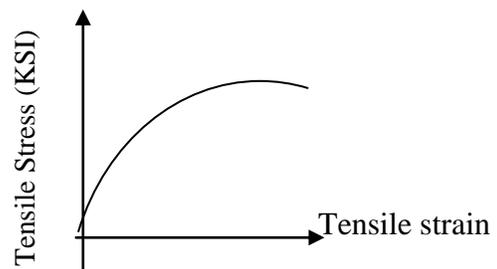
**Fig(6): Moment diagrams for two prestressing**



**Fig(7): stress strain relationship for steel beam**



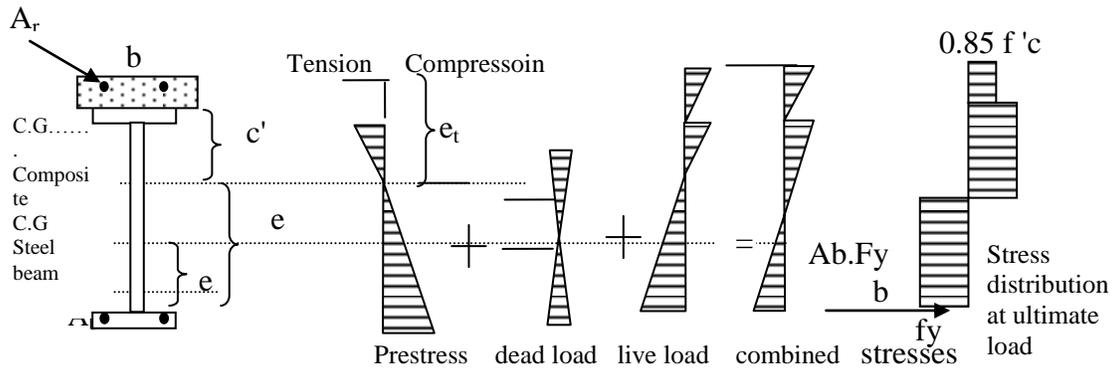
**Fig(8): Typical stress –strain relation ship for concrete slab**



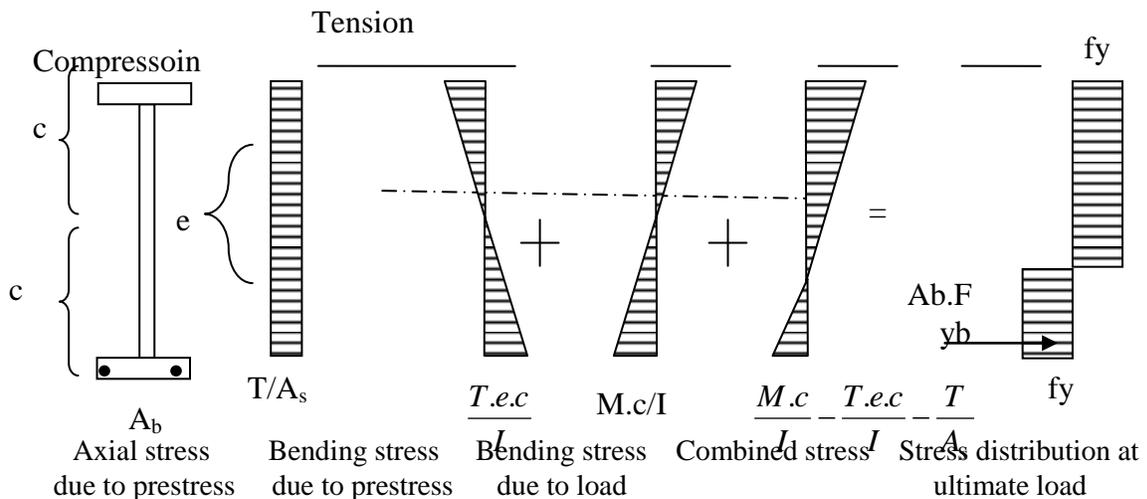
**Fig(9): stress-strain relationship for cable**

- 2) Linear strain distribution along the depth of composite beam.
- 3) Neglect the deformation by creep, shear, shrinkage and tendon relaxation.
- 4) No slip between concrete and beam flange.
- 5) Any residual stresses are neglected in the steel beam.
- 6) The prestressing tendons are restrained longitudinally at their ends and transversely to remain a constant distance from steel beam.

For simply supported beam subjected to uniform distributed load , prestressed composite beam which is internally statically indeterminate to the first degree because the tendon force can not determine by the equilibrium equation. The force in the tendon increase with the application of uniform load(Dead +Live). This increase related to the variation of moment a long the span. For the case shown in figs.(10,11) and it's bending stress distribution due to dead and live load and prestressing.



**Fig (10): Bending stress diagrams for various stages of loading for a prestressed composite steel beam**



**Fig(11): Bending stress diagrams for various stages of loading for a prestressed steel beam.**

If we take the moment at distance(x) from left support.

$$M(x) = \frac{wL}{2}x - \frac{w}{2}x^2 \quad \dots(6)$$

Where w=uniformly distributed load, L=span of beam.

The curvature is give by:

$$\frac{d^2y}{dx^2} = \frac{M(x)}{EI} = \frac{1}{EI} \left( \frac{wL}{2}x - \frac{w}{2}x^2 \right) \quad \dots(7)$$

Where EI= rigidity of the section.

By integrating equation (7), get:

$$\frac{dy}{dx} = \frac{w}{2EI} \left( \frac{Lx^2}{2} - \frac{x^3}{3} \right) + K_1 \quad \dots(8)$$

By integration equation (8) we obtain the vertical deflection:

$$y = \frac{w}{2EI} \left( \frac{Lx^3}{6} - \frac{x^4}{12} \right) + k_1x + k_2$$

By applying boundary conditions at both support, the vertical deflection as follow:

$$y = -\frac{w}{24EI} (x^4 - 2Lx^3 + L^3x) \quad \dots(9)$$

The slope angle equation is given by

$$\phi = \frac{dy}{dx} = -\frac{w}{24EI} (4x^3 - 6Lx^2 + L^3) \quad \dots(10)$$

The slope angle at support ( $\phi$  at  $x=0$ ) is given by

$$\phi = -\frac{wL^3}{24EI} \quad \dots(11)$$

The elongation of tendon due to external load is given by:

$$\Delta_1 = \phi e = \frac{wL^3 e}{24EI} \quad \dots(12)$$

Where e=eccentricity of tendon from centriod of steel beam. The moment due to prestress increase is givenby:

$$M_{\Delta T} = -\Delta T e \quad \dots(13)$$

Where  $\Delta T$ : increase in tendon force.

The curvature due to the prestress force increase is :

$$\frac{d^2y}{dx^2} = \frac{M}{EI} = -\frac{\Delta T e}{EI} \quad \dots(14)$$

The slope due to the prestress force increase is:

$$\frac{dy}{dx} = -\frac{\Delta T e x}{EI} + k_1 \quad \dots(15)$$

The vertical deflection due to the prestress force increase is:

$$y = -\frac{\Delta T e x^2}{2EI} + k_1 x + k_2 \quad \dots(16)$$

By applying the boundary condition at both supports, the vertical deflection can be shown:

$$y = -\frac{\Delta T e x^2}{2EI} + \frac{\Delta T e L x}{2EI} \quad \dots(17)$$

The slope angle equation is given by:

$$\phi = \frac{dy}{dx} = -\frac{\Delta T e}{EI} x + \frac{\Delta T e L}{2EI} \quad \dots(18)$$

The slope angle at the support ( $\phi$  at  $x=0$ ) is given by:

$$\phi = \frac{\Delta T e L}{2EI} \quad \dots(19)$$

The shortening of the tendon due to  $M_{\Delta T}$  is given by:

$$\Delta_2 = \phi e = \frac{\Delta T e^2 L}{2EI} \quad \dots(20)$$

Axial shortening due to  $\Delta T$  is given by:

$$\Delta_3 = \frac{\Delta T L}{2A_s E} \quad \dots(21)$$

Where  $A_s$  :area of steel beam.

Where elongation of the tendon is given by:

$$\Delta = \frac{\Delta T L}{2A_b E} \quad \dots(22)$$

Where  $A_b$  :area of tendon.

The compatibility of the deformation requirement (total axial displacement) is given by:

$$\Delta = \Delta_1 - \Delta_2 - \Delta_3 \quad \dots(23)$$

Substitute the value of  $\Delta_1, \Delta_2$  and  $\Delta_3$  in equation (23), results the following:

$$\frac{\Delta T L}{2A_b E} = \frac{w e L^3}{24EI} - \frac{\Delta T e^2 L}{2EI} - \frac{\Delta T L}{2A_s E} \quad \dots(24)$$

So the increase in tendon force is given by:

$$\Delta T = \frac{w e L^2}{12I \left( \frac{1}{A_s} + \frac{1}{A_b} + \frac{e^2}{I_s} \right)} \quad \dots(25)$$

The moment due to dead load is given by:

$$M_D = W_D L^2 / 8 \quad \dots(26)$$

Where  $W_D$ : dead load which includes steel beams, tendon and concrete slab.

$L$ : span of the beam.

Using Equation (25) the increase in tendon force due to the dead load is given by:

$$T_D = \frac{W_D e L^2}{12 I_s \left( \frac{1}{A_s} + \frac{1}{A_b} + \frac{e^2}{I_s} \right)} \quad \dots(27)$$

Where  $A_b$ : area of tendon. The moment increased due to the increase in tendon force is given by:

$$M_{TD} = T_D \cdot e \quad \dots(28)$$

The total stresses due to dead load at the top and bottom of the steel beam,  $f_2$  and  $f_3$  respectively are as follows:

$$f_2 = \frac{-M_D C}{I_s} + \frac{T_D \cdot e \cdot C}{I_s} - \frac{T_D}{A_s} \quad \dots(29)$$

$$f_3 = \frac{M_D \cdot C}{I_s} - \frac{T_D \cdot e \cdot C}{I_s} - \frac{T_D}{A_s} \quad \dots(30)$$

If the dead load and the prestressing are combined resisted by the steel beam only without the concrete slab, at this stage the stress at top and bottom of I-beam are  $f_{2i}$  and  $f_{3i}$  respectively:

$$f_{2i} = -\frac{T}{A_s} - \frac{T_D}{A_s} + \frac{M_T C}{I_s} + \frac{T_D e_c}{I_s} - \frac{M_D C}{I_s} \quad \dots(31)$$

$$f_{3i} = -\frac{T}{A_s} - \frac{T_D}{A_s} + \frac{M_T C}{I_s} - \frac{T_D e_c}{I_s} + \frac{M_D C}{I_s} \quad \dots(32)$$

The composite beam act as a composite section against external load, the area of the composite section  $A_c$

$$A_c = A_s + A_R + \frac{1}{n} b t_c \quad \dots(33)$$

As shown above, tendon area is not included in equation (28) because the lack of the shear transfers between the tendons and the beam. The neutral axis is at distance  $y_c$  from the bottom steel flange as follows:

$$y_c = \frac{\left(\frac{1}{n}bt_c + A_R\right)\left(d + \frac{t_c}{2}\right) + A_s \frac{d}{2}}{A_c} \quad \dots(34)$$

Where moment of inertia of cross-section:

$$I_c = I_s + A_s(y_c - \frac{d}{2})^2 + \frac{1}{12}\left(\frac{b}{n}\right)(t_c)^3 + \left(\frac{1}{n}bt_c + A_R\right)\left(d + \frac{t_c}{2} - y_c\right)^2 \quad \dots(35)$$

Where  $n$ : modular ratio,  $A_R$  : area of longitudinal reinforcement in concrete slab.

The prestress increase  $T_L$  due to Live load moment  $M_L$  is given :

$$T_L = \frac{PKL(1-K)(2-K)e_c}{3I_c\left(\frac{1}{A_c} + \frac{1}{A_b} + \frac{ec^2}{I_c}\right)} \quad \dots(36)$$

Where ( $P$ ) is the concentrated load at distance  $KL$  from the support. The Moment increased due to the prestress increase  $M_{TL}$  is  $M_{TL} = T_L \cdot e_c$

And the concrete stresses at top and bottom of the slab deck  $f_{1c}$  and  $f_{2c}$ , respectively are given:

$$f_{1c} = \frac{1}{n}\left(-\frac{T_L}{A_c} + \frac{T_L ec_t}{I_c} - \frac{M_L c_t}{I_c}\right) \leq 0.7 fc' \quad \dots(37)$$

$$f_{2c} = \frac{1}{n}\left(-\frac{T_L}{A_c} + \frac{T_L ec'}{I_c} - \frac{M_L c'}{I_c}\right) \quad \dots(38)$$

The steel stress at the top and bottom fiber of the I-beam  $f_2, f_3$  respectively are given.

$$f_2 = f_{2i} - \frac{T_L}{A_c} + \frac{T_L e_c c'}{I_c} - \frac{M_L c'}{I_c} \leq fy \quad \dots(39)$$

$$f_3 = f_{3i} - \frac{T_L}{A_c} - \frac{T_L e_c c_b}{I_c} + \frac{M_L c_b}{I_c} \leq fy \quad \dots(40)$$

If consider the procedure of determining the ultimate moment capacity by the ultimate strength analysis depends on whether the neutral axis occurs within the concrete slab or steel beam.

For N.A at concrete slab

Compressive force

$$(C) = 0.85 fc' a.b \quad \dots(41)$$

According to *whitne y's* stress block

$$T_s = A_s \cdot f_y + A_b \cdot f_{yb} \quad \dots(42)$$

so

$$soa = \frac{A_s \cdot f_y + A_b \cdot f_{yb}}{0.85 fc' \cdot b} \quad \dots(43)$$

$$so M_u = A_s \cdot f_y \cdot d_1 + A_b \cdot f_{yb} \cdot d_2$$

For N.A at steel beam:-

$$C_c = 0.85 fc' \cdot b \cdot t_c \quad \dots(44)$$

$$T' = A_s \cdot f_y + A_b \cdot f_{yb} - C_s \quad \dots(45)$$

From equilibrium condition

$$T' = C_s + C_c \quad \dots(46)$$

From equation (44,45and 46),find:

$$C_s = \frac{A_s \cdot f_y + A_b \cdot f_{yb} - C_c}{2} \quad \dots(47)$$

or

$$C_s = \frac{A_s \cdot f_y + A_b \cdot f_{yb} - 0.85 fc' \cdot b \cdot t_c}{2} \quad \dots(48)$$

Then the ultimate moment capacity  $M_u$  is given by

$$M_u = C_c \cdot d_3 + C_s \cdot d_4 \quad \dots(49)$$

### **Analysis and design example**

Twenty-two arbitrary sections of prestressed composite beams are examined, The concrete slab width and thickness, tendon type, i.e., cables versus bars and tendon eccentricity from the neutral axis are varied. One steel beam which is  $W_{14*30}(352mm*44.6kg/m)$  are examined for the specimen design. The results of the analysis of the section are shown in table (1),(2),(3) and (4), the general assumption as follows:

$$Fc'=4850 \text{ psi (33.5 MPa)}$$

$$fy=53300 \text{ psi (367.5 MPa)}$$

$$T=0.6 F_{pu} \cdot A_b=70.3 \text{ kip ( )}$$

$$F_{pu}=270 \text{ ksi (1861 MPa) for cables}$$

$$F_{pu}=160 \text{ ksi (1103 MPa) for bars}$$

$$F_{yp}=0.85 F_{pu}=229.5 \text{ ksi (1582 MPa) for cables}$$

$$F_{yp}=136 \text{ ksi (938 MPa) for bars}$$

$$A_r=0.33 \text{ in}^2 \text{ (213 mm}^2\text{)}$$

$$A_s=8.85 \text{ in}^2 \text{ (5710 mm}^2\text{)}$$

$A_b = 0.432 \text{ in}^2$  (279 mm<sup>2</sup>)  
 $2\Phi 0.6 \text{ in}$  (2 –  $\Phi 15.24 \text{ mm}$ ) high strength straight cables  
 $2\Phi 0.6 \text{ in}$  (2 –  $\Phi 15.24 \text{ mm}$ ) high strength drabed cables  
 $2\Phi \frac{3}{4} \text{ in}$  (2 –  $\Phi 18 \text{ mm}$ ) bars

**Table(1) analysis of pressed composite beams(straight cables 2-d5.25mm Ø)**

Concrete slab width (b)		Concrete slab thickness		Yield load (py)		Concrete stress at Yield load (fc)		Steel stress at yield load (fs)		Deflection (Δ)		Elastic neutral axis depth(ye)		plastic neutral axis depth(yp)		Ultimate load (pu)	
Inch	mm	inch	mm	Kip	kN	Ksi	Mpa *10 <sup>3</sup>	ksi	Mpa *10 <sup>3</sup>	in	mm	in	mm	in	mm	Kip	KN
32	813	3	76	100	444.8	-3.42	-23.6	53.04	365.7	0.557	14.15	11.94	303.2	13.81	350.7	124	551.5
32	813	4	102	104	462.6	-3.32	-22.9	54.1	373	0.501	12.72	12.82	325.6	13.84	351.5	149.2	663.6
36	915	3	76	100	444.8	-3.17	-21.85	53.71	370.3	0.542	13.76	12.17	309.1	13.82	351	131.3	584.0
36	915	4	102	102	453.7	-2.86	-19.7	54.0	372.3	0.479	12.16	13.05	331.5	13.84	351.5	156.5	696.1
40	1016	4	102	100	444.8	-2.63	-18.1	53.98	372.2	0.457	11.61	13.25	336.5	13.84	351.5	159.2	708.1
42	1067	3.5	89	98	435.9	-2.64	-18.2	53.53	369.1	0.477	12.11	12.92	328.2	13.84	351.5	152.6	678.7

**Table(2)analysis of prestressed composite beams(drapped cables 2-15.25mm Ø)**

Concrete slab width (b)		Concrete slab thickness		Yield load (py)		Concrete stress at Yield load (fc)		Steel stress at yield load (fs)		Deflection (Δ)		Elastic neutral axis depth(ye)		plastic neutral axis depth(yp)		Ultimate load (pu)	
inch	mm	inch	mm	Kip	kN	Ksi	Mpa *10 <sup>3</sup>	ksi	Mpa *10 <sup>3</sup>	in	mm	in	mm	in	mm	Kip	KN
32	813	3	76	102	453.7	-3.41	-23.51	45.88	316.3	0.547	13.89	11.94	303.2	13.81	350.7	134.7	599.1
32	813	4	102	116	516	-3.42	-23.58	52.54	362.3	0.539	13.69	12.82	325.6	13.83	351.3	159.1	707.7
36	915	3	76	110	489.2	-3.41	-23.51	51.25	353.4	0.574	14.58	12.17	309.1	13.82	351.0	141.2	628.0
36	915	4	102	116	516	-3.18	-21.93	53.54	369.2	0.525	13.33	13.05	331.5	13.84	351.5	166.1	738.8
40	1016	4	102	114	507	-2.93	-20.2	53.3	367.5	0.504	12.8	13.25	336.5	13.84	351.5	169.1	752.2

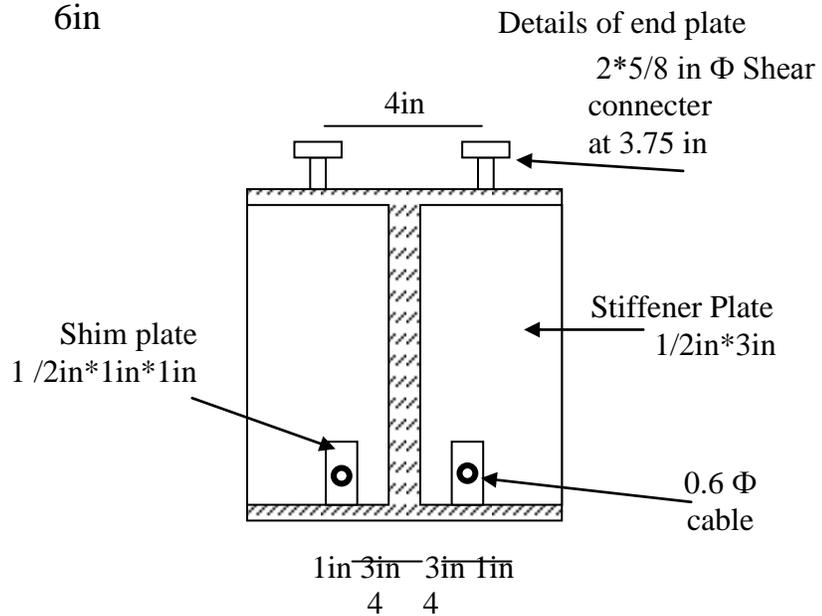
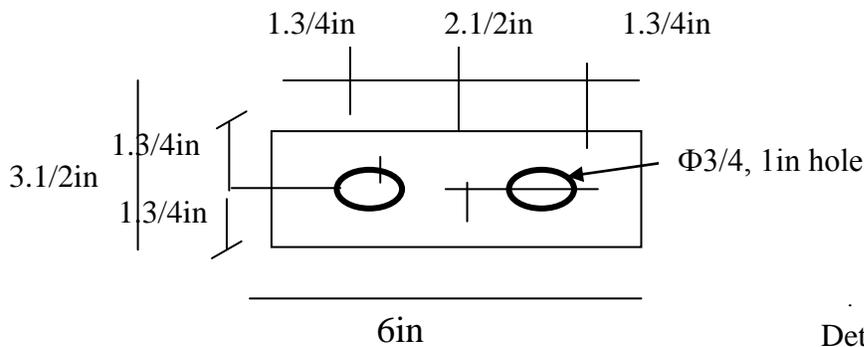
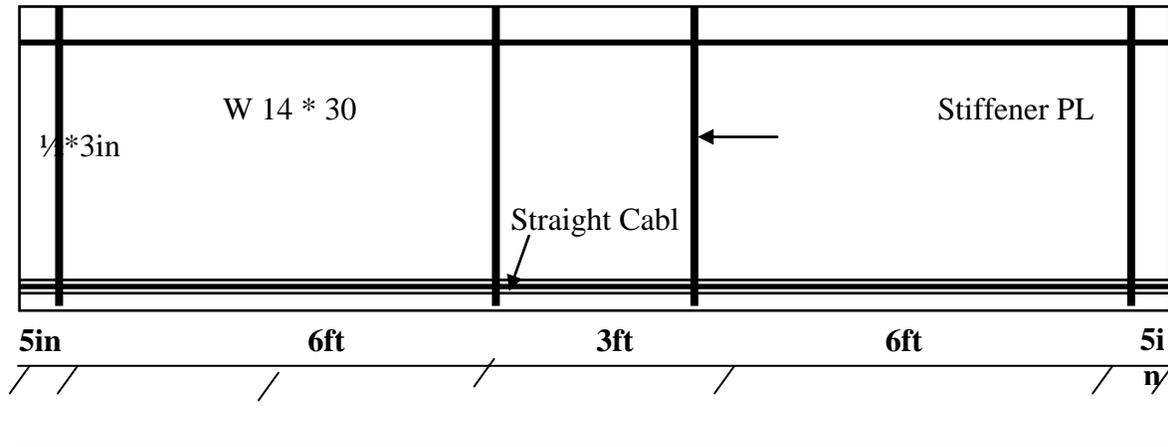
**Table (3) analysis of prestressed composite beams(straight cables 2- Ø 19 mm)**

Concrete slab width (b)		Concrete slab thickness		Yield load (py)		Concrete stress at Yield load (fc)		Steel stress at yield load (fs)		Deflection (Δ)		Elastic neutral axis depth(ye)		plastic neutral axis depth(yp)		Ultimate load (pu)	
inch	mm	inch	mm	Kip	kN	Ksi	Mpa *10 <sup>3</sup>	ksi	Mpa *10 <sup>3</sup>	in	mm	in	mm	in	mm	Kip	KN
32	813	3	76	102	453.7	-3.44	-23.72	51.44	354.7	0.543	13.79	11.94	303.2	13.81	350.7	130.3	579.6
32	813	4	102	108	480.4	-3.20	-22.06	53.65	369.9	0.497	12.62	12.82	325.6	13.83	351.3	154.7	688.1
36	915	3	76	104	462.6	-3.25	-22.41	53.31	367.6	0.538	13.66	12.17	309.1	13.82	351	136.7	608.0
36	915	4	102	106	471.5	-2.92	-20.13	53.6	369.6	0.474	12.04	13.05	331.5	13.84	351.5	163.6	727.7
40	1016	4	102	104	462.6	-2.68	-18.48	53.63	369.8	0.454	11.53	13.25	336.5	13.84	351.5	166.8	741.9

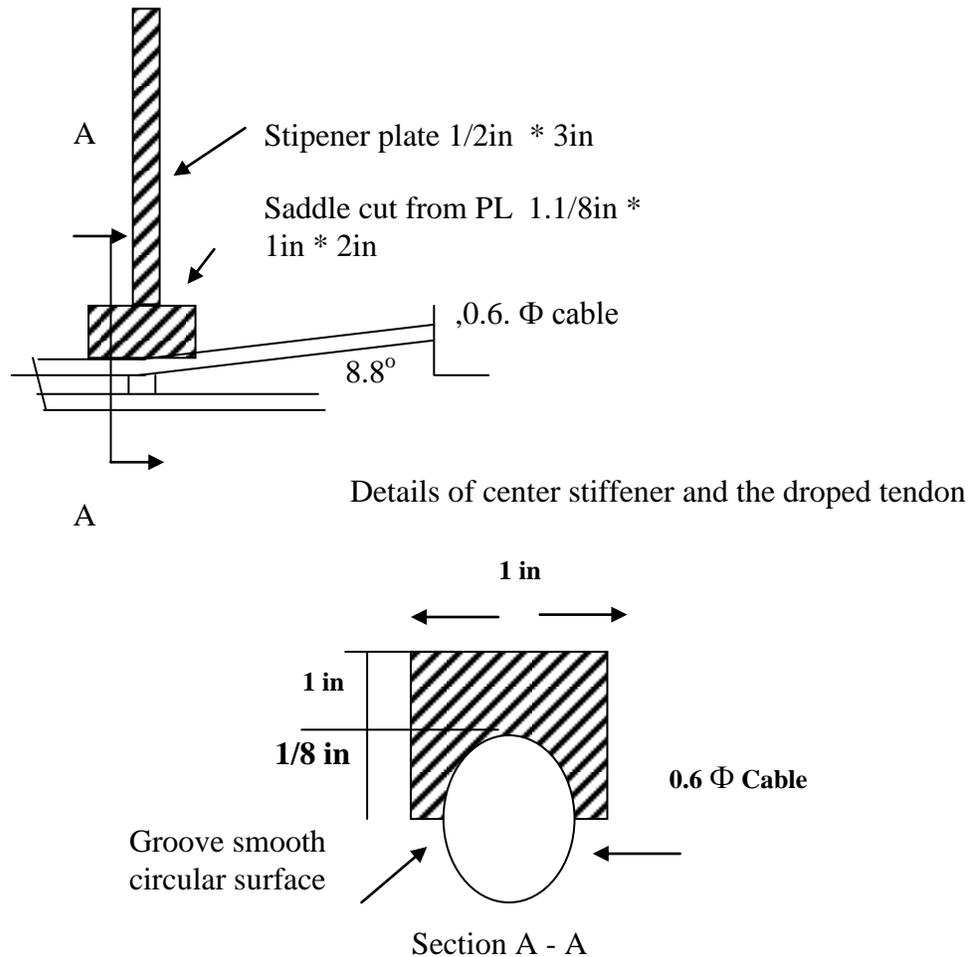
**Table (4) analysis of prestressed composite beams(drapped cables 2- Ø 19 mm)**

Concrete slab width (b)		Concrete slab thickness		Yield load (py)		Concrete stress at Yield load (fc)		Steel stress at yield load (fs)		Deflection (Δ)		Elastic neutral axis depth(ye)		plastic neutral axis depth(yp)		Ultimate load (pu)	
inch	mm	inch	mm	kip	kN	ksi	Mpa *10 <sup>3</sup>	ksi	Mpa *10 <sup>3</sup>	in	mm	in	mm	In	mm	kip	kN
32	813	3	76	106	471.5	-3.43	-23.65	43.88	302.5	0.526	13.36	11.94	303.2	13.81	350.7	142.4	633.4
32	813	4	102	120	533.8	-3.42	-23.58	50.29	346.75	0.516	13.11	12.82	325.6	13.83	351.3	166.9	742.4
36	915	3	76	114	507.0	-3.42	-23.58	48.96	337.58	0.55	13.97	12.17	309.1	13.82	351.0	148.9	662.3
36	915	4	102	124	551.6	-3.29	-22.68	53.35	367.85	0.52	13.21	13.05	331.5	13.84	351.5	175.7	781.5
40	1016	4	102	122	542.7	-2.03	-20.89	53.42	368.33	0.499	12.67	13.25	336.5	13.84	351.5	179.0	796.2

Span=L=4.6m (15 ft-10' ) Properties of sections of steel beam, shear connects, stiffener as shown in fig.(12)



Details stiffeners and sheaw connectors



**Fig(12): Detail of steel beam will straight and dropped tendon**

### **Analysis of result and Conclusions**

1-The tendon area is also an important factor to the behavior of the prestressed composite beams. Increasing the tendon area does increase the elastic and ultimate strength of the beam and reduce the vertical deflection of the beam.

2-Larger area for the tendons means larger increase in the tendon force which counteracts the external loads.

3-The advantage of the beam with draped tendons are that the moment produced by the variable eccentric prestressing force Cancels effectively the moment produce by the applied load, and again in shear resistance is obtained due to the vertical components of the prestressing force.

4-The end anchorage of the draped tendon may be more expensive and the holes in the web stiffeners should be drilled to pass the tendon.

- 5- Prestressing a conventional composite beam with tendons can significantly increase the yield load and the ultimate load.
- 6- The behavior of a prestressed composite beam is shown to be no very sensitive to variation in the slab thickness.
- 7- The anchorage in the web of steel beam may be need more care than the anchorage in the flange because the web is thinner than the flange of the beam and this is may be the disadvantages of using the drabed tendons in beams.
- 8- The advantages of using straight tendons in beams are like the hold-down device are not needed and may be simple fabrication and inexpensive anchorage system can be used, However, positioning devices may be needed. In some cases to keep the tendons in a fixed position relative to the beam.
- 9-The Ultimate strengths of both the beam with the bounded tendon and the beam with the unbounded tendon are the same.
- 10-The Prestressed composite steel beam with bounded tendons has more stiffness, and less deflection up to bond failure than the counterpart beam with unbounded tendons.
- 11-The tendon force in a prestressed composite steel beam with unbounded tendon is increased due to external loads. The increase in the tendon force counteracts the external load and reduces the deflection of the beam.
- 12- The tendon stress increase in the beam with unbounded tendon is averaged over the length of the tendon. Therefore, the yield load and ductility of the prestressed composite beam are increased.

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## **Notation**

- Ab: total area of prestressing tendons. ( $\text{mm}^2$ )  
Ac: transformed area of composite section. ( $\text{mm}^2$ )  
Ar: total area of longitudinal reinforcing bars in concrete slab. ( $\text{mm}^2$ )  
As: Area of steel beam. ( $\text{mm}^2$ )  
b: width of concrete slab. (mm)  
c: distance from centroidal axis of steel beam to top and bottom fiber of steel beam. (mm)  
 $c_{\circ}$ : distance from neutral axis of composite beam to top fiber of steel beam. (mm)  
 $c_b$ : distance from neutral axis of composite beam to bottom fiber of steel beam. (mm)  
 $c_t$ : distance from neutral axis of composite beam to top fiber to concrete slab. (mm)  
C: ultimate compressive force concrete slab. (Mpa)  
 $f_{s_{\circ}}$ : ultimate compressive force of steel beam. (Mpa)  
d: depth of steel beam. (mm)  
d1: distance from centroid of concrete ultimate compressive force to centroid of ultimate tensile force of steel beam. (mm)  
d2: distance from centroidal of concrete ultimate compressive force to centroid of prestressing tendon. (mm)  
d3: distance from centroid of concrete ultimate compressive force to centroid of ultimate tensile force of steel beam tendon. (mm)  
d4: distance from centroid of ultimate compressive force of steel beam to centroid of ultimate tensile force of steel beam and tendon. (mm)  
e: distance from tendon centroid to centroidal axis of steel beam. (mm)  
ec: distance from centroid of tendon to neutral axis of composite beam. (mm)  
 $E_c, E_s$ : modulus of elasticity of concrete and steel respectively. (Mpa)  
 $f_c$ : compressive strength of concrete. (Mpa)  
 $f_y$ : yield stress of steel beam. (Mpa)  
 $f_{yb}$ : yield stress of prestressing. (Mpa)  
 $F_{ub}$ : ultimate strength of prestressing tendon. (Mpa)  
 $F_{1c}$ : concrete stress at top of slab deck. (Mpa)  
 $F_{2c}$ : concrete stress at bottom of slab deck. (Mpa)  
 $f_2$ : steel stress at top fiber of steel beam due to total load. (Mpa)  
 $f_{2d}$ : stress at top fiber of steel beam due to dead load. (Mpa)  
 $f_{2i}$ : stress at top fiber of steel beam due to dead load and prestressing force . (Mpa)

$f_{3d}$ : stress at bottom fiber of steel beam due to dead load. (Mpa)  
 $f_{3i}$ : stress at bottom fiber of steel beam due to dead load and prestressing force. (Mpa)  
 $f_3$ : stress at bottom fiber of steel beam due to total load. (Mpa)  
 $f_r$ : modulus of rupture of concrete. (Mpa)  
 $I_c$ : moment of inertia of transformed composite beam. ( $\text{mm}^4$ )  
 $I_s$ : moment of inertia of transformed steel beam. ( $\text{mm}^4$ )  
 $K_1, k_2$ : constant of integration.  
 $L$ : span of beam. (m)  
 $M_d, m_l, m_t$ : moment due to dead load, live load and prestressing respectively. (KN.m)  
 $M_{td}$ : moment increased due to increase in tendon force by dead load. (KN.m)  
 $M_{tl}$ : moment increased due to increase in tendon force by live load. (KN.m)  
 $M_u$ : ultimate moment. (KN.m)  
 $n$ : modular ratio of steel to concrete .  
 $p$ : applied concentrated load. (KN)  
 $P_y$ : yield load of composite section. (KN)  
 $P_u$ : ultimate load of composite section. (KN)  
 $t_c$ : thickness of concrete slab. (mm)  
 $T$ : initial prestressing force in tendon. (KN)  
 $T_D, T_l$ : increase in tendon force due to dead and live load respectively. (KN)  
 $T_s$ : tension force in steel beam. (KN)  
 $W_d$ : uniformly distributed dead load. (KN/m)  
 $\Delta$ : net elongation of tendon due to external. (mm)  
 $\Delta_1$ : elongation of tendon due to external load. (mm)  
 $\Delta_2$ : axial shortening of tendon due to  $M_{DT}$ . (mm)  
 $\Delta_3$ : axial shortening of tendon due to  $T$ . (mm)  
 $\Delta t$ : increase in tendon force. (mm)  
 $\Phi$ : slope angle of beam

## دراسة تحليلية حول تصرف العتبات الحديدية المركبة المسبقة الجهد

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### الخلاصة

الجهد المسبق هو تحسين لخواص مواد متشابهة معينة وتبديل وقائي لمقاومة الاحمال الخدمية المسلطة وذلك من خلال تكوين اجهادات داخلية مضادة للاجهاد الذي يولده الحمل المسلط. فالخرسانة التي تمتاز بحملها العالي لقوى الانضغاط، فيتعرضها الى اجهاد انضغاط مسبق فإن ذلك سيولد مقاومة لاجهادات الشد الداخلية والتي تتعرض لها والتي هي بطبيعتها واهنه اتجاهاها . اما المقاطع الحديدية التي تمتاز بتفرداها اتجاه اجهادات الشد والانضغاط فان مفهوم الجهد المسبق سوف يعزز خواص منها المقاومة المرنة والمطيلية ومقاومة الكلال وذلك من خلال اعادة توزيع الاجهادات الداخلية في المقطع بالاضافة الى تقليل الانحراف الحاصل ووزن المقطع الحديدي. بالاعتماد على مبدا توازن القوى وتوافق التشوهات فلقد تم تطوير المعادلات التحليلية للحالة المرنة واللدنة. لقد اظهرت المقارنة للنماذج المختارة للمقاطع الحديدية المركبة المسبقة الجهد باسلاك منفصلة وغير منفصلة بان ذلك ليس له تاثير يذكر على تصرف المقطع المركب وذلك من خلال دراسة مقاومة المقطع والانحراف الذي ابداه . أن المقاطع الحديدية المركبة المسبقة الجهد باسلاك وقضبان مستقيمة قد ابدت مقاومة أقل لاجهادات القص من المقاطع المسبقة الجهد باسلاك او قضبان مائلة .